DOI: 10.24996/ijs.2026.67.1.43

Iraqi Journal of Science

ISSN: 0067-2904

Calculating the Effect of Third-Body Gravity on Orbits around the Moon

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Received: 29/6/2024 Accepted:13/1/2024 Published: 30/1/2026

Abstract:
This work includes calculating the orbital elements around the Moon with different heights, ranging from 100 to 1000 km, at an eccentricity of (e = 0.05 and 0.1) with an inclination=28.48 deg. Cowell's equation was used to calculate the effect of the Earth's attraction on a satellite orbiting around the Moon using the MATLAB program. In this paper, research was conducted to find the best orbit of a satellite orbiting the Moon, which has the least possible perturbation and the least possible change of orbital elements, in order to get the longer life of the sent satellites. The lowest height was found to be the best way to obtain more efficient images with the lower-cost camera. Newton-Raphson method was used to solve Kepler's equation for the ellipse orbit. Cowell's method was used to solve the perturbation with the equation of motion which was solved by the 4^{TH} order Runge–Kutta integration. Results showed that each orbital element changed across time due to the Earth's gravity only. The altitude change of an orbit had a relatively slight effect, while the eccentricity of the orbit had a greater effect on the orbital elements. When altitudes were changed between 100 to 1000 km, the results showed that the behavior of the orbital elements remained almost the same. Whereas when the value of eccentricity was changed from 0.1 to 0.05, it had a greater impact on the orbital elements. It was also found that inclination had an important effect on the orbital elements. It was found that the best orbit around the Moon had a height of 100km and an eccentricity of 0.05. It was also found that inclination had an important effect on the orbital elements.

Keywords: Moon orbit, satellite orbit, 3rd body attraction, orbital elements, perturbations.

حساب تأثير الجاذبية الجسم الثالث على المدارات حول القمر

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الخلاصه:
هذا العمل تضمن حساب العناصر المداريه حول القمربارتفاعات مختلفة تتراوح (من 100 الى 1000) كم عندما تكون قيمة
الانحراف المركزي (0.1 و0.05) وقيمة الميل = 28.48 . تم استخدام طريقة كول لحساب تأثيرالجاذبية الارضية على مدار قمر صناعي يدور حول القمر بتاريخ معين باستخدام برنامج الماتلاب . في هذا العمل، تم البحث عن افضل مدار لقمر اصطناعي يدور حول القمر باقل تأثير للاضطرابات و اقل تغيير ممكن

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للعناصرالمدارية و الحصول على عمر اطول للقمر الصناعي المرسل .وجدت الدراسه ان استخدام اقل ارتفاع هو أفضل للحصول على صور اكثر كفاءه بكامرات اقل كلفة . تم استخدام طريقة نيوتن – رفسن لحل معادلة كبلر للقطع الناقص . و تم استخدام طريقة كول لأضافة الاضطرابات على معادلة الحركة وحلها باستخدام طريقة رانج– كوتا من الدرجة الرابعة لحل معادلة الحركة المضطربة. بينت النتائج ان كل عنصر من العناصر المدارية يتغير مع الزمن بوجود تأثير جذب الارض , لكن تأثير الارتفاع المستخدم يكون ضئيلا نسبيا فيما كان للانحراف المركزي تاثير اكبر . و عندما تم تغيير الارتفاعات (من 100 الى 1000كم)، بينت النتائج ان تغير العناصر المدارية مع الزمن يبقى نفسه تقريبا . لكن عندما تم تغيير قيمة الانحراف المركزي (من 0.1 الى 0.05) ،فأنه اثر بصورة اكبر على العناصر المداريه. كما وجد ايضا ان ميل المدار له تأثير مهم على العناصر المدارية . وجد أن أفضل مدار حول القمر يكون ارتفاعه 100كم وانحرافه المركزي 0.05 . كما وجد ايضا ان ميل المدار له تأثير مهم على العناصر المدارية .

