UNSTEADY FREE CONVECTION FLOW OF THIRD GRAD ELECTRICALLY CONDUCTING FLUID PAST AN INFINITE VERTICAL PLATE

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Abstract

In this paper the problem of, unsteady, hydromagnatic free convective flow of viscous incompressible and electrically conducting third order fluids past in infinite vertical Porous plate in the presence of constant suction and heat absorbing sinks is considered. It is found that the velocity and temperature distribution equations are controlled by different dimensionless parameters, namely, Grashof number *Gr*, prandtl number *pr*, Eckert number *Ec*, sink strength s, material moduli β and coecostic parameter *α*. An analytic solution for each of the velocity and the temperature distribution is obtained. The velocity and temperature distributions are shown graphically taking many cases of *Gr, pr, Ec, s,* β and *α*.

الجريان الحر اللا مستقر لمائع قابل للتوصيل الكهربائي من الرتبة الثالثة خلال صحيفة عمودية لانهائية

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الخلاصة

في هذا البحث تم دراسة مسألة الجريان اللآمستقر لمائع قابل للتوصيل الكهربائي من الرتبة الثالثة في حقل مغناطيسي لصحيفة عمودية لانهائية في وسط بورس وبوجود أمتصاص حراري. لقد تبين ان معادلة السرعة ومعادلة الحرارة تحكمهما أعداد لأبعدية مثل كريشوف، برااندال، اكيورت، سنك سترنك، متيريل موديل وكوكاستك. تم الحصول على حل تحليلي لكل من معادلة السرعة ومعادلة انتقال الحرارة. رسمت كل من مركبة السرعة ودرجة الحرارة بيانيا وذلك بأخذ عدة قيم مختلفة لكل من *Gr , pr ,Ec*, β ,s و α.

Introduction

The problem of free convection flow of an electrically conducting third order fluid past a vertical plate under the influence of a magnetic field attracted many scientists, in view of its application in astrophysics, geophysics, engineeering and aerodynamics…etc.
The unsteady free convection flow past an infinite plate with constant suction and heat sources has been studied by Pop and Soundalgekar (١٩٧٥) [١]. The effect of magnetic field on the convective flow of electrically conducting fluid past a semi-infinite flat plate has been analyzed by Gupta (١٩٦١) [٢], Nanda and Mohanty (١٩٧٠) [٣]. Sacheti, Chardran and

Singh (١٩٩٤) [٤] have obtained an exact solution for the unsteady MHD problem Sahoo, Datta

and Biswal (٢٠٠٣) [٥] have been studied the heat transfer in mercuy (pr=٠.٠٢٥) and electrolytic solution (pr=١.٠) past an infinite porous plate with constant suction in the presence of uniform magnetic field and heat sink.

Sharma el. at (٢٠٠٤) [٦] has been studied unsteady MHD flow and heat transfer over continuous porous moving horizontal surface in the presence of an oscillating free stream and heat Source-Noushima et-al (٢٠٠٨) [٧] had extended the above problem to viscoelastic fluid. Mostafa, Rafiuddin and Ramaria (٢٠٠٨) [٨] have been studied unsteady MHD to extend the work of (Noushima et-al) with variable suction.

Finally, in this paper we will study unsteady free convection flow of third grade electrically conducting fluid, an analytic solution for each of the velocity and temperature distribution are obtained.

The governing equation

The Cauchy stress tenser T of a third grade fluid is defined by the Rivlin-Ericksen constitutive equation (Rivlin and Ericksen (١٩٥٥) [٩]

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + B_3 (trA_1^2) A_1$$

Where:

P is pressure, I is the identity tensor, μ is coefficient of shear viscosity , while $\alpha_١$, $\alpha_٢$ and $\beta_٢$ are the material constants which are satisfy $\mu \geq ٠$, $\alpha_j \geq ٠$, $\beta_٢ \geq ٠$ and $\alpha_١ + \alpha_٢ = ٠$

$A_١$ and $A_٢$ are the first and second Rivlin-Ericksen tensors defined as

$$A_1 = \nabla V + \nabla V^T$$

$$A_2 = \frac{\partial A_1}{\partial t} + V - \nabla A_1 + A_1 \nabla V + \nabla V^T A_1$$

Where: V is the velocity vector.

Let the x-axis be taken in the vertically upward direction along the infinite vertical plate and y-axis normal to it, neglecting the inducted magnetic field and applying Boussinesq's approximation, In this case the continuity equation is:

$$\frac{\partial v^*}{\partial y^*} = 0 \qquad (١)$$

$$v^* = -v_0 \text{ (constant)} \qquad (٢)$$

The momentum equation is:

$$p\left(\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*}\right) = \mu \frac{\partial^2 u^*}{\partial y^{*2}} + \alpha_1 \left(\frac{\partial^3 u^*}{\partial y^{*2} \partial t^*} + v^* \frac{\partial^3 v^*}{\partial y^{*2}}\right)$$
$$+ 6\beta_3 \left(\frac{\partial u^*}{\partial y^*}\right)^2 \frac{\partial^2 u^*}{\partial y^{*2}} + p_g \beta_g \left(T^* - T_0\right) \qquad (3)$$

An the energy equation

$$\frac{\partial \theta^*}{\partial t^*} + v^* \frac{\partial \theta^*}{\partial y^*} = k \frac{\partial^2 \theta^*}{\partial y^{*2}} + s^* \left(\theta^* - \theta_\infty^*\right)$$
$$+ \frac{v}{c_p} \left(\frac{\partial u^*}{\partial y^*}\right)^2 \qquad (4)$$

The associated boundary condition, of the problems are

$$u^* = 0 \,,\, \theta^* = 1 \;\; at \;\; y^* = 0 \qquad (٥\text{-a})$$

$$u^* \to 0 \,,\, \theta^* \to 1 \;\; at \;\; y^* \to \infty \qquad (٥\text{-b})$$

Method of Solution

To write the governing equation in dimensionless form, we introduce the following dimensionless quantities:

$$y = \frac{v_0 y^*}{v}, t = \frac{v_0^{*2}}{v} t^*, u = \frac{u^*}{v_0^*}, Gr = \frac{v_g \beta_3 \left(\theta_w^* - \theta_\infty^*\right)}{v_0^2},$$
$$pr = \frac{v}{k}, Ec = \frac{v_0^2}{c_p \left(\theta_w^* - \theta_\infty^*\right)}, \theta = \frac{\left(\theta^* - \theta_\infty^*\right)}{\left(\theta_w^* - \theta_\infty^*\right)}, \qquad (6)$$
$$\alpha = \frac{\alpha_1 v_0^2}{v^2}, \beta = \frac{\beta_3 v_0^4}{v^3}, \rho = \frac{v}{\mu} \; and \; s = \frac{s^* v}{v_0^2}$$

Where pr, Gr, s and Ec are the prandlt number, Grashof number, sink strength and Eckert number respectively.

