DOI: 10.24996/ijs.2026.67.1.35

ISSN: 0067-2904

Some New Characteristics of Topological Ring-Groupoids

Taghreed Hur Majeed
Mathematics Department, College of Education, Mustansiriyah University, Baghdad, Iraq

Received: 12/9/2024 Accepted: 9/1/2025 Published: xx

Abstract

The main goal of this work is to study some new characteristics of groupoids ring of algebraic topology, and we started with algebraic cases and simple topology, but it is vital as information background for comparing what we found as a result and if we focused in our study on the topological projective ring groupoid and topological projective cover. Taylor Robert, who talked about groupoid and topological groupoids was the start of the topological group groupoid and topological groupoid rings. The relationship between topological projective groupoid ring and topological group groupoid was developed. We used the definitions, previous relationships topological projective groupoids ring, topological group groupoid, projective cover and topological projective ring harmonious with our work. Many results regarding the topological projective cover for the topological projective groupoids ring and the partial topological projective groupoid ring and topological ring quotient groupoid are obtained. We applied some structures to clarify the relationships between those different spaces. Lastly, we studied the direction summation of equations that forms the topological projective cover of topological projective groupoid rings is the orderly highlighted guidelines such that we proved the final number of the direct sum for the topological groupoid covers ring is the cover of topological groupoid ring.

Keywords: Ring- groupoid, Topological ring- groupoid, Topological ring, Topological groupoid.

بعض الخصائص الجديدة لزمرويات الحلقة التبولوجية

تغريد حر مجيد
قسم الرياضيات, كلية التربية, الجامعة المستنصرية, بغداد, العراق

الخلاصة

الهدف الاساسي من العمل دراسة بعض الخصائص لزمرويات الحلقة الجبرية تبولوجيا ولقد بدأنا بقضايا جبرية وتبولوجية بسيطة ولكنها مهمة كخلفية معلوماتية وللمقارنة مع ما حصلنا علية من نتائج و ركزنا في بحثنا على زمرويات الحلقة الاسقاطية التبولوجية والغطاء الاسقاطي التبولوجي. روبرت تايلر تحدث عن الزمرويات والزمر التبولوجية كمقدمة لزمرويات الزمر التبولوجية وزمرويات الحلقة التبولوجية .تم تطوير علاقات بين زمرويات الحلقة الاسقاطية التبولوجية وزمرويات الحلقة التبولوجية . استخدمنا تعاريف ,علاقات سابقة بين الزمرويا ت والزمر التبولوجية ,زمرويات الحلقة الاسقاطية التبولوجية, الحلقات التبولوجية ,الغطاء الاسقاطي والغطاء الاسقاطي التبولوجي تتوافق مع عملنا. العديد من النتائج حول الغطاء الاسقاطي التبولوجي لزمرويات الحلقة الاسقاطية التبولوجية وزمرويات الحلقة الجزئية الاسقاطية التبولوجية وزمرويات القسمة للحلقة التبولوجية حصلنا عليها . *قدمنا* بعض المخططات لتوضيح العلاقات بين تلك الفضاءات المختلفة . واخيرا تم دراسة الجمع

Email: taghreedmajeed@uomustansiriyah.edu.iq

المباشر لدوال التي تشكل الغطاء الاسقاطي التبولوجي لزمرويات الحلقات الاسقاطية التبولوجية بالترتيب حيث
اثبتنا لعدد منتهي من الجمع المباشر لاغطية زمرويات الحلقة التبولوجية يكون غطاء لزمرويات الحلقة التبولوجية
.

1. Introduction

The originality and revival of the study of functional analysis is rooted in its connections with topological space. This area based on topological space has vastly gained interest by numerous investigators, see [1-11]. The interest started with studying the topological rings by Kaplansky [12], he used quotient rings as a main goal and metric topology and his interest was on the locally connected rings and he mentioned in his paper to the definition of topological quotient. Many researchers tried to explain topological rings like Nelson and Darken, their main target was metric spaces. In our paper we discussed the topological groupoid rings and precisely topological projective ring. We did not discuss countable spaces, metric spaces, and topological linear spaces, [13]. A ring groupoid R be an entity equipped with a ring configuration in which the maps that follow are groupoids' morphisms:

m: R×R→R, (r, s) →r +s, multiplication of groups.

