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Estimating Fixed Points for Suzuki Mappings via New Iterative Scheme

Raghad I.Sabri
Branch of Mathematics and Computer Applications, Department of Applied Sciences, University of Technology, Baghdad, Iraq

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Abstract
The main objective of this paper is to propose a new iterative method for approximating fixed points of a type of mapping known as Suzuki mappings. Important convergence results of the proposed method are also studied. To support the main results of this work, a numerical example is presented where the numerical results show that the new method is better than many other methods in terms of its convergence to the fixed point.

Keywords: Banach space, Converge sequence, Fixed point, Iteration Scheme, Suzuki mapping.

تقدير النقاط الصامدة لدوال سوزوكي باستخدام مخطط تكراري جديد

رغد ابراهيم صبري
فرع الرياضيات وتطبيقات الحاسوب, قسم العلوم التطبيقية, الجامعة التكنولوجية, بغداد, العراق

الخلاصة
في هذه الدراسة تم اقتراح مخطط تكراري جديد فعال لتقريب النقاط الصامدة لدوال سوزوكي. تم توضيح العديد من نتائج التقارب المهمة للنهج التكراري المقترح في إطار فضاء باناخ. لتوضيح فعالية النهج الجديد، تم تقديم مثال عددي. من النتائج العددية يمكن ملاحظة أن النهج المقترح يتقارب إلى النقطة الصامدة بشكل أسرع من العديد من الأساليب الأخرى.

1. Introduction

Functional analysis is a mathematical field that analyzes functions by investigating how a particular function works and uncovering connections and assumptions that might occur. It is also used to examine a variety of spaces [1-5]. One of the most difficult and quickly expanding subfields in nonlinear functional analysis is fixed-point (FP) theory. In [6] authors showed that there is an FP for α-η-fuzzy contractive mappings and extended β-ζ-fuzzy contractive mappings. Other efforts and outcomes on FPs might be mentioned in [7-13]. FPs are essential because they symbolize equilibrium, stability, and solutions to a variety of issues.

Email :raghad.i.sabri@uotechnology.edu.iq

If the presence of a solution to a fixed-point problem including an operator Ω is assured but a precise solution is not achievable, the demand to approximate the solution gets critical, necessitating the use of various iterative schemes. Numerous researchers have created and implemented many FP iteration systems in order to approximate the solution of equations, given the theoretical and practical relevance of these schemes. O. G. Amechi et al. [14] provided a new fixed point iterative strategy that approximates the solution of boundary value problems. It is based on Green's function. However, S. A. Khuri and I. Louhichi [15] introduced a unique method for the numerical solution of a wide class of third-order boundary value problems that are based on embedding Green's function within the Ishikawa fixed point iterative technique. Additional iterative techniques may be found in [16–23]. The rate of convergence of the iteration scheme is one of the most crucial factors in selecting one fixed-point iteration scheme over the other. Thus, in reality, a quicker fixed-point iteration approach is always favored. The well-known Banach contraction theorem employs the Picard iteration process to approximate FPs. Other well-known iterative procedures are E. Picard [24], S. Ishikawa [25], M. A. Noor [26], R. P. Agarwal et al. [27] and D. R. Sahu [28].
Recently, J. Ali et al. [29] presented a three-step iterative scheme to approximate FPs of Suzuki's generalized non-expansive mappings. Their iteration methodology demonstrates superior speed compared to established iteration methods using numerical examples. In 2021, J. Ahmad et al. [30] introduced a novel iterative approach for estimating the FPs of Suzuki mappings. They achieved significant convergence outcomes for their suggested iterative approach. S.Rezapour et al. [31] describe a modified F-iterative procedure to determine the FPs of three generalized α-nonexpansive mappings. N. Saleem et al. [32] provided a technique for determining the common fixed point of L-Lipschitzian and total asymptotically strictly pseudo-non-spreading self-mappings, and they established the proposed approach's weak convergence theorem. Some other basic iterative algorithms are discussed in [33-43].

