DIGITAL IMAGES EDGE DETECTION USING MATHEMATICAL MORPHOLOGY OPERATIONS

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Abstract

The field of mathematical morphology contributes a wide range of operators to image processing, all based around a few simple mathematical concepts from set theory. The two most basic operations in mathematical morphology are erosion and dilation, both of these operators take two pieces of data as input; an image to be eroded or dilated, and a structuring element. Opening and closing are two important operators from mathematical morphology; they are both derived from the fundamental operations of erosion and dilation. The primary application of mathematical morphology in this work occurs in binary images throw applying erosion and dilation operations to extract boundary edges, and then basic morphology operations are adopted to grey level images which are erosion residue and morphological gradient in order to achieve image edges detection.

كشف حافات الصور الرقمية باستخدام عمليات التشكّل الرياضية

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الخلاصة

ان حقل علم التشكّل الرياضي يسهم في جهد مشترك لمجال واسع من العوامل لمعالجة الصور ،جميعها تستند حول بعض المبادئ الرياضية البسيطة من نظرية المجموعة. العمليتين الأساسيتين في علم التشكل الرياضي هما التآكل و التوسيع ، يأخذ كلا العاملين جزئين من البيانات كمدخلات : الصورة لأجل تآكلها أو اتساعها ، وكذلك العنصر البنيويّ. الفتح و الأغلاق عاملين مهمين من علم التشكل الرياضي، و كلاهما يشتق من العمليات الأساسية للتآكل و الأتساع في عملنا التطبيق الأولي لعلم التشكل الرياضي يظهر في الصور الثنائية عبر تطبيق عمليات التآكل والتوسيع لاستخراج حافات الحدود ، ومن ثم عمليات التشكل الاساسية تم تبنيها لصور المستوى الرمادي وهي بقية التآكل والانحدار التشكلي من اجل انجاز كشف حواف الصورة.

Introduction

Mathematical morphology (MM) is a nonlinear branch of the signal processing field and concerns the application of set theory concepts to image analysis. Morphology refers to the study of shapes and structures from a general scientific perspective. Morphological filters or operators are nonlinear transformations, which modify geometric features of images. These operators transform the original image into another image through the iteration with other image of a certain shape and size which is known as structuring element [1]. According to Soille (1999), Mathematical Morphological (MM) can be defined such as a theory for analysis of the spatial structures. It is called Morphological because it examines the form of objects. It is mathematical in the sense that the analysis is based on an adjusted theory, geometry and algebra. However, the MM is not just a theory, but is also a powerful technique for the analysis of images [2].

In general, the structuring element is a set that describes a simple shape that probes an input image. The basic morphological filters are morphological opening and morphological closing with structuring elements [1].

The Mathematical Morphology is considered a non-linear approach in digital image processing of images that have made excellent results in the detection of cartographic features of interest in digital images. The Mathematical Morphology has two basic operators, the erosion and dilation, from which all others morphological operations are derived, the operators' erosion and dilation provide basic conditions for the construction of other morphological operators such as targeting; detection of features, pattern recognition etc [2].

The classical dilation rule, analogous to that for erosion, is to add any background pixel that touches another pixel that is already part of a foreground region. This will add a layer of pixels around the periphery of all features and regions, which will cause some increase in dimensions and may cause features to merge. It also fills in small holes within features. Because erosion and dilation cause a reduction or increase in the size of regions, respectively, they are sometimes known as etching and plating or shrinking and growing. There are a variety of rules for deciding which pixels to add or remove and for forming combinations of erosion and dilation [3]. Erosion and dilation are used to form opening and closing [4]. In most opening or closing operations, these are kept the same for both the erosion and the dilation.

The most widely used processing procedures for binary images are often collectively described as morphological operations by Serra, Soille, Coster and Chermant,and others. These include erosion and dilation as well as modifications and combinations of these operations [3].The binary morphology can be easily extended to gray-scale morphology. The only differences result from the definitions of dilation and erosion because other operations basically depend on them [1].

The morphological gradient operation is used to produce the edges of the object in an image. Morphological edge detection algorithm selects appropriate structuring element of the processed image and makes use of the basic theory of morphology including erosion, dilation, opening and closing operation and the synthesization operations of them to get clear image edge [5].