μ: R ×R→ R, r→-r group inverted map.

e:(*) →R, as well as (*) is one to one.

n: R×R→R, (r, s) →r. s, multiplying rings.

Thus, in the case of e, it equals 0 by addition and 1 by multiplication. [14], [15] and [16]. Let ring R, we can define the ring groupoid R×R over R. In the ring groupoid we know that ring operation by (r, s) (p, q) = (r p, s q) for each r, s, p, q ∈ R. Within a ring-groupoid R considering r, s ∈R, the symbol for the groupoid composite is through r ∘ s whenever α(r)=β(s) such that α is source and β is target in groupoids R. The multiplication of the group by r +s, and the multiplication of rings by r. s [17], [18] and [19]. Assume R additionally R´ be two groupoids in rings. A transformation f: R→ R´ from the R to R´ is an evolution of the underlying groupoids that keep the ring structure intact. A morphism of groupoids R, R´is a functor. Covering morphisms and universal covering of groupoids are defined in [20], [21] and [22]. The topological ring of groupoids be a ring object with topology within the class of topological groupies [23]. A ring with topological properties R is a ring with topology on the underlying set in such a way the mappings of the ring structure (i.e., collective multiplication), inverse group and multiplication of rings are ongoing [24], [25]. A morphism of topological rings (homomorphism in topology) of a ring along with topology into a different one is an abstract homomorphism of rings it is a continuous mapping, as well. In a ring groupoid R with topology, the interchange laws exist.:

(r ∘s) + (p ∘q) = (r +p) ∘ (s +q) and (r ∘s) (p ∘q) = (r p) ∘(sq).

Whenever both (r ∘s) and (p ∘q) are defined. Assume R be a topologically ring ,then a groupoid of topological rings R×R object set R is defined in the following way: The pairs are the morphisms (r ,s), then the mappings of the source and target are determined by α(r ,s)=r and β(r ,s)=s, so the groupoid composition is defined by (t ,s)∘(s ,r)=(t ,r), then group multiplication [26], is defined by (z ,t)+(r ,s)=(z +r, t +s) and ring multiplication is defined by (z ,t),(r ,s)=(z r ,t s). The R×R possesses a product topology. Thus, any ring-groupoid structure mapping R×R turns into a continuous, subsequently R×R possesses a ring-groupoid topology [26], [27] and [28]. A topological groupoid ring R is called connected space if and only if R is not able to be split up into a union of open, non-empty subsets that do not intersect, [29]. Let R be the topological ring ,then the topological ring groupoid R×R with object set be defined as :the morphisms are the pairs(r, t),the source and target maps are known by α(r ,s)=r and β(r ,s)=s ,the groupoid composition be defined by (p, s) ∘ (s, r)=(p, r), the group multiplication be defined by (p ,q)+(s ,r)=(p +s , q +r) and ring multiplication be known as (p ,q)(s ,r)=(p s ,q r). Hence, R×R has product topology. Then all structure maps for ring groupoid R×R will be continuous, then R×R is the topological ring groupoid

2. Some results of topological projective ring groupoids

A topological ring groupoids R is called topological projective ring groupoid if for all topological ring groupoid is epimorphism $g: A \to B$, and for all topological ring groupoid covering morphism [30], $f: q \to B$, there exists topological ring-groupoid covering morphism $f^*: q \to A$ for all q, A and B are topological ring groupoids, for which the following Figure 1, is commute:

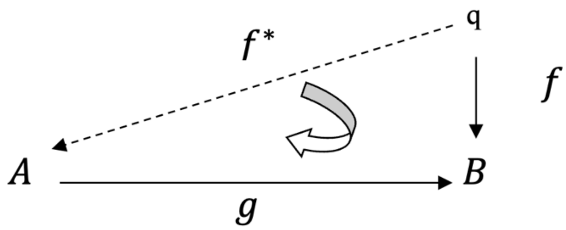

Figure 1: Topological projective ring groupoid

The topological subring groupoid is subring groupoid and topological subspace of topological space [30-33]. Allow H to be a topological subring groupoid of topological ring-groupoids R cost that of H for element r is the set of all $r + H$ where rR is a coset. Let H be topological subring groupoid of R, then set R/H be topological quotient ring groupoid if:

(i) R/H is ring-groupoid.