Influenced and prompted by ongoing research in this area, in this work a new iteration scheme for estimating FP of Suzuki mappings (Suz- maps) to obtain a faster convergence rate is presented. Consider $\Omega: E \to E$ be Suz- map where E is a nonempty subset of Banach space(BN-Space) $\mathcal{C}$. Then the sequence $\{\vartheta_n\}$ is produced by $\vartheta_1 \in E$ and

$$\sigma_n = \frac{\varpi\vartheta_n + \Omega\vartheta_n}{\varpi+1},$$

$$\delta_n = \frac{\varpi\vartheta_n + \Omega\sigma_n}{\varpi+1}, \tag{1}$$

$$\vartheta_{n+1} = \Omega\delta_n,$$

where $\varpi \geq 0$.

The goal of this work is to provide some convergence findings for the new iteration process (1). An example demonstrates how this new iterative scheme outperforms existing methods in terms of efficiency.

2. Preliminaries

A review of some lemmas and definitions that will be applied repeatedly throughout the work follows. Throughout this work, $\mathcal{F}_\Omega$ denotes the collection of all FPs of Ω.

Definition 2.1:[44] A mapping $\Omega: E \to E$ is termed as Suz-map if the following fact holds:

$$\frac{1}{2}\|e_1 - \Omega e_1\| \leq \|e_1 - e_2\| \Rightarrow \|\Omega e_1 - \Omega e_2\| \leq \|e_1 - e_2\| \quad \forall e_1, e_2 \in E.$$

Definition 2.2:[45] A BN-Space $\mathcal{C}$ fulfills the Opial's condition if for any sequence $\{\vartheta_n\}$ in $\mathcal{C}$ that converges weakly to $y \in \mathcal{C}$ i.e., $\vartheta_n \to y$ it can be stated that,
$\lim_{n\to\infty} sup\|\vartheta_n - y\| < \lim_{n\to\infty} sup\|\vartheta_n - w\|$ for each $w \in \mathcal{C}$ with $w \neq y$.

Definition 2.3:[46] Let $\{\vartheta_n\} \subseteq \mathcal{C}$ be a bounded sequence. If $\emptyset \neq E \subseteq \mathcal{C}$ (where E is convex and closed), then the asymptotic radius of $\{\vartheta_n\}$ which corresponds to E is given by
$\mathfrak{R}(E, \{\vartheta_n\}) = \inf\{\lim_{n\to\infty} sup\|\vartheta_n - e\| : e \in E\}$.
Likewise, $\{\vartheta_n\}$ corresponding to E has an asymptotic center that is specified and demonstrated via the formula
$\mathcal{A}(E, \{\vartheta_n\}) = \{e \in E: \lim_{n\to\infty} sup\|\vartheta_n - e\| = \mathfrak{R}(E, \{\vartheta_n\})\}$.

Lemma 2.4:[44] Suppose that $\mathcal{C}$ is BN-Space and $\emptyset \neq E \subseteq \mathcal{C}$. If $\Omega: E \to E$ is Suz-map, then for every $e \in E$ and $m \in \mathcal{F}_\Omega$, the fact $\|\Omega e - \Omega m\| \leq \|e - m\|$ holds.

Lemma 2.5:[44] Suppose that $\mathcal{C}$ is BN-Space and $\emptyset \neq E \subseteq \mathcal{C}$. Let $\Omega: E \to E$ be Suz-map, then for every elements $e_1, e_2 \in E$:

$$\|e_1 - \Omega e_2\| \leq 3\|e_1 - \Omega e_1\| + \|\|e_1 - e_2\|.$$

Lemma 2.6:[47] Let $0 < \rho \leq \vartheta_n \leq \sigma < 1$ and $\mathcal{C}$ is uniformly convex Banach space (UCB −space). If there is $\lambda \geq 0$ (real number) such that $\{d_n\}$ and $\{b_n\}$ in $\mathcal{C}$ fulfill $\lim_{n\to\infty} sup\|d_n\| \leq \lambda$, $\lim_{n\to\infty} sup\|b_n\| \leq \lambda$ and $\lim_{n\to\infty} sup\|\vartheta_n d_n + (1-\vartheta_n)b_n\| = \lambda$, then $\lim_{n\to\infty} sup\|d_n - b_n\| = 0$.

