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New Types of Perfectly Supra Continuous Functions

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Abstract

We wrote this paper to proffer new types of the perfectly supra continuous functions. We also introduced new types of supra continuous, supra open and supra closed functions.

Keywords: perfectly supra continuous, totally supra $\widehat{\omega}$-continuous, perfectly supra $\widehat{\omega}$-continuous, perfectly supra $\widehat{\omega}$-continuous, supra $\widehat{\omega}$-continuous.

أنواع جديدة من الدوال التامة المستمرة الفوقية

نادية كاظم حمدي، حيدر جبر علي
قسم الرياضيات، كلية العلوم، الجامعه المستنصريه، بغداد، العراق

الخلاصة

كتبنا هذا البحث لعرض انواع جديدة من الدوال التامة المستمرة الفوقية ، كذلك قدمنا انواع جديدة من الدوال الفوقية المستمرة، الفوقية المفتوحة والفوقية المغلقة.

1-Introduction:

The notion of supra spaces was submitted for the first time by A. S. Mashhour el al [1, 2] where he defined the supra space as a sub collection μ of $\mathcal{P}(X)$ which is called a supra topology on the space X if $X \in \mu$, and it is closed under the arbitrary union. The pair (X, μ) is called a supra space. Any set $\mathcal{W} \in \mu$ is called supra open set and its complement is supra closed. Also, he introduced the concept of supra continuous functions. After that, many groups studied the supra spaces and developed the research on these spaces [3,4]. In other studies [5-7], the researchers submitted the definition of supra closure and supra interior for a subset from a supra space. In 2013, Vidyarani and Vigneshwaran presented the perfectly supra N-continuous. In this paper, we introduced new types of continuous functions which are totally supra $\widehat{\omega}$-continuous, perfectly supra $\widehat{\omega}$-continuous, and perfectly supra $\widehat{\omega}$-irresolute functions. We also explained the relations among these types and clarified their connection to some types of the supra continuous function. Some theorems, propositions and examples were given to support our results.

2- Supra $\widehat{\omega}$-compact and supra $\widehat{\eta}$-compact spaces: In this section, using new supra open sets, we provide the concepts for new types of supra compact spaces which are supra $\widehat{\omega}$-compact space and supra $\widehat{\eta}$-compact space. In addition, we illustrate the relations among them and submit some properties, remarks, and examples. In this paper, we use the abbreviation "su" as an abbreviation of "supra".

Definition (2.1): A subset $\mathcal{W}$ from a su. space (X, μ) is called a su. $\widehat{\omega}$-open set if for any $s \in \mathcal{W}$, there is $\mathcal{V} \in \mu$ such that $s \in \mathcal{V}$ and $\mathcal{V}$-$\mathcal{W}$ is a countable set. $\mathcal{W}^c$ is called a su. $\widehat{\omega}$-closed set.

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Definition (2.2): In the previous definition, if $\mathcal{V}$-$\mathcal{W}$ is a finite set, then $\mathcal{W}$ is called a su. $\hat{\eta}$-open set and its complement is called a su. $\hat{\eta}$-closed.

Example (2.3): $\mathcal{Z}$-$\{x\}$ is a su. $\widehat{\omega}$-open, also a su. $\hat{\eta}$-open set in the co-finite su. space $(\mathcal{Z}, \mu_{cof})$.

Definition (2.4): 1- Let $G=\{U_i|i \in I\}$ be a collection of su. open sets in a su. space (X, μ). If $X \subseteq \bigcup_{i\in I} U_i$, then G is a su. open cover for X. When any su. open cover for X has a finite sub cover, then X is called a su. compact space [8], [9].

2- Let $G = \{U_i|i \in I\}$ be a collection of su. $\widehat{\omega}$-open sets in a su. space (X, μ). If $X \subseteq \bigcup_{i\in I} U_i$, then G is a su. $\widehat{\omega}$-open cover for X. When any su. $\widehat{\omega}$-open cover for X has a finite sub cover, then X is called a su. $\widehat{\omega}$-compact space.

3- Let $G = \{U_i|i \in I\}$ be a collection of su. $\hat{\eta}$-open sets in a su. space (X, μ). If $\subseteq \bigcup_{i\in I} U_i$, then G is a su. $\hat{\eta}$-open cover for X. When any su. $\hat{\eta}$-open cover for X has a finite sub cover, then X is called a su. $\hat{\eta}$-compact space

Example (2.5): $(\mathcal{R}, \mu_{cof})$ is a su. compact space. Moreover, it is a su. $\hat{\eta}$-compact and su. $\widehat{\omega}$-compact.

Remark (2.6) [10]: If (X, μ) is a finite su. space, then it is a su. compact.

Remark (2.7): If (X, μ) is a finite su. space, then it is a su. $\widehat{\omega}$-compact, since if $X=\{x_1, \dots, x_m\}$ and $G = \{G_i\}$ is a su. $\widehat{\omega}$-open cover for X, so any element in X belongs to at least one of the elements of G, say $x_1 \in G_{i_1}$, $x_2 \in G_{i_2}, \dots, x_m \in G_{i_m}$. Accordingly, $X \subseteq G_{i_1} \cup G_{i_2} \cup \dots \cup G_{i_m}$, so X is a su. $\widehat{\omega}$-compact space.

Example (2.8): The excluded point su. space (X, μ_{EX}), where X is a finite set and $\mu_{EX} = \{U \subseteq X, x_\circ \notin U$ for some $x_\circ \in X\} \cup \{X\}$, is su. compact and su. $\widehat{\omega}$-compact space.

Remark (2.9): 1. If $U \in \mu$, then it is su. $\widehat{\omega}$-open.

2. If $U \in \mu$, then it is su. $\hat{\eta}$-open.

Example (2.10): In (X, μ_{EX}) where X is a finite set, $\{x_\circ\} \notin \mu_{EX}$, but it is su. $\widehat{\omega}$-open and su. $\hat{\eta}$-open set.

Remark (2.11): Any su. $\hat{\eta}$-open set U is a su. $\widehat{\omega}$-open, since if (X, μ) is a su. space and U is a su. $\hat{\eta}$-open set in it. So, for every element x in U, there is $G \in \mu$, where $x \in G$ and satisfies that G-U is a finite set, and then it is a countable set. Therefore, U is a su. $\widehat{\omega}$-open.

Example (2.12): $\{1\}$ in $(\mathcal{Q}, \mu_{cof})$ is su. $\widehat{\omega}$-open set but not su. $\hat{\eta}$-open set.

Theorem (2.13): If (X, μ) is a su. $\widehat{\omega}$-compact space, hence it is a su. compact and a su. $\hat{\eta}$-compact space.

Proof: Let $G=\{U_i|\ i \in I\}$ be a su. open cover (resp. su. $\hat{\eta}$-open cover) for a su. space (X, μ), by remark (2.9) (and remark (2.11)), G will be a su. $\widehat{\omega}$-open cover for X and will has a finite sub cover for X (because X is a su. $\widehat{\omega}$-compact space). Therefore, X is a su. compact space (resp. su. $\hat{\eta}$-compact space).
By the same way, we can prove the following theorem.