(ii) R/H is topological groupoid on R with quotient topology.

(iii) The mapping $\lambda(x + H) \to \lambda x + H$ from $R \times R/H$ into R/H is continuous for all $x, \lambda \in R$ [34], [35]. The topological subring groupoid M of R is called small if for all topological subring groupoid U of R then $M + U = R$, thus $U = R$ [36]. The topological ring groupoid homomorphism $f: P \to B$ is called small if ker f is small topological subring groupoid of P. The surjective topological ring groupoid homomorphism $h: P \to M$ described as a topological projective cover of M if P be topological projective ring-groupoid and h is small, surjective topological ring groupoid [37], [38]. Every topological ring group R×R be a topological projective ring group of R×R.

Proposition 2.1:

Let *M* is a topological subring groupoid of *R, N* is a topological subring groupoid of *M* and *R/N* is a connected space then $\frac{R}{M}/\frac{R}{N}$ be connected space.

Proof:

Since, there exists a topological equivalent between $\frac{R}{N}$ additionally $\frac{R}{M} / \frac{N}{M}$ in a way that the official mapping q: $\frac{R}{N} \to \frac{R}{M}/\frac{N}{M}$, Figure 2, topological subring groupoid is commutative.

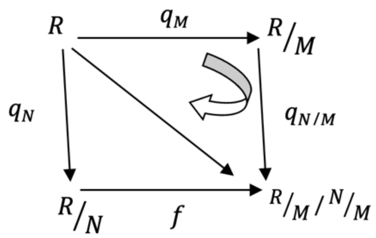

Figure 2: Topological subring groupoid

Such that $f \circ q_{N/M} \circ q_M = q_N$ thus f is continuous and open since $q_{N/M}, q_N, q_M$ are all continuous and open. Under the continuous mapping, the connected set's picture is connected as a topological ring groupoid R be referred to be a connected space only if R cannot be split up into a union of open, non-empty, non-intersecting subsets. Thus $\frac{R}{M}/\frac{N}{M}$ a place that is connected.

Assume $\frac{R}{M}/\frac{N}{M}$ is an unconnected area thus, there are two sets of $A, B \neq \phi$, $A = (x + M) + N/M$, $B = (Y + M) + N/M$ where $x, y \in R$, given that N/M be connected space because of topological ring groupoid R is called connected space if and only if R not able to be disconnected into a union of open, non-empty subsets that do not intersect, thus A and B is connected. Let M be connected then $x + M$ be connected and $A \cap B \neq \phi$, $A \cup B = z + M$, $z \in R$, but the canonical map $q: \frac{R}{M}/\frac{N}{M} \to \frac{R}{N}, q(A) \cap q(B) = \phi$, $q(A)$ and $q(B)$ are open set and also $q(A) \cup q(B) = \frac{R}{N}$ implies $\frac{R}{N}$ is disconnected which is a contradiction. Thus, $\frac{R}{M}/\frac{R}{N}$ is a connected collection.

Proposition 2.2:

Let M be a topological subring groupoid of topological ring groupoid R, $\frac{R}{M}/\frac{N}{M}$ be completely disengaged, $\frac{N}{M}$ be complete disconnection then $\frac{R}{N}$ be totally disconnected.