In the view of eqs (٥) and (٦), the governing equations (٢) and eqs (٣) can be written in dimensionless form as

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \alpha \left(\frac{\partial^3 u}{\partial y^2 \partial t} - \frac{\partial^3 u}{\partial y^3}\right) + 6\beta \left(\frac{\partial u}{\partial y}\right)^2 \left(\frac{\partial^2 u}{\partial y^2}\right) + Gr\,\theta \qquad (٧)$$

$$pr \frac{\partial \theta}{\partial t} - pr \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + pr\, s\, \theta + pr\, E \left(\frac{\partial u}{\partial y}\right)^2 \qquad (٨)$$

The corresponding boundary conditions, in non-dimensional form, are:

$$u = 0 \;,\, \theta = 1 \;\; at \;\; y = 0 \qquad (٩\text{-a})$$

and

$$u \to 0 \ , \theta \to 1 \ \ as \ \ y \to \infty \qquad (٩\text{-b})$$

To solve eqs (٧) and (٨), we assume

$$u(y,t) = u_0(y) + \in e^{i\omega t} u_1(y) \qquad (١٠\text{-a})$$

and

$$\theta(y,t) = \theta_0(y) + \in e^{i\omega t} \theta_1(y) \qquad (١٠\text{-b})$$

The substituting of eq (١٠.a,b) into equations (٧) and (٨) and equating the coefficients of o (∈), gives:
zero order equation

$$-u_0^{'} = u_0^{''} + Gr\,\theta_0 - \alpha\, u_0^{'''} + 6\beta\, u_0^{''} u_0^{'2} \qquad (١١)$$

$$-pr\,\theta_0^{'} = \theta_0^{'} + pr\,s\,\theta_0 + pr\,Ec\,u_0^{'2}$$
(١٢)

first order equation

$$i\omega\, u_1 - u_1^{'} = u_1^{''} + Gr\,\theta_1 + \alpha\, i\omega u_1^{''} - \alpha\, u_1^{'''} + 6\beta\left(u_0^{'2}\, u_1^{''} + 2u_0^{''}\, u_0^{'}\, u_1^{'}\right) \qquad (١٣)$$

$$pr\, i\omega\, \theta_1 - pr\,\theta_1^{'} = \theta_1^{''} + pr\,s\,\theta_1 + 2pr\,Ec\,u_0^{'} u_1^{'} \qquad (١٤)$$

and the boundary conditions are :

$$u_0 = 0, u_1 = 0, \theta_0 = 1, \theta_1 = 0\, at\ \ y \to 0$$
$$u_0 \to 0, u_1 \to 0, \theta_0 \to 0, \theta_1 \to 0\ \ at\ \ y \to \infty$$

Equations(١١-١٤) are nonlinear third order we are using multiparameters perturbation technique and assuming *Ec* is very small, therefore u_0, u_1, θ_0, θ_1 can be expanded in term of *Ec* ,as follows:

$$\left.\begin{aligned} u_0 &= u_{01} + Ec\ \ u_{02}\ \ ,\ \ u_1 = u_{11} + Ec\ \ u_{12} \\ \theta_0 &= \theta_{01} + Ec\ \ \theta_{02}\ \ ,\ \ \theta_1 = \theta_{11} + Ec\ \ \theta_{12} \end{aligned}\right\} (١٥)$$

Putting the last equation into equations (١١-١٤) and equating the power of *Ec* ,we obtain
Zero order of *Ec*

$$-pr\,\theta_{01}^{'} = \theta_{01}^{''} + pr\,s\,\theta_{01} \qquad (١٦)$$

$$-pr\,\theta_{02}^{'} = \theta_{02}^{''} + pr\,s\,\theta_{02} + pr\,u_{01}^{'2} \qquad (١٧)$$

$$-u_{01}^{'} = u_{01}^{''} + Gr\,\theta_{01} - \alpha\, u_{01}^{'''} + 6B\ u_{01}^{''}\ u_{01}^{'2} \qquad (١٨)$$

$$-u_{02}^{'} = u_{02}^{''} + Gr\,\theta_{02} - \alpha\, u_{02}^{'''} + 6B(u_{02}^{''}\, u_{01}^{'2} + 2u_{01}^{''}\, u_{01}^{'} u_{02}^{'}) \qquad (١٩)$$

First order of *Ec*

$$pr\,i\,\omega\,\theta_{11} - pr\,\theta_{11}^{'} = \theta_{11}^{''} + pr\,s\,\theta_{11} \qquad (٢٠)$$

$$pr\,i\,\omega\,\theta_{12} - pr\,\theta_{12}^{'} = \theta_{12}^{''} + pr\,s\,\theta_{12} + 2pr\ u_{01}^{'}\, u_{11}^{'} \qquad (٢١)$$

$$i\,\omega u_{11} - u_{11}^{'} = u_{11}^{''} + Gr\,\theta_{11} + \alpha\, i\,\omega u_{11}^{''} - \alpha\, u_{11}^{'''} + 6B\,(u_{01}^{'2}\ u_{11}^{''} + 2u_{01}^{''}\, u_{01}^{'}\, u_{11}^{'}) \qquad (٢٢)$$

$$i\omega u_{12} - u_{12}^{'} = u_{12}^{''} + Gr\,\theta_{12} + \alpha\ i\,\omega\ u_{12}^{''} - \alpha\ u_{12}^{'''} + 6B\,(u_{01}^{'2}\, u_{12}^{''} + 2u_{01}^{'}\, u_{02}^{'}\, u_{11}^{''} + 2u_{01}^{''}\ u_{01}^{'}\ u_{12}^{'} + u_{02}^{''}\, u_{01}^{'}\, u_{11}^{'} + u_{02}^{'}\ u_{01}^{''}\ u_{11}^{'}) \qquad (٢٣)$$

For small value of β , we can write

$$\left.\begin{aligned} \theta_{01} &= \theta_{011} + \beta\ \theta_{012}\ ,\ \ \theta_{02} = \theta_{021} + \beta\ \ \theta_{022} \\ u_{01} &= u_{011} + \beta u_{012}\ ,\ \ u_{02} = u_{021} + \beta u_{022} \\ \theta_{11} &= \theta_{111} + \beta\ \theta_{112}\ ,\ \ \theta_{12} = \theta_{121} +\ \beta\ \theta_{122} \\ u_{11} &= u_{111} + \beta\ u_{112}\ ,\ \ u_{12} = u_{121} + \beta\ u_{122} \end{aligned}\right\} (٢٤)$$

From equations (١٦-٢٣),we get Zero order of *B*

$$-pr\,\theta_{011}^{'} = \theta_{011}^{''} + pr\,s\,\theta_{011} \qquad (٢٥)$$

$$-pr\,\theta_{012}^{'} = \theta_{012}^{''} + pr\,s\,\theta_{012} \qquad (٢٦)$$

$$-pr\,\theta_{021}^{'} = \theta_{021}^{''} + pr\,s\,\theta_{021} + pr\,u_{011}^{'2} \qquad (٢٧)$$

$$-pr\,\theta_{022}^{'} = \theta_{022}^{''} + pr\,s\,\theta_{022} + 2pr\ u_{011}^{'}\, u_{012}^{'}$$
(٢٨)

$$-u_{011}^{'} = u_{011}^{''} + Gr\,\theta_{011} - \alpha\, u_{011}^{'''} \qquad (٢٩)$$

$$-u_{012}^{'} = u_{012}^{''} - \alpha\, u_{012}^{'''} + Gr\,\theta_{012} + 6(u_{011}^{''}\ u_{011}^{'2})$$
(٣٠)

$$-u_{021}^{'} = u_{021}^{''} + Gr\,\theta_{021} - \alpha\, u_{021}^{'''} \qquad (٣١)$$

$$-u_{022}^{'} = u_{022}^{''} + Gr\,\theta_{022} - \alpha\, u_{022}^{'''} + 6(u_{021}^{''}\, u_{011}^{'2} + 2u_{011}^{''}\, u_{011}^{'}\, u_{021}^{'})$$
(٣٢)