Theorem (2.14): If (X, μ) is a su. $\hat{\eta}$-compact space, hence it is a su. compact.

Example (2.15): The indiscrete su. space $(\mathcal{Z}, \mu_{ind})$ is a su. compact and a su. $\hat{\eta}$-compact space but not a su. $\widehat{\omega}$-compact.

Theorem (2.16): 1- Any su. closed set in a su. compact space is a su. compact set [11].

2- Any su. $\hat{\eta}$-closed set in a su. $\hat{\eta}$-compact space is a su. $\hat{\eta}$-compact set.

Lemma (2.17): Any su. $\widehat{\omega}$-closed set in a su. $\widehat{\omega}$-compact space is a su. $\widehat{\omega}$-compact.

Proof: If $\mathcal{M}$ is a su. $\widehat{\omega}$-closed set in a su. $\widehat{\omega}$-compact space X, and $C = \{U_\alpha | \alpha \in \wedge\}$ is a su. $\widehat{\omega}$-open covering to $\mathcal{M} \Longrightarrow \mathcal{M} \subseteq \bigcup_{\alpha\in\wedge} U_\alpha$, since $X = \mathcal{M} \cup \mathcal{M}^c \Longrightarrow X \subseteq \{\bigcup_{\alpha\in\wedge} U_\alpha\} \cup \mathcal{M}^c$, but $\mathcal{M}^c$ is a su. $\widehat{\omega}$-open set in X, thus $\{\{U_\alpha\}_{\alpha\in\wedge},\ \mathcal{M}^c\}$ is a su. $\widehat{\omega}$-open covering to X. So $X \subseteq \bigcup_{i=1}^{n} (U_{\alpha_i}) \cup \mathcal{M}^c$ (because X is a su. $\widehat{\omega}$-compact), but $\mathcal{M} \subseteq X \Longrightarrow \mathcal{M} \subseteq \bigcup_{i=1}^{n} (U_{\alpha_i}) \cup \mathcal{M}^c$, and since $\mathcal{M} \cap \mathcal{M}^c = \emptyset \Longrightarrow \mathcal{M} \subseteq \bigcup_{i=1}^{n} U_{\alpha_i}$, so $\{U_{\alpha_i}\}_{i=1}^{n}$ is a finite sub cover from C to $\mathcal{M}$,
therefore $\mathcal{M}$ is a su. $\widehat{\omega}$-compact.

Theorem (2.18): Let $\mathcal{A}$, $\mathcal{B}$ be a su. $\widehat{\omega}$-compact set and a su. $\widehat{\omega}$-closed set, respectively, in a su. space (X, μ), hence $\mathcal{A} \cap \mathcal{B}$ is a su. $\widehat{\omega}$-compact set in X.

Proof: Since $\mathcal{B}$ is a su. $\widehat{\omega}$-closed set in X, then $\mathcal{B}^c$ is a su. $\widehat{\omega}$-open set. Let $G=\{U_i|i \in I\}$ is a su. $\widehat{\omega}$-open cover for $\mathcal{A} \cap \mathcal{B}$, so $\{G \cup \mathcal{B}^c\}$ is a su. $\widehat{\omega}$-open cover for $\mathcal{A}$ which has a finite sub cover ($\mathcal{A}$ is a su. $\widehat{\omega}$-compact set), then $\mathcal{A} \subseteq (\bigcup_{i=1}^{n} U_i) \cup \mathcal{B}^c \Longrightarrow \mathcal{A} \cap \mathcal{B} \subseteq (\bigcup_{i=1}^{n} U_i) \cap \mathcal{B} \Longrightarrow (\bigcup_{i=1}^{n} U_i) \cap \mathcal{B}$ is a finite sub cover for $\mathcal{A} \cap \mathcal{B}$. Therefore, $\mathcal{A} \cap \mathcal{B}$ is a su. $\widehat{\omega}$-compact set.

Example (2.19): Let $\mathcal{B} \subseteq X$ be in the indiscrete su. space (X, μ_{ind}), then $X \cap \mathcal{B}$ is a su. $\widehat{\omega}$-compact.

3- New types of perfectly supra continuous and supra continuous functions.

In the next part, we provide definitions for some weaker functions, stronger functions, and functions that have no relation with perfectly supra continuous functions. We also introduce diagrams illustrate the relations among several types of functions.

Definition (3.1) [1,12]: A function $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is called a su. continuous function if $f^{-1}(\mathcal{W})$ is a su. open (resp. su. closed) set in X for any su. open (resp. su. closed) set $\mathcal{W}$ in Y.

Example (2.21): The identity function $I_{\mathcal{R}}$ from the co-countable su. space ($\mathcal{R}$, μ_{coc}) into the co-finite su. space ($\mathcal{R}$, μ_{cof}) is a su. continuous function.

Definition (3.2) [13]: Whenever $f^{-1}(\mathcal{W})$ is a su. clopen set in the su. space (X,μ_X) for any su. open set $\mathcal{W}$ in the su. space (Y,μ_Y), hence $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is called a perfectly su. continuous function.

Proposition (3.3): The function $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is called a perfectly su. continuous function iff $f^{-1}(\mathcal{M})$ is su. clopen set in X for each su. closed set $\mathcal{M}$ in Y.

Proof: Take f as a perfectly su. continuous function, $\mathcal{M}$ as a su. closed set in $Y \Longrightarrow \mathcal{M}^c \in \mu_Y \Longrightarrow f^{-1}(\mathcal{M}^c) = (f^{-1}(\mathcal{M}))^c$ is a su. clopen set in X (since f is a perfectly su. continuous) $\Longrightarrow f^{-1}(\mathcal{M})$ is also a su. clopen. Conversely, suppose that $\mathcal{M} \in \mu_Y \Longrightarrow \mathcal{M}^c$ is a su. closed subset from $Y \Longrightarrow f^{-1}(\mathcal{M}^c) = (f^{-1}(\mathcal{M}))^c$ is su. clopen set in X (given) $\Longrightarrow f^{-1}(\mathcal{M})$ is su. clopen set in X. Therefore, f is a perfectly su. continuous function.

Example (3.4): A function $f:(X,\mu_D) \longrightarrow (Y,\mu_Y)$ is a perfectly su. continuous, since the inverse image for any su. closed set in Y (at least Y is a su. closed set in (Y,μ_Y)) is su. clopen set in (X,μ_D)(since each set in (X, μ_D) is su. open and su. closed set), so f is a perfectly su. continuous function.

Remark (3.5): Whenever f is a perfectly su. continuous function, then it is a su. continuous, since if $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is a perfectly su. continuous function, and $U \in \mu_Y$, then U^C is su. closed set in Y, so $f^{-1}(U^C) = (f^{-1}(U))^C$ is a su. clopen subset from X (because f is a perfectly su. continuous). Hence, $f^{-1}(U)$ is a su. clopen set in X, which means that $f^{-1}(U) \in \mu_X$, therefore f is su. continuous function.

