DOI: 10.24996/ijs.2026.67.1.29

ISSN: 0067-2904

Supra Topological Spaces Via Delta-Semi-Open Sets

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Received: 28/7/2024 Accepted: 19/12/2024 Published: 30/1/2026

Abstract

The significance of supra topological spaces as a subject of study cannot be overstated, as they represent a broader framework than traditional topological spaces. Numerous scholars have proposed extensions to supra open sets, including supra semi-open sets, supra delta-open sets and others. In this paper, the concept of supra delta-semi-open set was introduced within the generalizations of the supra topology of sets. Our investigation involves harnessing this category of sets to introduce new notions in these spaces, specifically supra delta-semi-limit points, supra delta-semi-derive points and examining their relationship with supra semi-open. Building upon this set classification, we introduce several additional concepts such as supra delta-semi-symmetric, supra (delta, delta) semi-generalized closed, supra delta-semi-continuous functions, supra semi-kernel-delta sets, supra delta-semi-separation axioms, supra temperate delta-semi rho$_0$, rho$_1$ spaces and supra delta-semi rho$_0$, rho$_1$ spaces, and we have presented several theories that demonstrate cases of equivalence among these ideas under specific conditions. Additionally, we have proven a collection of useful relationships and properties for the aforementioned ideas. Furthermore, the research was enhanced with illustrative and refuting examples.

Keywords: Supra topology, δ-open, δ-limit point, supra semi open, δ-semi open.

الفضاءات التبولوجية الفوقية عبر المجموعات دلتا شبه المفتوحة

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الخلاصة

لا يمكن المبالغة في اهمية مواضيع الفضاءات التبولوجية الفوقية باعتبارها اوسع من الفضاءات التبولوجية التقليدية. اقترح العديد من الباحثين تعميمات للمجموعات المفتوحة الفوق تبولوجية، بما في ذلك المجموعات الشبه مفتوحة الفوق تبولوجية والمجموعات المفتوحة الفوق تبولوجية دلتا وغيرها. في هذا البحث تم تقديم المجموعات الشبه مفتوحة الفوق تبولوجية دلتا ضمن تعميمات التبولوجيا الفوقية للمجموعات. يتضمن بحثنا استغلال هذه الفئة من المجموعات لتقديم مفاهيم جديدة في هذه الفضاءات، لاسيما نقاط شبه الحد الفوق تبولوجية دلتا، ونقاط شبه الاشتقاق الفوق تبولوجية دلتا ودراسة علاقتها مع المجموعات شبه المفتوحة الفوق تبولوجية. بناءاً على هذا التصنيف للمجموعات، نقدم مفاهيم اضافية مثل المجموعات الفوق تبولوجية دلتا شبه المتناظرة، المجموعات شبه المغلقة العامة (دلتا، دلتا) الفوق تبولوجية، الدوال شبه المستمرة الفوق

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تبولوجية دلتا، مجموعة شبه النواة الفوق تبولوجية دلتا، شبه المحاور الفوق تبولوجية دلتا رو0، رو1 والفضاءات شبه فوق تبولوجية دلتا رو0، رو1، وقد قدمنا عدة نظريات تظهر حالات التكافؤ بين هذه المفاهيم في ظل ظروف معينة. بالاضافة الى ذلك، اثبتنا مجموعة من العلاقات والخصائص المفيدة لهذه المفاهيم المذكورة اعلاه. وعلاوة على ذلك، تم تعزيز البحث بأمثلة توضيحية وامثلة مضادة.

1. Introduction and preliminaries

It is known that the subset of the power of a set C forms a topology if certain conditions are met. The most important of which is its closed with respect to the union and the intersection of finite subsets of it. It is also known that every element in that set is called an open set and its complement is a closed set and that (C, τ_C) is called a topological space for instance [1], [2].