Fundamental Definitions

The fundamental operations associated with an object are the standard set operations union, intersection, and complement $\{\cup, \cap, {}^c\}$ plus translation[6]:

- Translation - Given a vector x and a set A, the translation, A + x, is defined as:

$$A+x=\{a+x \mid a\in A\} \qquad (1)$$

Note that, since we are dealing with a digital image composed of pixels at integer coordinate positions (Z^2), this implies restrictions on the allowable translation vectors x. The basic Minkowski set operations-addition and subtraction can now be defined. First we note that the individual elements that comprise B are not only pixels but also vectors as they have a clear coordinate position with respect to [0, 0]. Given two sets A and B [7]:

Minkowski addition :

$$A\oplus B=\bigcup_{\beta\in B}(A+B) \qquad (2)$$

Minkowski subtraction :

$$A\Theta B=\bigcap_{\beta\in B}(A+B) \qquad (3)$$

Where B are structure elements-B={-β| β∈ B}.

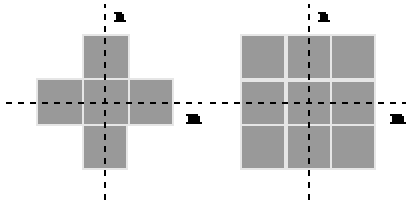
Mathematical Morphology Operations

Mathematical morphology deals with extracting image components that are useful in the representation and description of region shape, such as boundaries, skeletons, and the convex hull [4]. The language of mathematical morphology is set theory. As such, morphology offers a unified and powerful approach to numerous image processing problems. Sets in mathematical morphology represent objects in an image. For example, the set of all black pixels in a binary image is a complete morphological description of the image. In binary images, the sets in question are members of the 2-D integer space Z^2 where each element of a set is a tuple (2-D vector) whose coordinates are the (x, y) coordinates of a black (or white, depending on convention) pixel in the image. Gray-scale digital images can be represented as sets whose components are in Z^3. In this case, two components of each element of the set refer to the coordinates of a pixel, and the third corresponds to its discrete gray-level value. Sets in higher dimensional spaces can contain other image attributes, such as color and time varying components [7]. However, our interest initially is on binary images whose components are elements of Z^2 and later discuss extensions to gray-scale images.

The Mathematical Morphology has two basic operators, the erosion and dilation, from which all others morphological operations are derived [2] . These include erosion and dilation as well as modifications and combinations of these operations. The classic versions of these are fundamentally neighbor operations, where similar procedures that rank pixel values in a neighborhood are used to process gray-scale and color images in the spatial domain. Because the values of pixels in the binary images are restricted to black and white, the operations are simpler and usually involve counting rather than sorting. However, the basic ideas are the same. Mathematical Morphology has developed a specific language and notation for the operations and is generally discussed in terms of set theory. Operations can be described simply in terms of adding or removing pixels from the binary image according to certain rules, which depend on the pattern of neighboring pixels. Each operation is performed on each pixel in the original image, using the original pattern of pixels. In practice, it may not be necessary to create an entirely new image; the existing image can be replaced in memory by copying a few lines at a time. None of the new pixel values are used in evaluating the neighbor pattern [3].

1. Erosion and Dilation

Dilation and erosion. operations are fundamental to morphological processing. In fact, many of the morphological algorithms discussed, based on these two primitive operation [7]. Dilation, in general, causes objects to dilate or grow in size; erosion causes objects to shrink. The amount and the way that they grow or shrink depend upon the choice of the structuring element. Dilating or eroding without specifying the structural element makes no more sense than trying to lowpass filter an image without specifying the filter. The two most common structuring elements (given a Cartesian grid) are the 4-connected and 8-connected sets, N4 and N8. They are illustrated in Figure 1. [6]



(a) N_4 **(b)** N_8

Figure 1: The standard structuring elements N_4 and N_8..

With A and B as sets in Z^2, the dilation of A by B, denoted $A \oplus B$, is defined as [7,8] :

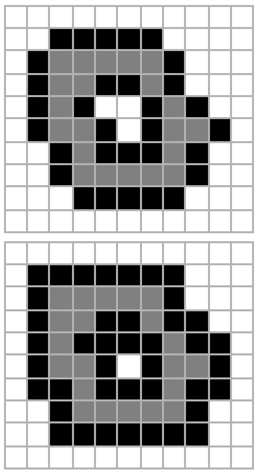
$$A \oplus B = \{\omega \in Z^2 \mid \omega - a + b, \text{ for some } a \in A \text{ and } b \in B\}. \quad (4)$$

Set B is commonly referred to as the structuring element in dilation, as well as in other morphological operations.