Example (3.6): The identity function from the co-countable su. space $(\mathcal{R},\mu_{coc})$ into the co-finite su. space $(\mathcal{R},\mu_{cof})$ is su. continuous function, but not perfectly su. continuous.

Remark (3.7): Whenever $\mathcal{M}$ is a su. $\hat{\eta}$-closed set in a su. space (X,μ) , then it is a su. $\hat{\omega}$-closed, because if $\mathcal{M}$ is a su. $\hat{\eta}$-closed set. Hence, $\mathcal{M}^c$ is a su. $\hat{\eta}$-open and, by remark (2.11), $\mathcal{M}^c$ is a su. $\hat{\omega}$-open set, which implies that $\mathcal{M}$ is a su. $\hat{\omega}$-closed.

Example (3.8): The set $\mathcal{Z}$- $\{x\}$ in $(\mathcal{Z},\mu_{ind})$ is su. $\hat{\omega}$-closed but not su. $\hat{\eta}$-closed.

Remark (3.9): It is clear from remark (2.11) and remark (3.7) that whenever a set is su. $\hat{\eta}$-clopen then it is su. $\hat{\omega}$-clopen set.

Example (3.10): $\{\frac{1}{2}\}$ in (Q,μ_{ind}) is su. $\hat{\omega}$-clopen set and not su. $\hat{\eta}$-clopen.

Remark (3.11): Whenever $\mathcal{M}$ is a su. closed subset from a su. space (X, μ), hence it is a su. $\hat{\omega}$-closed and a su. $\hat{\eta}$-closed, because $\mathcal{M}^c \in \mu$, then it is a su. $\hat{\omega}$-open and a su. $\hat{\eta}$-open subset from X (by remark (2.9)). Then, $\mathcal{M}$ is a su. $\hat{\omega}$-closed and a su. $\hat{\eta}$-closed subset from X.

Example (3.12): {3} in the indiscrete su. space (X, μ_{ind}), where X= {1, 2, 3} is su. $\hat{\omega}$-closed and su. $\hat{\eta}$-closed set which is not su. closed.

Definition (3.13): The function $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is called

1. Totally su. $\hat{\omega}$-continuous, if the inverse image of any su. closed set in the su. space (Y,μ_Y) is a su. $\hat{\omega}$-clopen set in the su. space (X,μ_X).
2. Totally su. $\hat{\eta}$-continuous, if the inverse image of any su. closed set in the su. space (Y,μ_Y) is a su. $\hat{\eta}$-clopen set in the su. space (X,μ_X).
3. Perfectly su. $\hat{\omega}$-continuous, if the inverse image of any su. $\hat{\omega}$-closed set in the su. space (Y,μ_Y) is a su. clopen set in the su. space (X,μ_X).
4. Perfectly su. $\hat{\eta}$-continuous, if the inverse image of any su. $\hat{\eta}$-closed set in the su. space (Y,μ_Y) is a su. clopen set in the su. space (X,μ_X).
5. Perfectly su. $\hat{\omega}$-irresolute, if the inverse image of any su. $\hat{\omega}$-closed set in the su. space (Y,μ_Y) is a su. $\hat{\omega}$-clopen set in the su. space (X,μ_X).

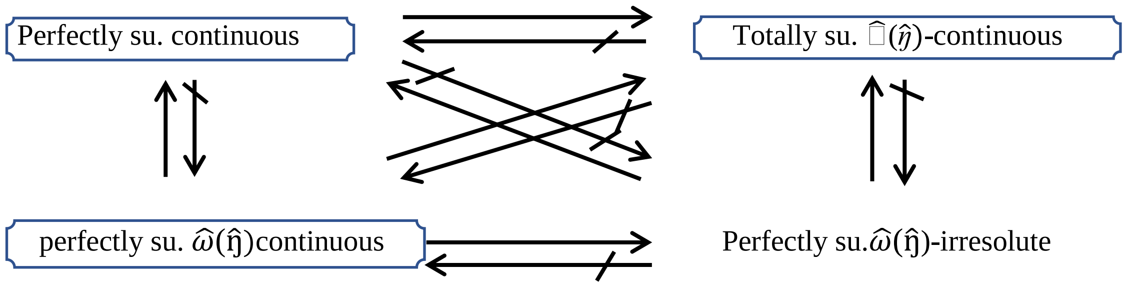
6. Perfectly su. $\hat{\eta}$-irresolute, if the inverse image of any su. $\hat{\eta}$-closed set in the su. space (Y, μ_Y) is a su. $\hat{\eta}$-clopen set in the su. space (X, μ_X).

Example (3.14): 1. $f\colon (\mathcal{Z}, \mu_{ind}) \longrightarrow (Y, \mu_Y)$ is a totally su. $\hat{\omega}$-continuous function.
2. $f\colon (X, \mu_D) \longrightarrow (Y, \mu_Y)$, where Y is a countable set, is a perfectly su. $\hat{\omega}$-continuous function.
3. $f\colon (X, \mu_X) \longrightarrow (Y, \mu_Y)$, where X is a finite set, is a totally su. $\hat{\eta}$-continuous function.
4. The function f from the discrete su. space $(\mathcal{R}, \mu_D)$ into the included point su. space (X, μ_{In}), in which $\mu_{In}= \{U \subseteq X$, where $x_\circ \in U$, for some $x_\circ$ in $X\} \cup \{\emptyset\}$, is a perfectly su. $\hat{\eta}$-continuous function.
5. The identity function from $(\mathcal{Q}, \mu)$ into the same su. space is a perfectly su. $\hat{\omega}$-irresolute function.
6. The identity function from (X, μ), where X is a finite set, into the same su. space is a perfectly su. $\hat{\eta}$-irresolute.

Remark (3.15): If f is a totally su. $\hat{\eta}$-continuous function, then it is a totally su. $\hat{\omega}$-continuous, because if $f\colon (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is a totally su. $\hat{\eta}$-continuous and $\mathcal{M}$ is a su. closed subset from Y. Then, $f^{-1}(\mathcal{M})$ is a su. $\hat{\eta}$-clopen set in X and, by remark (3.9), it will be a su. $\hat{\omega}$-clopen. Hence f is a totally su. $\hat{\omega}$-continuous function.

Example (3.16) $f\colon (\mathcal{Z}, \mu_{cof}) \longrightarrow (Y, \mu_Y)$ is totally su. $\hat{\omega}$-continuous function while it is not totally su. $\hat{\eta}$-continuous.

The following graph is beneficial.