N. Levine [3] and numerous researchers have generalized concepts in topology like open sets and closed sets and the ideas related to them such as interior, closure and other well-established concepts in topology, for more information see [4], [5]. In 1968 Velicko [6] introduced the notions of regular open and then defined delta-open for short (δ. open) and δ. closed sets as follows: A subset B is termed δ.open if for every $c \in B$ there is a regular open set O such that $c \in O \subseteq B$. The complement of δ.open is referred to δ.closed [7], [8]. A point $c \in C$ is referred to δ.cluster point of $B \subseteq C$ if $B \cap int(cl(O)) \neq \phi$ holds for every open set O of C that contains c. The collection of all δ.cluster points of B is defined as the δ.closure of B which denoted by $cl_\delta(B)$ [9]. The union of each regular open subsets contains in B is termed δ.interior of B which denoted by $int_\delta(B)$ [7], further B is deemed δ.closed if and only if $B = cl_\delta(B)$ applies [10]. The complement of δ.closed set is known as δ.open set. Recall that, B is considered a semi-open set if $B \subseteq cl_\delta(int(B))$, and the complement of a semi-open set is denoted as semi-closed set [8], [11]. Moreover, we denoted the semi-closure of B by ${}_{semi}cl(B)$, for an information of this concept see [11], [12]. A space (C, τ_C) is called semi-R_0 if for every semi-open set O and $c \in O$, ${}_{semi}cl(\{c\}) \subseteq O$ [13], [14] and it is called a semi-R_1 if for every $c_0, c_1 \in C$ such that ${}_{semi}cl(\{c_0\}) \neq {}_{semi}cl(\{c_1\})$, then there are disjoint semi-open sets O_0 and O_1 such that ${}_{semi}cl(\{c_0\}) \subseteq O_0$ and ${}_{semi}cl(\{c_1\}) \subseteq O_1$ [15].

In conjunction with the generalizations of open and closed sets mentioned above, the concept of topology itself was generalized by dispensing with the intersection condition as follows: A sub collection τ_C^S of the power set of C is referred to supra topology on C if $\phi, C \in \tau_C^S$ and it is closed under arbitrary union [9], [16]. The pair (C, τ_C^S) labeled supra topological space or supra space for short, where any member belongs to τ_C^S named supra open (briefly S.open), the complement of S.open is named supra closed (briefly S.closed). The intersection of all S.closed sets including B is labeled supra closure of B and symbolized by $\text{S}.cl(B)$, where B is any subset of C. The union of all S.open that contains in B is named supra interior of B and is denoted by $S.int(B)$ [17], [18]. A point c in a supra space (C, τ_C^S) is referred to supra δcluster point and symbolize is by S.δcluster of B if $O \cap \text{S}.int(\text{S}.cl(B)) \neq \phi$ for each S.open set O of C containing c [19]. The set of S.δcluster points of B is called supra δclosure of B and is symbolized by $\text{S}.cl_\delta(B)$. A subset B of supra space C is referred to supra δclosed whenever $B = \text{S}.cl_\delta(B)$, while the complement of supra δclosed set is referred to supra δopen [19]. The notations of δ-limit points [20], semi-cluster points, semi-derive points and semi-limit points [21], [22], semi-separation axioms [23], [24], semi-continuous functions [25], semi-symmetric [5] in supra topological spaces are defined in the same manner as the previous notations in topology, with the substitution of semi-open by supra semi open, for instance see [25].

In this paper we defined supra δsemi open which is stronger than supra semi open. The concepts related to S.δsemi open are introduced, like supra δsemi closure, supra δsemi interior, supra δsemi limit points, supra δsemi neighborhood, supra δsemi kernel, supra δsemi continuous functions, supra temperate δsemi ρ_0, ρ_1 and supra δsemi ρ_0, ρ_1. Many results were proved as well as we investigated the relationship involving supra semi open and supra δsemi open. Also, the connection among the concepts mentioned above have been highlighted through numerous theories and properties, supported by examples that illustrate the differences between these concepts and the concept of supra δsemi sets.

2. Supra δ.semi. derived Sets

In this section we will acquaint supra topological space via δsemi open, examples and verified some important results and properties associated with previous concepts, and we will be beginning with the following definitions.

Definition 2.1: A subset B of supra space C is referred to supra-delta semi open (briefly S.δsemi. open) set if there is an S.δopen set O of C such that $O \subseteq B \subseteq \mathrm{S}.cl(O)$. The complement of S.$\delta$semi. open set is referred to supra-delta semi closed (briefly S.δsemi. closed) set.

Definition 2.2: A point c in a subset B of a supra space (C, τ_C^S) is referred to supra- δsemi limit (briefly S.δsemi. limit) point of B if every S.δsemi. open subset O of C containing c satisfies the condition $O \cap (B - \{c\}) \neq \phi$. The collection of all S.δsemi. limit points of B is said to be supra-δsemi derived (briefly S.δsemi. derived) set of B and is symbolized by $\mathrm{S}._{semi}\, D_\delta(B)$.

