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Investigation of Nuclear Structure and Symmetry Energy of Some Nuclei

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Abstract

The symmetry energy accounts for the rise in nuclear matter's energy as the ratio of protons to neutrons deviates from equality. Using the Coherent Density Fluctuation Model (CDFM), the nuclear symmetry energy (S), neutrons pressure (Po), and asymmetric compressibility (ΔK) in argon isotopes were calculated. The in-depth study also investigates the correlation between the neutron skin thickness (ΔR_n) and the properties of the nuclear symmetry energy's density dependence for Ar isotope chains. In addition, the significance of the neutron-proton asymmetry, the nuclear symmetry energy's mass dependency, and the neutron skin's thickness are investigated. Analysis of density-dependent nuclear symmetry energy characteristics reveals unique pathways taken by the nuclei of (A = 35–41) and nuclei (A = 29–34) of Ar isotope chains. It is evident from the result that the nuclear symmetry energies of Ar isotope chains (A = 29–41) and their neutron skin thickness have an approximately linear correlation. It is also observed that, for Ar isotope chains, the neutron skin thickness obtained agrees rather well with the experimental results.

Keywords: Neutron skin thickness, pressure, symmetry energy, nuclear structure, asymmetric compressibility, coherent density.

التحقيق في التركيب النووي وطاقة التناظر لبعض النوى

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الخلاصة

تمثل طاقة التناظر ارتفاع طاقة المادة النووية حيث تنحرف نسبة البروتونات إلى النيوترونات عن المساواة. باستخدام نموذج تذبذب الكثافة المتماسك (CDFM) ، يتم حساب طاقة التناظر النووي (S) و ضغط النيوترونات (P_o) والانضغاط غير المتماثل (ΔK) في نظائر الاركون. يبحث التحليل العالمي أيضا في العلاقة بين سمك الجلد النيوتروني (ΔR_n) والخصائص المتعلقة بالاعتماد على كثافة طاقة التناظر النووي لسلاسل نظائر الاركون بالإضافة إلى ذلك ، يتم التحقيق في أهمية عدم تناسق النيوترون والبروتون ،

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والاعتماد على كتلة طاقة التناظر النووي ، وسمك القشرة النيوترون. يكشف تحليل خصائص طاقة التناظر النووي المعتمدة على الكثافة عن مسارات فريدة اتخذتها النوى $(A = 35-41)$ والنوى $(A = 29-34)$ لسلاسل نظائر الاركون. يتضح من النتيجة أن طاقات التناظر النووي لسلاسل نظائر الاركون $(A = 29-41)$ وسمك قشرتها النيوتروني لهما ارتباط خطي تقريبا. ويلاحظ أيضا انه بالنسبة لسلاسل نظائرالاركون فأن سمك القشرة النيوتروني الذي تم الحصول عليه يتفق جيدا مع النتائج التجريبية.

1. INTRODUCTION

Seeking to solve fundamental problems regarding nuclear interactions, stability, and uncommon nuclear states, modern nuclear physics is extending its reach and addressing both its practical uses and obstacles. Nuclear physics investigates the underlying structural features of nuclei and provides crucial knowledge on a wide spectrum of fundamental topics in several branches of physics [1]. From the development of minute patterns in atomic nuclei to the unique composition of neutron stars, the complex character of the nuclear force generates a wide variety of occurrences [2].