And for sets A and B in Z^2 the erosion of A by B, denoted A Θ B, is defined as :

$$A \Theta B = \{\omega \in Z^2 \mid \omega + b \in A, \text{ for every } b \in B\}.. \quad (5)$$

The simplest kind of erosion is to remove any pixel touching another pixel that is part of the background . This removes a layer of pixels from around the periphery of all features and regions, which will cause some shrinking of dimensions and may create other problems if it causes a feature to break up into parts. Erosion can entirely remove extraneous pixels representing point noise or line defects (e.g., scratches) because these defects are frequently only 1 or 2 pixels wide. Instead of removing pixels from features, a complementary operation known as dilation (or sometimes dilatation) can be used to add pixels. The classical dilation rule, analogous to that for erosion, is to add any background pixel that touches another pixel that is already part of a foreground region. This will add a layer of pixels around the periphery of all features and regions, which will cause some increase in dimensions and may cause features to merge. It also fills in small holes within features. Because erosion and dilation cause a reduction or increase in the size of regions, respectively, they are sometimes known as etching and plating or shrinking and growing. There are a variety of rules for deciding which pixels to add or remove and for forming combinations of erosion and dilation [3] . The basic effect of erosion on a binary image is to erode away the boundaries of regions of foreground pixels (resulting in the image being shrunk). The basic effect of dilation on a binary image is to gradually enlarge the boundaries of regions of foreground pixels (resulting in the image being expanded) [4]. Figure 2 shows that they are equivalent to the formal definitions for dilation and erosion. The procedure is illustrated for *dilation* where $\boldsymbol{B} = \boldsymbol{N}_{C=4}$ or $\boldsymbol{N}_{C=8}$ [7].



(a) $\boldsymbol{B} = \boldsymbol{N}_4$ **(b)** $\boldsymbol{B} = \boldsymbol{N}_8$

Figure 2: Illustration of *dilation*. Original object pixels are in gray; pixels added through *dilation* are in black.

A gray-scale image can be considered as a three-dimensional set where the first two elements are the *x* and *y* coordinates of a pixel and the third element is gray-scale value. It can be also applied to the gray-scale structuring element. With this concept, gray-scale dilation of f by b, denoted $f \oplus$ b, can be defined as follows [1] :

$$(f \oplus b)(s,t) = \max\{ f(s-x,t-y) + b(x,y) \mid (s-x),(t-y) \in D_f \ ; \ (x,y) \in D_b \} \qquad (6)$$

Where D_f and D_b are the domains of f and b, respectively.

Gray-scale erosion, denoted $f \Theta$b , is defined as:

$$(f \,\Theta\, b)(s,t) = \min\{ f(s+x,t+y) - b(x,y) \mid (s+x),(t+y) \in D_f \ ; \ (x,y) \in D_b \} \qquad (7)$$

Where D_f and D_b are the domains of each image or function, that is the minimum operator (eq. 7) will interrogate a neighborhood with a certain domain and select the smallest pixel value to become the output value. This has the effect of causing the bright areas of an image to shrink or erode. Similarly, gray-scale dilation is performed in (eq. 6) to select the greatest value in a neighborhood [7].

2. Opening and Closing

The combination of an erosion followed by a dilation is called an opening, referring to the ability of this combination to open up gaps between just-touching features, as shown in Figure 3. It is one of the most commonly used sequences for removing pixel noise from binary images. Performing the same operations in the opposite order (dilation followed by erosion) produces a different result. This sequence is called a closing because it can close breaks in features. There are several parameters that can be used to adjust erosion and dilation operations, particularly the neighbor pattern or rules for adding or removing pixels and the number of iterations, as discussed below. In most opening or closing operations, these are kept the same for both the erosion and the dilation [3].