Example (3.17): 1. $I_{\mathcal{Z}}\colon (\mathcal{Z}, \mu_{ind}) \longrightarrow (\mathcal{Z}, \mu_{cof})$ is totally su. $\hat{\omega}$-continuous function but not perfectly su. continuous.
2. $I_X\colon (X, \mu_{ind}) \longrightarrow (X, \mu_D)$ where X is finite, is totally su. $\hat{\eta}$-continuous and perfectly su. $\hat{\eta}$-irresolute function, but neither perfectly su. continuous nor perfectly su. $\hat{\eta}$-continuous.
3. The identity function from $(\mathcal{R}, \mu_{ind})$ into the same space is perfectly su. continuous Function but not perfectly su. $\hat{\omega}$-continuous and not perfectly su. $\hat{\eta}$-continuous.
4. $I_{\mathcal{Q}}\colon (\mathcal{Q}, \mu_{ind}) \longrightarrow (\mathcal{Q}, \mu_D)$ is totally su. $\hat{\omega}$-continuous function but not perfectly su. $\hat{\omega}$-continuous. Also, it is perfectly su. $\hat{\omega}$-irresolute function but not perfectly su.continuous.
5. A function f from $(\mathcal{Z}, \mu_{ind})$ into (X, μ_{ind}) where X is finite and $f(x) = \begin{cases} y_1 & if\ x < 0 \\ y_2 & if\ x \geq 0 \end{cases}$ is perfectly su. continuous function but not perfectly su. $\hat{\eta}$-irresolute.
6. $I_{\mathcal{Z}}$ from $(\mathcal{Z}, \mu_{ind})$ into $(\mathcal{Z}, \mu_{ind})$ is perfectly su. $\hat{\omega}$-irresolute function but not perfectly su. $\hat{\omega}$-continuous.

Theorem (3.18): If a function f from a su. compact space (X, μ_X) into any su. space (Y, μ_Y) is surjective and perfectly su. continuous function, then Y is su. compact space.

Proof: Suppose that $G = \{U_i \mid i \in I\}$ is a su. open cover for Y, hence $Y = \bigcup_{i\in I} U_i$. Then, $\bigcap_{i\in I} {U_i}^C = \emptyset$, where each ${U_i}^C$ is su. closed set in Y for any $i \in I$, $\Longrightarrow f^{-1}(\bigcap_{i\in I} {U_i}^c) = f^{-1}(\emptyset) \Longrightarrow \bigcap_{i\in I} f^{-1}\left({U_i}^C\right) = \emptyset$, in which each $f^{-1}({U_i}^C)$, $i \in I$ is a su. clopen set in X (f is perfectly su. continuous). But $\bigcap_{i\in I} f^{-1}({U_i}^C) = \bigcap_{i\in I}(f^{-1}(U_i))^{\,C} \Longrightarrow \bigcap_{i\in I}(f^{-1}(U_i))^C = \emptyset \Longrightarrow X\text{-}\bigcap_{i\in I} (f^{-1}(U_i))^C = X\text{-}\emptyset \Longrightarrow$ $\bigcup_{i\in I} \left(f^{-1}(U_i)\right) = X$, which means that $\bigcup_{i\in I} f^{-1}(U_i)$ is su. open cover for X, and since X is su. compact, then this su. open cover has a finite sub cover, so $X = \bigcup_{i=1}^{n} f^{-1}(U_i) \Longrightarrow f(X) = \bigcup_{i=1}^{n} f\left(f^{-1}(U_i)\right) = \bigcup_{i=1}^{n} U_i \Longrightarrow Y = \bigcup_{i=1}^{n} U_i$ (f is surjective). Hence, Y is a su. compact space.

Example (3.19): $I_{\mathcal{R}}\colon (\mathcal{R}, \mu_{ind}) \longrightarrow (\mathcal{R}, \mu_{ind})$ is satisfying this theorem since $(\mathcal{R}, \mu_{ind})$ is a compact space, $I_{\mathcal{R}}$ is a surjective and perfectly su. continuous function (because the only closed sets in the co-domain are $\emptyset, \mathcal{R}$ and ${I_{\mathcal{R}}}^{-1}(\emptyset) = \emptyset, {I_{\mathcal{R}}}^{-1}(\mathcal{R}) = \mathcal{R}$ are su. clopen sets in the domain)

Corollary (3.20): When the function $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is a surjective and:
1. Perfectly su. continuous, where X is su. $\widehat{\omega}$-compact, then Y is su. compact.
2. Perfectly su. continuous, where X is su. ῆ-compact, then Y is su. compact.
3. Totally su. $\widehat{\omega}$-continuous, where X is su. $\widehat{\omega}$-compact, then Y is su. compact.
4. Totally su. ῆ-continuous, where X is su. ῆ-compact, then Y is su. compact.
5. Perfectly su. $\widehat{\omega}$-continuous, where X is su. $\widehat{\omega}$-compact, then Y is su. $\widehat{\omega}$-compact.
6. Perfectly su. ῆ-continuous, where X is su. ῆ-compact, then Y is su. ῆ-compact.
7. Perfectly su. $\widehat{\omega}$-continuous, where X is su. compact, then Y is su. compact.
8. Perfectly su. ῆ-continuous, where X is su. compact, then Y is su. compact.
9. Perfectly su. $\widehat{\omega}$-continuous, where X is su. compact, then Y is su. $\widehat{\omega}$-compact.
10. Perfectly su. ῆ-continuous, where X is su. compact, then Y is su. ῆ-compact.
11. Perfectly su. $\widehat{\omega}$-irresolute, where X is su. $\widehat{\omega}$-compact, then Y is su. $\widehat{\omega}$-compact.
12. Perfectly su. ῆ-irresolute, where X is su. ῆ-compact, then Y is su. ῆ-compact.

Example (3.21): $I_{\mathcal{R}}:(\mathcal{R}, \mu_{ind}) \longrightarrow (\mathcal{R}, \mu_{ind})$ satisfies (1, 2) and $I_X: (X, \mu_{ind}) \longrightarrow (X, \mu_{ind})$ where X is a finite set, which satisfies (3, 4) from the previous proposition. While the function $I_X: (X, \mu_D) \longrightarrow (X, \mu_{ind})$, where X is a finite set, satisfies the rest.

Remark (3.22): Whenever $f: (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is perfectly su. $\widehat{\omega}$-continuous function, then it is perfectly su. ῆ-continuous. Since if $\mathcal{M}$ is su. ῆ-closed set in Y, hence it is su. $\widehat{\omega}$-closed (by remark (3.7)). So $f^{-1}(\mathcal{M})$ is su. clopen set in X. Therefore, f is perfectly su. ῆ-continuous function.

Remark (3.23): There is no relation between perfectly su. $\widehat{\omega}$-irresolute function and perfectly su. ῆ-irresolute.

Example (3.24): $I_{\mathcal{Z}} : (\mathcal{Z}, \mu_{coc}) \longrightarrow (\mathcal{Z}, \mu_{cof})$ is perfectly su. $\widehat{\omega}$-irresolute function while it is not perfectly su. ῆ-irresolute.