Lemma 2.7:[44] Assume that E is any nonempty subset of a BN-Space having the Opial property. If $\Omega: E \to E$ is Suz-map, then the following requirement holds:
$\{\vartheta_n\} \subseteq E, \vartheta_n \to m, \|\vartheta_n - \Omega\vartheta_n\| \to 0 \Rightarrow \Omega m = m.$

3. Main results

In the current study part, extremely interesting and relevant Suz-map results are analyzed using the newly presented method A required lemma for the major conclusions is presented and demonstrated below, which will play a key part in each of the subsequent section's results.

Lemma 3.1: Let $\mathcal{C}$ be BN-Space and $\emptyset \neq E \subseteq \mathcal{C}$ (where E closed and convex). Consider $\Omega: E \to E$ Suz-map with $\mathcal{F}_\Omega \neq \emptyset$. If $\{\vartheta_n\}$ a sequence produced by (1), then $\lim_{n\to\infty} \|\Omega\vartheta_n - m\|$ exists for all $m \in \mathcal{F}_\Omega$.
Proof: Consider $m \in \mathcal{F}_\Omega$. Based on Lemma 2.4, one gets

$$\begin{aligned}\|\sigma_n - m\| &= \left\|\frac{\varpi\vartheta_n + \Omega\vartheta_n}{\varpi + 1} - m\right\| \\ &= \left\|\tfrac{\varpi}{\varpi+1}(\vartheta_n - m) + \tfrac{1}{\varpi+1}(\Omega\vartheta_n - m)\right\| \\ &\leq \tfrac{\varpi}{\varpi+1}\|\vartheta_n - m\| + \tfrac{1}{\varpi+1}\|\Omega\vartheta_n - \Omega m + \Omega m - m\| \\ &\leq \tfrac{\varpi}{\varpi+1}\|\vartheta_n - m\| + \tfrac{1}{\varpi+1}\|\vartheta_n - m\| \pm \|\Omega m - m\| \\ &= \|\vartheta_n - m\|. \end{aligned} \quad (2)$$

Moreover,

$$\|\delta_n - m\| = \left\|\frac{\varpi\vartheta_n + \Omega\sigma_n}{\varpi + 1} - m\right\|$$
$$= \left\|\frac{\varpi}{\varpi + 1}(\vartheta_n - m) + \frac{1}{\varpi + 1}(\Omega\sigma_n - m)\right\|$$
$$\leq \frac{\varpi}{\varpi+1}\|\vartheta_n - m\| + \frac{1}{\varpi+1}\|\Omega\sigma_n - \Omega m + \Omega m - m\|$$
$$\leq \frac{\varpi}{\varpi+1}\|\vartheta_n - m\| + \frac{1}{\varpi+1}\|\sigma_n - m\|\pm\|\Omega m - m\|$$
$$= \|\vartheta_n - m\|. \quad (3)$$

It is evident from (3) that

$$\|\vartheta_{n+1} - m\| = \|\Omega\delta_n - m\|$$
$$\leq \|\delta_n - m\|$$
$$\leq \|\vartheta_n - m\|.$$

Thus $\lim_{n\to\infty}\|\vartheta_n - m\|$ exists for all $m \in \mathcal{F}_\Omega$.

Theorem 3.2: Assume $\mathcal{C}$ is UCB-Space and $\emptyset \neq E \subseteq \mathcal{C}$ (where E closed and convex). Let $\Omega: E \to E$ Suz-map and $\{\vartheta_n\}$ produced by (1), then $\mathcal{F}_\Omega \neq \emptyset$ if and only if $\lim_{n\to\infty}\|\Omega\vartheta_n - \vartheta_n\| = 0$ and $\{\vartheta_n\}$ is bounded.