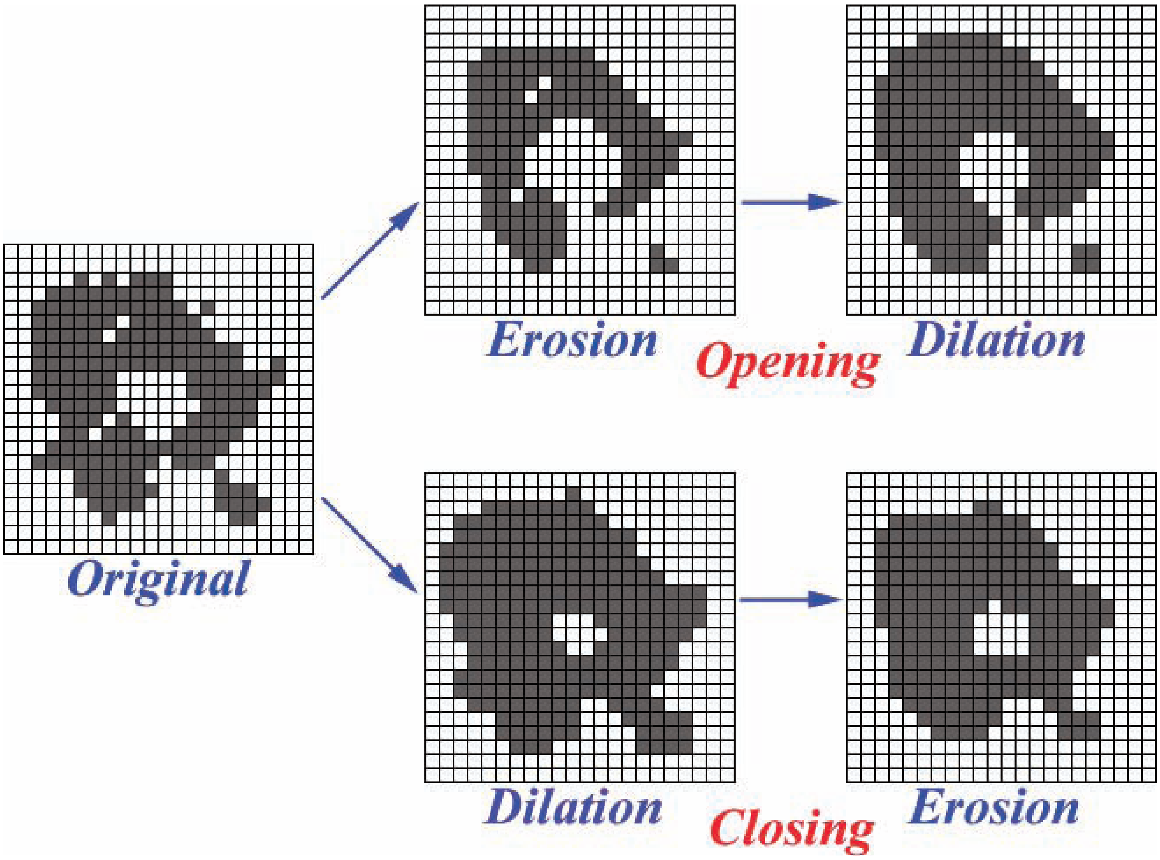


Figure 3: Combining erosion and dilation to produce an opening or a closing.

As we have seen, dilation expands an image and erosion shrinks it. Opening generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions. Closing also tends to smooth sections of contours but, as opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour .

The opening of set A by structuring element B, denoted A ° B, is defined as [7]:

$$A \circ B = (A \Theta B) \oplus B \qquad (8)$$

Thus, the opening A by B is the erosion of A by B, followed by a dilation of the result by B .

Similarly, the closing of set A by structuring element B, denoted A • B, is defined as [7] :

$$A \bullet B = (A \oplus B) \Theta B \qquad (9)$$

which, in words, says that the closing of A by B is simply the dilation of A by B , followed by the erosion of the result by B.

The expressions for opening and closing of gray-scale images have the same form as their binary counterparts.

Some Applications of Mathematical Morphology

Some practical uses of morphology can be considered .When dealing with binary images, the principal application of morphology is extracting image components that are useful in the representation and description of shape. In particular, we consider morphological algorithms for extracting boundaries. In addition gray-scale morphology are used to compute the morphological gradient of an image:

1. Boundary Extraction

The boundary of a set A, denoted by β (A), can be obtained by first eroding A by β and then performing the set difference between A and its erosion. That is [7]:

$$\beta(A) = A - (A \Theta B) \qquad (10)$$

where B is a suitable structuring clement.

Figure 4 illustrates the mechanics of boundary extraction. It shows a simple binary object, a structuring element B, and the result of using Eq. (10). Although the structuring element shown in Figure. 4 is among the most frequently used, it is by no means uniq.