Definition (3.25): The function $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is:
1. Su. $\widehat{\omega}$-continuous function, if the inverse image for any su. open subset from Y is a su. $\widehat{\omega}$-open subset from X.
2. Strongly su. $\widehat{\omega}$-continuous function, if the inverse image for any su. $\widehat{\omega}$-open subset Y is a su. open subset from X.
3. Su. $\widehat{\omega}$-irresolute function, if the inverse image for any su. $\widehat{\omega}$-open subset from Y is a su. $\widehat{\omega}$-open subset from X.

Theorem (3.26): The composition between
1. Su. $\widehat{\omega}$-continuous function and su. continuous function is su. $\widehat{\omega}$-continuous.
2. Su. ῆ-continuous function and su. continuous function is su. ῆ-continuous.
3. Su. $\widehat{\omega}$-continuous function and strongly su. $\widehat{\omega}$-continuous function is su. $\widehat{\omega}$-irresolute.
4. Su. ῆ-continuous function and strongly su. ῆ-continuous function is su. ῆ-irresolute.
5. Strongly su. $\widehat{\omega}$-continuous function and su. $\widehat{\omega}$-continuous function is su. continuous.
6. Strongly su. ῆ-continuous function and su. ῆ-continuous function is su. continuous.
7. Su. $\widehat{\omega}$-irresolute function and su. $\widehat{\omega}$-continuous function is su. $\widehat{\omega}$-continuous.
8. Su. ῆ-irresolute function and su. ῆ-continuous function is su. ῆ-continuous.
9. Strongly su. $\widehat{\omega}$-continuous function and su. continuous function is su. continuous.
10. Strongly su. ῆ-continuous function and su. continuous function is su. continuous.
11. Su. continuous function and strongly su. $\widehat{\omega}$-continuous function is strongly su. $\widehat{\omega}$-continuous.
12. Su. continuous function and strongly su. ῆ-continuous function is strongly su. ῆ-continuous.
13. Strongly su. $\widehat{\omega}$-continuous function and strongly su. $\widehat{\omega}$-continuous function is strongly su. $\widehat{\omega}$-continuous.
14. Strongly su. ῆ-continuous function and strongly su. ῆ-continuous function is strongly su. ῆ-continuous.
15. Strongly su. $\widehat{\omega}$-continuous function and su. $\widehat{\omega}$-irresolute function is strongly su. $\widehat{\omega}$-continuous.
16. Strongly su. ῆ-continuous function and su. ῆ-irresolute function is strongly su. ῆ-continuous.
17. Su. $\widehat{\omega}$-irresolute function and strongly su. $\widehat{\omega}$-continuous function is su. $\widehat{\omega}$-irresolute.
18. Su. ῆ-irresolute function and strongly su. ῆ-continuous function is su. ῆ-irresolute.
19. Su. $\widehat{\omega}$-irresolute function and su. continuous function is su. $\widehat{\omega}$-continuous.
20. Su. ῆ-irresolute function and su. continuous function is su. ῆ-continuous.
21. Su. $\widehat{\omega}$-irresolute function and su. $\widehat{\omega}$-irresolute function is su. $\widehat{\omega}$-irresolute.

22. Su. ŋ̂-irresolute function and su. ŋ̂-irresolute function is su. ŋ̂-irresolute.

Proof: 1 and 2- If $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is su. $\widehat{\omega}$(ŋ̂)-continuous function, $h:(Y,\mu_Y) \longrightarrow (\mathcal{Z},\mu_{\mathcal{Z}})$ is su. continuous function and U is a su. open subset from $\mathcal{Z}$, so $h^{-1}(U)$ is su. open subset from Y and $f^{-1}(h^{-1}(U)) = (h\circ f)^{-1}(U)$ is su. $\widehat{\omega}$(ŋ̂)-open subset from X. Hence, $h\circ f$ is su. $\widehat{\omega}$(ŋ̂)-continuous function.

3 and 4- If $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is su. $\widehat{\omega}$(ŋ̂)-continuous function, $h:(Y,\mu_Y) \longrightarrow (\mathcal{Z},\mu_{\mathcal{Z}})$ is strongly su. $\widehat{\omega}$(ŋ̂)-continuous function and U is a su. $\widehat{\omega}$(ŋ̂)-open subset from $\mathcal{Z}$. So $h^{-1}(U)$ is su. open subset from Y and $f^{-1}(h^{-1}(U)) = (h\circ f)^{-1}(U)$ is su. $\widehat{\omega}$(ŋ̂)-open subset from X. Hence, $h\circ f$ is su. $\widehat{\omega}$(ŋ̂)-irresolute function.

5 and 6- If $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is strongly su. $\widehat{\omega}$(ŋ̂)-continuous function, $h:(Y,\mu_Y) \longrightarrow (\mathcal{Z},\mu_{\mathcal{Z}})$ is su. $\widehat{\omega}$(ŋ̂)-continuous function and U is a su. open subset from $\mathcal{Z}$, so $h^{-1}(U)$ is su. $\widehat{\omega}$(ŋ̂)-open subset from Y and $f^{-1}(h^{-1}(U)) = (h\circ f)^{-1}(U)$ is su. open subset from X. Hence, $h\circ f$ is su. continuous function.

7 and 8- If $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is su. $\widehat{\omega}$(ŋ̂)-irresolute function, $h:(Y,\mu\_Y) \longrightarrow (\mathcal{Z},\mu_{\mathcal{Z}})$ is su. $\widehat{\omega}$(ŋ̂)-continuous function and U is a su. open subset from $\mathcal{Z}$, so $h^{-1}(U)$ is su. $\widehat{\omega}$(ŋ̂)-open subset from Y and $f^{-1}\left(h^{-1}(U)\right) = (h\circ f)^{-1}(U)$ is su. $\widehat{\omega}$(ŋ̂)-open subset from X. Hence, $h\circ f$ is su. $\widehat{\omega}$(ŋ̂)-continuous function.

9 and 10- If $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is strongly su. $\widehat{\omega}$(ŋ̂)-irresolute function, $h:(Y,\mu_Y) \longrightarrow (\mathcal{Z},\mu_{\mathcal{Z}})$ is su. continuous function and U is a su. open subset from $\mathcal{Z}$, so $h^{-1}(U)$ is su. open subset from Y and then it is su. $\widehat{\omega}$(ŋ̂)-open set (by remark (2.9)). Hence, $f^{-1}\left(h^{-1}(U)\right) = (h\circ f)^{-1}(U)$ is su. open subset from X. Hence, $h\circ f$ is su. continuous function.

11 and 12- If $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is su. continuous function, $h:(Y,\mu_Y) \longrightarrow (\mathcal{Z},\mu_{\mathcal{Z}})$ is strongly su. $\widehat{\omega}$(ŋ̂)-continuous function and U is a su. $\widehat{\omega}$(ŋ̂)-open subset from $\mathcal{Z}$, so $h^{-1}(U)$ is su. open subset from Y and $f^{-1}(h^{-1}(U)) = (h\circ f)^{-1}(U)$ is su. open subset from X. Hence, $h\circ f$ is strongly su. $\widehat{\omega}$(ŋ̂)-continuous function.