The study of symmetry energy in nuclear matter is among the most fascinating research directions in modern nuclear physics, as it precisely characterizes the part of the asymmetric equation of the state of nuclear matter that depends on the isospin [3]. Understanding many facets of nuclear physics and astronomy depends on knowledge of symmetry energy [4]. It also plays a major part in investigating intriguing issues regarding possible revolutionary physics outside of the conventional paradigm [5]. A basic concept in nuclear physics, nuclear symmetry energy greatly influences the structure, reactivity, and stability of nuclear matter. The energy difference between symmetric nuclear matter and neutron-rich matter is distinguished by the Equation of State (EOS) of nuclear matter, particularly under severe conditions [6, 7, 8]. Knowing this difference is essential to comprehending how nuclear matter behaves in harsh environments. When looking at nuclear structure, the symmetry energy affects the binding energies, neutron skin, and group excitations of strange and neutron-rich nuclei [9]. Furthermore, the symmetry energy's impact on nuclear reactions in heavy-ion collisions, such as isospin diffusion, particle emission, and reaction dynamics, restrict its dependence on density [10]. The neutron star's proton proportion and Equation of State (EOS) [11] are largely under control by nuclear symmetry energy. Nuclear symmetry energy also significantly affects neutron star stability, thereby altering variables such as mass, radius, cooling rates, and tidal deformability [12]. Although it is of great relevance in uniform matter, the density dependence of the symmetry energy is yet unknown in many different fields [13]. According to Li et al. [14], an increasing number of theoretical studies point out that an increasing number of theoretical studies provide an understanding of how density affects this dependence and associated nuclear properties. Researchers who used phenomenological or microscopic many-body models have come up with very different estimates of the density-dependent nuclear symmetry energy [15]. Despite a tremendous deal of research, the density dependence of the symmetry energy is still mostly unknown because of its changeable isotope chain. To understand the limits of nuclear stability and make predictions about how exotic nuclei will behave, we need to know how symmetry energy changes with density. Argon is a great gas for studying this [16]. Moreover, simulating neutron-rich nuclei in astronomical applications like supernovae and neutron stars depends on exact knowledge of argon's symmetry energy [12]. Experiments with isotopes can give us useful information that can improve theoretical models and help us get a better idea of the symmetry energy parameters [17]. Brown [18] recently conducted a substantial link between the EOS density derivative of neutron matter, almost reaching saturation density, and the neutron skin: $\Delta R = Rn - Rp$. As a ground state parameter of the finite nucleus, the neutron-skin thickness is a good predictor of the isovector features of efficient nuclear exchanges

[19]. Neutron skin thickness provides more insights into the properties of nuclei rich in neutrons [20], neutron stars, and asymmetric nuclear matter's equation of state. To establish the neutron-skin thickness directly, precise measurements of the root mean square (rms) radii of the charge and mass distributions are usually necessary [21]. The spatial distribution of the nucleon affects the finite nuclei's symmetry energy. Two significant characteristics of nuclear matter that can be calculated are the energy of nuclear symmetry and the thickness of the neutron skin. These can be done with the Fluctuation Model of Coherent Density (CDFM) [22], a complex theoretical model that considers fluctuations in nuclear density. Since this model is based on the coherent density fluctuation concept, it comprehensively explains density-dependent fluctuations [23]. This idea makes the description of nucleon distributions inside nuclei possible. With the aid of the CDFM, a crucial aspect of nuclear physics that influences the stability and structure of atomic nuclei, nuclear symmetry energy, may be better understood at the microscopic level [24]. Utilizing this model, scientists can establish a direct connection between the density distributions of protons and neutrons and measurable characteristics, such as neutron skin thickness [25], which describes the degree to which neutrons protrude beyond the protons in a nucleus. According to a specific theoretical framework and accounting for the nuclei in particular isotopic chains, this model enables the estimation of multiple surface parameters, including symmetry energy, neutron pressure, and symmetry energy curvature coefficient, for a broad range of nuclei from the proton to the neutron drip lines [26]. Although symmetrical energy expansion around saturation density (ρ_0) has been the subject of numerous studies, the ultimate goal is to determine the symmetry energy function along a broad spectrum of densities and study the relationship between the density-dependent parameters: the symmetry energy (S), pressure (P_o), and the asymmetric compression parameter (ΔK) and the thickness of neutron skin (ΔR_n) within the nuclei of an Ar isotopic chain.

The primary goal of this study is to explore the connection between the symmetry energy (S), pressure (P_o), and the asymmetric compression parameter (ΔK). These parameters are derived from the density dependence of the symmetry energy near saturation density and the neutron skin thickness (ΔR_n) of nuclei in the argon isotope chain.