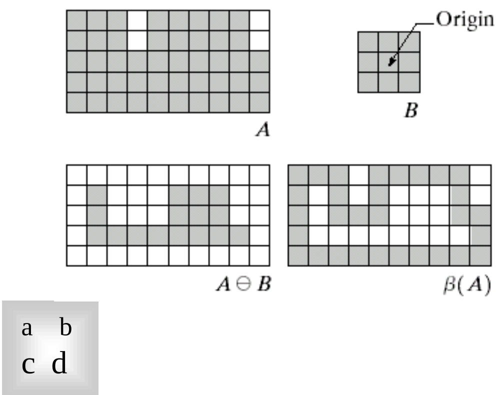


Figure 4: (a) Set A .(b) Structuring element B . (c) A eroded by B . (d) Boundary given by the set difference between A and its erosion.

2. Morphological Gradient

Morphological edge detection algorithm selects appropriate structuring element of the processed image and makes use of the basic theory of morphology including erosion, dilation, opening and closing operation and the synthesization operations of them to get clear image edge. The following algorithms are used for image edge detection. The edge of image F, which is denoted by Ed (F), is defined as the difference set of the dilation domain of F and the domain of F . This is also known as dilation residue edge detector [5] :

$$E_d(F) = (F \oplus B) - F \qquad (11)$$

Accordingly, the edge of image F , which is denoted by Ee (F) , can also be defined as the difference set of the domain of F and the erosion domain of F . This is also known as erosion residue edge detector :

$$E_e(F) = F - (F \Theta B) \qquad (12)$$

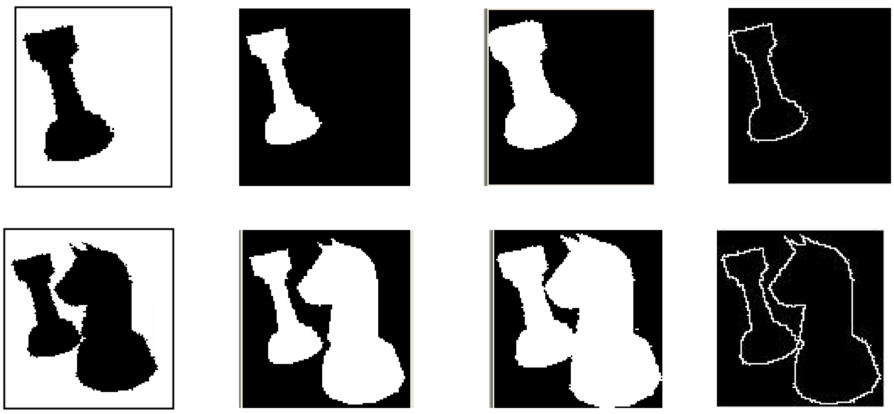
The dilation and erosion often are used to compute the morphological gradient of image F, denoted by G(F) [7] :

$$G(F) = (F \oplus B) - (F \Theta B) \qquad (13)$$

The morphological gradient highlights sharp gray-level transition in the input image. As opposed to gradients obtained using the First Derivatives for Enhancement, morphological gradients obtained using symmetrical structuring elements tend to depend less on edge directionality .