Proof:

Let $\sqcup = x + N$, $x \in R, N$ be a subset of R and let $q: \frac{R}{N} \to \frac{R}{M}/\frac{N}{M}$ be canonical mapping, continuous and surjective, then $q(\sqcup)$ is connected that contains one element since $\frac{R}{M}/\frac{N}{M}$ be totally disconnected hence $\sqcup$ is included in a collection that of $\frac{N}{M}$ where $\frac{N}{M}$ be subgroup, but the coset of $\frac{N}{M}$ is a topological homomorphism with $\frac{N}{M}$ that means $\frac{N}{M}$ is completely cut off and $\sqcup$ has one component, thus $\frac{R}{N}$ be disengaged.

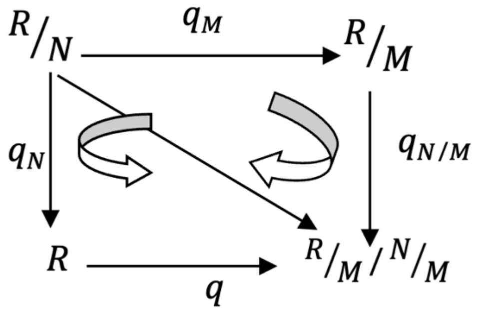

Figure 3: Topological ring groupoid

Proposition 2.3:

Let N and M be topological subring groupoids of R, then $\frac{M+N}{N}$ be connected space if and only if $\frac{M}{M\cap N}$ be connected space.

Proof:

There exists a ring homomorphism $P: M + N \to \frac{M+N}{N}$. Then P is surjective ring homomorphic where $ker\ ker\ P = N$. $\alpha: M \to \frac{M+N}{N}$ be a continuous ring homomorphism. As $ker\ ker\ \alpha = M \cap N$. Thus, there exists a continuous homomorphic between $\frac{M+N}{N}$ and $\frac{M}{M\cap N}$. Every image of connected set inside a continuous mapping connected since a topological ring -groupoid R is referred to as connected space if and only if R cannot be able to be disconnected into a non-empty union open subsets .That do not cross subsequently $M\ /M \cap N$ maintain a connection.

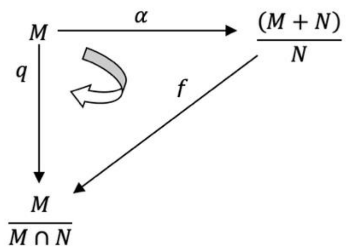

Figure 4: Connected spaces

Suppose $\frac{M}{M\cap N}$ is not connected space, then there exists two set $\sqcup$ and $V \neq \emptyset$ to the extent that $U \cap V = \emptyset, U\sqcup \quad \cup V = \frac{M}{M\cap N}$ when $U = a + (M \cap N), V = b + (M \cap N),\ a, b \in M$. Considering $M \cap N$ is not connected, thus the coset is not connected. Notice, a topological ring groupoid R is called connected space if and only if R is not split into a union of non-empty. The canonical mapping and open non-intersecting subsets $P: \frac{M}{M\cap N} \to \frac{M+N}{N}$
$P(U) \cap P(V) = \emptyset$ and $P(U) + P(V)$ are open set that $P(U) \cup P(V) = \frac{M+N}{N}$ where $\frac{M+N}{N}$ be disconnected, but that is a contradiction then $\frac{M}{M\cap N}$ is connected space.

Proposition 2.4:

Let $\frac{M}{M\cap N}$ and $\frac{M+N}{N}$ be topological ring groupoids, f be topological ring groupoid cover morphism from $\frac{M}{M\cap N}$ to $\frac{M+N}{N}$ to the extent that $M \cap N \subset ker\ ker\ f$, then there is a unique cover morphism of topological ring groupoids $g: \frac{M}{M\cap N} \to \frac{M+N}{N}$.

Proof:

From the commutative diagram g is continuous and f is continuous as well, and g is an open map since f is an open map, thus g is open surjective morphism. Then a unique topological ring groupoid exists cover morphism $g: \frac{M}{M\cap N} \to \frac{M+N}{N}$.