2. FRAMEWORK OF THEORY

1.1. NUCLEAR MATTER: ESSENTIAL EOS PARAMETERS

The binding energy per nucleon is commonly used to define the Equation of State (EOS) for nuclear matter. By extending the expression for the energy per particle in nuclear matter to include the isospin asymmetry $\delta = (\rho_n - \rho_{p)})/\rho$, where ρ_n and ρ_p are the neutron and proton densities, respectively, one obtains the symmetry energy S(ρ), which is often approximated as [27]:

$$\boldsymbol{E(\rho,\delta) = E(\rho,O) + S^{ANM}(\rho)\delta^2 + O(\delta^4) + \cdots} \tag{1}$$

It has been demonstrated [28] that $\rho = (\rho_n + \rho_p)$ represents the baryon density, $O(\delta^4)$ supports the inclusion of the fourth power of the system's isospin asymmetry. Isospin symmetry prevents the appearance of odd powers of δ, and terms of the order of δ^4 and higher have been shown to be insignificant.

Isospin-symmetric matter's energy, E (ρ, δ = 0), and the energy of symmetry, $S^{ANM}(\rho)$, for Asymmetric Nuclear Matter (ANM) can be expanded when the density reaches saturation, ρ_0, as follows [29]:

$$\boldsymbol{E(\rho,O) = E_O + \frac{K}{18\rho_O^2}(\rho - \rho_O)^2 + \cdots} \tag{2}$$

$$S^{ANM}(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\delta)}{\partial\delta^2}|_{\delta=0} = a_4 + \frac{P_O{}^{ANM}}{\rho_O^2}(\rho - \rho_O) + \frac{\Delta K^{ANM}}{18\rho_O^2}(\rho - \rho_O)^2 + \cdots \quad (3)$$

The equilibrium symmetry energy (ρ = ρo) is represented by the parameter **a4**. The pressure in asymmetric nuclear matter P_O^{ANM} [30], defined by the following equation offers insights into how energy shifts with changes in the density of individual nucleons

$$P_O{}^{ANM} = \rho_O^2 \frac{\partial S^{ANM}(\rho)}{\partial\rho}|_{\rho=\rho_O} \quad (4)$$

Consequently, the way symmetry energy varies with density is influenced by the pressure exerted by the surplus neutrons. The curvature of the nuclear symmetry energy (ΔK^{ANM}) at ρ_0 provides crucial insights into its properties at both high and low densities, as it determines how the energy depends on density:

$$\Delta K^{ANM} = 9\rho_O^2 \frac{\partial^2 S^{ANM}(\rho)}{\partial\rho^2} \quad (5)$$

It plays a role in determining the frequently used "slope" parameter (L^{ANM}):

$$L^{ANM} = \frac{3P_O^{ANM}}{\rho_O} \quad (6)$$

1.2. FINITE NUCLEI SYMMETRY ENERGY PARAMETERS IN CDFM

The delta-function limit of the generator coordinate method is associated with the CDFM model [31]. It suggests that the nucleus's single body density matrix, $\rho(r,r')$, can be represented as a cohesive superposition of the density matrices for single body, $\rho_x^{NM}(r,r')$, for nuclear matter with specific densities that is spherically symmetric and commonly referred to as "fluctons" [32]:

$$\rho_x(r) = \rho_o(x)\Theta(x - |r|) \quad (7)$$

Where

$$\rho_o(x) = \frac{3\,A}{4\pi X^3}$$

The radius of the sphere, where the Fermi gas of all A nucleons is uniformly spread, serves as the generator coordinate, denoted as x, and is a body's density matrix that has a kernel $\rho(r,r')$:

$$\rho(r,r') = \int_0^\infty dx\,|\mathfrak{F}(x)|^2 \rho_x^{NM}(r,r') \quad (8)$$