1-Introduction:

The planets revolve around the Sun according to Kepler's laws, as well as the orbit of the Moons around the planets [1]. In the same way, artificial satellites orbit around the planets and moons [2]. Studying satellite orbits has significant importance because it has become one of the most important means used by the world in managing many fields [3]

We must invest in what satellites provide by transmitting information about the Earth, and its atmosphere and exploit outer space for peaceful purposes where satellites have made a great scientific breakthrough in many fields [4]. Two-body problem is the movements of two huge bodies around the center of a mass of each other [5]. In (2003) de Almeida Prado and A. F. B. were among the researchers who studied the evolution of orbits for natural satellites in the solar system [6]. Al-Ali (2011) studied Kepler's equation to solve the problem of the two bodies with the presence and absence of perturbation of the ellipse orbit around the Moon [2]. Lara, Martin, et al (2012) investigated the long-term consequences of a far-off third-body on a satellite orbiting an oblate body for a high-order expansion of the third-body disturbing function [7]. R. H. Ibrahim and A. H. Saleh (2024) found a solution to Kepler's equation by using a function called the first kind Bessel, for an elliptical orbiting satellite [8]. The satellite's orbits were classified into several types according to inclination, altitude, eccentricity, and missions for which the satellite was sent. The closer satellite was better in terms of imaging and exploration [9], but it was exposed to higher perturbation due to the gravity of the other bodies and the non-sphericity of the Moon [8]. In this research, the effect of the third body was taken, which is the Earth, on a satellite orbiting around the Moon. For the purpose of describing the orbit, six parameters must be known. These are [10]: a semi-major axis (a), eccentricity (e), inclination (i), angle of ascending node (Ω), argument of perigee (ω), and the true anomaly (ta). One of Kepler's main problems or mistakes is the assumption that the orbital motion of the body is the result of gravity between two objects only [11]. This is an ideal situation that does not occur because there are other forces caused by perturbations, which affect the movement of the orbital body that must be taken into account. There are two types of methods to determine the satellite motion. These methods are general perturbation and special perturbation. The types of general perturbation are [12]: Atmospheric drag (in this work, the effect of atmospheric perturbation will be neglected because the Moon doesn't have an atmosphere which can affect the orbits) [13], non-spherical of the Moon (the satellite's orbit around the Moon is exposed to major perturbations due to the Moon's non-spherical as it is flatter than the Earth) [14] and third body attractions (any two bodies in the universe attract each other by the forces of gravity) [15], but in reality there are other bodies that affect the object, they are called " third term "[6]. The dominant third body that affects the satellite orbit around the Moon is the Earth, therefore it's used in this research. The Earth's attraction on the orbits around the Moon depends on the distance between the satellite and the Earth more precisely. This effect depends on the Earth's coordination in relevance to the satellite at any moment. Therefore, the position of the Moon and the position of the Earth must be calculated at any time. There are also other perturbations that affect the body such as solar radiation, pressure perturbation (SRP), the Moon's magnetic field, and the solar wind perturbation [16].

2. Theoretical background:

Six parameters are typically required to define the position of any physical object in three dimensions. Three-dimensional elements x, y, and z are present on each axis. and three-angle elements. This, however, is not very helpful in terms of seeing the spacecraft in its orbit. It is difficult to determine the size, shape, and position of the satellite within the orbit. In this regard, Kepler has made a huge contribution by defining a set of six classical orbital elements, often known as Keplerian elements. The semi-major axis measures 50% of the major axis length of the orbit. The relationship which will make it possible for us to calculate the semi-major axis is the following equation [17]:

$$a = \frac{(ra+rp)}{2} \quad or \quad a = -\mu/2\varepsilon \tag{1}$$

Where r_a: is the radius to apogee (km), r_p: is the radius to perigee (km)[20,15], a: is the semi-major axis (km), ε:is specific mechanical energy (km^2/s^2), and μ is the central body's gravitational parameter (km^3/s^2).

The value of the semi-major axis varies depending on the shape of the orbit or the eccentricity of the orbit [18].

The second orbital element is eccentricity (e). It is the relationship between the two foci's distance and the major axis' length [19]. ;

$$e = \frac{ra-rp}{ra+rp} \tag{2}$$

r_a: is the radius to apogee (km) and it's equal a (1+e) and

r_p: is the radius to perigee (km) and it's equal a (1-e).

The shape of the orbit depends on the value of the eccentricity. The third orbital element is inclination (i), it is the angular distance, expressed in degrees, between the orbital plane and the plane of reference, which is often the ecliptic or the planet's equator. It is one of the six orbital parameters that characterize the form and direction of a celestial orbit. The fourth element is the longitude of the ascending node (Ω). It is the angle between the node direction and the vernal equinox direction on the reference plane. The other orbital element is the argument of the perihelion (ω)it is the angle between the node direction and the perigee direction on the orbit plane [20].

