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Application of the Two Rowed Weyl Module in the Case of Partitions (7,7) and (7, 7) / (1, 0)

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Abstract
The purpose of this paper is to study the application of Weyl module’s resolution in the case of two rows which will be specified in the partitions (7, 7) and (7, 7) / (1, 0), using the homological Weyl (i.e. the contracting homotopy and place polarization).

Keywords: Divided power algebra, resolution of Weyl module, place polarization, mapping Cone.

تطبيق مقاس وايل لصفين في حالة التجزئة (7,7) و(7, 7) / (1, 0)

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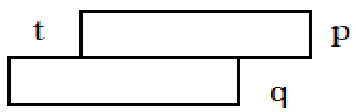
الخلاصة
الغرض من هذا البحث هو دراسة تطبيق تحلل مقاس وايل في حالة الصفين والتي ستكون محددة بالتجزئة (7,7) و (7, 7) / (1, 0) وذلك بإستخدام متشاكلة وايل (أي التوافق الهوموتوبي ودالة المكان).

1. Introduction

Let $\mathcal{R}$ be a commutative ring with identity “1” and $\mathbb{F}$ be a free $\mathcal{R}$ -module by $\mathcal{D}_b\mathbb{F}$ which is the divided power of degree b.

Consider the theory associated to the resolution of the two-rowed Weyl module $K_{\lambda/\mu}\mathbb{F}$, that was previously described [1], as in the following:

λ/μ =



(1)

where $K_{\lambda/\mu}\,\mathbb{F} = \mathrm{Im}(d'_{\lambda/\mu})$ and $d'_{\lambda/\mu}\colon \mathcal{D}\mathbb{F} \longrightarrow \Lambda\mathbb{F}$ is the Weyl map whose images will be called ″Weyl module″.

We have:

$$\sum \mathcal{D}_{p+k}\otimes\mathcal{D}_{q-k} \xrightarrow{\;\square\;} \mathcal{D}_{p}\otimes\mathcal{D}_{q} \xrightarrow{\;d'_{\lambda/\mu}\;} \mathbb{K}_{\lambda/\mu} \to 0 \tag{2}$$

and by using letter place, the maps will be explained now as follows:

$$\begin{pmatrix} w \\ w' \end{pmatrix}\!\!\left|\begin{matrix} 1^{(p+k)} \\ 2^{(q-k)} \end{matrix}\right) \xrightarrow{\;\partial_{21}^{(k)}\;} \begin{pmatrix} w \\ w' \end{pmatrix}\!\!\left|\begin{matrix} 1^{(p)}2^{(k)} \\ 2^{(q-k)} \end{matrix}\right) \longrightarrow \sum_{w} \begin{pmatrix} w_{(1)} \\ w'w_{(2)} \end{pmatrix}\!\!\left|\begin{matrix} (t+1)'(t+2)'\ldots(p+t)' \\ 1'2'3'\ldots q' \end{matrix}\right) \tag{3}$$

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where $w \otimes w' \in D_{p+k} \otimes D_{q-k}$, $\square = \sum_{k=t+1}^{q} \partial_{21}^{(k)}$ is the box map and $d'_{\lambda/\mu} = \partial_{q'2} \ldots \partial_{1'2}\partial_{(p+t)'1} \ldots \partial_{(t+1)'1}$ is the composition of place polarization, from positive places {1,2} to negative places {1', 2', … , (p+t)'}.

Also, as shown in [2], $\square$ is deliver a component $x \otimes y$ of $D_{p+k} \otimes D_{q-k}$ to $\sum x_p \otimes x'_k y$, where $\sum x_p \otimes x'_k$ is the element of the diagonal of $\varkappa$ in $D_p \otimes D_k$.

Let Z_{21} be the free generator of divided power algebra $D(Z_{21})$ in one generator, then the divided power component $Z_{21}^{(k)}$ of degree k of the free generator Z_{21} acts on $D_{p+k} \otimes D_{q-k}$ by place polarization of degree k from place 1 to place 2.

Particularly, the 'graded' algebra 'with identity' $A = \mathcal{D}(Z_{21})$ acts on the graded module $\mathcal{M} = \sum D_{p+k} \otimes D_{q-k} = \sum \mathcal{M}_{q-k}$, where the degree of the 2nd factor dictates the grading [3] .

Therefore, $\mathcal{M}$ is a 'graded' left A;module, where for $w = \mathbb{Z}_{21}^{(k)} \in A$ and $v \in D_{\beta_1} \otimes D_{\beta_2}$, by definition, we have:

$$w(v) = Z_{21}^{(k)}(v) = \partial_{21}^{(k)}(v). \tag{4}$$

And if we have (ς^{+}) which is the graded strand of degree q

$$\mathcal{M}_{\bullet} : 0 \to \mathcal{M}_{q-t} \xrightarrow{\partial_s} \ldots \to \mathcal{M}_l \xrightarrow{\partial_s} \ldots \mathcal{M}_1 \xrightarrow{\partial_s} \mathcal{M}_0 \tag{5}$$

of the normalized bar complex Bar($\mathcal{M}$, A; S, •), and S = {x},

By definition, we have that $\acute{M}_{\bullet}$ is the complex:

$$\begin{aligned} &\sum_{k_i \geq 0} Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \ldots x Z_{21}^{(k_l)} x D_{p+t+|k|} \otimes D_{q-t-|k|} \xrightarrow{d_l} \\ &\quad \sum_{k_i \geq 0} Z_{21}^{(t+k_1)} x Z_{21}^{(k_2)} x \ldots x Z_{21}^{(k_l-1)} x D_{p+t+|k|} \otimes D_{q-t-|k|} \xrightarrow{d_{l-1}} \\ &\ldots \xrightarrow{d_1} \sum_{k_i \geq 0} Z_{21}^{(t+k)} x D_{p+t+k} \otimes D_{q-t-k} \xrightarrow{d_0} D_p \otimes D_q \end{aligned} \tag{6}$$

where $|k| = \sum k_i$ and d_l is the boundary operator $\partial_{\varkappa}$. Notice that (6) illustrates a left complex ($\partial_{\varkappa}^2 = 0$) over the Weyl module in terms of bar complex and letter-place algebra. Furthermore, when it occurs in (6) that the separator $\varkappa$ disappears between $Z_{ab}^{(t)}$ and the components in the tensor product of the divided powers, this means that $\partial_{ab}^{(t)}$ is applied to the tensor product [1,4].

2. Application of Weyl Module Resolution in the Case of Partition (7,7)

In this section we define the terms of Weyl module resolution in the case of partition (7, 7) and give the proof of its exactness.