$$\rho_x^{NM}(r,r') = 3\rho_o(x)\frac{j_1(k_F(x)|r-r'|)}{(k_F(x)|r-r'|)} \times \Theta\left(x - \frac{|r+r'|}{2}\right) \quad (9)$$

where the original sphere j_1 is the symbol for the Bessel function, and

$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_o(x)\right)^{1/3} \equiv \frac{\beta}{x} \quad (10)$$

with

$$\beta = \left(\frac{9\pi A}{8}\right)^{1/3} \cong 1.52\,A^{1/3} \quad (11)$$

where β is the nucleons' Fermi momentum in the flucton. In the CDFM, the nucleon density distribution takes the following form, according to Equations (8) and (9):

$$\boldsymbol{\rho(r) = \int_0^\infty dx\, |\mathfrak{F}(x)|^2 \rho_o(x) \Theta(x - |r|)} \tag{12}$$

A known density, whether obtained through theoretical or experimental means, can be used to derive the weight function $|\mathfrak{F}(x)|^2$, as discussed by Myers and Swiatecki [33]. This is applicable when the local density is monotonically decreasing $(d\rho(r)/dr \leq 0)$, as can be seen from Eq. (12)

$$|\mathfrak{F}(x)|^2 = -\frac{1}{\rho_o(x)} \frac{d\rho(r)}{dr}\Big|_{r=x} \tag{13}$$

$$\int_0^\infty dx\, |\mathfrak{F}(x)|^2 = \mathbf{1} \tag{14}$$

By applying the weight function $|\mathfrak{F}(x)|^2$, the symmetry energy, pressure, and curvature for finite nuclei can be defined, and these quantities are weighted for nuclear matter at a specific density, $\rho_0(x)$. This forms the basis of our approach within the CDFM. Consequently, the corresponding nuclear matter quantities are represented as an infinite superposition in the CDFM framework. According to the CDFM scheme, the expressions for curvature, pressure, and symmetry energy, given in detail by Gaidarov et al. [34], are:

$$\boldsymbol{S = \int_0^\infty dx\, |\mathfrak{F}(x)|^2 S^{ANM}(x)} \tag{15}$$

$$\boldsymbol{P_O = \int_0^\infty dx\, |\mathfrak{F}(x)|^2 P_O^{ANM}(x)} \tag{16}$$

$$\Delta \boldsymbol{K} = \int_0^\infty dx\, |\mathfrak{F}(x)|^2 \Delta \boldsymbol{K}^{ANM}(x) \tag{17}$$

3. RESULTS AND DISCUSSION

The weight functions from Equation (13) and Equations (15-17) were applied to calculate the energy of nuclear symmetry, neutron pressure, and asymmetric compressibility within the context of the CDFM. Often, the neutron skin thickness(ΔR_n) is determined by evaluating the difference in the root mean square (rms) radii between protons and neutrons, as discussed by Ding et al. [35] and Abdullah [36]:

$$\boldsymbol{\Delta R_n = < r_n^2 >^{1/2} - < r_p^2 >^{1/2}} \tag{18}$$

Figure 1(a) illustrates an almost linear correlation between the symmetry energy (S) and the neutron skin thickness (ΔR_n) for Ar isotopes of A = 29–41. The symmetry energy showed a positive increase for the isotopes from ^{29}Ar to ^{33}Ar (N = 15) nucleus; since the nucleus is less stable and the nuclear force must successfully counterbalance the proton-proton repulsion, the greater asymmetry (more protons than neutrons) results in higher symmetry energy. Conversely, at the ^{34}Ar (N = 16) nucleus, a slow decrease in this energy was observed. For the isotopes from ^{34}Ar to ^{41}Ar, the thickness of the neutron (ΔR_n) increases sharply with the symmetry energy (S). Excess neutrons help stabilize neutron-rich nuclei, where the neutron skin is more pronounced by reducing the repulsive Coulomb energy between protons. Nevertheless, adding more neutrons causes instability after a certain amount. Additionally, if the neutron-to-proton ratio is far from the ideal value, the probability of decay processes like fission, beta decay, and neutron emission rises.