Example 2.3: Let $C = \{a, b, c, d, e\}$ and $\tau_C^S = \{\phi, C, \{\mathrm{a}\}, \{\mathrm{d, c}\}, \{\mathrm{b, e, a}\}, \{\mathrm{b, e, d, c}\}, \{\mathrm{c, d, a}\}\}$. Then, the family of all S.δopen sets is $\{\phi, C, \{\mathrm{b, e, d, c}\}, \{\mathrm{b, e, a}\}, \{\mathrm{c, d, a}\}, \{\mathrm{d, c}\}, \{\mathrm{a}\}\}$. The family of S.$\delta$semi. open set is $\{\phi, C, \{\mathrm{b, e, d, c}\}, \{\mathrm{b, e, a}\}, \{\mathrm{c, d, a}\}, \{\mathrm{c, b, d, a}\}, \{\mathrm{c, e, d, a}\}, \{\mathrm{d, c}\}, \{\mathrm{a}\}\}$. It is clear that $\{c, b, d, a\}$ is S.δsemi. open and $\{b, a\}$ is not S.δsemi. open set. On the other hand, the elements (e) and (a) are examples of S.δsemi. limit point and not S.δsemi. limit point respectively for the subset $\{b, c\}$.

Definition 2.4: Let B be a subset of (C, τ_C^S), A point c in C is referred to supra-δsemi cluster (briefly S.δsemi. cluster) point of B whenever $O \cap B \neq \phi$ for each S.δsemi. open set O of C containing c. The set of each S.δsemi. cluster points of B is called supra-δsemi closure (briefly S.δsemi. closure) symbolized by $\mathrm{S}._{semi}\, cl_\delta(B)$. We symbolized the collection of S.δsemi. open (resp., S.δsemi. closed) sets by $\mathrm{S}.\delta semi.O(C, \tau_C^S)$ (resp., $\mathrm{S}.\delta semi.C(C, \tau_C^S)$).

Definition 2.5: A subset O of a supra space (C, τ_C^S) is referred to supra-δsemi neighborhood (briefly S.δsemi. neighborhood) of a point c if whenever O contains a S.δsemi. open set to which c belongs.

Example 2.6: In Example 2.3, let $B = \{b, c\}, H = \{d, c\}$ then $\mathrm{S}._{semi}\, cl_\delta(B) = \{b, e, c, d\}$, $S._{semi}\, cl_\delta(H) = \{d, c\}$. Let $O = \{c, e, d\}$ is $S.\delta$semi. neighborhood of a point d, but O is not $S.\delta$semi. neighborhood of a point e since there is not S.δsemi. open set that contains in O which e belongs.

Now, we proof the following result which is important in our work.

Proposition 2.7: The intersection of any collection of S.δsemi. closed sets in (C, τ_C^S) is S.δsemi. closed.

Proof: Assume that $\{F_i : i \in I\}$ is a family of S.δsemi. closed sets, we want to prove that $\bigcap_{i \in I} F_i$ is S.δsemi. closed. Since $\bigcap_{i \in I} F_i \subseteq F_i$ for each $i \in I$, so $\mathrm{S}.cl(\bigcap_{i \in I} F_i) \subseteq S.cl(F_i)$ for each $i \in I$, [22]. By Definition 2.1 we have S.δopen sets B_i such that $B_i \subseteq F_i^c \subseteq S.cl(B_i)$ for

each $i \in I$ implies $\bigcup_{i\in I} B_i \subseteq \bigcup_{i\in I} F_i^c \subseteq \bigcup_{i\in I} S.cl(B_i)$, [22]. Hence, $\bigcup_{i\in I} B_i \subseteq \bigcup_{i\in I} F_i^c \subseteq$ $S.cl(\bigcup_{i\in I} B_i)$, [22]. Since $\bigcup_{i\in I} B_i$ is an S.δopen, hence $\bigcup_{i\in I} F_i^c$ is an S.δsemi. open, and hence $\bigcap_{i\in I} F_i = \left(\bigcup_{i\in I} F_i\right)^c$ is an S.δsemi. closed set, which complete the proof.
The following case highlights some properties of the derive and closure operator.

Proposition 2.8: Let B be a subset of a supra topological space (C,τ_C^S), then $S._{semi}\, D_\delta(B) \subseteq$ $S._{semi}\, cl_\delta(B)$.

Proof: Suppose $c \notin S._{semi}\, cl_\delta(B)$, so there is an S.$\delta$semi. open subset say O containing c with $O \cap B = \phi$ implies $c \notin S._{semi}\, D_\delta(B)$, hence $S._{semi}\, D_\delta(B) \subseteq S._{semi}\, cl_\delta(B)$.

Proposition 2.9: For a subset B of a supra space (C,τ_C^S) the following properties are hold:
1. $S._{semi}\, D_\delta(B) \cup B \subseteq S._{semi}\, cl_\delta(B)$;
2. $B \subseteq S._{semi}\, cl_\delta(B)$.