Proof: Consider that $\lim_{n\to\infty}\|\Omega\vartheta_n - \vartheta_n\| = 0$ and $\{\vartheta_n\}$ is bounded**.** Let $m \in \mathcal{A}(E, \{\vartheta_n\})$ then according to Lemma 2.5, obtain,

$$\mathcal{A}(\Omega m, \{\vartheta_n\}) = \lim_{n\to\infty} sup\|\{\vartheta_n - \Omega m\|$$
$$\leq 3 \lim_{n\to\infty} \sup \|\Omega\vartheta_n - \vartheta_n\| + \|\vartheta_n - m\|$$
$$\leq \lim_{n\to\infty} \sup \|\{\vartheta_n - m\|$$
$$= \mathcal{A}(m, \{\vartheta_n\}).$$

This illustrates that $\Omega m \in \mathcal{A}(E, \{\vartheta_n\})$. Thus $\Omega m = m$ and $\mathcal{F}_\Omega \neq \emptyset$.

In contrast, assume that $\mathcal{F}_\Omega \neq \emptyset$, and consider $m \in \mathcal{F}_\Omega$. Conclusions of Lemma 3.1 provide that $\lim_{n\to\infty}\|\vartheta_n - m\|$ exists and $\{\vartheta_n\}$ is bounded. Let

$$\lim_{n\to\infty}\|\vartheta_n - m\| = \tau. \quad (4)$$

Subsequently, by looking at the proof of Lemma 3.1 and taking consideration (4), one gets

$$\lim_{n\to\infty} sup\|\sigma_n - m\| \leq \lim_{n\to\infty} sup\|\vartheta_n - m\| = \tau. \quad (5)$$

According to Lemma 2.4, obtain

$$\lim_{n\to\infty} sup\|\Omega\vartheta_n - m\| \leq \lim_{n\to\infty} sup\|\vartheta_n - m\| = \tau. \quad (6)$$

Along with (4), it provides (7)

Making use of (5) and (7), acquire

$$\lim_{n\to\infty}\|\sigma_n - m\| = \tau. \quad (8)$$

According to (8), obtain

$$\tau = \lim_{n\to\infty}\|\sigma_n - m\|$$
$$= \left\|\frac{\varpi\vartheta_n + \Omega\vartheta_n}{\varpi + 1} - m\right\|$$
$$= \left\|\frac{\varpi}{\varpi+1}(\vartheta_n - m) + \frac{1}{\varpi+1}(\Omega\vartheta_n - m)\right\|. \quad (9)$$

Thus,
$$\tau = \lim_{n\to\infty}\left\|\frac{\varpi}{\varpi+1}(\vartheta_n - m) + \frac{1}{\varpi+1}(\Omega\vartheta_n - m)\right\|. \quad (10)$$

Based on (4), (6), (10), and Lemma 2.6, conclude that $\lim_{n\to\infty}\|\Omega\vartheta_n - \vartheta_n\| = 0$.

Theorem 3.3: Assume $\mathcal{C}$ is UCB-Space where $\mathcal{C}$ fulfills Opial's condition and $\emptyset \neq E \subseteq \mathcal{C}$ (where E closed and convex). Let $\Omega: E \to E$ Suz-map with $\mathcal{F}_\Omega \neq \emptyset$ and consider $\{\vartheta_n\}$ produced by (1), then $\{\vartheta_n\}$ weakly converges in $\mathcal{F}_\Omega$.