13 and 14- If $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is strongly su. $\widehat{\omega}$(ŋ̂)-continuous function, $h:(Y,\mu_Y) \longrightarrow (\mathcal{Z},\mu_{\mathcal{Z}})$ is strongly su. $\widehat{\omega}$(ŋ̂)-continuous function and U is a su. $\widehat{\omega}$(ŋ̂)-open subset from $\mathcal{Z}$, so $h^{-1}(U)$ is su. open subset from Y and then it is su. $\widehat{\omega}$(ŋ̂)-open set (by remark (2.9)). Hence, $f^{-1}\left(h^{-1}(U)\right) = (h\circ f)^{-1}(U)$ is su. open subset from X. Hence, $h\circ f$ is strongly su. $\widehat{\omega}$(ŋ̂)-continuous function.

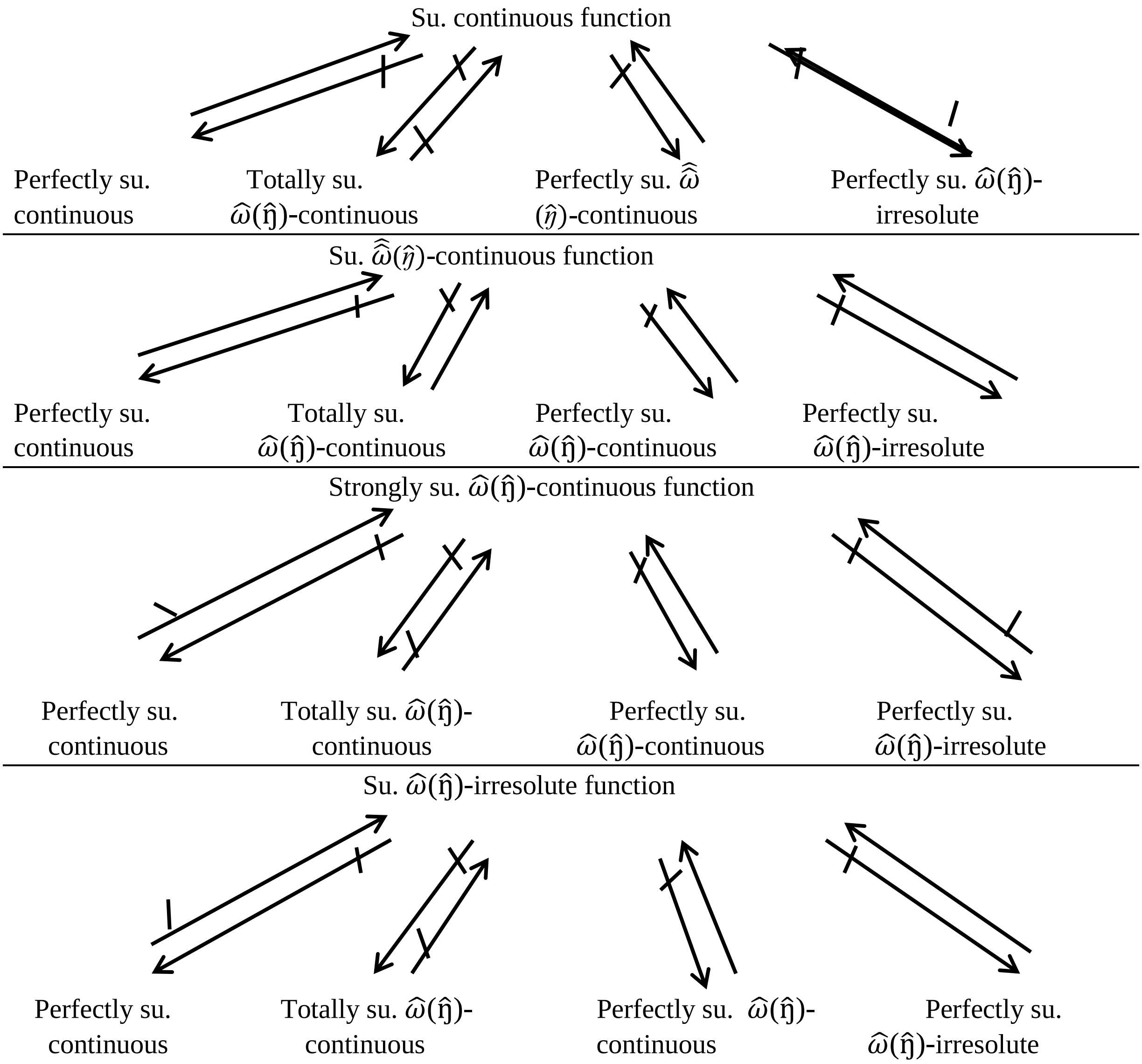
15 and 16- If $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is strongly su. $\widehat{\omega}$(ŋ̂)-continuous function, $h:(Y,\mu_Y) \longrightarrow (\mathcal{Z},\mu_{\mathcal{Z}})$ is su. $\widehat{\omega}$(ŋ̂)-irresolute function and U is a su. $\widehat{\omega}$(ŋ̂)-open subset from $\mathcal{Z}$, so $h^{-1}(U)$ is su. $\widehat{\omega}$(ŋ̂)-open subset from Y, so $f^{-1}(h^{-1}(U)) = (h\circ f)^{-1}(U)$ is su. open subset from X. Hence, $h\circ f$ is strongly su. $\widehat{\omega}$(ŋ̂)-continuous function.

17 and 18- If $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is su. $\widehat{\omega}$(ŋ̂)-irresolute function, $h:(Y,\mu_Y) \longrightarrow (\mathcal{Z},\mu_{\mathcal{Z}})$ is strongly su. $\widehat{\omega}$(ŋ̂)-continuous function and U is a su. $\widehat{\omega}$(ŋ̂)-open subset from $\mathcal{Z}$, so $h^{-1}(U)$ is su. open subset from Y and then it is su. $\widehat{\omega}$(ŋ̂)-open set (by remark (2.9)). Hence, $f^{-1}\left(h^{-1}(U)\right) = (h\circ f)^{-1}(U)$ is su. $\widehat{\omega}$(ŋ̂)-open subset from X. Hence, $h\circ f$ is su. $\widehat{\omega}$(ŋ̂)-irresolute function.

19 and 20- If $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is su. $\widehat{\omega}$(ŋ̂)-irresolute function, $h:(Y,\mu_Y) \longrightarrow (\mathcal{Z},\mu_{\mathcal{Z}})$ is su. continuous function and U is a su. open subset from $\mathcal{Z}$, so $h^{-1}(U)$ is su. open subset from Y and then it is su. $\widehat{\omega}$(ŋ̂)-open set (by remark (2.9)). Hence, $f^{-1}\left(h^{-1}(U)\right) = (h\circ f)^{-1}(U)$ is su. $\widehat{\omega}$(ŋ̂)-open subset from X. Hence, $h\circ f$ is su. $\widehat{\omega}$(ŋ̂)-continuous function.