Proof:

1. Let $c \in S._{semi}\, D_\delta(B) \cup B$, so either $c \in B$ or $c \in S._{semi}\, D_\delta(B)$. Now, if $c \in B$ and since $B \subseteq S._{semi}\, cl_\delta(B)$, thus $c \in S._{semi}\, cl_\delta(B)$. If $c \in S._{semi}\, D_\delta(B) \subseteq S._{semi}\, cl_\delta(B)$ (Proposition 2.8), hence $c \in S._{semi}\, cl_\delta(B)$, and hence $S._{semi}\, D_\delta(B) \cup B \subseteq S._{semi}\, cl_\delta(B)$.
Conversely: Let $d \notin S._{semi}\, D_\delta(B) \cup B$, then $d \notin S._{semi}\, D_\delta(B)$ and $d \notin B$ which means there exists an S.δsemi. open set O containing d with $O \cap B = \phi$ implies $d \notin S._{semi}\, cl_\delta(B)$, hence $S._{semi}\, cl_\delta(B) \subseteq S._{semi}\, D_\delta(B) \cup B$, which complete the proof.

2. Follows from Definition 2.4.

Corollary 2.10: Let B be a subset of a supra space (C,τ_C^S), then $S._{semi}\, cl_\delta(B) = \bigcap\{K: K \in S.\delta semi.C(C,\tau_C^S), B \subseteq K\}$.

Proof: Let $c \in C$, then either $c \in B$ or, $c \notin B$. If $c \in B$ and $c \in S._{semi}\, cl_\delta(B)$, then $c \in$ $\bigcap\{K: B \subseteq K, C \in S.\delta semi.C(C,\tau_C^S)\}$. If $c \notin B$ and $c \in S._{semi}\, cl_\delta(B)$, then $c \in S._{semi}\, D_\delta(B)$ (Proposition 2.9 part 1), hence $O \cap B \neq \phi$ for every S.δsemi. open subset O of C containing c. Now, it is clear that $K^c \cap B = \phi$ for each K in $\bigcap\{K: K \in S.\delta semi.C(C,\tau_C^S), B \subseteq K\}$. If $c \notin K$ for some K then $c \in K^c$, hence $K^c \cap B \neq \phi$ which is a contradiction. Thus, $c \in K$ for all K containing B which leads to $c \in \bigcap\{K: K \in S.\delta semi.C(C,\tau_C^S), B \subseteq K\}$.
Conversely: Suppose $c \notin S._{semi}\, cl_\delta(B)$, so there is an S.$\delta$semi. open O containing b such that $O \cap B = \phi$ implies $B \subseteq O^c$ and since O^c is an S.δsemi. closed with $b \notin O^c$, hence $b \notin \bigcap\{K: K \in S.\delta semi.C(C,\tau_C^S), B \subseteq K\}$ implies $\bigcap\{K: B \subseteq K, C \in S.\delta semi.C(C,\tau_C^S)\} \subseteq$ $S._{semi}\, cl_\delta(B)$.

Corollary 2.11: Let B be a subset of a supra space (C,τ_C^S), then $S._{semi}\, cl_\delta(B)$ is S.δsemi. closed, that is $S._{semi}\, cl_\delta(S._{semi}\, cl_\delta(B)) = S._{semi}\, cl_\delta(B)$.

Proof: Follows from Proposition 2.7 and Corollary 2.10.

Theorem 2.12: Let $A_i, i \in I$ be a subsets of a supra space (C,τ_C^S), then the following statements are holds.
1. If $A_i \subseteq A_j, i,j \in I$, then $S._{semi}\, cl_\delta(A_i) \subseteq S._{semi}\, cl_\delta(A_j)$;
2. $S._{semi}\, cl_\delta(\bigcap\{A_i, i \in I\}) \subseteq \bigcap\{S._{semi}\, cl_\delta(A_i), i \in I\}$;
3. $\bigcup\{S._{semi}\, cl_\delta(A_i), i \in I\} \subseteq S._{semi}\, cl_\delta(\bigcup\{A_i, i \in I\})$;
4. A_i is S.δsemi. closed if and only if $A_i = S._{semi}\, cl_\delta(A_i)$.

Proof:

1. Let $a \notin S._{semi}\, cl_\delta(A_i)$, so there is an S.$\delta$semi. closed set O such that $a \notin O$ with $A_i \subseteq O$. Since, $A_i \subseteq A_j$ implies $A_i \subseteq O$, hence $a \notin S._{semi}\, cl_\delta(A_j)$.