Proof: By Theorem 3.2, $\lim_{n\to\infty}\|\Omega\vartheta_n - \vartheta_n\| = 0$ and $\{\vartheta_n\}$ is bounded. Since $\mathcal{C}$ is UCB-Space, $\mathcal{C}$ is reflexive. As a result, there is a subsequence $\{\vartheta_{n_k}\}$ of $\{\vartheta_n\}$, with $\vartheta_{n_k} \to \vartheta$ where $\vartheta \in E$. By Lemma 2.7, $\vartheta \in \mathcal{F}_\Omega$. To prove that ϑ is the weak limit of $\{\vartheta_n\}$. Let ϑ not be the weak limit of $\{\vartheta_n\}$. Then, one could find another subsequence, $\{\vartheta_{n_j}\}$ of $\{\vartheta_n\}$ with $\vartheta_{n_j} \to \vartheta^*$ and $\vartheta^* \neq \vartheta$. Once more via Lemma 2.7, $\vartheta^* \in \mathcal{F}_\Omega$. Now, employing Opial's property and Lemma 3.1, obtain

$$\begin{aligned}\lim_{n\to\infty}\|\vartheta_n - \vartheta\| &= \lim_{k\to\infty}\|\vartheta_{n_k} - \vartheta\| \\ &< \lim_{k\to\infty}\|\vartheta_{n_k} - \vartheta^*\| \\ &= \lim_{n\to\infty}\|\vartheta_n - \vartheta^*\| \\ &= \lim_{j\to\infty}\left\|\vartheta_{n_j} - \vartheta^*\right\| \\ &< \lim_{j\to\infty}\left\|\vartheta_{n_j} - \vartheta\right\| \\ &= \lim_{n\to\infty}\|\vartheta_n - \vartheta\|. \quad (11)\end{aligned}$$

Thus, $\lim_{n\to\infty}\|\vartheta_n - \vartheta\| < \lim_{n\to\infty}\|\vartheta_n - \vartheta\|$, resulting in a contradiction, and so ϑ is a weak limit of $\{\vartheta_n\}$.

Next, the following results demonstrate the strong convergence.

Theorem 3.4: Suppose that $\mathcal{C}$ is UCB-Space and $\emptyset \neq E \subseteq \mathcal{C}$ (where E is compact and convex). Let $\Omega: E \to E$ be Suz-map and assume $\{\vartheta_n\}$ is given in (1), then $\{\vartheta_n\}$ strongly onverges in $\mathcal{F}_\Omega$.

Proof: By Theorem 3.2, $\lim_{n\to\infty}\|\Omega\vartheta_n - \vartheta_n\| = 0$. A strongly convergent subsequence $\{\vartheta_{n_k}\}$ of $\{\vartheta_n\}$ with a limit, say y, may be found with ease since E is compact. Using Lemma 2.5, the subsequent assertion is valid:

$$\|\vartheta_{n_k} - \Omega\, y\| \leq 3\|\vartheta_{n_k} - \Omega\vartheta_{n_k}\| + \|\|\vartheta_{n_k} - y\|.$$

Therefore, $\vartheta_{n_k} \longrightarrow \Omega\, y$ whenever $k \longrightarrow \infty$, so $\Omega\, y = y$. According to Lemma 3.1, $\lim_{n\to\infty}\|\vartheta_n - y\|$ exists. Hence, y represents the strong limit of $\{\vartheta_n\}$.

4. Numerical results

This section offers an example to confirm the convergence result covered in the section before it. The iterative scheme given in Equation (1) converges to Fp more quickly than different iterative methods, as this example's numerical and graphical analysis shows.

Example 4.1: Let $C = \mathbb{R}$ and consider $E = [0,1]$. Set a map $\Omega: E \to E$ as follows:

$$\Omega(e) = \begin{cases} 1 - e, & if\ e < \frac{1}{6} \\ \frac{e+4}{5} & if\ e \geq \frac{1}{6}. \end{cases}$$

Initially, to demonstrate that Ω on E possesses the Suzuki property. The partition of the proof is as follows:

Case1: Choose $e_1 \in [0,\frac{1}{6})$ and , $e_2 \in [\frac{1}{2},1]$ then

$\frac{1}{2}\|e_1 - \Omega e_1\| = \frac{1-2e_1}{2}$. For $\frac{1}{2}\|e_1 - \Omega e_1\| \leq \|e_1 - e_2\|$ one has,

$\frac{1-2e_1}{2} \leq e_2 - e_1$ that is mean $e_2 \geq \frac{1}{2}$. Thus one gets,