Results

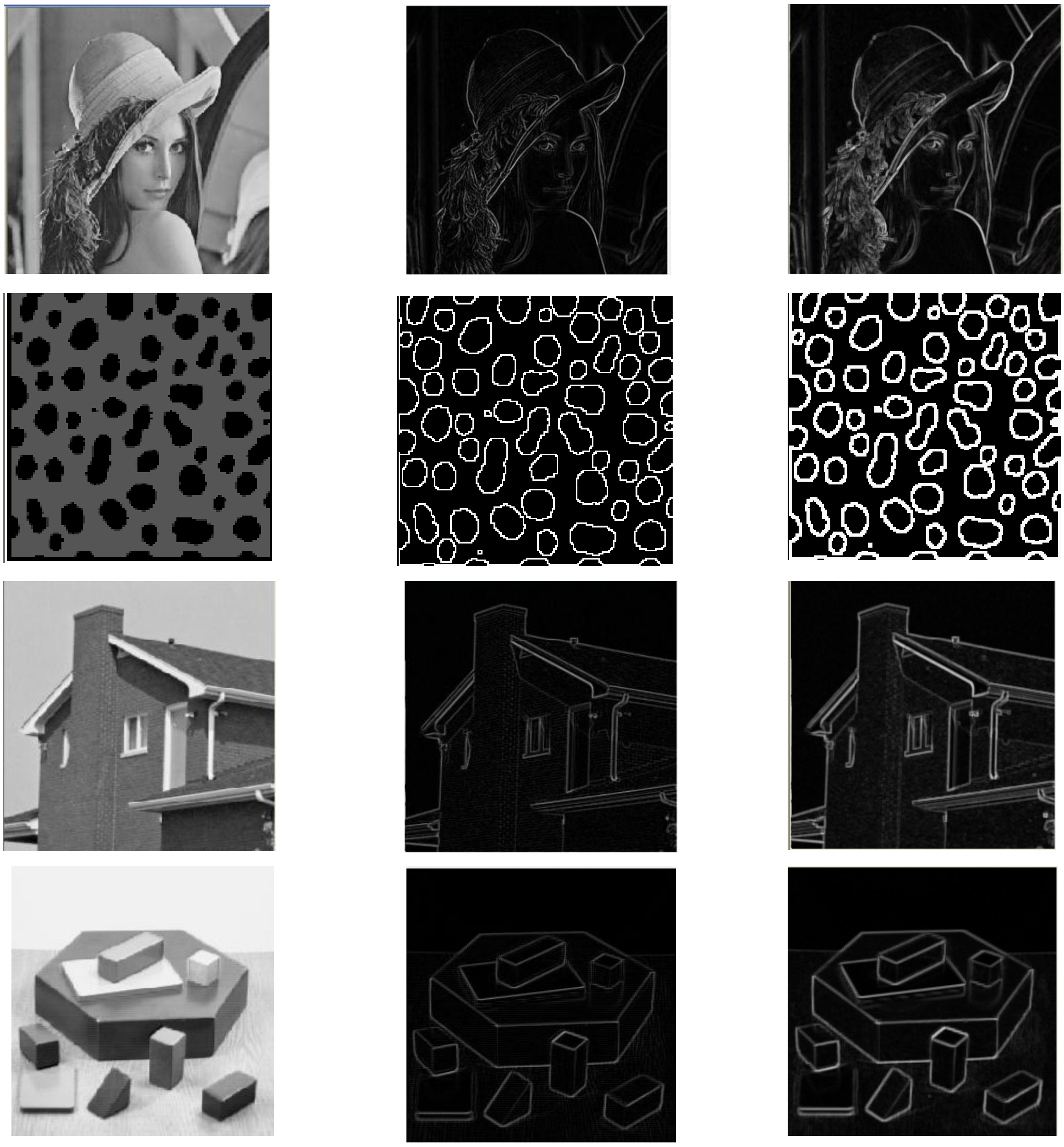
In this work, the Mathematical Morphology technique was applied on binary and gray level images by utilizing 8-directional structuring elements. Figure.(5-a) shows the original binary images while Figure.(5-b&c) shows the eroded and dilated images, respectively after applying erosion and dilation operations which demonstrated in section (3-1). Figure.(5-d) shows outlining operation operates on binary images by subtracting the eroded version of the image from the original one which presented in eq. (10).



(a) **(b)** **(c)** **(d)**

Figure 5: (a)original images. (b)eroded images. (c)dilated images. (d)boundary extraction

Figure.(6-a) show the original gray images while Figure.(6-b) shows erosion residue edge detector which denoted by eq.(12), and Figure.(6-c)



(a) **(b)** **(c)**

Figure 6 :(a) original images. (b) erosion residue. (c) Morphological gradient.

Discussion and Conclusions

The results indicated that erosion operation uniformly reduces the size of white objects on a black background in an image. The reduction is by one pixel around the object's perimeter, so erosion can be used to remove small anomalies such as single-pixel objects, called speckles, and shows gray images processed by Morphological gradient operation denoted by eq.(13).

single pixel wide spurs from the image, while multiple erosion operation on an image shrinks touching objects until they finally separate. This can be useful in image segmentation operations. The dilation operation is inverse of erosion, so binary dilation operations are used to remove small anomalies such as single pixels holes in objects and single pixel wide gaps from an image, while multiple dilation process expands broken objects until they finally merge into one.

Both the eroded and dilated version is created, and then the eroded version of image is subtracted from the dilated version. The resulting image shows the edges of the objects in the original image obviously. According to the results, Morphological gradient operation is detect edges successfully than erosion residue edge detector, therefore the edges are clearer than the edges detected by erosion residue edge detector.

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