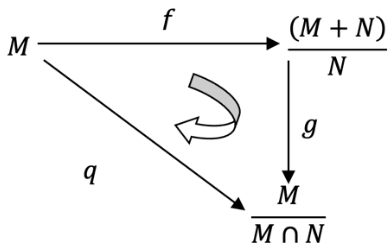

Figure 5: A unique cover morphism of topological ring-groupoids

Proposition 2.5:

Let R be ring-groupoid topology and $\frac{R}{M}$ be a topological quotient ring groupoid of R then closure $(\frac{R}{M})$ be topological quotient ring groupoid of R.

Proof:

We show that $x \in closure\ (\frac{R}{M})$, $y \in closure\ (\frac{R}{M})$ subsequently $x - y \in closure\ (\frac{R}{M})$ and $r \in R$ then $r.x \in closure\ (\frac{R}{M})$ that we demonstrate closure $(\frac{R}{M})$ be ring groupoid composed of R and $closure\ (\frac{R}{M})$ be topological ring-groupoid of R. Assume W be arbitrary neighborhood of x and y ,respectively such that $A - B \subseteq W$ that a, b in $\frac{R}{M}$ such that $a \in A$ and $b \in B$ but $(a - b) + M \in \frac{R}{M}, a - b \in W$ that
$W \cap \frac{R}{M} \neq \{0\}, (x - y) + M \in closure\ (\frac{R}{M})$ and let

$closure\, W$ neighborhood of $r.x$. That $closure\, A$ and $closure\, B$ are closed to x and r, respectively such that $closure\, A$, $closure\, B \subseteq W$, $closure\, x$ and $closure\, y$ in $\frac{R}{M}$ such that closure $a \in w$, closure $b \in w$, but closure $(ab) + M \in \frac{R}{M}$. Closure $ab \in w$ closure $w \cap \frac{R}{M} \neq \{0\}$. That $r - x \in$ closure $\frac{R}{M}$. Then the closure of topological subgroup of topological group be topological group.

Proposition 2.6:

Let $h_1: p_1 \to M_1$ and $h_2: p_2 \to M_2$ be topological evocative cover of topological ring groupoids M_1 and M_2, respectively then $h_1 \times h_2: p_1 \times p_2 \to M_1 \times M_2$ be represented as topological evocative cover of topological ring groupoids $M_1 \times M_2$.

Proof:

We prove:

$P_1 \times P_2$ is topological projective ring groupoid, since P_1 , P_2 are ring groupoids of topological projectivity then $P_1 \times P_2$ possess a ring groupoid topological projectivity. Then h is a small surjective topological homomorphism.

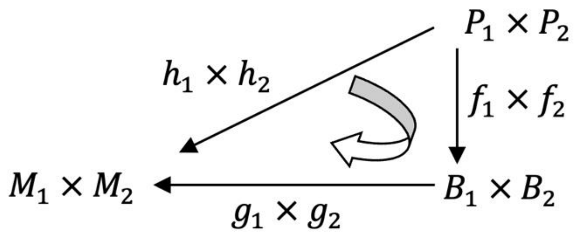

Figure 6: Topological evocative cover of topological ring-groupoids M_1 and M_2

The surjective topological ring groupoid homomorphism $h^*: P \to M$ is referred to topological projective cover M. Let P be projective ring groupoid in topology. Then h^* is a small, surjective topological ring groupoid homomorphism. Then $ker\, ker\, h^*$ be small topological subring groupoid of $P_1 \times P_2$, $h: P \to B$ be small and the $ker\, ker\, h$ be small topological subring groupoid of P. Thus, for all P_1 and P_2 ,there exists $u_1 < P_1, u_2 < P_2$ and $(ker\, ker\, h_1 \times ker\, ker\, h_2) + (u_1 \times u_2) = P_1 \times P_2$

$\Rightarrow u_1 \times u_2 = P_1 \times P_2$ we get

$$ker\, ker\, h_1 + u_1 = P_1 \Rightarrow u_1 = P_1,$$
$$ker\, ker\, h_2 + u_2 = P_2 \Rightarrow u_2 = P_2.$$

Thus

$ker\, ker\, h_1 \times ker\, ker\, h_2 + (u_1 \times u_2) = P_1 \times P_2 \Rightarrow u_1 \times u_2 = P_1 \times P_2$.