$\|\Omega e_1 - \Omega e_2\| = \left|\frac{e_2+4}{5} - (1-e_1)\right| = \left|\frac{e_2+5e_1-1}{5}\right| < \frac{1}{6}$

$\|e_1 - e_2\| = |e_1 - e_2| > \left|\frac{1}{6} - \frac{1}{2}\right| = \frac{1}{3}.$

Hence, $\frac{1}{2}\|e_1 - \Omega e_1\| \leq \|e_1 - e_2\| \Rightarrow \|\Omega e_1 - \Omega e_2\| \leq \|e_1 - e_2\|$.

Case2: Select $e_1 \in [\frac{1}{6},1]$ then

$\frac{1}{2}\|e_1 - \Omega e_1\| = \left|\frac{e_1+4}{5} - e_1)\right| = \frac{4-4e_1}{10}$. For $\frac{1}{2}\|e_1 - \Omega e_1\| \leq \|e_1 - e_2\|$ one has,

$\frac{4-4e_1}{10} \leq |e_2 - e_1|$ .Consequently, the following possibilities might arise:

(i) If $e_1 < e_2$, $\frac{4-4e_1}{10} \leq e_2 - e_1 \Rightarrow e_2 \geq \frac{4+6e_1}{10}$ then $e_2 \in [\frac{1}{2},1]$. So,

$\|\Omega e_1 - \Omega e_2\| = \left|\frac{e_1+4}{5} - \frac{e_2+4}{5}\right| = \frac{1}{5}\|e_1 - e_2\| \leq \|e_1 - e_2\|$.

Hence, $\frac{1}{2}\|e_1 - \Omega e_1\| \leq \|e_1 - e_2\| \Rightarrow \|\Omega e_1 - \Omega e_2\| \leq \|e_1 - e_2\|$.

(ii) If $e_1 > e_2$, $\frac{4-4e_1}{10} \leq e_1 - e_2 \Rightarrow e_2 \leq e_1 - \left(\frac{4-4e_1}{10}\right) = \frac{14e_1-4}{10}$ then $e_2 \in [-\frac{1}{6},1]$.

Since $e_2 \in [0,1]$, $e_2 \leq \frac{14e_1-4}{10} \Rightarrow e_1 \in \left[\frac{4}{14},1\right]$. So, the case is $e_1 \in \left[\frac{4}{14},1\right]$ and $e_2 \in [0,1]$.

Assume $e_1 \in \left[\frac{4}{14},\frac{1}{2}\right]$ and $e_2 \in [0,\frac{1}{6})$ then,

$\|\Omega e_1 - \Omega e_2\| \leq \frac{1}{6}$. and $\|e_1 - e_2\| > \frac{5}{42}$.

Hence, $\|\Omega e_1 - \Omega e_2\| \leq \|e_1 - e_2\|$.

Next consider $e_1 \in \left[\frac{1}{2},1\right]$ and $e_2 \in [0,\frac{1}{6})$ then,

$\|\Omega e_1 - \Omega e_2\| \leq \frac{1}{6}$. and $\|e_1 - e_2\| > \frac{1}{3}$.

Hence, $\|\Omega e_1 - \Omega e_2\| \leq \|e_1 - e_2\|$. So,

$\frac{1}{2}\|e_1 - \Omega e_1\| \leq \|e_1 - e_2\| \Rightarrow \|\Omega e_1 - \Omega e_2\| \leq \|e_1 - e_2\|$. Thus in all cases, Ω satisfies Suzuki property on E.

Table 1 and Figure 1 show the rate convergence of the new iteration scheme (1). It is evident that iteration scheme (1) iterates approach $e = 1$ more quickly than other schemes like E.

Picard [24], M. A. Noor [26], R. P. Agarwal et al. [27], D. R. Sahu [28] and S. Rawat et al. [48].