Figure 1(b) illustrates that within the Ar isotope chain, the relationship between pressure (P_o) and neutron skin thickness (ΔR_n) was weaker than the symmetry energy (S) up to ^{34}Ar (N=16). For the proton-rich nuclei ranging from A = 29 to 34, a gradual decrease in P_0 was observed, where the influence of Coulomb repulsion among protons complicated the relationship. Beyond this, for A=34-41, the correlation between P_0 and ΔR_n increased linearly, a steady rise in P_0 as ΔR_n becoming larger was noted. The interaction between nuclear attraction and Coulomb repulsion greatly affects neutron skin thickness and pressure in these nuclei. For neutron-rich materials, a larger symmetry energy results in a more rigid equation of state (EOS) and increased pressure. Increased pressure causes the neutrons dispersing more apart, which thickens the neutron skin. Knowing nuclear structure, astronomical events, and the nuclear matter equation of state requires an understanding of the link between pressure and neutron skin thickness in the Ar isotope chain.

Figure 1(c) shows the relationship between asymmetric compressibility (ΔK) and neutron skin thickness (ΔR_n) in the argon isotope chain and related variables (P_o). ΔK decreases as ΔRn increases; it reaches a minimum for the nucleus ^{34}Ar. Despite having more protons than neutrons (Z>N), ^{34}Ar has a nearly balanced neutron-to-proton ratio, making it less likely to develop a pronounced neutron or proton skin. This balance and nuclear shell effects enhance the symmetry energy's contribution to stability and reduce asymmetry, thereby lowering compressibility. After this point, ΔK sharply increased with increasing ΔR_n in the nuclei with A = 35–41. Generally, a greater thickness of neutron skin is associated with a stiffer equation of state and lower asymmetric compressibility in neutron-rich nuclei.

The behavior of the curves in Figure 1(b, c), particularly the transition at the ^{34}Ar (N = 16) nucleus, is the key to these correlations. It suggests that kinks in the argon isotope chain are due to structural changes within these nuclei, influencing their densities and weight functions. Ar isotope experimental investigations yield useful information that can improve theoretical frameworks and help determine the symmetry energy parameters more precisely [37].

Figure 1(d) illustrates the convergence of neutron skin thickness for Ar isotopes ranging from ^{29}Ar to ^{36}Ar, where the accuracy varies between −0.439±0.025 fm and −0.046±0.007 fm, depending on the experimental setup [38]. Our theoretical calculations, which predicted ΔR_n to be between −0.497 fm and −0.0397 fm, align well with the experimental range for nuclei with mass numbers A = 32 to A = 36. However, a significant discrepancy exists when comparing our results to the reported experimental values of Sammarruca [38] for isotopes with A = 29 to A = 31. Conversely, Ozawa et al. [39] provided experimental predictions for the neutron skin thickness of ^{32}Ar to ^{40}Ar isotopes, which ranged from −0.362±0.221 fm to 0.044±0.067 fm, depending on the specific experimental conditions. Our theoretical predictions, which yield ΔR_n between −0.255 fm and 0.1206 fm, show good agreement with the observed experimental data for isotopes with mass numbers A=32, 33, 34, 35, 36, 39, and 40 but differ significantly from the reported values of Ozawa et al. [39] for the isotopes with A = 37 and A = 38. Since the theoretical models used to calculate neutron skin have limitations, there are inherent inaccuracies in estimating neutron skin thickness.

Measurement errors can significantly impact interpretations of neutron skin thickness regarding nuclear stability. The theoretical framework utilized to ascertain the neutron skin thickness may have affected the results and data. To address this concern and identify any potential biases, the findings of this study were compared with those of Sammarruca [38] and Ozawa et al. [39].