First order of β

$$pr\,i\,\omega\theta_{111} - pr\,\theta_{111}^{'} = \theta_{111}^{''} + pr\,s\,\theta_{111} \qquad (٣٣)$$

$$pr\,i\,\omega\theta_{112} - pr\,\theta_{112}^{'} = \theta_{112}^{''} + pr\,s\,\theta_{112} \qquad (٣٤)$$

$$pr\,i\,\omega\theta_{121} - pr\,\theta_{121}^{'} = \theta_{121}^{''} + pr\,s\,\theta_{121} + 2pr\,u_{011}^{'}\, u_{111}^{'} \qquad (٣٥)$$

$$pr\,i\,\omega\theta_{122} - pr\,\theta_{122}^{'} = \theta_{122}^{''} + pr\,s\,\theta_{122}^{'} + 2pr(u_{011}^{'}\, u_{112}^{'} + u_{012}^{'}\, u_{111}^{'}) \qquad (٣٦)$$

$$i\omega u_{111} - u'_{111} = u''_{111} + Gr\,\theta_{111} + \alpha\, i\omega u''_{111} - \alpha\, u'''_{111} \tag{٣٧}$$

$$i\omega u_{112} - u'_{112} = u''_{112} + \alpha\, i\omega u''_{112} - \alpha\, u'''_{112} + 6(u'^2_{011} u''_{111} + 2u''_{011} u'_{011} u'_{111}) \tag{٣٨}$$

$$i\omega u_{121} - u'_{121} = u''_{121} + Gr\,\theta_{121} + \alpha\, i\omega u''_{121} - \alpha\, u'''_{121} \tag{٣٩}$$

$$i\omega u_{122} - u'_{122} = u''_{122} + Gr\,\theta_{122} + \alpha\, i\omega u''_{122} - \alpha\, u'''_{122} + 6(u'^2_{011}\, u''_{121} + 2u'_{011} u'_{021} u''_{111} + 2u''_{011} u'_{011} u'_{121} + u''_{021} u'_{011} u'_{111} + u'_{021} u''_{011} u'_{111}) \tag{٤٠}$$

Finally if α is very small, we can write

$$\left.\begin{array}{ll}
\theta_{011} = q + \alpha\, q_0 & u_{011} = w + \alpha\, v \\
\theta_{012} = p + \alpha\, p_0 & u_{012} = f + \alpha\, g \\
\theta_{021} = q_1 + \alpha\, q_2 & u_{021} = w_1 + \alpha\, v_1 \\
\theta_{022} = p_1 + \alpha\, p_2 & u_{022} = f_1 + \alpha\, g_1 \\
\theta_{111} = q_3 + \alpha\, q_4 & u_{111} = w_2 + \alpha\, v_2 \\
\theta_{112} = p_3 + \alpha\, p_4 & u_{112} = f_2 + \alpha\, g_2 \\
\theta_{121} = q_5 + \alpha\, q_6 & u_{121} = w_3 + \alpha\, v_3 \\
\theta_{122} = p_5 + \alpha\, p_6 & u_{122} = f_3 + \alpha\, g_3
\end{array}\right\} \tag{٤١}$$

And from equations (٢٥-٣٩), we get zero order of α

$$-pr\,q = q'' + pr\,s\,q \tag{٤٢}$$

$$-pr\,p = p'' + pr\,s\,p \tag{٤٣}$$

$$-pr\,q'_1 = q''_1 + pr\,s\,q_1 + pr\,w'^2 \tag{٤٤}$$

$$-w' = w'' + Gr\,q \tag{٤٥}$$

$$-f' = f'' + Gr\,p + 6(w''\,w'^2) \tag{٤٦}$$

$$-w'_1 = w''_1 + Gr\,q_1 \tag{٤٧}$$

$$-pr\,p'_1 = p''_1 + pr\,s\,p_1 + 2pr\,w'\,f' \tag{٤٨}$$

$$-f'_1 = f''_1 + Gr\,p_1 + 6(w''_1\,w'^2 + 2w''\,w'\,w'_1) \tag{٤٩}$$

$$pr\,i\omega q_3 - pr\,q'_3 = q''_3 + pr\,s\,q_3 \tag{٥٠}$$

$$pr\,i\omega p_3 - pr\,p'_3 = p''_3 + pr\,s\,p_3 \tag{٥١}$$

$$pr\,i\omega q_5 - pr\,q'_5 = q''_5 + pr\,s\,q_5 + 2pr\,w'\,w'_2 \tag{٥٢}$$

$$pr\,i\omega p_5 - pr\,p'_5 = p''_5 + pr\,s\,p'_5 + 2pr(w'\,f'_2 + f'\,w'_2) \tag{٥٣}$$

$$i\omega w_2 - w'_2 = w''_2 + Gr\,q_3 \tag{٥٤}$$

$$i\omega f_2 - f'_2 = f''_2 + 6(w'^2\,w''_2 + 2w''\,w'\,w'_2) \tag{٥٥}$$

$$i\omega w_3 - w'_3 = w''_3 + Gr\,q_5 \tag{٥٦}$$

$$i\omega f_3 - f'_3 = f''_3 + Gr\,p_5 + 6[w'^2\,w''_3 + 2w'\,w'_1\,w''_2 + 2w''\,w'\,w'_3 + w''_1\,w'\,w'_2 + w'_1\,w''\,w'_2] \tag{٥٧}$$