The last element of the orbit is the time of perigee or the mean anomaly (M)

$$M = n * (t - tp) \tag{3}$$

Where *M*: is the mean anomaly in radian, *n*: is the mean motion in rad/sec, *t*: is epoch time, and tp: is the time at perigee.

The eccentric anomaly calculated by Kepler's equation of an ellipse, solved by the Newton-Rafson method [21]:

$$E = M + e \sin E \tag{4}$$

E: Eccentric anomaly in radian

e: Eccentricity of orbit.

The orbit of a satellite is determined by the state vector of the satellite. This vector is described based on six parameters in the equatorial plane, velocity vectors (vx, vy, vz) and position vectors (x, y, z). The eccentric anomaly or true anomaly is needed to calculate the following [[22],[12]].

$$x` = a(\cos(E) - e) \tag{5}$$

$y` = a\sqrt{1-e^2}\sin(E)$ (6)

$z` = 0$ (7)

$v`x = -\sqrt[2]{(\mu_m/p)\sin(ta)}$ (8)

$v`y = \sqrt[2]{(\mu_m/p)(e+\cos(ta))}$

(9)

Where:

$$\tan\left(\frac{ta}{2}\right) = \sqrt{(1+e)/(1-e)} * \tan\left(\frac{E}{2}\right)$$

$v`z = 0$ (10)

ta is a true anomaly and the mass gravity constant of the Moon μ_m=4904.8695 km^3/s^2 at distance in km, mass in kg, and time in second [22]

$p = a(1-e^2)$ (11)

The period, which is determined by the third Kepler's law, is the amount of time, that a satellite takes to complete one orbit [21].

$T^2 = \frac{4\pi}{\mu}a^3$ (12)

Convert the coordinates of the satellite's position from the plane of its orbit to the plane of the equator by using the Gauss matrix [23].

$$R=\begin{bmatrix} Px & Qx & Wx \\ Py & Qy & Wy \\ Pz & Qz & Wz \end{bmatrix} \quad (13)$$

$Px = cos\omega\, cos\Omega - sin\omega\, sin\Omega\, cos\, i$

$Py = cos\omega\, sin\Omega + sin\omega\, cos\Omega\, cos\, i$

$Pz = sin\omega\, sin\, i$

$Qx = -sin\omega\, cos\Omega - cos\omega\, sin\Omega\, cos\, i$

$Qy = -\, sin\omega\, sin\Omega + cos\omega cos\Omega cos\, i$

$Qz = cos\, \omega\, sin\, i$

$Wx = sin\Omega\, sin\, i$

$Wy = -\, cos\Omega\, sin\, i$

$Wz = cos\, i$ (14)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R^{-1}\begin{bmatrix} x` \\ y` \\ z` \end{bmatrix} \quad ; \quad \begin{bmatrix} vx \\ vy \\ vz \end{bmatrix} = R^{-1}\begin{bmatrix} v`x \\ v`y \\ v`z \end{bmatrix} \quad (15)$$

The equation of motion without perturbations can be written as:

$\ddot{\bar{r}} = \mu(\frac{\bar{r}}{r^3})$ (16)

To add the perturbations' effects on the orbit, the equatorial coordinate of the Earth from the Moon center must be calculated at any moment across the time.

The Calculation of the Earth's position from the Moon as references [24,25]. The obliquity angle of the Earth's orbit with the equatorial plane was calculated using:

$\epsilon = 23.452294 - 0.0130125 * T - 0.00000164 * T^2 - 0.000000503 * T^3 + 0.00256 * cos(\Omega)$ (17)

The input date and time (year, month, day, hour, min, sec), and the date and time used are (2023,9,15,0,0,0) to determine the Julian date (J.D.) as in reference [24,25].