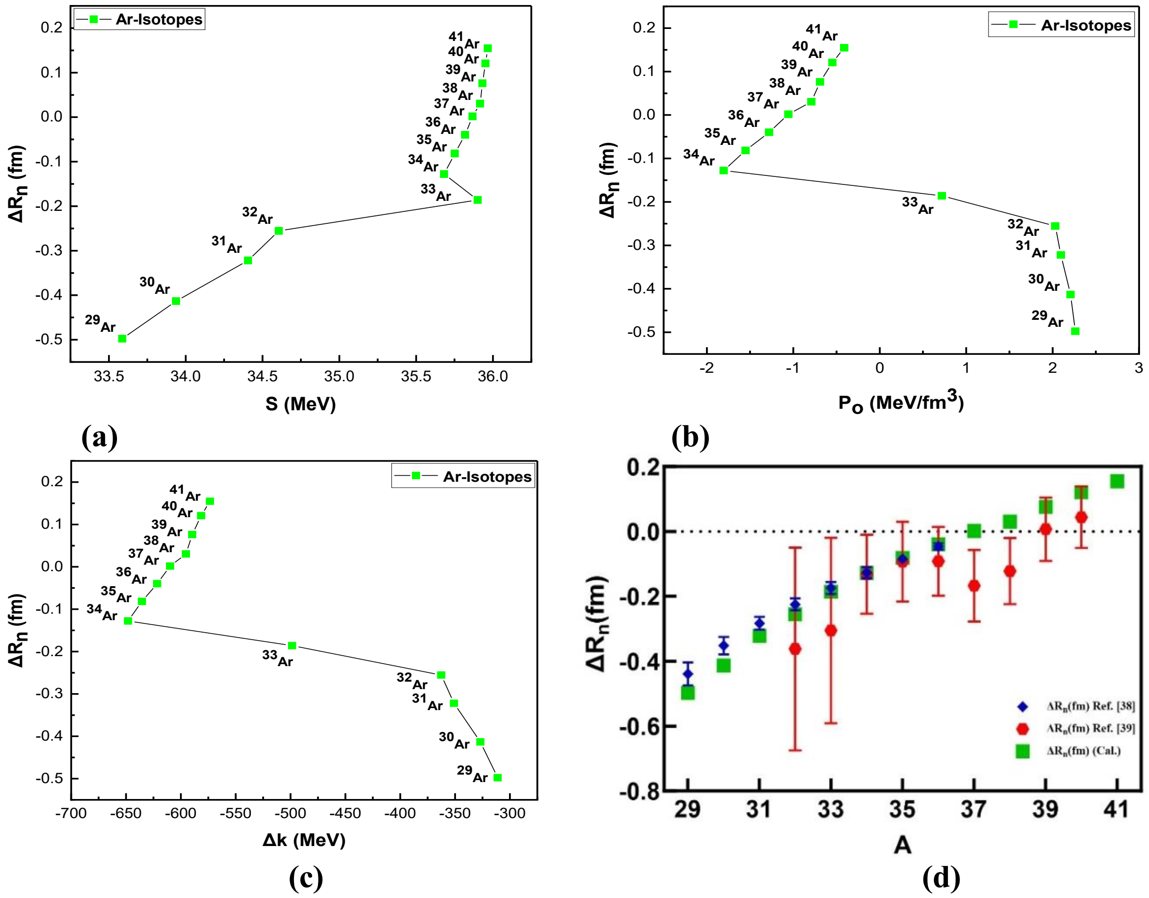


Figure 1. Neutron skin thicknesses (ΔR_n) for Ar isotopes (A = 29- 41) as a function of: (a) nuclear symmetry energy (S), (b) neutron pressure (P_o), (c) asymmetric compressibility (ΔK), and (d) mass number (A).