2.1 The Terms of Weyl Module Resolution in the Case of Partition (7, 7)

We define the terms of Weyl module resolution in the case of partition (7, 7), as follows:

$\acute{M}_0 = D_7 \otimes D_7$

$$\begin{aligned} \acute{M}_1 = {} & Z_{21}\varkappa D_8 \otimes D_6 \oplus Z_{21}^{(2)}\varkappa D_9 \otimes D_5 \oplus Z_{21}^{(3)}\varkappa D_{10} \otimes D_4 \oplus Z_{21}^{(4)}\varkappa D_{11} \otimes D_3 \\ & \oplus Z_{21}^{(5)}\varkappa D_{12} \otimes D_2 \oplus Z_{21}^{(6)}\varkappa D_{13} \otimes D_1 \oplus Z_{21}^{(7)}\varkappa D_{14} \otimes D_0 \end{aligned}$$

$$\begin{aligned} \acute{M}_2 = {} & Z_{21}\varkappa Z_{21}\varkappa D_9 \otimes D_5 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{10} \otimes D_4 \oplus Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{10} \otimes D_4 \\ & \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{11} \otimes D_3 \oplus Z_{21}\varkappa Z_{21}^{(3)}\varkappa D_{11} \otimes D_3 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa D_{11} \otimes D_3 \\ & \oplus Z_{21}^{(4)}\varkappa Z_{21}\varkappa D_{12} \otimes D_2 \oplus Z_{21}\varkappa Z_{21}^{(4)}\varkappa D_{12} \otimes D_2 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa D_{12} \otimes D_2 \\ & \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa D_{12} \otimes D_2 \oplus Z_{21}^{(5)}\varkappa Z_{21}\varkappa D_{13} \otimes D_1 \oplus Z_{21}\varkappa Z_{21}^{(5)}\varkappa D_{13} \otimes D_1 \\ & \oplus Z_{21}^{(4)}\varkappa Z_{21}^{(2)}\varkappa D_{13} \otimes D_1 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(4)}\varkappa D_{13} \otimes D_1 \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(3)}\varkappa D_{13} \otimes D_1 \\ & \oplus Z_{21}^{(6)}\varkappa Z_{21}\varkappa D_{14} \otimes D_0 \oplus Z_{21}\varkappa Z_{21}^{(6)}\varkappa D_{14} \otimes D_0 \oplus Z_{21}^{(5)}\varkappa Z_{21}^{(2)}\varkappa D_{14} \otimes D_0 \\ & \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(5)}\varkappa D_{14} \otimes D_0 \oplus Z_{21}^{(4)}\varkappa Z_{21}^{(3)}\varkappa D_{14} \otimes D_0 \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(4)}\varkappa D_{14} \otimes D_0 \end{aligned}$$

$$\begin{aligned} \acute{M}_3 = {} & Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{10} \otimes D_4 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{11} \otimes D_3 \oplus Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{11} \otimes D_3 \\ & \oplus Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{11} \otimes D_3 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12} \otimes D_2 \oplus Z_{21}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{12} \otimes D_2 \\ & \oplus Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(3)}\varkappa D_{12} \otimes D_2 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{12} \otimes D_2 \oplus Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa D_{12} \otimes D_2 \\ & \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{12} \otimes D_2 \oplus Z_{21}^{(4)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13} \otimes D_1 \oplus Z_{21}\varkappa Z_{21}^{(4)}\varkappa Z_{21}\varkappa D_{13} \otimes D_1 \\ & \oplus Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(4)}\varkappa D_{13} \otimes D_1 \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{13} \otimes D_1 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{13} \otimes D_1 \\ & \oplus Z_{21}\varkappa Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa D_{13} \otimes D_1 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{13} \otimes D_1 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(3)}\varkappa D_{13} \otimes D_1 \end{aligned}$$

$$\oplus\, Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(3)}\varkappa Ɗ_{13}\otimes Ɗ_{1} \;\oplus\; Ƶ_{21}^{(5)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0} \;\oplus\; Ƶ_{21}\varkappa Ƶ_{21}^{(5)}\varkappa Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$
$$\oplus\, Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(5)}\varkappa Ɗ_{14}\otimes Ɗ_{0} \;\oplus Ƶ_{21}^{(4)}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0} \;\oplus Ƶ_{21}^{(4)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$
$$\oplus\, Ƶ_{21}\varkappa Ƶ_{21}^{(4)}\varkappa Ƶ_{21}^{(2)}\varkappa Ɗ_{14}\otimes Ɗ_{0} \;\oplus\; Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(4)}\varkappa Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0} \oplus Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(4)}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$
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$$\oplus\; Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(3)}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$

$$\acute{M}_4 = \; Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{11}\otimes Ɗ_{3} \;\oplus\; Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{12}\otimes Ɗ_{2}$$
$$\oplus\, Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{12}\otimes Ɗ_{2} \;\oplus\; Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ɗ_{12}\otimes Ɗ_{2}$$
$$\oplus\, Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ɗ_{12}\otimes Ɗ_{2} \;\oplus\; Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{13}\otimes Ɗ_{1}$$
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$$\oplus\, Ƶ_{21}\varkappa Ƶ_{21}^{(3)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{13}\otimes Ɗ_{1} \;\oplus\; Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(3)}\varkappa Ƶ_{21} x Ɗ_{13}\otimes Ɗ_{1}$$
$$\oplus\, Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(3)}\varkappa Ɗ_{13}\otimes Ɗ_{1} \;\oplus\; Ƶ_{21}^{(4)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$
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$$\oplus\, Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(3)}\varkappa Ɗ_{14}\otimes Ɗ_{0} \;\oplus\; Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$
$$\oplus\, Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ɗ_{14}\otimes Ɗ_{0} \;\oplus\; Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(2)}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$
$$\oplus\, Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(2)}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$

$$\acute{M}_5 = \; Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{12}\otimes Ɗ_{2} \;\oplus\; Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{13}\otimes Ɗ_{1}$$
$$\oplus Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{13}\otimes Ɗ_{1} \;\oplus\; Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{13}\otimes Ɗ_{1}$$
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$$\oplus\; Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa\, Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0} \;\oplus\; Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa\, Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$
$$\oplus\; Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa\, Ɗ_{14}\otimes Ɗ_{0} \;\oplus\; Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa\, Ɗ_{14}\otimes Ɗ_{0}$$
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$$\oplus\; Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ɗ_{14}\otimes Ɗ_{0} \;\oplus\; Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}^{(2)}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$

$$\acute{M}_6 = \; Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{13}\otimes Ɗ_{1} \;\oplus Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$
$$\oplus\, Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0} \oplus Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$
$$\oplus\, Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0} \oplus Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$
$$\oplus\, Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}^{(2)}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$

$$\acute{M}_7 = \; Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ƶ_{21}\varkappa Ɗ_{14}\otimes Ɗ_{0}$$