Next, compute the Julian century starting on January 1, 1900

T = (JD – 2415020) / 36525 (18-a)

The following formula can be applied after the year 2000

T2 = (JD - 2451545) / 36525 (18-b)

The ecliptic geocentric coordinate of the Moon or the Moon's ecliptic longitude and latitude can be calculated at that moment by utilizing these values:

$\lambda m = 218.32 + 481267.883T2 + 6.29 \sin(134.9 + 477198.85T2) - 1.27 \sin(259.2 - 413335.38) + 0.66 \sin(235.7 + 890534.23) + 0.21 \sin(269.9 + 954397.7) - 0.19 \sin(357.5 + 35999.05) - 0.11 \sin(186.6 + 966404.05\,T2)$ (19)

$\beta m = 5.13\, sin(93.3 + 483202.03T2) + 0.280606\, sin(228.2 + 960400.87\,T2) -0.28 * sin(318.3 + 6003.18\,T2) - 0.17 * sin(217.6 - 407332.2\,T2)$ (20)

The following formula can be used to get the Moon's distance from the Earth's center

$Rm = 385000 - 20905\, cos\, M - 3699\, cos\,(2D - M) - 2956\, cos\,(2D) - 570\, cos\,(2M) + 246\, cos\,(2M - 2D) - 171 * cos(M + 2D) - 152\, cos\,(M + M' - 2D)\,(km)$ (21)

M: mean anomaly of the Moon
M': mean anomaly of the Sun
D: The difference between the Sun's and the Moon's mean longitudes

$$M = 134°.96292 + 477198°.86753\,T$$
$$M' = 358°.42543 + 35999°.04944\,T$$
$$D = 297°.85027 + 445267°.11135\,T$$

The equation of perturbed motion of the satellite on the orbit around the Moon is

$\ddot{r} = \mu_m(\bar{r}lr^3) + a3\_body$ (22)

This equation was solved by integration using the 4th order Rang-Kotta method [22]. To get the state vector in the equatorial plane (x,y,z,v_x,v_y,v_z) and the momentum components (h_x,h_y,h_z), the following formula can be used to determine the perturbing acceleration brought on by a third body's gravitational attraction [2]:

Form can be expressed as acceleration due to forces of third-body as:

$a_d = -(G(m_e + m_m)/\, r^3_M) * r_M + Gm_3(\frac{r_{dm}}{r^3_{dm}} - \frac{r_d}{r^3_d})$ (23)

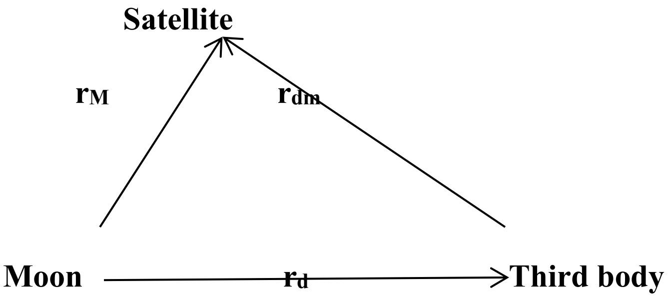

Fig ure1: Definitions of third body attraction vectors.

3. The program algorithm:

1- Input the information hp=(100-1000)km, e1=0.1, e2=0.05, the Moon mean radius (R m)=1737.4, pi=3.141592653589793 ,μ_{Moon}=4904.8695, μ_{earth}=398602, Ω=20 ; ω=80, *i*=28.48°; the values of **Ω** and ω assumed but the used value of *i* equal the inclination of the Moon at used date and time
2- Use the formula to determine the semi-major axis without perturbation a=rp/(1-e).
3- The program begins on the date and time (2023/9/15).
4- Calculate the position and velocity component of the satellite by solving Kepler's equation.
5- Calculate the position values of the Earth from the Moon. $x_E = -x_m$, $y_E = -y_m$, $z_E = z_m$
6- Calculate the satellite state vectors with perturbation by solving the equation of motion using the 4th order Range-Kotta integration.
7- Calculate angular momentum, semi-major axis, eccentricity longitude of ascending node, eccentric anomaly, true anomaly, and the velocity with perturbation.

8- Use the next time to recalculate all steps 4 to 8.
9- Use other height or other eccentricity and recalculate all the above steps.

4-The Results and discussion:

All cases were calculated and plotted, but not all figures were used in this study due to similar behaviors in some cases.

Case 1: The orbital elements variation with i=28.48°, e=0.05, and through 1000 period on the orbits around the Moon, due to the effect of the Earth's attraction.