2. Let $a \notin \bigcap\{S._{semi}\, cl_\delta(A_i, i \in I)\}$, then there is $i \in I$ such that $a \notin S._{semi}\, cl_\delta(A_i)$. Since $\bigcap_{i\in I} A_i \subseteq A_i$ for each $i \in I$, by (1) $a \notin S._{semi}\, cl_\delta(\bigcap_{i\in I} A_i)$, hence we are done.

3. Since $A_i \subseteq \bigcup_{i\in I} A_i$ for each $i \in I$, by (1) $S._{semi}\, cl_\delta(A_i) \subseteq S._{semi}\, cl_\delta(\bigcup\{A_i, i \in I\})$, so $\bigcup\{S._{semi}\, cl_\delta(A_i), i \in I\} \subseteq S._{semi}\, cl_\delta(\bigcup\{A_i, i \in I\})$.

4. Follows from Corollaries 2.10 and 2.11.

Remark 2.13: The converse of Theorem 2.12 parts 2 and 3 are not necessarily holds in general. The following examples explain that.

Examples 2.14: Consider:

1. $C = \{a, b, c\}$ and $\tau_C^S = \{\phi, C, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Then, the family of all S.δopen sets is the same family of τ_C^S and S.$\delta semi.O(C, \tau_C^S) = \{\phi, C, \{a\}, \{a, b\}\}$. Let $A = \{a, b\}$ and $B = \{a, c\}$. We see that S.$_{semi}\, cl_\delta(A \cap B) = \{a\}$ and S.$_{semi}\, cl_\delta(A) \cap$ S.$_{semi}\, cl_\delta(B) = C \cap C = C$. So S.$_{semi}\, cl_\delta(A) \cap$ S.$_{semi}\, cl_\delta(B) \nsubseteq$ S.$_{semi}\, cl_\delta(A \cap B)$ and S.$_{semi}\, cl_\delta(A) = C \nsubseteq A$, also A is not S.δsemi. closed with S.$_{semi}\, cl_\delta(A) \neq A$.

2. $Z = \{a, b, c, d\}$ and $\tau_Z^S = \{\phi, Z, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. So, the family of all S.δopen sets is the same family of τ_Z^S and S.$\delta semi.O(Z, \tau_Z^S) = \{\phi, Z, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Let $A = \{a\}$ and $B = \{b\}$. It is clear that S.$_{semi}\, cl_\delta(A \cup B) = Z$ and S.$_{semi}\, cl_\delta(A) \cup$ S.$_{semi}\, cl_\delta(B) = \{a\} \cup \{b\} = \{a, b\}$. So, S.$_{semi}\, cl_\delta(A \cup B) \nsubseteq$ S.$_{semi}\, cl_\delta(A) \cup$ S.$_{semi}\, cl_\delta(B)$.

3. Supra δ. semi. separation axioms

In this section we will introduce three concepts of separation axioms, and the three other concepts which are supra-δsemi symmetric, supra- δ, δsemi generalized closed and supra-δcontinuous functions by using an S.δsemi. open set, where some characteristics related to these concepts have been investigated and supported by illustrative examples.

Definition 3.1: Let (C, τ_C^S) be a supra topological space, then it is called supra-δsemi T_0 (briefly S.$\delta semiT_0$) if for any distinct pair of points in C, there is an S.δsemi. open set containing one of the points but not the other.

Example 3.2: Let $Z = \{a, b, c\}$ and $\tau_Z^S = \{\phi, Z, \{c\}, \{c, b\}, \{c, a\}, \{b, a\}\}$. Then, the family of all S.δ open sets is the same family of τ_Z^S and S.$\delta semi.O(C, \tau_Z^S) = \{\phi, Z, \{c\}, \{c, b\}\}$. Then, (Z, τ_Z^S) is S.$\delta semiT_0$. But (C, τ_C^S) in Example 2.3 is not an S.$\delta semiT_0$ since the elements d and c are disjoint, but there is not an S.δsemi. open contains d but not c or conversely.

Definition 3.3: Let (C, τ_C^S) be a supra space, then it is called a supra-δsemi T_1 (briefly S.$\delta semiT_1$) if for any distinct pair of points w, z in C, there is an S.δsemi. open set O in C containing w but not z and an S.δsemi. open set V in C containing z but not w.

Definition 3.4: Let (C, τ_C^S) be a supra space, then it is called supra-δsemi T_2 (briefly S.$\delta semiT_2$) if for any distinct pair of points w, z in C, there are S.δsemi. open sets O and V in C containing w and z respectively with $O \cap V = \phi$.