2.2 The Exactness of Weyl Resolution in the Case of Partition (7,7)

Define the $\mathbb{S}_i$ map as follows:

$\mathbb{S}_0 : \acute{M}_0 \longrightarrow \acute{M}_1$

$$\mathbb{S}_0\left(\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7)}2^{(\text{ƙ})}\\ 2^{(7-\text{ƙ})}\end{matrix}\right)\right)=\begin{cases}Z_{21}^{(\text{ƙ})}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+\text{ƙ})}\\ 2^{(7-\text{ƙ})}\end{matrix}\right) & ;if\ \text{ƙ}=1,2,3,4,5,6,7\\ 0 & ;if\ \text{ƙ}=0\end{cases}$$

And

$\mathbb{S}_1: \acute{M}_1 \longrightarrow \acute{M}_2$

$$\mathbb{S}_1\left(Z_{21}^{(\text{ƙ})}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+\text{ƙ})}2^{(\text{ɱ})}\\ 2^{(7-\text{ƙ}-\text{ɱ})}\end{matrix}\right)\right)=\begin{cases}Z_{21}^{(\text{ƙ})}\varkappa Z_{21}^{(\text{ɱ})}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+\text{ƙ}+\text{ɱ})}\\ 2^{(7-\text{ƙ}-\text{ɱ})}\end{matrix}\right) & ;if\ \text{ɱ}>0\\ 0 & ;if\ \text{ɱ}=0\end{cases}$$

And

$\mathbb{S}_2: \acute{M}_2 \longrightarrow \acute{M}_3$

$$\mathbb{S}_2\left(Z_{21}^{(\text{ƙ}_1)}\varkappa Z_{21}^{(\text{ƙ}_2)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\text{ƙ}|)}2^{(\text{ɱ})}\\ 2^{(7-|\text{ƙ}|-\text{ɱ})}\end{matrix}\right)\right)=\begin{cases}Z_{21}^{(\text{ƙ}_1)}\varkappa Z_{21}^{(\text{ƙ}_2)}\varkappa Z_{21}^{(\text{ɱ})}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\text{ƙ}|+\text{ɱ})}\\ 2^{(7-|\text{ƙ}|-\text{ɱ})}\end{matrix}\right) & ;if\ \text{ɱ}>0\\ 0 & ;if\ \text{ɱ}=0\end{cases}$$

And

$\mathbb{S}_3: \acute{M}_3 \longrightarrow \acute{M}_4$

$$\mathbb{S}_3\left(Z_{21}^{(\text{ƙ}_1)}\varkappa Z_{21}^{(\text{ƙ}_2)}\varkappa Z_{21}^{(\text{ƙ}_3)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\text{ƙ}|)}2^{(\text{ɱ})}\\ 2^{(7-|\text{ƙ}|-\text{ɱ})}\end{matrix}\right)\right)$$

$$=\begin{cases}Z_{21}^{(\text{ƙ}_1)}\varkappa Z_{21}^{(\text{ƙ}_2)}\varkappa Z_{21}^{(\text{ƙ}_3)}\varkappa Z_{21}^{(\text{ɱ})}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\text{ƙ}|+\text{ɱ})}\\ 2^{(7-|\text{ƙ}|-\text{ɱ})}\end{matrix}\right) & ;if\ \text{ɱ}>0\\ 0 & ;if\ \text{ɱ}=0\end{cases}$$

And

$\mathbb{S}_4: \acute{M}_4 \longrightarrow \acute{M}_5$

$$\mathbb{S}_4\left(Z_{21}^{(\text{ƙ}_1)}\varkappa Z_{21}^{(\text{ƙ}_2)}\varkappa Z_{21}^{(\text{ƙ}_3)}\varkappa Z_{21}^{(\text{ƙ}_4)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\text{ƙ}|)}2^{(\text{ɱ})}\\ 2^{(7-|\text{ƙ}|-\text{ɱ})}\end{matrix}\right)\right)$$

$$=\begin{cases}Z_{21}^{(\text{ƙ}_1)}\varkappa Z_{21}^{(\text{ƙ}_2)}\varkappa Z_{21}^{(\text{ƙ}_3)}\varkappa Z_{21}^{(\text{ƙ}_4)}\varkappa Z_{21}^{(\text{ɱ})}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\text{ƙ}|+\text{ɱ})}\\ 2^{(7-|\text{ƙ}|-\text{ɱ})}\end{matrix}\right) & ;if\ \text{ɱ}>0\\ 0 & ;if\ \text{ɱ}=0\end{cases}$$

And

$\mathbb{S}_5: \acute{M}_5 \longrightarrow \acute{M}_6$

$$\mathbb{S}_5\left(Z_{21}^{(\text{ƙ}_1)}\varkappa Z_{21}^{(\text{ƙ}_2)}\varkappa Z_{21}^{(\text{ƙ}_3)}\varkappa Z_{21}^{(\text{ƙ}_4)}\varkappa Z_{21}^{(\text{ƙ}_5)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\text{ƙ}|)}2^{(\text{ɱ})}\\ 2^{(7-|\text{ƙ}|-\text{ɱ})}\end{matrix}\right)\right)$$

$$=\begin{cases}Z_{21}^{(\text{ƙ}_1)}\varkappa Z_{21}^{(\text{ƙ}_2)}\varkappa Z_{21}^{(\text{ƙ}_3)}\varkappa Z_{21}^{(\text{ƙ}_4)}\varkappa Z_{21}^{(\text{ƙ}_5)}\varkappa Z_{21}^{(\text{ɱ})}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\text{ƙ}|+\text{ɱ})}\\ 2^{(7-|\text{ƙ}|-\text{ɱ})}\end{matrix}\right) & ;if\ \text{ɱ}>0\\ 0 & ;if\ \text{ɱ}=0\end{cases}$$