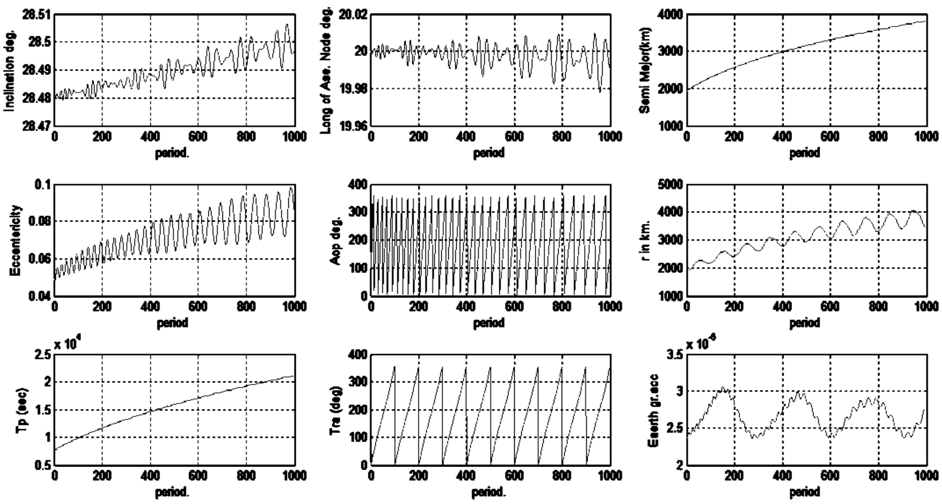

Figure 2 : The orbital elements variation at height=100km.