Example 3.5: Let $Y = \{a, b, c\}$ and $\tau_Y^S = \{\phi, Y, \{c, d, f, e\}, \{c, b, f, e\}, \{c, a, f, e\}, \{c, f, e\}, \{a, b, e\}, \{b, e\}, \{a, e\}, \{e\}, \{a, b, f, e\}, \{b, f, e\}, \{a, f, e\}, \{f, e\}, \{d, b, f\}, \{a, d, f\}, \{a, b, f\}, \{d, f\}, \{b, f\}, \{a, f\}, \{f\}, \{a, b\}, \{b\}, \{a\}\}$, then the family of S.$\delta semiO(Y, \tau_Y^S)$ is $\{\phi, Y, \{c, d, b, f, e\}, \{c, a, d, f, e\}, \{c, d, f, e\}, \{c, a, d, b, e\}, \{c, d, b, e\}, \{c, a, d, e\}, \{c, a, b, e\}, \{c, b, e, a, f\}, \{c, b, e\}, \{c, b, f, e\}, \{c, a, e\}, \{c, a, f, e\}, \{d, a, b\}, \{c, d, a, b\}, \{d, a, b, e\}, \{d, a, b, f\}, \{d, b\}, \{d, b, f\}, \{a, d\}, \{a, d, f\}, \{a, b\}\}$. It is clear that (Y, τ_Y^S) is an S.$\delta semiT_1$ but not S.$\delta semiT_2$ since the elements e and c are disjoint but there are not two S.δsemi. open sets O and V in Y containing e and c, respectively with $O \cap V = \phi$.

Example 3.6: Let $C = \{a, b, c\}$ and $\tau_C^S = \{\phi, C, \{c, d\}, \{b, a, c, d\}, \{b, c\}, \{a, d\}, \{a, c\}, \{b, a\}, \{d\}, \{c\}, \{b\}, \{a\}\}$, then the family of S.$\delta semiO(C, \tau_C^S)$ is $\{\phi, C, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, a, d\}, \{a, d\}, \{b, a, c\}, \{b, c\}, \{b, a\}\}$, one can show that (C, τ_C^S) is an S.$\delta semiT_2$ space.

Remark 3.7: If (C,τ_C^S) is an S.$\delta semiT_i$, then it is an S.$\delta semiT_{i-1}$, $i=1,2$. Additively, the converse is not true in general.
Example 3.8: In Example 2.14, (Z,τ_Z^S) is not an S.δT_i and is not an S.T_1 space, $i=0,1,2$. But (C,τ_C^S) is an S.$\delta semiT_i$ for $i=0,1,2$.

The following two theorems give some properties satisfies in supra-δsemi T_0, supra-δsemi T_1 spaces.
Theorem 3.9: Let (C,τ_C^S) be a supra space, then (C,τ_C^S) is an S.$\delta semiT_0$ if and only if for any pair of distinct points y,z in C, S.$_{semi}\,cl_\delta(\{y\})\neq$ S.$_{semi}\,cl_\delta(\{z\})$.
Proof: Let (C,τ_C^S) be an S.$\delta semiT_0$ space and let y,z be any two distinct points in C. So, there is an S.δsemi. open set O containing y or z, say y but not z. So, O^c is an S.δsemi. closed set which does not contain y but contains z. Since, S.$_{semi}\,cl_\delta(\{z\})$. is the smallest S.$\delta$semi. closed set containing z (Corollary 2.10), S.$_{semi}\,cl_\delta(\{z\})\subseteq O^c$, hence $y\notin$ S.$_{semi}\,cl_\delta(\{z\})$, and therefore S.$_{semi}\,cl_\delta(\{y\})\neq$ S.$_{semi}\,cl_\delta(\{z\})$.
Conversely: Assume that $y,z\in C$, $y\neq z$ and S.$_{semi}\,cl_\delta(\{y\})\neq$ S.$_{semi}\,cl_\delta(\{z\})$. Let w be a point of C such that $w\in S._{semi}\,cl_\delta(\{y\})$ but $w\notin$ S.$_{semi}\,cl_\delta(\{z\})$. We claim that $y\notin$ S.$_{semi}\,cl_\delta(\{z\})$. If $w\in$ S.$_{semi}\,cl_\delta(\{z\})$ implies S.$_{semi}\,cl_\delta(\{y\})\subseteq$ S.$_{semi}\,cl_\delta(\{z\})$, and this a contradiction with $w\notin$ S.$_{semi}\,cl_\delta(\{z\})$. Consequently y belongs to the S.δsemi. open set (S.$_{semi}\,cl_\delta(\{z\}))^c$ to which z does not belong.
Theorem 3.10: A supra space (C,τ_C^S) is an S.$\delta semiT_1$ if and only if the singletons are $S.\delta$semi. closed sets.
Proof: Assume that (C,τ_C^S) is an S.$\delta semiT_1$ and let w be any point of C. Let $z\in\{w\}^c$, so $w\neq z$ implies there is an S.δsemi. open set O_z such that $z\in O_z$ but $z\notin O_z$. Thus, $z\in O_z\subseteq\{w\}^c$ that means $\{w\}^c=\bigcup\{O_z: z\in\{w\}^c\}$ which is an S.δsemi. open.
Conversely: From assumption we have $\{w\}$ is an S.δsemi. closed for each $w\in C$. Now assume that $y,z\in C$ with $y\neq z$, hence $z\in\{y\}^c$. Thus, $\{y\}^c$ is an S.δsemi. open set containing z but not y. Similarly, $\{z\}^c$ is an S.δsemi. open containing y but not z and we are done.
Definition 3.11: The (C,τ_C^S) is termed supra-δsemi symmetric (briefly S.δsemi. symmetric) if for $w,z\in C$ with $w\in$ S.$_{semi}\,cl_\delta(\{z\})$ implies $z\in$ S.$_{semi}\,cl_\delta(\{w\})$.