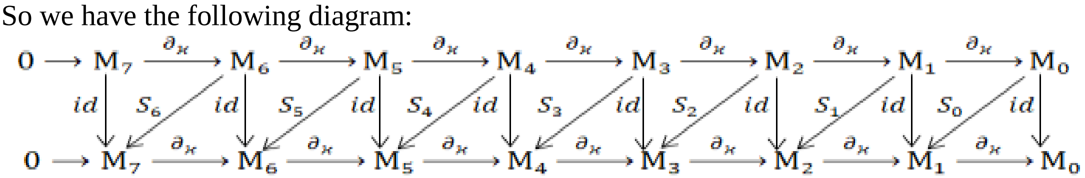
And

$\mathbb{S}_6: \acute{M}_6 \longrightarrow \acute{M}_7$

$$\mathbb{S}_6\left(Z_{21}^{(\text{ƙ}_1)}\varkappa Z_{21}^{(\text{ƙ}_2)}\varkappa Z_{21}^{(\text{ƙ}_3)}\varkappa Z_{21}^{(\text{ƙ}_4)}\varkappa Z_{21}^{(\text{ƙ}_5)}\varkappa Z_{21}^{(\text{ƙ}_6)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\text{ƙ}|)}2^{(\text{ɱ})}\\ 2^{(7-|\text{ƙ}|-\text{ɱ})}\end{matrix}\right)\right)$$

$$=\begin{cases}Z_{21}^{(\text{ƙ}_1)}\varkappa Z_{21}^{(\text{ƙ}_2)}\varkappa Z_{21}^{(\text{ƙ}_3)}\varkappa Z_{21}^{(\text{ƙ}_4)}\varkappa Z_{21}^{(\text{ƙ}_5)}\varkappa Z_{21}^{(\text{ƙ}_6)}\varkappa Z_{21}^{(\text{ɱ})}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\text{ƙ}|+\text{ɱ})}\\ 2^{(7-|\text{ƙ}|-\text{ɱ})}\end{matrix}\right) & ;if\ \text{ɱ}>0\\ 0 & ;if\ \text{ɱ}=0\end{cases}$$

So we have the following diagram:



(1)

2.2.1 Proposition

In diagram (1), we can see that $\mathbb{S}_n\partial_\varkappa + \partial_\varkappa\mathbb{S}_{n+1} = id$ where n = 0, 1, 2, 3, 4, 5.

Proof:

$$\mathbb{S}_0\partial_\varkappa\left(Z_{21}^{(ƙ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ)}2^{(ɱ)}\\ 2^{(7-ƙ-ɱ)}\end{matrix}\right)\right)=\mathbb{S}_0\partial_{21}^{(ƙ)}\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ)}2^{(ɱ)}\\ 2^{(7-ƙ-ɱ)}\end{matrix}\right)=\binom{ƙ+ɱ}{ɱ}\,Z_{21}^{(ƙ+ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ+ɱ)}\\ 2^{(7-ƙ-ɱ)}\end{matrix}\right)$$

And

$$\partial_\varkappa\mathbb{S}_1\left(Z_{21}^{(ƙ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ)}2^{(ɱ)}\\ 2^{(7-ƙ-ɱ)}\end{matrix}\right)\right)=\partial_\varkappa\left(Z_{21}^{(ƙ)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ+ɱ)}\\ 2^{(7-ƙ-ɱ)}\end{matrix}\right)\right)=-\binom{ƙ+ɱ}{ɱ}\,Z_{21}^{(ƙ+ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ+ɱ)}\\ 2^{(7-ƙ-ɱ)}\end{matrix}\right)$$
$$+\,Z_{21}^{(ƙ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ)}2^{(ɱ)}\\ 2^{(7-ƙ-ɱ)}\end{matrix}\right)$$

It is obvious that $\mathbb{S}_0\partial_\varkappa+\partial_\varkappa\mathbb{S}_1=id.$

$$\mathbb{S}_1\partial_\varkappa\left(Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\right)=\mathbb{S}_1\left(-\binom{|ƙ|}{ƙ_2}Z_{21}^{|ƙ|}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+\right.$$
$$\left.Z_{21}^{(ƙ_1)}\varkappa\,Z_{21}^{(ƙ_2)}\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\right)$$
$$=-\binom{|ƙ|}{ƙ_2}Z_{21}^{|ƙ|}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+\binom{ƙ_2+ɱ}{ɱ}Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right),$$

And

$$\partial_\varkappa\mathbb{S}_2\left(Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\right)=\partial_\varkappa\left(Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ+ɱ)}\\ 2^{(7-ƙ-ɱ)}\end{matrix}\right)\right)$$
$$=$$
$$\binom{|ƙ|}{ƙ_2}Z_{21}^{|ƙ|}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)-\binom{ƙ_2+ɱ}{ɱ}Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$Z_{21}^{(ƙ_1)}\varkappa\,Z_{21}^{(ƙ_2)}\left(\begin{matrix}W\\ W'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right),$$

where $|ƙ|=ƙ_1+ƙ_2$.

It is obvious that $\mathbb{S}_1\partial_\varkappa+\partial_\varkappa\mathbb{S}_2=id.$

$$\mathbb{S}_2\partial_\varkappa\left(Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\right)$$
$$=\mathbb{S}_2\left(\binom{ƙ_1+ƙ_2}{ƙ_2}Z_{21}^{(ƙ_1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)-\binom{ƙ_2+ƙ_3}{ƙ_3}Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+\right.$$
$$\left.Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\right)$$
$$=\binom{ƙ_1+ƙ_2}{ƙ_2}Z_{21}^{(ƙ_1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)-\binom{ƙ_2+ƙ_3}{ƙ_3}Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_3+ɱ}{ɱ}Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right),$$

And

$$\partial_\varkappa\mathbb{S}_3\left(Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\right)=\partial_\varkappa\left(Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\right)$$
$$=-\binom{ƙ_1+ƙ_2}{ƙ_2}Z_{21}^{(ƙ_1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+\binom{ƙ_2+ƙ_3}{ƙ_3}Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_3+ɱ}{ɱ}Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ɱ)}x\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+\quad Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ɱ)}\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)$$
$$=-\binom{ƙ_1+ƙ_2}{ƙ_2}Z_{21}^{(ƙ_1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+\binom{ƙ_2+ƙ_3}{ƙ_3}Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_3+ɱ}{ɱ}Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ɱ)}x\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+\quad Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right),$$

where $|ƙ|=ƙ_1+ƙ_2+ƙ_3$.

It is obvious that $\mathbb{S}_2\partial_\varkappa+\partial_\varkappa\mathbb{S}_3=id.$

$$\mathbb{S}_3\partial_\varkappa\left(Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\right)$$

$$= \mathbb{S}_3\Bigg(-\binom{ƙ_1+ƙ_2}{ƙ_2} Z_{21}^{(ƙ_1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right) + \binom{ƙ_2+ƙ_3}{ƙ_3} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa\Bigg)$$
$$\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\binom{ƙ_3+ƙ_4}{ƙ_4} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ƙ_4)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)$$
$$= -\binom{ƙ_1+ƙ_2}{ƙ_2} Z_{21}^{(ƙ_1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right) +$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_3+ƙ_4}{ƙ_4} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ƙ_4)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_4+ɱ}{ɱ} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4+ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right),$$

And

$$\partial_\varkappa\mathbb{S}_4\left(Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)} x\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\right)$$
$$=\partial_\varkappa\left(Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\right)$$
$$= \binom{ƙ_1+ƙ_2}{ƙ_2} Z_{21}^{(ƙ_1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right) -$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_3+ƙ_4}{ƙ_4} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ƙ_4)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_4+ɱ}{ɱ} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4+ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa\partial_{21}^{(ɱ)}\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)$$
$$= \binom{ƙ_1+ƙ_2}{ƙ_2} Z_{21}^{(ƙ_1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right) -$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_3+ƙ_4}{ƙ_4} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ƙ_4)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_4+ɱ}{ɱ} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4+ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right),$$

where $|ƙ| = ƙ_1+ƙ_2+ƙ_3+ƙ_4$.