Table 1: Comparison of convergence rates for various iteration approaches

step	New iteration	S**-Iteration [48]	Normal S[28]	Agrawal [27]	Picard[24]	Noor[26]
1	0.9	0.9	0.9	0.9	0.9	0.9
2	0.9971900826	0.9980480000	0.9928000000	0.9902400000	0.9800000000	0.9758784000
3	0.9999210436	0.9999618969	0.9994816000	0.9990474240	0.9960000000	0.9941814841
4	0.9999977813	0.9999992562	0.9999626752	0.9999070285	0.9992000000	0.9985964808
5	0.9999999376	0.9999999854	0.9999973126	0.9999909259	0.9998400000	0.9996614487
6	0.9999999982	0.9999999997	0.9999998065	0.9999991143	0.9999680000	0.9999183360
7	0.9999999999	0.9999999999	0.9999999860	0.9999999135	0.9999936000	0.9999803013
8	0.9999999999	0.9999999999	0.9999999989	0.9999999915	0.9999987200	0.9999952483
9	0.9999999999	0.9999999999	0.9999999999	0.9999999991	0.9999997440	0.9999988538
10	0.9999999999	0.9999999999	0.9999999999	0.9999999999	0.9999999488	0.9999997235
11	1	0.9999999999	0.9999999999	0.9999999999	0.9999999897	0.9999999333
12		1	0.9999999999	0.9999999999	0.9999999979	0.9999999839
13			0.9999999999	0.9999999999	0.9999999995	0.9999999961
14			1	0.9999999999	0.9999999999	0.9999999990
15				0.9999999999	0.9999999999	0.9999999997
16				1	0.9999999999	0.9999999999
17					0.9999999999	0.9999999999
18					0.9999999999	0.9999999999
19					0.9999999999	0.9999999999
20					0.9999999999	0.9999999999
21					0.9999999999	0.9999999999
22					1	0.9999999999
23						0.9999999999
24						0.9999999999
25						1

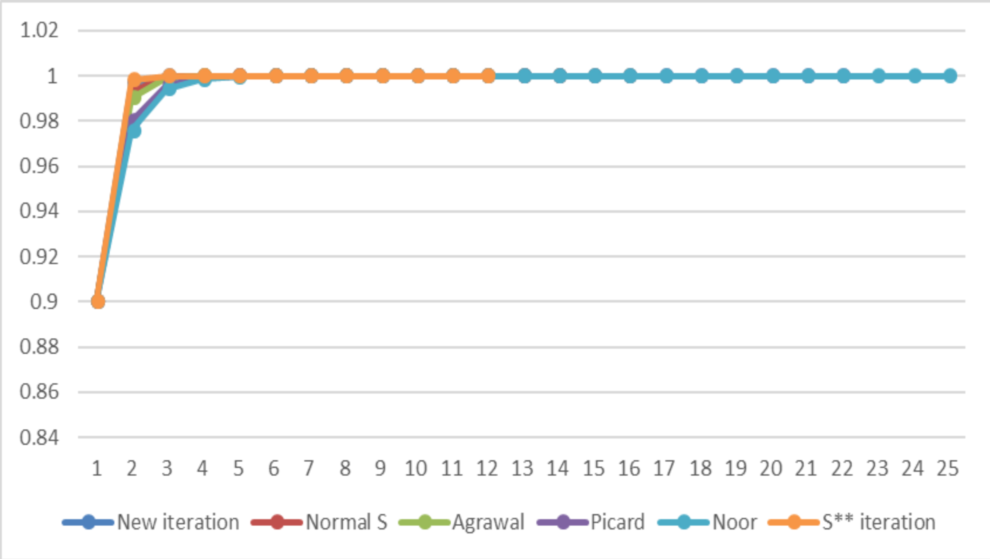

Figure 1: Graph corresponding to Table 1

5. Conclusions

This paper introduces an effective new iterative approach for estimating the FP of Suz-maps. The proposed iterative strategy produces strong convergence results in the Banach space. An example is offered to demonstrate the convergence of different results. This study provides an excellent foundation for additional debate and research into iteration approaches for approximating fixed points and their applications in various equations.

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