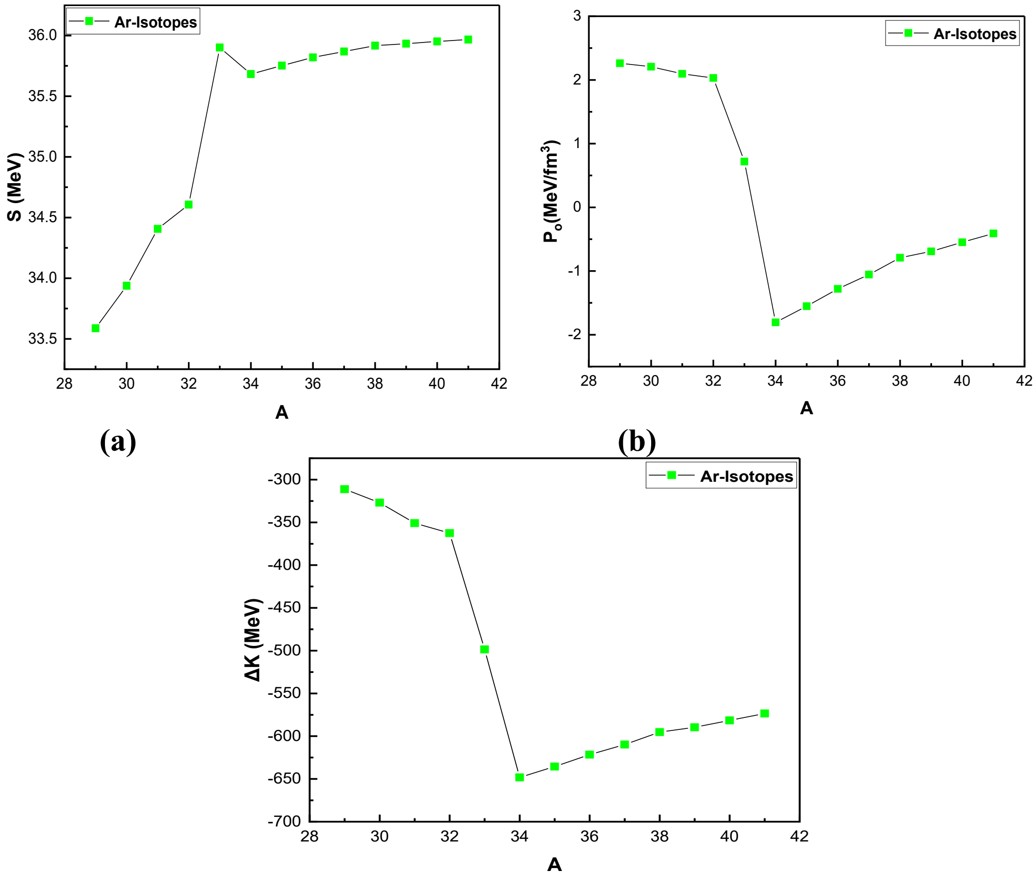
Furthermore, a more linear relationship between neutron skin thickness (ΔRn) and mass number (A) is seen in Figure 1(d). For the chain of argon isotopes, the neutron skin thickness is generally small or even negative in proton-rich nuclei (A = 29-35) because there are more protons than neutrons. In such cases, the attractive nuclear force pulls neutrons closer to the protons. In contrast, in neutron-rich nuclei (A = 37–41), the neutron skin thickness increases as the excess neutrons are less tightly bound and tend to form a "skin" around the nucleus's surface. Understanding how neutron skin thickness varies with mass number is crucial for studying nuclear structure, as it provides valuable insights into neutron distribution, nuclear forces, and the nuclear matter equation of state.

The relationship between the mass number (A) and various macroscopic properties of nuclear matter in finite nuclei was explored. Figure 2 presents the findings for a series of Ar isotopes. Figure 2(a) shows the symmetry energy (S) vs. the mass number (A) for Ar isotopes, which predominantly follows a linear trend, with the exception of nucleus Ar-34. As the mass number A increases, the symmetry energy S initially rises until it peaks at the nucleus ^{33}Ar (with N = 15). After this point, S decreases slightly up to the nucleus ^{34}Ar, likely due to a more balanced interaction between protons and neutrons in a more symmetric nucleus, leading to greater stability and a reduced requirement for symmetry energy. Beyond ^{34}Ar, the symmetry energy increases smoothly in a linear manner. This pattern suggests that excess neutrons can help stabilize neutron-rich nuclei by lowering the repulsive Coulomb energy among protons. Still, it also implies that such nuclei are positioned further from the valley of stability.