It is obvious that $\mathbb{S}_3\partial_\varkappa + \partial_\varkappa\mathbb{S}_4 = id$.

$$\mathbb{S}_4\partial_\varkappa\left(Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\right)$$
$$=\mathbb{S}_4\Bigg(\binom{ƙ_1+ƙ_2}{ƙ_2} Z_{21}^{(ƙ_1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right) -$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right) +$$
$$\binom{ƙ_3+ƙ_4}{ƙ_4} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right) -$$
$$\binom{ƙ_4+ƙ_5}{ƙ_5} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4+ƙ_5)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right) +$$
$$Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)} Z_{21}^{(ƙ_4)}\varkappa\partial_{21}^{(ƙ_5)}\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\Bigg)$$

$$= \binom{\mathfrak{k}_1+\mathfrak{k}_2}{\mathfrak{k}_2} Z_{21}^{(\mathfrak{k}_1+\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) -$$
$$\binom{\mathfrak{k}_2+\mathfrak{k}_3}{\mathfrak{k}_3} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2+\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) +$$
$$\binom{\mathfrak{k}_3+\mathfrak{k}_4}{\mathfrak{k}_4} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3+\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) -$$
$$\binom{\mathfrak{k}_4+\mathfrak{k}_5}{\mathfrak{k}_5} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4+\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) +$$
$$= \binom{\mathfrak{k}_4+\mathfrak{m}}{\mathfrak{m}} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5+\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right),$$

And

$$\partial_{\varkappa}\mathbb{S}_5\left(Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|)}2^{(\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right)\right)$$
$$=\partial_{\varkappa}\left(Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right)\right)$$
$$= -\binom{\mathfrak{k}_1+\mathfrak{k}_2}{\mathfrak{k}_2} Z_{21}^{(\mathfrak{k}_1+\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) +$$
$$\binom{\mathfrak{k}_2+\mathfrak{k}_3}{\mathfrak{k}_3} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2+\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) -$$
$$\binom{\mathfrak{k}_3+\mathfrak{k}_4}{\mathfrak{k}_4} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3+\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) +$$
$$\binom{\mathfrak{k}_4+\mathfrak{k}_5}{\mathfrak{k}_5} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4+\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) -$$
$$\binom{\mathfrak{k}_4+\mathfrak{m}}{\mathfrak{m}} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5+\mathfrak{m})} \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) +$$
$$Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa \partial_{21}^{(\mathfrak{m})} \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right)$$
$$= -\binom{\mathfrak{k}_1+\mathfrak{k}_2}{\mathfrak{k}_2} Z_{21}^{(\mathfrak{k}_1+\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) +$$
$$\binom{\mathfrak{k}_2+\mathfrak{k}_3}{\mathfrak{k}_3} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2+\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) -$$
$$\binom{\mathfrak{k}_3+\mathfrak{k}_4}{\mathfrak{k}_4} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3+\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) +$$
$$\binom{\mathfrak{k}_4+\mathfrak{k}_5}{\mathfrak{k}_5} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4+\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|+\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) -$$
$$\binom{\mathfrak{k}_5+\mathfrak{m}}{\mathfrak{m}} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5+\mathfrak{m})}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|)}2^{(\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) +$$
$$Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|)}2^{(\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right),$$

where $|\mathfrak{k}| = \mathfrak{k}_1+\mathfrak{k}_2+\mathfrak{k}_3+\mathfrak{k}_4+\mathfrak{k}_5$.

It is obvious that $\mathbb{S}_4\partial_{\varkappa} + \partial_{\varkappa}\mathbb{S}_5 = id$.

$$S_5\partial_{\varkappa}\left(Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{k}_6)}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|)}2^{(\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right)\right)$$
$$=S_5\left(-\binom{\mathfrak{k}_1+\mathfrak{k}_2}{\mathfrak{k}_2} Z_{21}^{(\mathfrak{k}_1+\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{k}_6)}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|)}2^{(\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) +\right.$$
$$\binom{\mathfrak{k}_2+\mathfrak{k}_3}{\mathfrak{k}_3} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2+\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{k}_6)}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|)}2^{(\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) -$$
$$\binom{\mathfrak{k}_3+\mathfrak{k}_4}{\mathfrak{k}_4} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3+\mathfrak{k}_4)} Z_{21}^{(\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{k}_6)}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|)}2^{(\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) +$$
$$\binom{\mathfrak{k}_4+\mathfrak{k}_5}{\mathfrak{k}_5} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)} Z_{21}^{(\mathfrak{k}_4+\mathfrak{k}_5)}\varkappa Z_{21}^{(\mathfrak{k}_6)}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|)}2^{(\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) -$$
$$\binom{\mathfrak{k}_5+\mathfrak{k}_6}{\mathfrak{k}_6} Z_{21}^{(\mathfrak{k}_1)}\varkappa Z_{21}^{(\mathfrak{k}_2)}\varkappa Z_{21}^{(\mathfrak{k}_3)}\varkappa Z_{21}^{(\mathfrak{k}_4)}\varkappa Z_{21}^{(\mathfrak{k}_5+\mathfrak{k}_6)}\varkappa \left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|\mathfrak{k}|)}2^{(\mathfrak{m})}\\ 2^{(7-|\mathfrak{k}|-\mathfrak{m})}\end{matrix}\right) +$$

$$Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa \partial_{21}^{(ƙ_6)}\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\Bigg)$$
$$= -\binom{ƙ_1+ƙ_2}{ƙ_2} Z_{21}^{(ƙ_1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right) +$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right) -$$
$$\binom{ƙ_3+ƙ_4}{ƙ_4} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ƙ_4)} Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right) +$$
$$\binom{ƙ_4+ƙ_5}{ƙ_5} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)} Z_{21}^{(ƙ_4+ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right) -$$
$$\binom{ƙ_5+ƙ_6}{ƙ_6} Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5+ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_6+ɱ}{ɱ} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6+ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\Bigg),$$

And

$$\partial_\varkappa S_6\left(Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\right)$$
$$=\partial_\varkappa\left(Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)\right)$$
$$= \binom{ƙ_1+ƙ_2}{ƙ_2} Z_{21}^{(ƙ_1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right) -$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_3+ƙ_4}{ƙ_4} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_4+ƙ_5}{ƙ_5} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4+ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_5+ƙ_6}{ƙ_6} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5+ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_6+ɱ}{ɱ} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6+ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa \partial_{21}^{(ɱ)}\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)$$
$$= \binom{ƙ_1+ƙ_2}{ƙ_2} Z_{21}^{(ƙ_1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right) -$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa \partial_{21}^{(ɱ)}\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_3+ƙ_4}{ƙ_4} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_4+ƙ_5}{ƙ_5} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4+ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_5+ƙ_6}{ƙ_6} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5+ƙ_6)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_6+ɱ}{ɱ} Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6+ɱ)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$Z_{21}^{(ƙ_1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ƙ_6)}\varkappa\left(\begin{matrix}𝕨\\𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\2^{(7-|ƙ|-ɱ)}\end{matrix}\right),$$

where $|ƙ| = ƙ_1+ƙ_2 + ƙ_3+ƙ_4+ƙ_5+ƙ_6$.