Thus ,$ker\, ker\, h_1 \times ker\, ker\, h_2$ is a topological subring groupoid smallest of $P_1 \times P_2$.

Then

$h_1 \times h_2: P_1 \times P_2 \to M_1 \times M_2$ is small, surjective topological ring groupoid homomorphism, that means:

$h_1 \times h_2: P_1 \times P_2 \to M_1 \times M_2$ be topological projective cover of $M_1 \times M_2$.

Proposition 2.7:

Let $\oplus_{1 \le i \le n} h_i: \oplus_{1 \le i \le n} p_i \to \oplus_{1 \le i \le n} M_i$

and $\oplus_{1 \le j \le n} h_j: \oplus_{1 \le j \le n} p_j \to \oplus_{1 \le j \le n} M_j$

be projective topological the front of topological ring groupoids $\oplus_{1 \le i \le n} M_i$ and $\oplus_{1 \le j \le n} M_j$, respectively then

$\oplus_{1\le i\le n}h_i \times \oplus_{1\le j\le n}h_j : \oplus_{1\le i\le n}p_i x \oplus_{1\le j\le n}p_j \to \oplus_{1\le j\le n}M_i \times \oplus_{1\le j\le n}M_j$ be the topological projective cover represented by topological ring-groupoid $\oplus_{1\le i\le n}M_i \times \oplus_{1\le j\le n}M_j$.

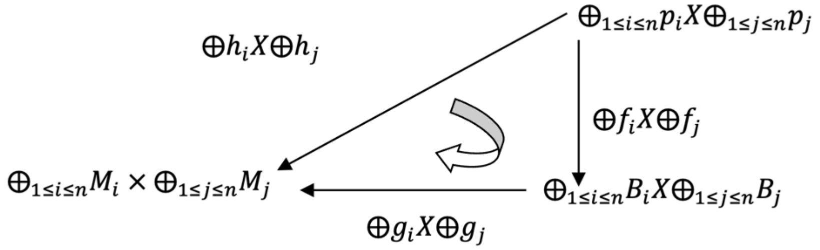

Figure 7: A projective topological the front of topological ring-groupoids

Proof:

The proof is clear by Proposition 2.5.

Proposition 2.8:

Let $h_i : p_i \to M_i (1 \le i \le n)$ and $h_j : p_j \to M_j$ $(1 \le j \le n)$ be topological projective cover of topological ring groupoid $M_i \times M_j$ then

$$\oplus_{1\le i\le n\ 1\le j\le n}\left(h_i \times h_j\right) : \oplus_{1\le i\le n\ 1\le j\le n}\left(P_i \times P_j\right) \to \oplus_{1\le i\le n\ 1\le j\le n}\left(M_i \times M_j\right)$$

projective cover of topology of topological ring groupoid $\oplus_{1\le i\le n\ 1\le j\le n}\left(M_i \times M_j\right)$

Proof:

By Propositions 2.5 and 2.6, then we have

$\oplus_{1\le i\le n\ 1\le j\le n}\left(h_i \times h_j\right) : \oplus_{1\le i\le n\ 1\le j\le n}\left(P_i \times P_j\right) \to \oplus_{1\le i\le n\ 1\le j\le n}\left(M_i \times M_j\right)$ topological projective cover of topological ring groupoid $\oplus_{1\le i\le n\ 1\le j\le n}\left(M_i \times M_j\right)$.

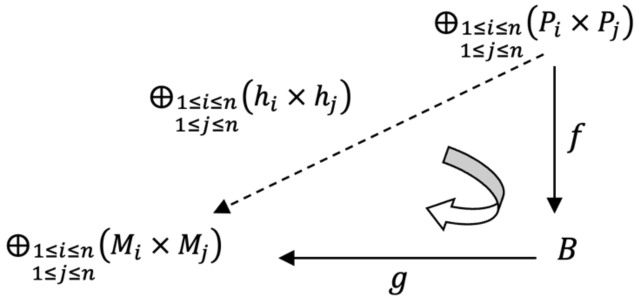

Figure 8: Topological projective cover of topological ring groupoids

3. Conclusions

In this work, we study a morphism ring groupoid, subring groupoid topological structure and ring-groupoid topological quotients. New results have been gotten about topological ring groupoid and topological quotients ring groupoid were written in the form of new propositions.