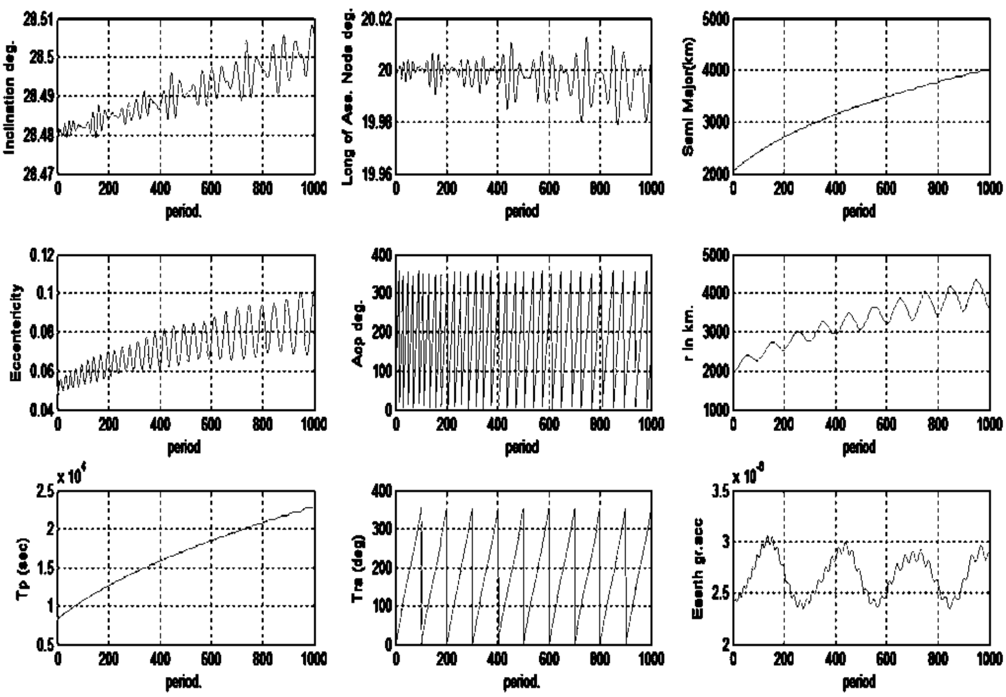

Figure 3 :The orbital elements variation at height=200km

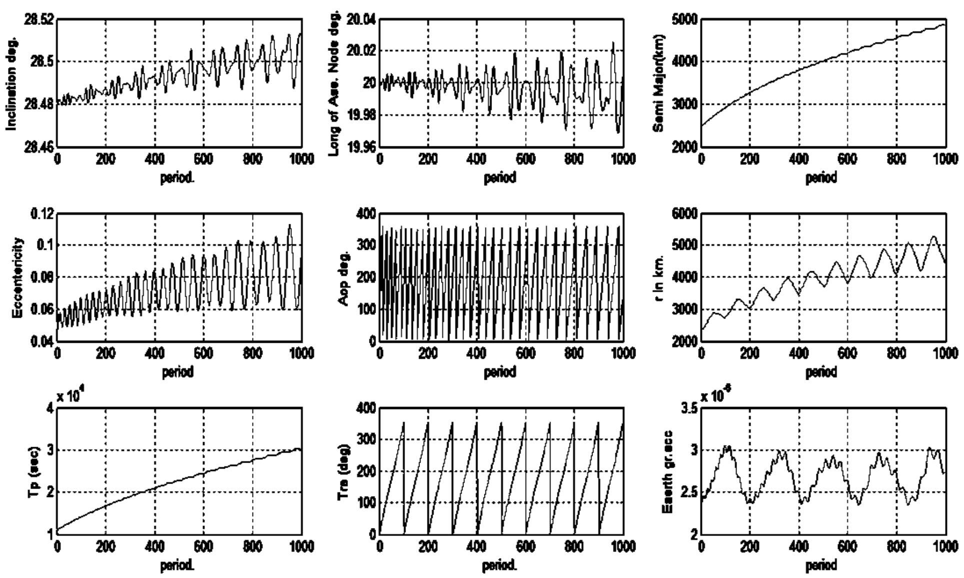

Figure 4 : The orbital elements variation at height=600km

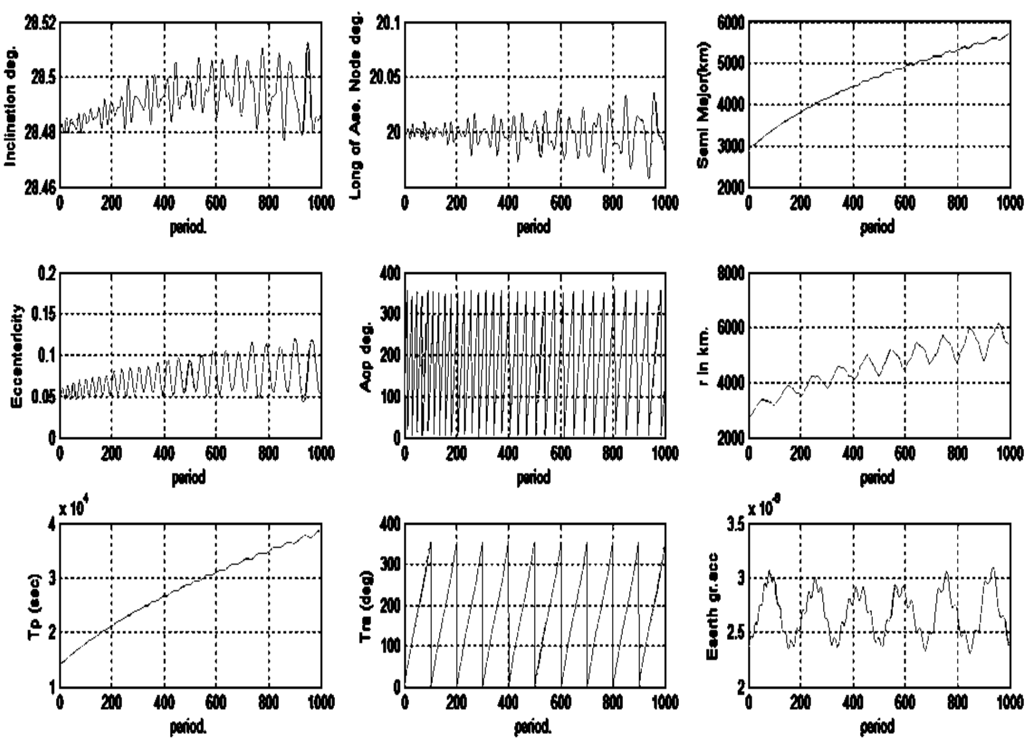

Figure 5 : The orbital elements variation at height=1000km.

Case (2): The orbital variation through the 1000 period with i=28.48°, eccentricity=0.1 on the orbits around the Moon, due to effect of the Earth's attraction.