It is obvious that $\mathbb{S}_5\partial_\varkappa + \partial_\varkappa\mathbb{S}_6 = id$.

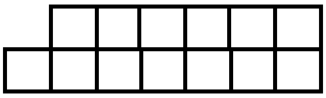
From the above we conclude that $\{\mathbb{S}_0, \mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3, \mathbb{S}_4, \mathbb{S}_5, \mathbb{S}_6\}$ is a contracting homology [5], which implies that our complex is exact.

3. Application of Weyl Module Resolution in the Case of Partition (7, 7) / (1, 0)

In this section we define the terms of Weyl module resolution in the case of partition (7, 7) / (1, 0) and give the proof of its exactness.

3.1 The Terms of Weyl Module Resolution in the Case of Partition (7, 7) / (1, 0)

If the Skew-shape (7, 7) / (1, 0) , then the characteristic free resolution terms will be as follows:



$\acute{M}_0 = D_6 \otimes D_7$

$\acute{M}_1 = Z_{21}^{(2)}\varkappa D_8 \otimes D_5 \oplus Z_{21}^{(3)}\varkappa D_9 \otimes D_4 \oplus Z_{21}^{(4)}\varkappa D_{10} \otimes D_3$
$\oplus Z_{21}^{(5)}\varkappa D_{11} \otimes D_2 \oplus Z_{21}^{(6)}\varkappa D_{12} \otimes D_1 \oplus Z_{21}^{(7)}\varkappa D_{13} \otimes D_0$

$\acute{M}_2 = Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_9 \otimes D_4 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{10} \otimes D_3 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa D_{10} \otimes D_3$
$\oplus Z_{21}^{(4)}\varkappa Z_{21}\varkappa D_{11} \otimes D_2 \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa D_{11} \otimes D_2 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa D_{11} \otimes D_2$
$\oplus Z_{21}^{(5)}\varkappa Z_{21}\varkappa D_{12} \otimes D_1 \oplus Z_{21}^{(4)}\varkappa Z_{21}^{(2)}\varkappa D_{12} \otimes D_1 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(4)}\varkappa D_{12} \otimes D_1$
$\oplus Z_{21}^{(3)}\varkappa Z_{21}^{(3)}\varkappa D_{12} \otimes D_1 \oplus Z_{21}^{(6)}\varkappa Z_{21}\varkappa D_{13} \otimes D_0 \oplus Z_{21}^{(5)}\varkappa Z_{21}^{(2)}\varkappa D_{13} \otimes D_0$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}^{(5)}\varkappa D_{13} \otimes D_0 \oplus Z_{21}^{(4)}\varkappa Z_{21}^{(3)}\varkappa D_{13} \otimes D_0 \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(4)}\varkappa D_{13} \otimes D_0$

$\acute{M}_3 = Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{10} \otimes D_3 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{11} \otimes D_2 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{11} \otimes D_2$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{11} \otimes D_2 \oplus Z_{21}^{(4)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12} \otimes D_1 \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{12} \otimes D_1$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{12} \otimes D_1 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{12} \otimes D_1 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(3)}\varkappa D_{12} \otimes D_1$
$\oplus Z_{21}^{(5)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13} \otimes D_0 \oplus Z_{21}^{(4)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{13} \otimes D_0 \oplus Z_{21}^{(4)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{13} \otimes D_0$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}^{(4)}\varkappa Z_{21}\varkappa D_{13} \otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(4)}\varkappa D_{13} \otimes D_0 \oplus Z_{21}^{(3)}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{13} \otimes D_0$
$\oplus Z_{21}^{(3)}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{13} \otimes D_0$

$\acute{M}_4 = Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{11} \otimes D_2 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12} \otimes D_1$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12} \otimes D_1 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{12} \otimes D_1$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{12} \otimes D_1 \oplus Z_{21}^{(4)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13} \otimes D_0$
$\oplus Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13} \otimes D_0 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{13} \otimes D_0$
$\oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{13} \otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13} \otimes D_0$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa D_{13} \otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(3)}\varkappa D_{13} \otimes D_0$

$\acute{M}_5 = Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12} \otimes D_1 \oplus Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13} \otimes D_0$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13} \otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13} \otimes D_0$
$\oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa D_{13} \otimes D_0 \oplus Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa D_{13} \otimes D_0$

$\acute{M}_6 = Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13} \otimes D_0$

3.2 The Exactness of Skew- Shape in the case of partition (7, 7) / (1, 0)

In this section we illustrate the construction of the homotopies $\{\mathbb{S}_i\}$ in the case of partition (7, 7) / (1, 0) where i= 1, 2, …, 5.

As for homologies we have:

$\mathbb{S}_0 : \acute{M}_0 \longrightarrow \acute{M}_1$

$$\mathbb{S}_0\left(\left(\begin{array}{c|l} w & 1^{(7)}2^{(k)} \\ w' & 2^{(6-k)} \end{array}\right)\right) = \begin{cases} Z_{21}^{(k)}\varkappa\left(\begin{array}{c|l} w & 1^{(7+k)} \\ w' & 2^{(6-k)} \end{array}\right) & ; if\ k = 1,2,3,4,5,6,7 \\ 0 & ; if\ k = 0 \end{cases}$$

And

$\mathbb{S}_1 : \acute{M}_1 \longrightarrow \acute{M}_2$

$$\mathbb{S}_1\left(Z_{21}^{(k+1)}\varkappa\left(\begin{array}{c|l} w & 1^{(7+k)}2^{(m)} \\ w' & 2^{(6-k-m)} \end{array}\right)\right) = \begin{cases} Z_{21}^{(k+1)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{array}{c|l} w & 1^{(7+k+m)} \\ w' & 2^{(6-k-m)} \end{array}\right) & ; if\ m > 0 \\ 0 & ; if\ m = 0 \end{cases}$$