4. Acknowledgments

The author thanks and appreciates Mustansiriyah University is in Baghdad. (www.edu.iq/mustansiriyah) for providing cooperation and support in this submitted work.

References

[1] G.A. Edgar and R.F. Wheeler, "Topological properties of Banach spaces, " *Pacific Journal of Mathematics*, vol. 115, no.2, pp. 318-350, 1984.

[2] J.G. Llavona, "Weak topologies on bounded sets of a Banach space. associated function spaces", *North-Holland Mathematics Studies*, vol. 170, pp. 307-318, 1992.
[3] N.M. Atheab and S.D. Mohsen, "Some Results on (N, k)-Hyponormal Operators," *Journal of Physics: Conference Series,* vol. 1591, pp. 1-8, 2020.
[4] R. Pradeep, P.M. Naik, S.A.O.A. Mahmoud, "On m-quasi-totally– (α, β)-normal operators, " *Operators and Matrices*, vol. 15, no. 3, pp. 1055–1072, 2021.
[5] S. D. Mohsen, "Solvability Of (λ ,u)-Commuting Operator equations for bounded generalization of hypo normal operators," *Iraqi Journal Science*, vol. 63, no. 9, pp. 3854-3860, 2022.
[6] A. Adhikari and M.R. Adhikari, "Basic Topology2, Topological groups, Topology of Manifolds and Lie Groups," *Springer, pte, ltd. Singapore*, 2022.
[7] R.J. Aumann, "Survey of Repeated Game," *Essays in Game Theory and Mathematical Economics in Honor of Oscar Morgenstern*, pp.259-271, 1981.
[8] S.D. Mohsen, "Some generalizations of fuzzy soft $(\hat{k}^* - \hat{A})$-quasinormal operators in fuzzy soft Hilbert spaces," *Journal of Interdisciplinary Mathematics*, vol. 26, no. 6, pp. 1133–1143, 2023.
[9] S.D. Mohsen and Y.H. Thiyab, "Some characteristics of completeness property in fuzzy soft b−metric space, " *Journal of Applied Science and Engineering*, vol. 27, no. 3, pp. 2227-2232, 2023.
[10] S.D. Mohsen, "On (n ,D)-quasi operators," *Iraqi Journal for Computer Science and Mathematics*, vol. 5, no. 1, pp.175-180, 2024.
[11] T.H. Majeed, "Action of Topological Groupoid on Topological Space," *International Journal on Nonlinear Analysis and Application*, vol.13, no. 1, pp. 85-89, 2022
[12] T.O. Mucuk, H.F. Akiz, B.C. Plaut, "Covering morphisms of topological internal groupoids," *Hacettepe Journal of Mathematics and Statistics*, vol. 49, no. 3, pp. 1020-1029, 2020.
[13] M.Q. Mann'a, " Groupoid and c Topological Quotient Group, *"Global Journal of pure and applied Mathematics*, vol.13, no.7, pp. 3173-3191, 2017.
[14] H. Sahleh, "On Tensor Square of Topological Group and its Subgroups," *World Applied Sciences Journal Technology, Transaction A*, vol. 19, no. 12, pp. 1772-1775, 2012.
[15] A. M. Kumar, P. Gnana Chandra and B. A. Priya, "Quotient on Some Generalization of Topological group," *Baghdad Science Journal,* vol. 20, no. 1, pp. 400-408, 2023.
[16] A. R. Sadek and R. B. Esmaeel, "Some games via (D, DL) Compact topological groups," *Iraqi Journal of Science,* vol. 65, no. 6, pp. 3395-3402, 2024.
[17] I. Icen, A. F. Özcan, and M. H. Gursoy, "Topological Group- Groupoids and their Coverings," *Indian Journal of pure and applicated Mathematics,* vol. 36, no. 9, pp.72-83, 2005.
[18] M. Q. Mann'a, "Some Properties of Topological Ring- Groupoids," *International Journal Contemporary Mathematical Sciences*, vol. 7, no. 11, pp. 517-529, 2012.
[19] O. Mucuk, "Coverings and Ring- Groupoids," *Georgian Mathematical Journal,* vol. 5, no. 5, pp. 475-482, 1998.
[20] T.H. Majeed, "Algebraic topology :On some results of topological group space," *Periodicals of Engineering and Natural Sciences*, vol.9, no.3, pp.297-301, 2021.
[21] W.W. Comfort, "Topological Groups," *Hand Book of Set-Theoretic Topology. North-Holland,* pp. 1143-1263, 1948.
[22] M.S. Bosan, M. U .Khan and D.R. Kocinac," On s-Topological Groups, "*Mathematical Moravica*, vol.18, no.2, pp35-44, 2014.
[23] S. Findik, L.Centrone "The image of Lie polynomials on real Lie algebras of dimension up to 3," *Journal of Algebra*, vol.659, no.1, pp. 344-360, 2024.
[24] M.Q. Mann'a, "Topological Subgroup-Groupoid," *Far East Journal of Mathematical Sciences*, vol.62, no.1, pp.135-144, 2012.