First order of α

$$-pr\,q_0 = q''_0 + pr\,s\,q_0 \tag{٥٨}$$

$$-pr\,p_0 = p''_0 + pr\,s\,p_0 \tag{٥٩}$$

$$-pr\,q'_2 = q''_2 + pr\,s\,q_2 + 2pr\,w'\,v' \tag{٦٠}$$

$$-pr\,p'_2 = p''_2 + pr\,s\,p_2 + 2pr(w'\,g' + v'\,f') \tag{٦١}$$

$$-v' = v'' + Gr\,q_0 - w''' \tag{٦٢}$$

$$-g' = g'' - f''' + Gr\,p_0 + 6[w'^2\,v'' + 2w'\,v'\,w''] \tag{٦٣}$$

$$-v'_1 = v''_1 + Gr\,q_2 - w'''_1 \tag{٦٤}$$

$$-g'_1 = g''_1 + Gr\,p_2 - f'''_1 + 6[(w'^2\,v''_1 + 2w'v'\,w''_1) + 2\left(w''w'v'_1 + v''w'w'_1 + v'w''w'_1\right)] \tag{٦٥}$$

$$pr\,i\omega q_4 - pr\,q'_4 = q''_4 + pr\,s\,q_4 \tag{٦٦}$$

$$pr\,i\omega p_4 - pr\,p'_4 = p''_4 + pr\,s\,p_4 \tag{٦٧}$$

$$pr\,i\omega q_6 - pr\,q'_6 = q''_6 + pr\,s\,q_6 + 2pr(w'\,v'_2 + v'\,w'_2) \tag{٦٨}$$

$$pr\,i\omega p_6 - pr\,p'_6 = p''_6 + pr\,s\,p'_6 + 2pr[w'\,g'_2 + v'\,f'_2 + f'\,v'_2 + g'\,w'_2] \tag{٦٩}$$

$$i\omega v_2 - v_2' = v_2'' + Gr\, q_4 + i\omega w_2'' - w_2''' \qquad (٧٠)$$

$$i\omega g_2 - g_2' = g_2'' + i\omega f_2'' - f_2''' + 6[w'^2 v_2'' + 2w' v' w_2'' + 2w'' w' v_2' + v'' w' w_2' + v' w'' w_2'] \qquad (٧١)$$

$$i\omega v_3 - v_3' = v_3'' + Gr\, q_6 + i\omega w_3'' - w_3''' \qquad (٧٢)$$

$$i\omega g_3 - g_3' = g_3'' + Gr\, p_6 + i\omega f_3'' - f_3''' + 6[w'^2 v_3'' + 2w' v' w_3'' + 2(w' w_1' v_2'' + v' w_1' w_2'' + v_1' w' w_2'') + 2(w'' w' v_3' + v'' w' w_3' + v' w'' w_3' + w_1'' w' v_2' + v' w_1'' w_2' + v_1'' w' w_2' + w_1' w'' v_2' + v'' w_1' w_2' + v_1' w'' w_2'] \qquad (٧٣)$$

Solving these differential equations (٤٢-٧٣) with the aid of the corresponding boundary conditions and then substituting these solutions into relation (١٥, ٢٤, ٤١), we obtain the velocity distribution u and temperature distribution θ as

$$u = \frac{A_1 Gr}{pr^{3/2}(-2+pr+\sqrt{pr}\sqrt{pr-4s})^4} e^{-y} + \frac{4Gr\alpha}{\sqrt{pr}(\sqrt{pr}+\sqrt{pr-4s})} e^{-y} y + (A_2 Gr + \alpha Gr^3 + \beta\Big(Gr^3 \alpha A_3 e^{-3y} + A_4 e^{-\frac{1}{2}(4+pr-\sqrt{pr}\sqrt{pr-4s})y} + \frac{2}{3} Gr^2 \alpha (A_5 e^{\frac{1}{2}(-8+3pr+3\sqrt{pr}\sqrt{pr-4s})y} + \frac{A_6 e^{(-1-pr-\sqrt{pr}\sqrt{pr-4s})y}}{(-2+pr+\sqrt{pr}\sqrt{pr-4s})^4 (pr+\sqrt{pr}\sqrt{pr-4s})^3} + (\frac{32pr^3\sqrt{pr-4s}\, s}{\sqrt{pr}(-2+pr+\sqrt{pr}\sqrt{pr-4s})} + A_7)e^{-y}y^2\Big) + Ec\Big(\Big(A_8 e^{-y} y + A_9 e^{-\frac{1}{2}(pr+\sqrt{pr}\sqrt{pr-4s})y}\Big) + \alpha\big(A_{10} e^{-y} y^2\big)\Big) \beta\Big(A_{11} e^{\frac{1}{2}(-4-pr+\sqrt{pr}\sqrt{pr-4s})y} + A_{12} e^{-\frac{1}{2}(-2+pr+\sqrt{pr}\sqrt{pr-4s})y} + A_{13} e^{-\frac{1}{2}(-12+7pr+7\sqrt{pr}\sqrt{pr-4s})y}\Big) + \in e^{i\omega t}$$

$$\Big\{\Big(\frac{(2(e^{-\frac{1}{2}(1+\sqrt{1+4i\omega})y} - e^{-\frac{1}{2}\sqrt{pr}(\sqrt{pr}+\sqrt{pr-4s+4i\omega})y)})Gr)}{(pr^2 + pr(-1-2s+2i\omega) - \sqrt{pr}\sqrt{pr-4s+4i\omega} + pr^{3/2}} + \alpha(A_{14} e^{-\frac{1}{2}(1+\sqrt{1+4i\omega})y} + A_{15} e^{-\frac{1}{2}\sqrt{pr}(\sqrt{pr}+\sqrt{pr-4s+4i\omega})y)}) + \beta(A_{16} e^{-\frac{1}{2}(5+\sqrt{1+4i\omega})y} + \alpha A_{17}) e^{-\frac{1}{2}(3-pr-\sqrt{pr-4s}+\sqrt{1+4i\omega})y} + (A_{18} + \alpha A_{19}) e^{\frac{1}{2}(-2-\sqrt{pr}(\sqrt{pr-4s}+\sqrt{pr-4s+4i\omega}))y} + A_{20} e^{(-1+\frac{1}{2}(pr+\sqrt{pr}\sqrt{pr-4s})+\frac{1}{2}(-1-\sqrt{1+4i\omega}))y} + (A_{21} + \alpha A_{22}) e^{\frac{1}{2}(-2+\sqrt{pr}(\sqrt{pr-4s})-\sqrt{pr-4s+4i\omega}))y} + (A_{23} + \alpha A_{24}) e^{\frac{1}{2}(-3+pr+(\sqrt{pr-4s}+\sqrt{1+4i\omega}))y} + (A_{25} + \alpha A_{26}) e^{-\frac{1}{2}(1-2pr+2\sqrt{pr}\sqrt{pr-4s}+\sqrt{1+4i\omega})y} + Ec((A_{27} + \alpha A_{28}) e^{-\frac{1}{2}(1-pr-\sqrt{pr}\sqrt{pr-4s}+\sqrt{1+4i\omega})y} + (A_{29} + \alpha A_{30}) e^{-\frac{1}{2}(3+\sqrt{1+4i\omega})y} + A_{31} e^{-\frac{1}{2}(2+pr+\sqrt{pr}\sqrt{pr-4s+4i\omega})y}\Big) + \eta\Big\} \qquad (٧٤)$$

where $A_١, A_٢, A_٣, \ldots, A_{٣١}$ and η is a function of $pr, Gr, \alpha, \beta, s, y, i, \omega$