And

$\mathbb{S}_2: \acute{M}_2 \longrightarrow \acute{M}_3$

$$\mathbb{S}_2\left(Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)=\begin{cases}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right) & ;if\ ɱ>0\\ 0 & ;if\ ɱ=0\end{cases}$$

And

$\mathbb{S}_3: \acute{M}_3 \longrightarrow \acute{M}_4$

$$\mathbb{S}_3\left(Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)=$$

$$\begin{cases}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right) & ;if\ ɱ>0\\ 0 & ;if\ ɱ=0\end{cases}$$

And

$\mathbb{S}_4: \acute{M}_4 \longrightarrow \acute{M}_5$

$$\mathbb{S}_4\left(Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)$$

$$=\begin{cases}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right) & ;if\ ɱ>0\\ 0 & ;if\ ɱ=0\end{cases}$$

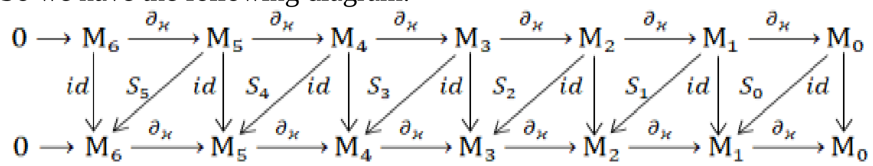
And

$\mathbb{S}_5: \acute{M}_5 \longrightarrow \acute{M}_6$

$$\mathbb{S}_5\left(Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)$$

$$=\begin{cases}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ƙ_5)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right) & ;if\ ɱ>0\\ 0 & ;if\ ɱ=0\end{cases}$$

So we have the following diagram:



(2)

3.2.1 Proposition

In diagram (2), we can see that $\mathbb{S}_n\partial_\varkappa + \partial_\varkappa\mathbb{S}_{n+1} = id$ where n = 0, 1, 2, 3, 4.

Proof:

$$\mathbb{S}_0\partial_\varkappa\left(Z_{21}^{(ƙ+1)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ)}2^{(ɱ)}\\ 2^{(6-ƙ-ɱ)}\end{matrix}\right)\right)=\mathbb{S}_0\partial_{21}^{(ƙ+1)}\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7)}2^{(ƙ+ɱ)}\\ 2^{(6-ƙ-ɱ)}\end{matrix}\right)=$$

$$=\binom{ƙ+1+ɱ}{ɱ}Z_{21}^{(ƙ+1+ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ+ɱ)}\\ 2^{(6-ƙ-ɱ)}\end{matrix}\right),$$

And

$$\partial_\varkappa\mathbb{S}_1\left(Z_{21}^{(ƙ+1)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ)}2^{(ɱ)}\\ 2^{(6-ƙ-ɱ)}\end{matrix}\right)\right)=\partial_\varkappa\left(Z_{21}^{(ƙ+1)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ+ɱ)}\\ 2^{(6-ƙ-ɱ)}\end{matrix}\right)\right)$$

$$=-\binom{ƙ+1+ɱ}{ɱ}Z_{21}^{(ƙ+1+ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ+ɱ)}\\ 2^{(6-ƙ-ɱ)}\end{matrix}\right)+Z_{21}^{(ƙ+1)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ)}2^{(ɱ)}\\ 2^{(6-ƙ-ɱ)}\end{matrix}\right),$$

It is obvious that $\mathbb{S}_0\partial_\varkappa + \partial_\varkappa\mathbb{S}_1 = id$.

$$S_1\partial_\varkappa\left(Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)$$

$$=S_1\left(-\binom{|ƙ|+1}{ƙ_2}Z_{21}^{|ƙ|+1}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+Z_{21}^{(ƙ_1+1)}\varkappa\,\partial_{21}^{(ƙ_2)}\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)$$

$$=-\binom{|ƙ|+1}{ƙ_2}Z_{21}^{|ƙ|+1}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+\binom{ƙ_2+ɱ}{ɱ}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2+ɱ)}\chi\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right),$$

And

$$\partial_\varkappa \mathbb{S}_2\left(Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)=\partial_\varkappa\left(Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+ƙ+ɱ)}\\ 2^{(6-ƙ-ɱ)}\end{matrix}\right)\right)$$

$$=\binom{|ƙ|+1}{ƙ_2}Z_{21}^{|ƙ|+1}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)-\binom{ƙ_2+ɱ}{ɱ}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2+ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$Z_{21}^{(ƙ_1+1)}\varkappa\, Z_{21}^{(ƙ_2)}\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right),$$

where $|ƙ|= ƙ_1+ƙ_2$.

It is obvious that $\mathbb{S}_1\partial_\varkappa + \partial_\varkappa\mathbb{S}_2 = id$.

$$\mathbb{S}_2\partial_\varkappa\left(Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)$$
$$=\mathbb{S}_2\left(\binom{ƙ_1+1+ƙ_2}{ƙ_2}Z_{21}^{(ƙ_1+1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)-\right.$$
$$\left.\binom{ƙ_2+ƙ_3}{ƙ_3}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)$$
$$=\binom{ƙ_1+1+ƙ_2}{ƙ_2}Z_{21}^{(ƙ_1+1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_3+ɱ}{ɱ}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right),$$

And

$$\partial_\varkappa\mathbb{S}_3\left(Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)=\partial_\varkappa\left(Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)$$
$$=-\binom{ƙ_1+1+ƙ_2}{ƙ_2}Z_{21}^{(ƙ_1+1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_3+ɱ}{ɱ}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa\partial_{21}^{(ɱ)}\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)$$
$$=-\binom{ƙ_1+1+ƙ_2}{ƙ_2}Z_{21}^{(ƙ_1+1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_3+ɱ}{ɱ}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right),$$

where $|ƙ| = ƙ_1+ƙ_2+ƙ_3$.