21 and 22- If $f:(X,\mu_X) \longrightarrow (Y,\mu_Y)$ is su. $\widehat{\omega}$(ŋ̂)-irresolute function, $h:(Y,\mu_Y) \longrightarrow (\mathcal{Z},\mu_{\mathcal{Z}})$ is su. $\widehat{\omega}$(ŋ̂)-irresolute function and U is a su. $\widehat{\omega}$(ŋ̂)-open subset from $\mathcal{Z}$, so $h^{-1}(U)$ is su. $\widehat{\omega}$(ŋ̂)-open subset from Y. Hence, $f^{-1}\left(h^{-1}(U)\right) = (h\circ f)^{-1}(U)$ is su. $\widehat{\omega}$(ŋ̂)-open subset from X. Hence, $h\circ f$ is su. $\widehat{\omega}$(ŋ̂)-irresolute function.

The following diagrams are advantageous.



Example (3.27):

1. $I_{\mathcal{Z}}: (\mathcal{Z}, \mu_{coc}) \longrightarrow (\mathcal{Z}, \mu_{cof})$ is su. continuous, su. $\widehat{\omega}$-continuous, and su. ŋ̂-continuous function, but not perfectly su. continuous, not totally su. $\widehat{\omega}$ -continuous, not totally su. ŋ̂-continuous, and not perfectly su. ŋ̂-irresolute.
2. $I_{\mathcal{R}}: (\mathcal{R}, \mu_{coc}) \longrightarrow (\mathcal{R}, \mu_{cof})$ is su. continuous, su. $\widehat{\omega}$-continuous, su. ŋ̂-continuous, strongly su. $\widehat{\omega}$-continuous, strongly su. ŋ̂-continuous, su. $\widehat{\omega}$-irresolute and su. ŋ̂-irresolute function, but not totally su. $\widehat{\omega}$-continuous, not totally su. ŋ̂-continuous, not perfectly su. $\widehat{\omega}$-irresolute, not perfectly su. $\widehat{\omega}$-continuous, not perfectly su. ŋ̂-continuous, not perfectly su. ŋ̂-irresolute, and not perfectly su. continuous.
3. $I_{\mathcal{Z}}: (\mathcal{Z}, \mu_{ind}) \longrightarrow (\mathcal{Z}, \mu_{EX})$ is totally su. $\widehat{\omega}$-continuous function, but not su. continuous.
4. $I_X: (X, \mu_{ind}) \longrightarrow (X, \mu_D)$ where X is finite, is totally su. ŋ̂-continuous, and perfectly su. ŋ̂-irresolute function, but neither su. continuous nor strongly su. ŋ̂-continuous. On the other hand, it is su. $\widehat{\omega}$-continuous, su. ŋ̂-continuous and su. ŋ̂-irresolute function, but not perfectly su. continuous and not perfectly su. ŋ̂-continuous function.
5. The identity function $I_{\mathcal{R}}$ from $(\mathcal{R}, \mu_{ind})$ into the same space is su. continuous function, but not perfectly su. $\widehat{\omega}$-continuous, not perfectly su. ŋ̂-continuous, and not perfectly su. ŋ̂-irresolute.
6. The identity function $I_{\mathcal{Z}}: (\mathcal{Z}, \mu_{ind}) \longrightarrow (\mathcal{Z}, \mu_D)$ is perfectly su. $\widehat{\omega}$-irresolute function, but neither su. continuous nor strongly su. $\widehat{\omega}$-continuous. Also, it is su. $\widehat{\omega}$-irresolute not perfectly su. $\widehat{\omega}$-continuous function.

7. The identity function $I_{\mathcal{R}}: (\mathcal{R}, \mu_{cof}) \longrightarrow (\mathcal{R}, \mu_{coc})$ is su. $\widehat{\omega}$-continuous and su. $\widehat{\omega}$-irresolute function, while it is not perfectly su. $\widehat{\omega}$-irresolute, not totally su. $\widehat{\omega}$-continuous, and not totally su. ŋ̂-continuous.
8. The identity function $I_{\mathcal{R}}$ from $(\mathcal{R}, \mu_{coc})$ to the same space is strongly su. $\widehat{\omega}$-continuous and strongly su. ŋ̂-continuous function, but not perfectly su. $\widehat{\omega}$-irresolute and not perfectly su. ŋ̂-irresolute.
9. The identity function $I_{\mathcal{Z}}$ from $(\mathcal{Z}, \mu_{ind})$ into the same space is perfectly su. $\widehat{\omega}$-irresolute, totally su. $\widehat{\omega}$-continuous, and totally su. ŋ̂-continuous function, but not strongly su. $\widehat{\omega}$-continuous and not strongly su. ŋ̂-continuous.
10. $I_X: (X, \mu_{ind}) \longrightarrow (X, \mu_{ind})$ in which X is a finite set, is perfectly su. ŋ̂-irresolute function, but not strongly su. ŋ̂-continuous.
11. The function f from $(\mathcal{R}, \mu_{ind})$ into $(\mathcal{Z}, \mu_{ind})$, in which $f(x)=\begin{cases} x & when\ x \in \mathcal{Z} \\ 0 & o.w \end{cases}$ is perfectly su. continuous function, while it is not su. $\widehat{\omega}$-irresolute and not su. ŋ̂-irresolute function.

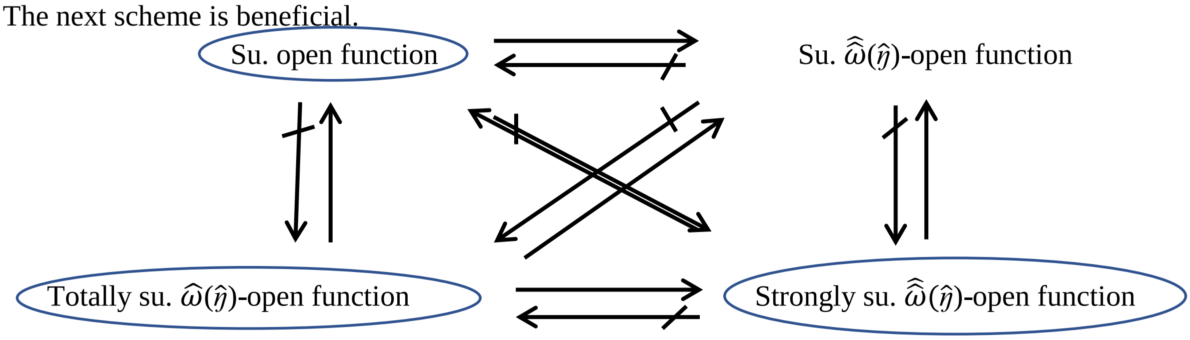
4- supra $\widehat{\omega}$-open and supra ŋ̂-open functions.