The following definition is crucial in our work to achieve certain conclusions regarding the relationship of the concept of supra-δsemi symmetric with the concepts of supra-δsemi T_0, supra-δsemi T_1 spaces.
Definition 3.12: A subset B of (C,τ_C^S) is said to be supra-δ,δsemi generalized closed (briefly S.$(\delta,\delta)semi.GC$) set if S.$_{semi}\,cl_\delta(B)\subseteq O$, whenever $B\subseteq O$ and O is an S.δsemi. open.
Remark 3.13: Let (C,τ_C^S) be a supra space. Easley from Definitions 2.1, 2.4 and Theorem 2.12, show that each S.δsemi. closed set is an S.$(\delta,\delta)semi.GC$.
Theorem 3.14: A supra space (C,τ_C^S) is an S.δsemi. symmetric if and only if $\{w\}$ is S.$(\delta,\delta)semi.GC$ set for each $w\in C$.
Proof: Suppose $z,w\in C$ such that $w\in$ S.$_{semi}\,cl_\delta(\{z\})$ and $z\notin$ S.$_{semi}\,cl_\delta(\{w\})$ implies S.$_{semi}\,cl_\delta(\{z\})\subseteq$ (S.$_{semi}\,cl_\delta(\{w\}))^c$ (Theorem 2.12 part 1). Now, (S.$_{semi}\,cl_\delta(\{w\}))^c$ contains w which is a contradiction.
Conversely: Suppose $\{w\}\subseteq O\in S.\delta semi.O(C,\tau_C^S)$ and S.$_{semi}\,cl_\delta(\{w\})$ is not subset of O. Thus, and O^c are not disjoint, so let z belongs to their intersection. But (C,τ_C^S) is S.δsemi. symmetric, hence $w\in$ S.$_{semi}\,cl_\delta(\{z\})$ which is subset of O^c a contradiction with assumption.
Corollary 3.15: A supra space (C,τ_C^S) is an S.δsemi. symmetric if it is an S.$\delta semiT_1$ space.

Proof: By Theorem 3.10 the singleton sets are S.δsemi. closed, hence they are S.$(\delta,\delta)semi.GC$ sets (Remark 3.13) and hence (C,τ_C^S) is an S.δsemi. symmetric (Theorem 3.14).

Corollary 3.16: For a supra space (C,τ_C^S) the properties are equivalent:
1. (C,τ_C^S) is an S.δsemi. symmetric and S.$\delta semiT_0^S$;
2. (C,τ_C^S) is S.$\delta semiT_0^S$.

Proof: (2→1) Follows immediately from Remark 3.7 and Corollary 3.15.
(1→2) Let w,z be any distinct points in (C,τ_C^S). Since, (C,τ_C^S) is an S.$\delta semiT_0^S$ then there is O_w such that $w\in O_w\subseteq\{z\}^c$ for some $O_w\in$ S.$\delta semi.O(C,\tau_C^S)$. Thus, $w\notin$ S.$_{semi}\,cl_\delta(\{z\})$, hence $z\notin$ S.$_{semi}\,cl_\delta(\{w\})$ that means there is $O_z\in$ S.$\delta semi.O(C,\tau_C^S)$ such that $z\in O_z\subseteq\{w\}^c$ and we are done.