As shown in Figure 2(b), the relationship between pressure (P_o) and mass number (A) is initially nonlinear but becomes more linear around the nucleus of Ar-34 (with N = 16). For isotopes with mass numbers between 29 and 34, the pressure (P_o) decreases steadily, reaching a minimum at Ar-34. This decrease is attributed to an increase in the number of neutrons (from 11 to 16), which reduces the Coulomb repulsion between protons and decreases nuclear asymmetry, leading to greater stability. This occurs because a more symmetric nucleus balances the forces between protons and neutrons, resulting in lower internal pressure. For nuclei with mass numbers from 35 to 41, the pressure (P_o) gradually increases. In neutron-rich isotopes, where the neutron-to-proton ratio is high ($N > Z$), the pressure is higher due to increased symmetry energy from the neutron excess.

Although neutron-rich isotopes are generally more stable than proton-rich ones, they are still less stable than isotopes with nearly equal numbers of neutrons and protons ($N \approx Z$). If the neutron-to-proton ratio becomes too high, these isotopes can undergo beta-minus decay (β^- decay) or neutron emission to reach a more stable state, placing them beyond the line of stability and making them susceptible to rapid decay.

Figure 2(c) shows a relatively weak correlation between the asymmetric compressibility, ΔK, and the mass number (A) within the argon isotope chain up to the nucleus ^{34}Ar (N = 16). Beyond this point, ΔK increases steadily, and a more linear relationship between ΔK and A emerges. The behavior of the curves shown in Figures 2(b) and 2(c) reflects this pattern, highlighting the "kink" or point of inflexion occurring at the ^{34}Ar (N = 16) nucleus, which is crucial for understanding these correlations. These kinks within the argon isotope chains arise due to internal structural changes in these nuclei, impacting their density and weight functions.



(a) **(b)**

(C)

Figure 2: Variation of: (a) symmetry energy (S), (b) pressure (P_o), and (c) asymmetric compressibility (ΔK) with mass number (A) for Ar isotopes (A = 29- 41).

4. CONCLUSION

The evaluation of the Coherent Density Fluctuation Model (CDFM) gives us significant new information on argon isotope symmetry energy. Therefore, what we study, nuclear structure, stability, and reactivity, is more visible. This significantly advances our understanding of nuclear matter in the fields of nuclear physics and astrophysics. The increasing argon isotope series shows how crucial density-dependent symmetry energy is for both anticipating the stability of nuclei and their behavior when they are out of the ordinary. These consist of neutron pressure (P_o), symmetry energy (S), and asymmetric compressibility (ΔK). Explicit EOS linkages, using a pragmatic weight function methodology, achieved a significant advancement.

The study found a strong link between the thickness of the neutron skin and the nuclear symmetry energy in argon isotopes (A = 29–41). A stiffer equation of state with a higher slope leads to a thicker neutron skin, which stabilizes nuclei that are rich in neutrons. However, too many neutrons cause instability. Because of isospin symmetry, the neutron skin in nuclei with a lot of protons is either very small or negative. The connection between neutron skin thickness, pressure, and asymmetric incompressibility is not linear for A = 29–34 but linear for A = 35–41. At Ar-34, a change was seen that suggests structural changes. Variations in densities and weight functions result in observable discontinuities. The neutron skin thickness in argon isotopes exhibited close agreement with the experimental data for isotopes with A= 32, 33, 34, 35, 36, 39, and 40. Conversely, it showed a significant deviation from the previously reported values for isotopes with A= 29, 30, 31, 37, and 38. To study exotic nuclei and neutron stars, you need to understand how density affects symmetry energy. This shows that this theoretical framework can be used to make predictions in nuclear structure analysis.

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