And

$$\theta = e^{-\frac{1}{2}(pr+\sqrt{pr}\sqrt{pr-4s})y} + Ec\Big(\Big(\frac{-2\,Gr\,pr^3 e^{-y}}{(pr^2 + pr^{3/2}\sqrt{pr-4s})(1+pr(-1+s))} + \alpha\Big(\frac{8192\,Gr^2\,pr^{19/2}\sqrt{pr-4s}}{((-1+pr)^{4^2}(4-pr+\sqrt{pr}\sqrt{pr-4s})^2} + D_1 e^{-\frac{1}{2}\sqrt{pr}(\sqrt{pr}+\sqrt{pr-4s}))y}\Big) + \beta((D_2 e^{-\frac{1}{2}(pr+\sqrt{pr}\sqrt{pr-4s}} + D_3 e^{-\frac{1}{2}(-6+pr+\sqrt{pr}\sqrt{pr-4s})y} + D_4 e^{-4y} + D_5 e^{-\frac{1}{2}(-2+3pr+3\sqrt{pr}\sqrt{pr-4s})y}) + \alpha(D_6 e^{-\frac{1}{2}(-2+\sqrt{pr}\sqrt{pr-4s})y} + D_7 e^{-\frac{1}{2}\sqrt{pr}(\sqrt{pr}+\sqrt{pr-4s})y+\frac{1}{2}(2(-3+\frac{1}{2}(pr+\sqrt{pr}\sqrt{pr-4s}))+\sqrt{pr}\sqrt{pr-4s})y} + D_8 e^{-\frac{1}{2}\sqrt{pr}(\sqrt{pr}+\sqrt{pr-4s})y+\frac{1}{2}(pr+2(-2+pr+\sqrt{pr}\sqrt{pr-4s})+\sqrt{pr-4s})y}$$

$$+D_9 e^{(-8+pr-4(-pr-\sqrt{pr}\sqrt{pr-4s})y+(5-pr+\frac{9}{2}(-pr-\sqrt{pr}\sqrt{pr-4s})y}))$$

$$+\in e^{-i\omega t}(D_{10} e^{-\frac{1}{2}(pr+\sqrt{pr}\sqrt{pr-4s+4i\omega}} + Ec(($$

$$+D_{11} e^{-\frac{1}{2}(-pr-\sqrt{pr}+\sqrt{pr-4s})y-\frac{1}{2}\sqrt{pr}(\sqrt{pr}+\sqrt{pr-4s+4i\omega})y})$$

$$+\alpha(e^{-\frac{1}{2}(\sqrt{pr}(\sqrt{pr}+(\sqrt{pr-4s-4i\omega})y} \qquad (٧٥)$$

$$+D_{12} e^{-\frac{1}{2}(-2pr+\sqrt{1+4i\omega}-\sqrt{pr}(\sqrt{pr-4s}+(\sqrt{pr-4s-4i\omega}))y}$$

$$+D_{13} e^{-\frac{1}{2}(1-2pr+\sqrt{1+4i\omega}-\sqrt{pr}(\sqrt{pr-4s}+(\sqrt{pr-4s-4i\omega}))y}$$

$$+D_{14} e^{-\frac{1}{2}(pr+\sqrt{pr}+\sqrt{pr-4s+4i\omega}+2(-1+\frac{1}{2}(-pr-\sqrt{pr}\sqrt{pr-4s+4i\omega})))y}$$

$$+D_{15} e^{-\frac{1}{2}(1-2pr+\sqrt{1+4i\omega}-\sqrt{pr}(\sqrt{pr-4s}+(\sqrt{pr-4s-4i\omega}))y}))+\psi$$

where $D_١$, $D_٢$,…………., $D_{١٥}$ and ψ is a function of $pr, Gr, \alpha, \beta, s, \omega, y, i$

Skin Friction and Nusselt number

We can find the coefficient of Skin Friction by using the formula

$$C_p = -\left(\frac{\partial u}{\partial y}\right)_{y\to 0}$$

$$= \frac{2Gr}{-pr-\sqrt{pr}\sqrt{pr-4s}} + \frac{2Gr\alpha}{(\sqrt{pr}(\sqrt{pr}+\sqrt{pr-4s}))}$$

$$\frac{(-2-pr-\sqrt{pr}\sqrt{pr-4s}\,(-pr^2-pr^{3/2}\sqrt{pr-4s}))}{(-2+pr+\sqrt{pr}\sqrt{pr-4s})^2}$$

$$\frac{4Gr\alpha(-2+pr+pr(pr+\sqrt{pr}\sqrt{pr-4s}-2s)}{\sqrt{pr}(\sqrt{pr}+\sqrt{pr-4s})(-2+pr+\sqrt{pr}\sqrt{pr-4s})^2}+$$

$$+\beta(\frac{4Gr^3}{(-2+pr+\sqrt{pr}\sqrt{pr-4s})^3}(24\sqrt{pr}(\sqrt{pr-4s})$$

$$(-1-pr-\sqrt{pr}\sqrt{pr-4s})(1+pr+\sqrt{pr}\sqrt{pr-4s})^3-$$

$$\frac{(6\sqrt{pr}(\sqrt{pr}\sqrt{pr-4s})(4+pr+\sqrt{pr}\sqrt{pr-4s})^3)}{(2+pr+\sqrt{pr}\sqrt{pr-4s})}$$

$$Ec(-(4Gr^3(9pr^3(-1+s)+9pr^{5/2}\sqrt{pr-4s}(-1+s)$$

$$-40s+\sqrt{pr}(1+pr(-1+s))(pr^{3/2}$$

$$+pr\sqrt{pr-4s}-3\sqrt{pr}s-\sqrt{pr-4s}s)$$

$$-9pr^2(-3+s+3s^2)+pr(-20-7s+72s^2)+$$

$$2(\sqrt{pr}(10-23s)\sqrt{pr-4s}+20s-pr^3(-5+5s)$$

$$+\in e^{-i\omega t}(\beta(6Gr^3((Gr(+\sqrt{pr}\sqrt{pr-4s+4i\omega})^2$$

$$(\frac{1}{2}(pr+\sqrt{pr-4s+4i\omega})))/(2(pr(-1-2s+2i\omega)`$$

$$-\sqrt{pr}\sqrt{pr-4s+4i\omega}+pr^{3/2})+$$

$$(2iGr(pr+\sqrt{pr}\sqrt{pr-4s+4i\omega})^2+$$

$$i\omega)\omega)/(pr^2-\sqrt{pr}\sqrt{pr-4s+4i\omega}+$$

$$pr^{3/2}\sqrt{pr-4s+4i\omega}-2i\omega)))/$$

$$((-1+\frac{1}{2}(-\sqrt{pr}\sqrt{pr-4s}))^2$$

$$(+(pr+\sqrt{pr}\sqrt{pr-4s+4i\omega})^2-i\omega)(1+4i\omega)$$

$$+pr(10+5s-36s^2)+\sqrt{pr-4s}(-14+5s^2)$$

$$+pr^2(-14+3s+15s^2))))/$$

$$(\sqrt{pr}(\sqrt{pr}+\sqrt{pr-4s})^3(1+pr(-1+s))$$

$$(40+9pr^3+9pr^{5/2}\sqrt{pr-4s}$$

$$-9pr^{3/2}\sqrt{pr-4s}(2+s)pr(-7+36s)))+\in e^{i\omega t}($$

$$(\beta(-(Gr^3\ pr^{12}s^3)/((-1+\frac{1}{2}(-pr-\sqrt{pr}\sqrt{pr-4s}))$$

$$(pr+(-1+\sqrt{1+4i\omega})(+\sqrt{1+4i\omega}-\sqrt{pr-4s+4i\omega})^2$$

$$(1-2pr-\sqrt{pr}\sqrt{pr-4s}+pr^{3/2}\sqrt{pr-4s+4i\omega})))+$$

$$\alpha(\frac{(Gr^2\ pr(pr+\sqrt{pr}\sqrt{pr-4s}))}{(pr^2++pr^{3/2}\sqrt{pr-4s+4i\omega}-2i\omega)}$$

$$+\frac{Gr\ pr((\sqrt{pr}+\sqrt{pr-4s+4i\omega})+i\omega)}{2(pr^2+pr^{3/2}\sqrt{pr-4s+4i\omega}-2i\omega}))+\zeta$$

where ζ is a function of $pr, Gr, \alpha, \beta, s, \omega, y, i$. The rate of heat transfer in terms of Nusselt number at the plate is