It is obvious that $\mathbb{S}_2\partial_\varkappa + \partial_\varkappa\mathbb{S}_3 = id$.

$$\mathbb{S}_3\partial_\varkappa\left(Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)$$
$$=\mathbb{S}_3\left(-\binom{ƙ_1+1+ƙ_2}{ƙ_2}Z_{21}^{(ƙ_1+1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+\right.$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa\Big)\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_3+ƙ_4}{ƙ_4}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ƙ_4)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa\partial_{21}^{(ƙ_4)}\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)$$
$$=-\binom{ƙ_1+1+ƙ_2}{ƙ_2}Z_{21}^{(ƙ_1+1+ƙ_2)}\varkappa Z_{21}^{(ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2+ƙ_3)}\varkappa Z_{21}^{(ƙ_4)}\varkappa\ \ Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_3+ƙ_4}{ƙ_4}Z_{21}^{(ƙ_1+1)}\varkappa Z_{21}^{(ƙ_2)}\varkappa Z_{21}^{(ƙ_3+ƙ_4)}\varkappa Z_{21}^{(ɱ)}\varkappa\left(\begin{matrix}\mathbb{W}\\ \mathbb{W}'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+$$

$$\binom{ƙ_4+ɱ}{ɱ} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4+ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)$$

And

$$∂_ϰ\mathbb{S}_4\left(Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)$$

$$= ∂_ϰ\left(Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰZ_{21}^{(ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)$$

$$= \binom{ƙ_1+1+ƙ_2}{ƙ_2} Z_{21}^{(ƙ_1+1+ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰZ_{21}^{(ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right) -$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2+ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰZ_{21}^{(ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_3+ƙ_4}{ƙ_4} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3+ƙ_4)}ϰZ_{21}^{(ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_4+ɱ}{ɱ} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4+ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰ∂_{21}^{(ɱ)}\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)$$

$$= \binom{ƙ_1+1+ƙ_2}{ƙ_2} Z_{21}^{(ƙ_1+1+ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰZ_{21}^{(ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right) -$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2+ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰZ_{21}^{(ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_3+ƙ_4}{ƙ_4} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3+ƙ_4)}ϰZ_{21}^{(ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_4+ɱ}{ɱ} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4+ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰ∂_{21}^{(ɱ)}\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)$$

$$= \binom{ƙ_1+1+ƙ_2}{ƙ_2} Z_{21}^{(ƙ_1+1+ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰZ_{21}^{(ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right) -$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2+ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰZ_{21}^{(ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$\binom{ƙ_3+ƙ_4}{ƙ_4} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3+ƙ_4)}ϰZ_{21}^{(ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)-$$
$$\binom{ƙ_4+ɱ}{ɱ} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4+ɱ)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)+$$
$$Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right),$$

where $|ƙ| = ƙ_1+ƙ_2 + ƙ_3+ƙ_4$.

It is obvious that $\mathbb{S}_3∂_x + ∂_x\mathbb{S}_4 = id$.

$$\mathbb{S}_4∂_ϰ\left(Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰZ_{21}^{(ƙ_5)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)$$

$$=\mathbb{S}_4\left(\binom{ƙ_1+1+ƙ_2}{ƙ_2} Z_{21}^{(ƙ_1+1+ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰZ_{21}^{(ƙ_5)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right) -\right.$$
$$\binom{ƙ_2+ƙ_3}{ƙ_3} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2+ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰZ_{21}^{(ƙ_5)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right) +$$
$$\binom{ƙ_3+ƙ_4}{ƙ_4} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3+ƙ_4)}Z_{21}^{(ƙ_5)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right) -$$
$$\binom{ƙ_4+ƙ_5}{ƙ_5} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3)}Z_{21}^{(ƙ_4+ƙ_5)}ϰ\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right) +$$
$$\left.Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2)}ϰZ_{21}^{(ƙ_3)}Z_{21}^{(ƙ_4)}ϰ∂_{21}^{(ƙ_5)}\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|)}2^{(ɱ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right)\right)$$

$$=\binom{ƙ_1+1+ƙ_2}{ƙ_2} Z_{21}^{(ƙ_1+1+ƙ_2)}ϰZ_{21}^{(ƙ_3)}ϰZ_{21}^{(ƙ_4)}ϰZ_{21}^{(ƙ_5)}ϰZ_{21}^{(ɱ)}x\left(\begin{matrix}𝕨\\ 𝕨'\end{matrix}\middle|\begin{matrix}1^{(7+|ƙ|+ƙ)}\\ 2^{(6-|ƙ|-ɱ)}\end{matrix}\right) - \binom{ƙ_2+ƙ_3}{ƙ_3} Z_{21}^{(ƙ_1+1)}ϰZ_{21}^{(ƙ_2+ƙ_3)}ϰ$$

$$Z_{21}^{(k_4)}\varkappa Z_{21}^{(k_5)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)+\binom{k_3+k_4}{k_4}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+k_4)}\varkappa Z_{21}^{(k_5)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)$$

$$-\binom{k_4+k_5}{k_5}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4+k_5)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)+\binom{k_4+m}{m}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa$$

$$Z_{21}^{(k_4)}\varkappa Z_{21}^{(k_5+m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right),$$

And

$$\partial_{\varkappa}\mathbb{S}_5\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(k_5)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)\right)$$

$$=\partial_{\varkappa}\left(Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(k_5)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)\right)$$

$$=-\binom{k_1+1+k_2}{k_2}Z_{21}^{(k_1+1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(k_5)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)+$$

$$\binom{k_2+k_3}{k_3}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(k_5)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)-$$

$$\binom{k_3+k_4}{k_4}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+k_4)}\varkappa Z_{21}^{(k_5)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)+\binom{k_4+k_5}{k_5}$$

$$Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4+k_5)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)-\binom{k_5+m}{m}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa$$

$$\partial_{21}^{(k_5+m)}\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)+Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(k_5)}\varkappa\partial_{21}^{(m)}\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)$$

$$=-\binom{k_1+1+k_2}{k_2}Z_{21}^{(k_1+1+k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(k_5)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)+\binom{k_2+k_3}{k_3}Z_{21}^{(k_1+1)}\varkappa$$

$$Z_{21}^{(k_2+k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(k_5)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)-$$

$$\binom{k_3+k_4}{k_4}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3+k_4)}\varkappa Z_{21}^{(k_5)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)+\binom{k_4+k_5}{k_5}$$

$$Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4+k_5)}\varkappa Z_{21}^{(m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|+m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)-\binom{k_5+m}{m}Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa$$

$$Z_{21}^{(k_5+m)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\ 2^{(6-|k|-m)}\end{matrix}\right)+Z_{21}^{(k_1+1)}\varkappa Z_{21}^{(k_2)}\varkappa Z_{21}^{(k_3)}\varkappa Z_{21}^{(k_4)}\varkappa Z_{21}^{(k_5)}\varkappa\left(\begin{matrix}\mathbb{w}\\ \mathbb{w}'\end{matrix}\middle|\begin{matrix}1^{(7+|k|)}2^{(m)}\\ 2^{(6-|k|-m)}\end{matrix}\right),$$

where $|k| = k_1+k_2+k_3+k_4+k_5$.

It is obvious that $\mathbb{S}_4\partial_{\varkappa}+\partial_{\varkappa}\mathbb{S}_5 = id$.

From the above we conclude that $\{\mathbb{S}_0,\mathbb{S}_1,\mathbb{S}_2,\mathbb{S}_3,\mathbb{S}_4,\mathbb{S}_5\}$ is a contracting homology [5], which means that our complex is exact.

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