[25] G.N. Malleswari, S. Sreenivasulu, "On skew Jordan product and generalized derivations in prime rings with involution," *JP Journal of algebra, number theory and applications*, vol. 63, no. 4, pp. 329-334, 2024.
[26] M.H. Gursoy, "Generalized ring-groupoids," *Annals of the University of Craiova, Mathematics and Computer Science Series*, vol. 44, no. 1, pp. 126-136, 2017.
[27] R. A .Hussain, "On strong and weak sets in topological spaces, *"Journal of interdisciplinary Mathematics*, vol.23, no.3, pp.765-773, 2021.
[28] M.H. Gursoy, I. Icen,"The Homomorphisms of Topological Groupoids," *Novi Sad Journal of Mathematics,* vol.44, no.1, pp.129-141, 2014.
[29] S.H. Mayerova,"Topological Hypergroupoids," *Computers and Mathematics with Applications*, vol.64, no.9, pp.2845-2949, 2012.
[30] G. Ivan, "Special Morphisms and Groupoids,"," *Novi Sad Journal of Mathematics*, vol.31, no.2, pp. 23-37, 2002.
[31] G. Ivan, "Strong Morphism 0f Groupoids," *Balkan Journal of Geometry and Its Applications*, vol.4, no.1, pp.91-102,1999.
[32] A.R. Sadek and J.H. Hussein, "Regularity via Semi-Generalized Open Set," *Journal of Xi, a University of Architectur & Technology,* vol. 2, no. 4, pp.4631-4636, 2020.
[33] L .Peng, Y. He "A note On Topological Groups and Their Remainders, *"Czechoslovak Mathematical Journal*, vol.62, no.1, pp.197-214, 2012.
[34] S. Nag, "A study on Topological Groups and Their Separation Axioms," *International Journal of Mathematics and Physical Sciences Research*, vol.4, no.2, pp.119-126, 2017.
[35] M.M. Abed, A.M. Farhan and F.G.AL-Sharqi, "Some mathematical structures in a topological group," Journal *of Algebra and Applied Mathematics*, vol.16, no.2, pp.99-117, 2018.
[36] R .K. Manfe ,M. M. Abed, "Some results of topological rings," *Journal of Interdisciplinary Mathematics,*" vol. 25, no.6, pp.1869-1873, 2022.
[37] A.A. Ali and A.R. Sadek, "On Regular δ- Semi-Open Spaces." *Journal of interdisciplinary Mathematics*, vol. 24, no.4, pp. 953-960, 2022
[38] T. H. Majeed, "Algebraic Topology: On Some Results of category for source proper groupoid space," Journal *of Interdisciplinary Mathematics,* vol. 25, no. 2, pp.521-525, 2022.