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{\theta\to 0}$$

$$= -(\frac{1}{2}(-pr-\sqrt{pr}\sqrt{pr-4s})+Ec($$

$$(-1+pr+\frac{1}{2}(4-3pr-3\sqrt{pr}\sqrt{pr-4s})$$

$$(\frac{1}{2}(-pr-\sqrt{pr}\sqrt{pr-4s})+$$

$$pr(+\frac{1}{2}(-3pr-3\sqrt{pr}\sqrt{pr-4s})s))/((\sqrt{pr}\sqrt{pr-4s})$$

$$+\beta(-(8192Gr^4\ pr(\frac{(-pr-\sqrt{pr}\sqrt{pr-4s})}{2\,pr(\sqrt{pr}+\sqrt{pr-4s})^4}+$$

$$\alpha(\ pr^4\ (-\frac{6Gr^4}{((-1+\frac{1}{2}(-pr-\sqrt{pr}\sqrt{pr-4s}))^5))})$$

$$+\alpha(6Gr^3(1+\sqrt{1+4i\omega}))/((-1+\frac{1}{2}(-\sqrt{pr}\sqrt{pr-4s}))^2$$

$$(pr^2+(-2s+2i\omega)+pr^{3/2}\sqrt{pr-4s+4i\omega}-2i\omega))$$

$$+(3Gr^3(\sqrt{1+4i\omega})^2)/((-1+\frac{1}{2}(-pr-\sqrt{pr-4s}))^2$$

$$(pr^2+pr(-1-2s+2i\omega)-\sqrt{pr}\sqrt{pr-4s}))^2-$$

$$\sqrt{pr}\sqrt{pr-4s+4i\omega}-2i\omega))+\varphi$$

Where φ is a function of $pr\ ,Gr\ ,\alpha\ ,\beta\ ,s,\omega,y,i$

Results and discussions

In this section, we will study the effects of different dimensionless numbers upon velocity and temperature distribution.

Figures (١-٦), illustrate the effect of each of material moduli, coecostic parameter, prandtl number, Grashof number, sink strength and Eckerts number, upon the velocity component respectively.

Figure (١) exhibit the effect of material moduli β upon the velocity component , the velocity increases as material moduli (β =٠.٠٠٠٥ , ٠.١ , ١.٣ , ٢) increase . Figures (٢ , ٣) show the effects of the coecostic parameter upon the velocity component , we observe that there is small change in velocity values even for large value of α .

From figure (٤), it is observed that the velocity is increases when there is increasing in Grashof number . However, figure (٥), shows opposite effect with prandtl number . The sink strength has the same effects as Grashof number with some different in velocity values, see figure (٦).

The effect of dimensionless parameters upon temperature are shown in figures (٧ -١٣).

We note that there is increasing in temperature values as the material parameter β decreases, but this is true only for very small values of β , see figures (٧, ٨).

Figure (٩, ١٠), shows the effect of coecostic parameter α upon the temperature, this effect is clear for large values of α. Prandtl number and Grashof number have opposite effects upon temperature distribution , see figures(١١) and (١٢) .Figure (١٣) shows the effect of s upon the temperature distribution.

Table١ shows that as material moduli β increase there is increasing in the value of the skin frication coefficient C_p , also as α increases there is a small increasing in the C_p value .Table ٢ shows that as *Gr* or s increasing there is increasing in C_p . However, the increasing in prandtle number resulting a decreasing in C_p .Table ٣ shows that as α or *B* increasing there is increasing in the Nusselte number *Nu*. Table ٤ shows that as *Gr* increasing there is increasing in Nusselte number *Nu* , however as sink strength s or Prandtl number *pr* increase loads to a decreasing in Nusselte number *Nu.*

Velocity distribution

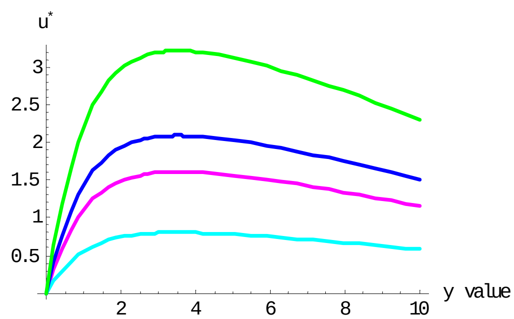

Figure ١: *Gr*=٥ , *Ec*=٠.٠٠١ , ω=٥ , *pr*=٠.٠٢٥, s=٠.١٥ , α =٠.٠١ , t=Pi/١٠ , Є=٠.٢
β=٠.٠٠٠٥ , β=٠.١ , β=١.٣ , β=٢

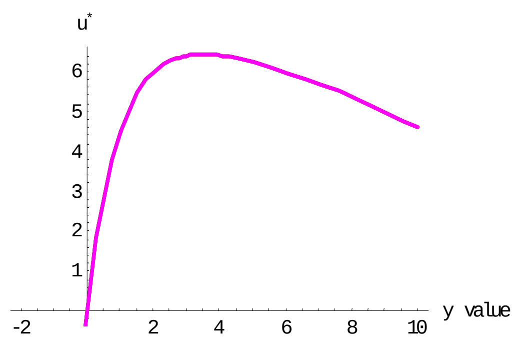

Figure ٢: *Gr*=٥ , Ec=٠.٠٠١ , ω=٥ , p=٠.٠٢٥, S=٠.١٥ , β=٠.٤ , t=Pi/١٠ , Є=٠.٢
α =٠.٠١ , α =٠.٠٠١ , α =٠.٠٠٠١ , α =٠.٠٠٠٠١

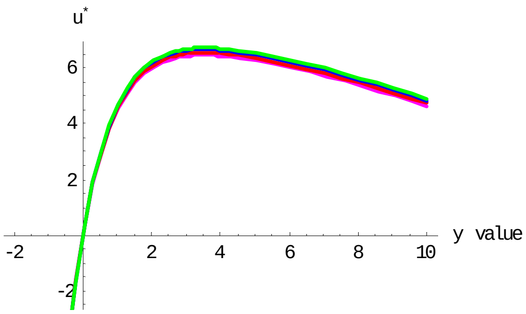

Figure ٣: *Gr*=٥,Ec=٠.٠٠١, ω=٥ , *pr*=٠.٠٢٥, s=٠.١٥ , β=٠.٤ , t=Pi/١٠ , Є=٠.٢
α=١ , α=٤ , α=٨ , α=١٠