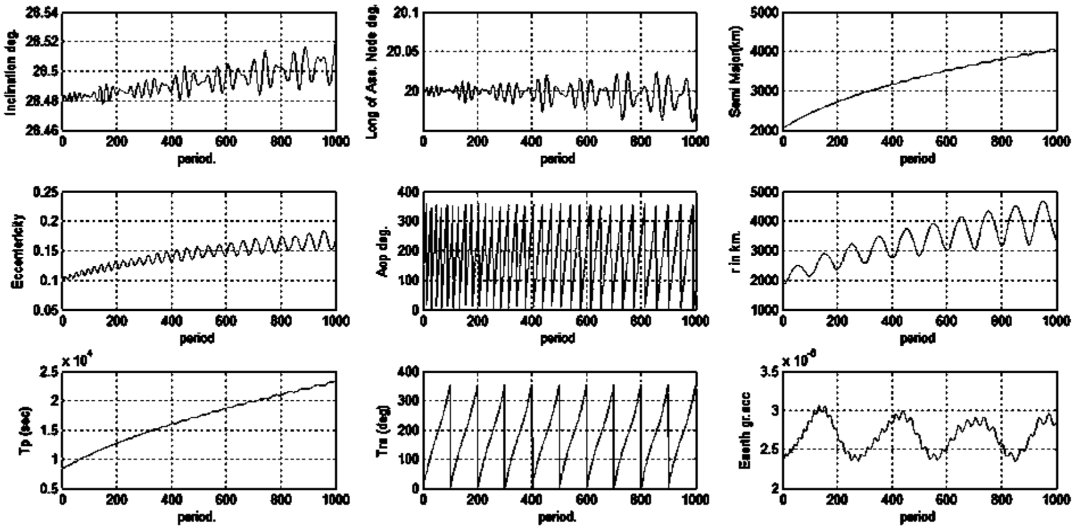

Figure 6 : Shows the orbital variation at height=100km

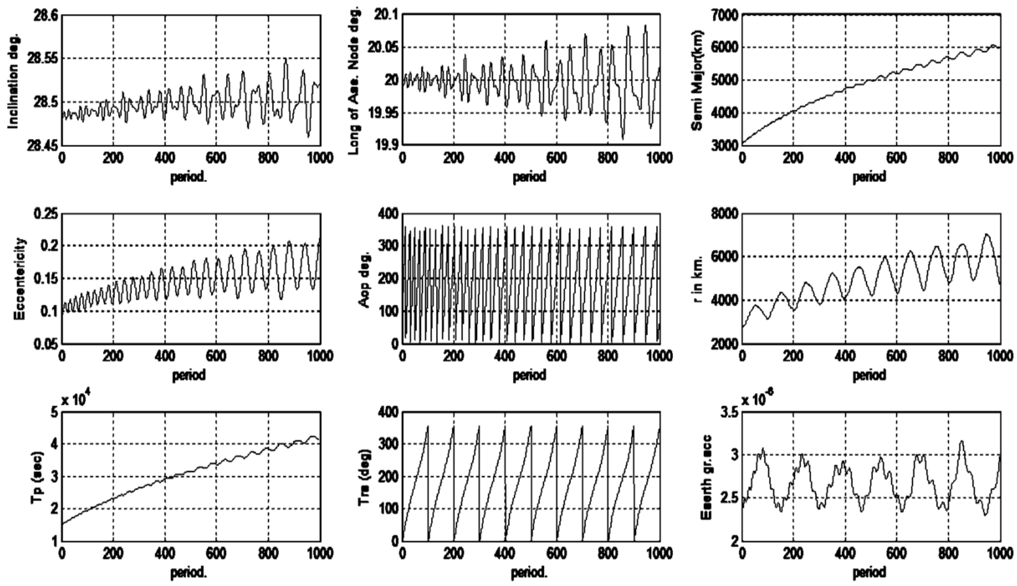

Figure 7 :Shows the orbital variation at height=1000km

Case 1: e=0.05

Table 1: The orbital variation through the 1000 period with the Earth's gravity at eccentricity=0.05

h(km)	100	200	400	600	800	1000
Δi(deg)	0.01	0.02	0.009	0.03	0.005	0.002
ΔΩ(deg)	0.011	0.01	0.02	0.1	0.015	0.02
Δω(deg)	0.50	0.50	0.48	0.47	0.47	0.48
Δe	0.04	0.05	0.02	0.04	0.04	0.001
Δa(m)	1999	2000	2300	2499	2250	2900
Δr(m)	1501	1600	2000	2300	2100	2100
ΔTp(sec)	1.3*10^4	1.5*10^4	1.8*10^4	1.9*10^4	2.3*10^4	2.6*10^4