In this part we will use supra $\widehat{\omega}$-open and supra ŋ̂-open sets to define new forms of supra open and supra closed functions.

Definition (4.1): The function $f : (X, \mu_X) \longrightarrow (Y, \mu_Y)$ is called:
1. Su. open (resp. su. closed) function, if $f(V)$ is su. open (resp. su. closed) set in the su. space Y, for any su. open (resp. su. closed) set V in the su. space X [14].
2. Su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) function, if $f(V)$ is su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set in the su. space Y, for any su. open (resp. su. closed) set V in the su. space X.
3. Totally su. $\widehat{\omega}$-open (resp. totally su. $\widehat{\omega}$-closed) function, if $f(V)$ is su. open (resp. su. closed) set in the su. space Y, for any su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set V in the su. space X.
4. Strongly su. $\widehat{\omega}$-open (resp. strongly su. $\widehat{\omega}$-closed) function, if $f(V)$ is su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set in the su. space Y, for any su. $\widehat{\omega}$-open (resp. su. $\widehat{\omega}$-closed) set V in the su. space X.
5. Su. ŋ̂-open (resp. su. ŋ̂-closed) function, if $f(V)$ is su. ŋ̂-open (resp. su. ŋ̂-closed) set in the su. space Y, for any su. open (resp. su. closed) set V in the su. space X.
6. Totally su. ŋ̂-open (resp. su. ŋ̂-closed) function, if $f(V)$ is su. open (resp. su. closed) set in the su. space Y, for any su. ŋ̂-open (resp. su. ŋ̂-closed) set V in the su. space X.
7. Strongly su. ŋ̂-open (resp. strongly su. ŋ̂-closed) function, if $f(V)$ is su. ŋ̂-open (resp. su. ŋ̂-closed) set in the su. space Y, for any su.ŋ̂-open (resp. su. ŋ̂-closed) set V in the su. space X.

Example (4.2): 1. The identity function from $(\mathcal{Z}, \mu_{ind})$ into the same space is su. open and su. closed function.
2. The identity function I_Q from (Q, μ_D) into (Q, μ_{ind}) is su. $\widehat{\omega}$-open, su. $\widehat{\omega}$-closed, strongly su. $\widehat{\omega}$-open and strongly su. $\widehat{\omega}$-closed function.
3. $I_X: (X, \mu_D) \longrightarrow (X, \mu_{ind})$ in which X is finite set, is su. ŋ̂-open, su. ŋ̂-closed, strongly su. ŋ̂-open and strongly su. ŋ̂-closed function.
4. A function $f: (X, \mu_X) \longrightarrow (X, \mu_D)$ is totally su. $\widehat{\omega}$-open, totally su. $\widehat{\omega}$-closed, totally su. ŋ̂-open and totally su. ŋ̂-closed function.

The next scheme is beneficial.



Example (4.3): 1. The identity function from the discrete su. space (X, μ_D) into the indiscrete su. space (X, μ_{ind}), where X is finite set, is su. ŋ̂-open, strongly su. $\widehat{\omega}$-open, strongly su. ŋ̂-open and su. $\widehat{\omega}$-open function, while it is not su. open, not totally su. $\widehat{\omega}$-open and not totally su. ŋ̂-open function.
2. The identity function $I_Q: (Q, \mu_D) \longrightarrow (Q, \mu_{ind})$ is su. $\widehat{\omega}$-open, but not su. open function.
3. The identity function from (X, μ_{EX}) into the same space, in which X is finite set, is su. open function, but neither totally su. $\widehat{\omega}$-open nor totally su. ŋ̂-open function.

4. $I_{\mathcal{R}}: (\mathcal{R}, \mu_{coc}) \longrightarrow (\mathcal{R}, \mu_{cof})$ is strongly su. $\widehat{\omega}$-open function, but not strongly su. ŋ̂-open.
5. $I_{\mathcal{Z}}: (\mathcal{Z}, \mu_{ind}) \longrightarrow (\mathcal{Z}, \mu_{ind})$ is strongly su. $\widehat{\omega}$-open and strongly su. ŋ̂-open function, while it is neither totally su. $\widehat{\omega}$-open nor totally su. ŋ̂-open function.

Theorem (24.5): The composition between:

1. Su. $\widehat{\omega}$-closed function and totally su. $\widehat{\omega}$-closed function is su. closed function.
2. Su. ŋ̂-closed function and totally su. ŋ̂-closed function is su. closed function.
3. Su. $\widehat{\omega}$-closed function and strongly su. $\widehat{\omega}$-closed function is su. $\widehat{\omega}$ -closed function.
4. Su. ŋ̂-closed function and strongly su. ŋ̂-closed function is su. ŋ̂-closed function
5. Totally su. $\widehat{\omega}$-closed function and su. closed function is totally su. $\widehat{\omega}$-closed function.
6. Totally su. ŋ̂-closed function and su. closed function is totally su. ŋ̂-closed function.
7. Totally su. $\widehat{\omega}$-closed function and su. $\widehat{\omega}$-closed function is strongly su. $\widehat{\omega}$-closed function.
8. Totally su. ŋ̂-closed function and su. ŋ̂-closed function is strongly su. ŋ̂-closed function.
9. Totally su. $\widehat{\omega}$-closed function and totally su. $\widehat{\omega}$-closed function is totally su. $\widehat{\omega}$-closed function.
10. Totally su.ŋ̂-closed function and totally su. ŋ̂-closed function is totally su. ŋ̂-closed function.
11. Totally su. $\widehat{\omega}$-closed function and strongly su. $\widehat{\omega}$-closed function is strongly su. $\widehat{\omega}$-closed function.
12. Totally su. ŋ̂-closed function and strongly su. ŋ̂-closed function is strongly su. ŋ̂-closed function.
13. Strongly su. $\widehat{\omega}$-closed function and totally su. $\widehat{\omega}$-closed function is totally su. $\widehat{\omega}$-closed function.
14. Strongly su. ŋ̂-closed function and totally su. ŋ̂-closed function is totally su. ŋ̂-closed function.
15. Strongly su. $\widehat{\omega}$-closed function and strongly su. $\widehat{\omega}$-closed function is strongly su. $\widehat{\omega}$-closed function.
16. Strongly su. ŋ̂-closed function and strongly su. ŋ̂-closed function is strongly su. ŋ̂-closed function

Proof: If $f: (X, \mu_X) \longrightarrow (Y, \mu_Y)$is su. $\widehat{\omega}$-closed function, $g: (Y, \mu_Y) \longrightarrow (\mathcal{Z}, \mu_{\mathcal{Z}})$ is totally su. $\widehat{\omega}$-closed function and $\mathcal{M}$ is a su. closed set in X, so $f(\mathcal{M})$ is su. $\widehat{\omega}$-closed set in Y. Then, $g(f(M))$=$(g \circ f)(M)$ is su. closed set in $\mathcal{Z}$. Therefore, $g \circ f$ is su. closed function. In the same manner, we can prove the rest.

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