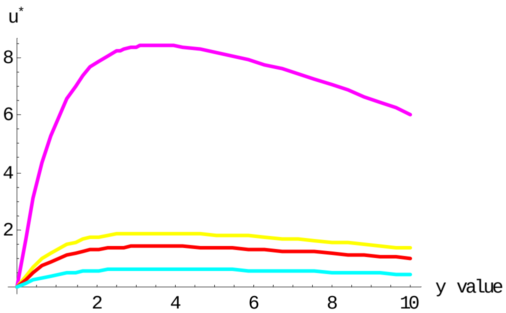

Figure ٤: *Ec*=٠.٠٠١ , ω =٥ , *pr*=٠.٠٢٥, S=٠.١٥ , α=٠.٠١ , β=٠.٤ , t=Pi/١٠ , Є=٠.٢
Gr=٥ , *Gr*=١٠ , *Gr*=١٥, , *Gr*=٢٠

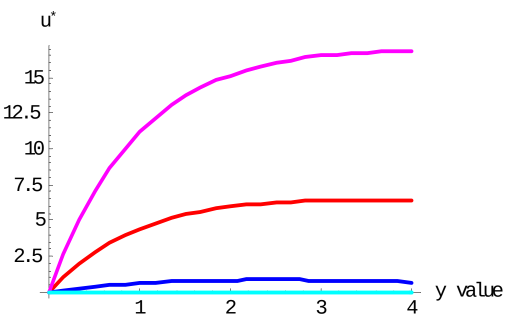

Figure ٥: *Gr*=٥ , Ec= ٠.٠٠١ , ω=٥ , s=٠.١٥, α=٠.٠١ , β=٠.٤ , t=Pi/١٠ , Є=٠.٢
Pr=٠.٠٠١ , *pr*=٠.٠٢٥ , *pr*=٠.٣٥ , *pr*= ٥

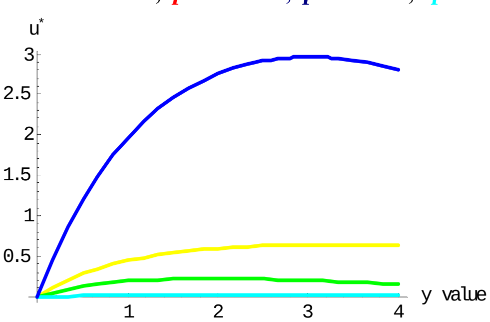

Figure ٦: *Gr*=٥ , Ec=٠.٠٠١ , ω=٥ , *pr*=٠.٠٢٥ α=٠.٠١ , β=٠.٤ , t=Pi/١٠ , Є=٠.٢
s=٠.١٥ , s=٠.١٠ , s=٠.٠٢ , s=٣

Temperature distribution

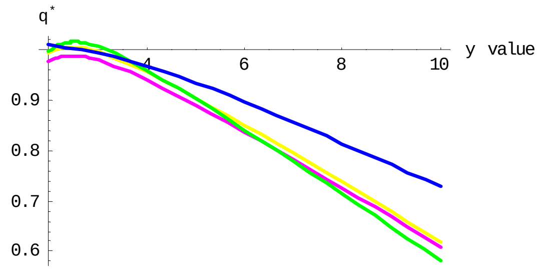

Figure ٧: *Gr*=٥ , *Ec*=٠.٠٠١ , ω =٥ , *pr*=٠.٠٢٥, s=٠.١٥ , α=٠.١ , t=Pi/١٠ , Є=٠.٢
β= ٠.٠٠٠١ , β =٠.٠٠٠٠٠٠٢ ,
β = ٠.٠٠٠٠٠٠٠٠٣ , β= ٠.٠٠٠٠٠٠٠٠٠٠٤

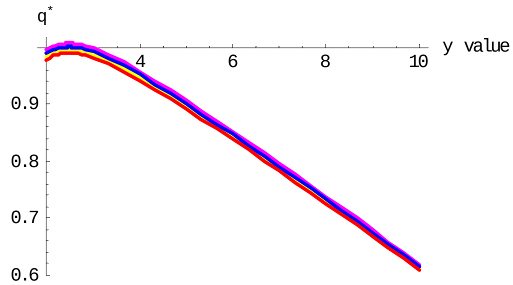

Figure ٨ : *Gr*=٥ , *Ec*=٠.٠٠١ , ω=٥ , *pr*=٠.٠٢٥, s=٠.١٥ , α=٠.١ , t=Pi/١٠ , Є=٠.٢
β=١ , β=١.٢, , β =١.٦ , β =٢

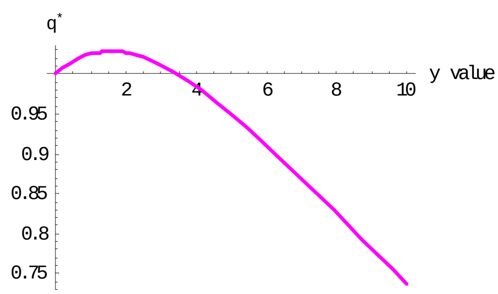

Figure ٩: *Gr*=٥ , *Ec*=٠.٠٠١ , ω=٥ , *pr*=٠.٠٢٥ , s=٠.١٥ , β=١٠^-٩ , t=Pi/١٠ , Є=٠.٢
,α=٠.٠١ , α=٠.٠٠١ , α=٠.٠٠٠١ , α=٠.٠٠٠٠١