Case 2: e=0.1

Table 2: The orbital variation through 1000 periods with the Earth's gravity at eccentricity=0.1

h(km)	100	200	400	600	800	1000
Δi(deg)	0.04	0.025	0.046	0.006	0.039	0.041
ΔΩ(deg)	0.022	0.014	0.01	0.007	0.04	0.01
Δω(deg)	15.42	14.76	16.097	14.855	15.691	17.179
Δe	0.06	0.053	0.087	0.071	0.038	0.11
Δa(m)	1900	2139	2265	2557	2858	2898
Δr(m)	1500	1646	1899	1900	2250	2100
ΔTp(sec)	1.9*10^4	1.62*10^4	1.8*10^4	2*10^4	2.6*10^4	2.7*10^4

In figure (2), the inclination variation increases throughout seven big periods; all seven have five small periods. The scalar variation has made a linear increase from 28.48 to 28.5 degrees. The longitude of ascending node variation has slightly decreased. This decrease was with seven periods including five small periods through 1000 periods (rotation) of the satellite. The reason for the difference between the amount of variation of inclination and (long) of ascending nodes is due to the change in the location and the distance of the Earth in relation to the satellite. The distance from the Moon to the Earth changes according to the changes taking place in time and date. Semi-major axis increases curvature from 2000 to 4300 km throughout 1000 periods. This means that the satellite orbit and its period will increase. The eccentricity's increase vibrated with the range of period increase too. The argument of perigee depends on all orbital elements. Semi-major axis, the distance between the satellite and the Moon, ranges from 2000 to 3500 km. This Semi-major axis has a wave increase variation of about 10 periods through 1000 periods. The time of period (Tp) depends on semi-major axis increase curvature ranging from 0.8*10^4 to 2.1*10^4 sec according to the 3rd Kepler's law. True anomaly increased throughout 1000 period and changed from (0 to 360) degree due to perturbation. The Earth's acceleration amount is not constant and it changes through 1000 periods due to the gravity of the Earth depending on the Moon's position from the Earth.
At 200km, as shown in Figure (3), the same beaver variation of all elements slightly increased in variation values.

In other heights, as shown in Figure (4), it was found that the increase in height increases the variation values because the period of satellite increased. The increase in variation and acceleration of the Earth's period will result in the same variation for all other elements along with an increase in variation value, which means it shows the same behaviors. In Figure (5), at 1000 km, the orbital elements have the same behavior described in Figures (2,3,4), but at this height, the final values are greater, whereas the maximum value of the semi-major axis at 100 km is 3700 km (Figure 2), while at 600 km the final value of the semi-major axis is 4700 km (Figure 4). The final value at 1000 km is 5800 km. This implies that at a height of 1000 km, the orbit becomes less stable. That is, the satellite in this orbit is prone to escape because increases the Earth's gravity with respect to the Moon's gravity.

In Figure (6), at eccentricity 0.1 this leads to an increase in the perigee height, which increases the gravitational effect of the third body (the Earth) relative to the gravitational effect of the Moon, so the orbital elements have the same behavior but the amount change will increase because the satellite is at apogee position, as shown in Figure (7). This means that the Earth's gravity will increase. Therefore, the Earth's gravity effect is greater on the satellite.

Other heights have the same behavior for all elements, but the variations are more because the effect of the Earth's attraction is more near the apogee, which is higher at e=0.1 .
Tables (1, 2) show the orbital elements, distance, and period variations for all studied orbits. These orbits have different perigee heights, for e=0.05 Table (1) and for e=0.1, Table (2).
In this research, it is found that the orbital elements are changed by the Earth's attraction for all orbits around the Moon. The best height of the lower orbit is 100 km when the eccentricity is 0.05.

5. Conclusions:

1. The satellite's orbit around the Moon is highly influenced by the gravity of the Earth, especially the semi-major axis after many periods. The effects are not constant in relation to date. This is due to the change in distance between the Earth and the Moon as well as the variation of the Moon's inclination angle.
2. The Earth's gravity perturbation on the orbit around the Moon has a very slight effect at the higher orbit.
3. The Earth's gravity effect on the orbit around the Moon has a greater effect when eccentricity increases.
4. The Earth's gravity perturbation on the orbit around the Moon is affected by the inclination of the orbit's change.
5. The best height of the lower orbit around the Moon is 100 km when the eccentricity is 0.05 and the inclination is 28.45 degrees.

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