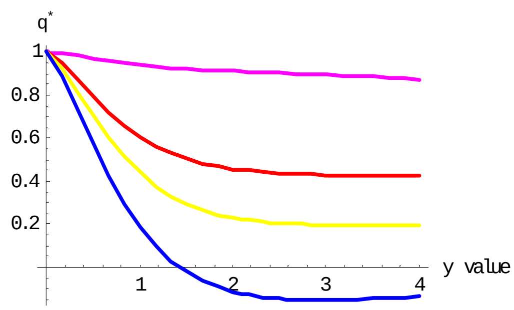

Figure١٠: *Gr*=٥ , *Ec*=٠.٠٠١ , ω=٥ , *pr*=٠.٠٢٥

,s=٠.١٥ , β=١٠^-٩ , t=Pi/١٠ , €=٠.٢

α=١ α = ٤ , ,α=٨ ,α=١٠

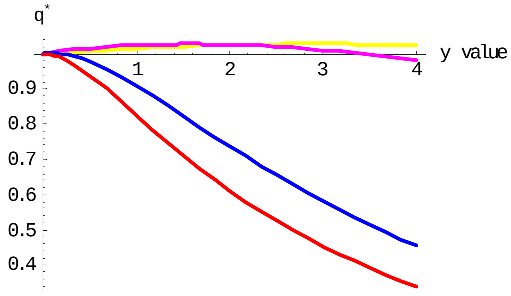

Figure ١١: *ω*=٥ , *Gr*=٥ , *Ec*=٠.٠٠١, α=٠.١ β=١٠^-٩ , t=Pi/١٠ , €=٠.٢,

Pr=٠.٠١ , *pr*=٠.٠٢٥ , *pr*=٠.٣ , *pr*=٠.٥,

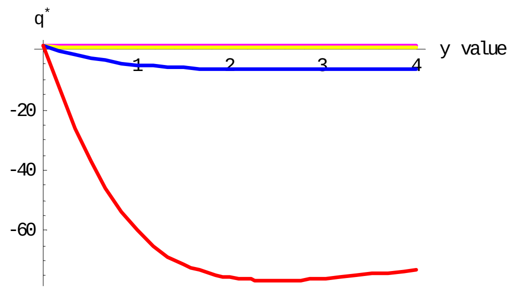

Figure ١٢: *Ec*=٠.٠٠١ , *ω*=٥ , *pr*=٠.٠٢٥ , s=٠.١٥,

α=٠.١ , β=١٠^-٩ , t=Pi/١٠ , €=٠.٢

Gr=٥ , *Gr*=١٠ , *Gr*=١٥ , *Gr*=٢٠

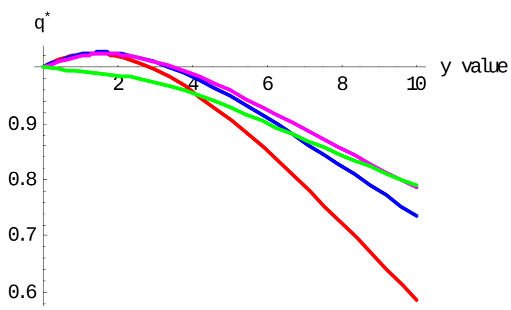

Figure ١٣: *Gr*=٥, *Ec*=٠.٠٠١, *ω*=٥, *pr*=٠.٠٢٥, α=٠.١ , β=١٠^-٩, t=Pi/١٠ , €=٠.٢

S=٠.٠٥ s=٠.٣ , s=٠.١٥ , s=٠.١٠

Table ١:Values of Sink Friction Cp when t=Pi/١٠ , €=٠.٢ , Gr=٥ , Ec = ٠.٠٠١, *ω* =٥ , pr =٠.٠٢٥, s=٠.١٥.

α	β	C_p
٠.٠١	٠.٠٠٠٥	٠.١٧٧٧٢٤
٠.٠١	٠.٠٠٤	٠.٥٣٣١٧١
٠.٠١	٠.٠٣	٠.٧١٠٨٩٤
٠.٠١	٠.١	٠.٨٨٨٦١٨
٠.٠١	١	١.٧٧٧٩٧
٠.٠١	١.٣	٢.٣١١٣٧
٠.٠١	١.٨	٣.٢٠٠٣٥
٠.٠١	٢	٣.٥٥٥٩٥
٠.٠٠٠٠١	٠.٤	٠.١٧٧٧٣٢
٠.٠٠٠١	٠.٤	٠.١٧٧٧٣٠٥
٠.٠٠١	٠.٤	٠.١٧٧٧٢٢٩
٠.٠١	٠.٤	٠.١٧٧٧٢٢٨
١	٠.٤	٠.٣٥٦٩٣٨
٤	٠.٤	٠.٣٦١٤١٦
٨	٠.٤	٠.٣٦٧٣٨٧
١٠	٠.٤	٠.٣٧٠٣٧٢

Table ٢:Values of Sink Friction C_p when t=Pi/١٠ , €=٠.٢ , *Ec* = ٠.٠٠١, *ω* =٥ , α= ٠.٠١ , β= ٠.٤.

Gr	*pr*	s	C_p
٥	٠.٠٢٥	٠.١٥	٠.٣٥٥٤٦١
١٠	٠.٠٢٥	٠.١٥	٠.٤٦٥٩٠٩
١٥	٠.٠٢٥	٠.١٥	٠.٥٢٧٨٩٩
٢٠	٠.٠٢٥	٠.١٥	٠.٦١٠٦٧٧
٥	٠.٠٠١	٠.١٥	٠.٤٤٤٧٨٣
٥	٠.٠٢٥	٠.١٥	٠.٣٥٥٥٩٥
٥	٠.٣٥	٠.١٥	٠.٢٤٩٥٦٨
٥	٥	٠.١٥	٠
٥	٠.٠٢٥	٠.٠٢	٠.١١٧٥٧٣
٥	٠.٠٢٥	٠.١٠	٠.٢٤٤٥٧٥
٥	٠.٠٢٥	٠.١٥	٠.٣٥٥٤٦١
٥	٠.٠٢٥	٣	٠.٣٦٨٤٦٢

٥	٠.٠٢٥	٠.١٥	٤.٠٢٤٥٥
٥	٠.٠٢٥	٣	٣.٧٦٤٣٨

Table ٣:Values of Rate of Heat Transfer (Nusselt number Nu) when t=Pi/١٠ , Є=٠.٢ , *Gr*=٥ , *Ec* = ٠.٠٠١ , *ω* =٥ , *pr* =٠.٠٢٥ , s=٠.١٥.

α	β	Nu
٠.١	٠.٠٠٠٠٠٠٠٠٠٠٤	١.٣٣٣١٥
٠.١	٠.٠٠٠٠٠٠٠٠٣	١.٥٩٩٧٨
٠.١	٠.٠٠٠٠٠٢	١.٩٩٩٧٣
٠.١	٠.٠٠٠١	٢.١٦٦٣٠
٠.١	١	٢.٥٥٦٤٨
٠.١	١.٢	٢.٦٨٩٦٧
٠.١	١.٦	٤.٠٢٢٨٥
٠.١	٢	٥.٣٥٦٠٣
٠.٠٠٠٠١	٠.١	١.٣٠٢٣٠
٠.٠٠٠١	٠.١	١.٢٦٩٤٥
٠.٠٠١	٠.١	١.٢٦٦١٠
٠.٠١	٠.١	١.٢٦٥٠٧
١	٠.١	١.١٨٧٩١
٤	٠.١	١.٢٦١٤١
٨	٠.١	٢.٠٢٠٥٦
١٠	٠.١	٢.٥٢٦٦٦

Table ٤: Values of Rate of Heat Transfer (Nusselt number Nu) when t=Pi/١٠ , Є=٠.٢, *Ec* = ٠.٠٠١ , *ω* =٥ , α= ٠.١ , *β* = ١٠^-٩

Gr	*pr*	s	*Nu*
٥	٠.٠٢٥	٠.١٥	١.٣٥٦٤٨
١٠	٠.٠٢٥	٠.١٥	١.٩٦١٠١
١٥	٠.٠٢٥	٠.١٥	٢.٦٩٢٥٣
٢٠	٠.٠٢٥	٠.١٥	٣.٢٩٩٤٧
٥	٠.٠٠١	٠.١٥	٤.٢٨٤٧١
٥	٠.٠٢٥	٠.١٥	٣.٤٣٨٥٣
٥	٠.٣٥	٠.١٥	٣.٣٦٠٦١
٥	٥	٠.١٥	١.٣٥٧٥٢
٥	٠.٠٢٥	٠.٠٢	٤.٧٥٠٨٨
٥	٠.٠٢٥	٠.١٠	٤.٢٢٢٩٨

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