Definition 3.17: Let $(C_1,\tau_{C_1}),(C_2,\tau_{C_2})$ be two topological spaces. A map $f:(C_1,\tau_{C_1})\longrightarrow(C_2,\tau_{C_2})$ is called δ-semi. continuous map if for every $c\in C_1$ and every δ-semi. open set O containing $f(c)$, there is a δ-semi. open set V in C_1 containing c such that $f(v)\subseteq O$, [5]. In a similar manner, we define a new type of continuous maps as given by the following definition.

Definition 3.18: Let (C,τ_C^S) and (Z,τ_Z^S) be two supra spaces, then $f:(C,\tau_C^S)\longrightarrow(Z,\tau_Z^S)$ is said to be supra-δcontinuous (briefly S.δsemi. continuous) function if for each $w\in C$ and each S.δsemi. open set O containing $f(w)$, there is an S.δsemi. open set V in C containing w such that $f(w)\subseteq O$.

Proposition 3.19: Let (C,τ_C^S) and (Z,τ_Z^S) be two supra spaces, then a function $f:(C,\tau_C^S)\longrightarrow(Z,\tau_Z^S)$ is an S.δsemi. continuous if and only if the inverse image of each S.δsemi. open set is an S.δsemi. open.

Proof: Let f be an S.δsemi. continuous and let $O\in$ S.$\delta semi.O(Z,\tau_Z^S)$, if $O\cap f(C)=\phi$, then $f^{-1}(C)=\phi$ and hence is an S.δsemi. open set in C. If $O\cap f(C)\neq\phi$, then O is S.δsemi. neighborhood of each of its points in z implies $f^{-1}(O)$ must be an S.δsemi. neighborhood of each of its points in C, hence $f^{-1}(O)$ is an S.δsemi. open set in C (Proposition 2.7).
Conversely: Let $w\in C$ and V be a S.δsemi. neighborhood of $f(w)$ in Z. Then, $w\in f^{-1}(V)$, hence $f(w)\in f\left(f^{-1}(V)\right)\subseteq V$, [8] and since $f^{-1}(V)$ is an S.δsemi. open implies f is an S.δsemi. continuous.

Example 3.20: Let (M,τ_M^S) and (K,τ_K^S) be two supra spaces such that $M=\{m_1,m_2,m_3\}$, $K=\{k_1,k_2,k_3\}$, $\tau_M^S=\{\phi,M,\ \{m_1\},\ \{m_2\},\ \{m_1,m_2\},\ \{m_1,m_3\},\ \{m_2,m_3\}\}$, $\tau_K^S=\{\phi,K,\{k_1\},\{k_1,k_2\},\{k_1,k_3\},\{k_2,k_3\}\}$. Consider $f,g:(M,\tau_M^S)\longrightarrow(K,\tau_K^S)$ such that $f(m_1)=k_3$, $f(m_2)=k_1$, $f(m_3)=k_2$, $g(m_1)=k_2$, $g(m_2)=k_3$, $g(m_3)=k_1$, then f is an S.δsemi. continuous function but g is not $S.\delta$semi. continuous since $g^{-1}(\{k_1\})=\{m_3\}$ which is not S.δsemi. open set.

4. Supra Temperate $\boldsymbol{\delta}$. semi $\boldsymbol{\rho_0}$-spaces

In this section, we introduce two new concepts, namely supra-δsemi kernel and supra temperate δsemi ρ_0-spaces. Some theorems and properties related to the relationship between these two concepts have been proved, as well as the relationship between supra temperate δsemi ρ_0-spaces and supra spaces in the case of cartesian product of supra spaces was investigated. Additionally, we shed light on the connection between the concepts of supra-δsemi kernel and supra-δsemi closure.

Definition 4.1: Let B be a subset of a supra space (C,τ_C^S), the supra-δsemi kernel (briefly S.δsemi. kernel) of B, symbolized by S.$_{semi}\,ker_\delta(B)$ is defined by S.$_{semi}\,ker_\delta(B)=\bigcap\{O\in$ S.$\delta semi.O(C,\tau_C^S):B\subseteq O\}$.

Theorem 4.2: Let (C,τ_C^S) be a supra space and $w\in C$, then $S._{semi}\, ker_\delta(A) = \cap\{w \in A\colon S._{semi}\, cl_\delta(\{w\}) \cap A \neq \phi\}$.
Proof: Let $w \in S._{semi}\, ker_\delta(A)$ and $S._{semi}\, cl_\delta(\{w\}) \cap A = \phi$, so $w \notin C/S._{semi}\, ker_\delta(\{w\})$ which is an $S.\delta$semi. open set containing A, but this a contradiction with assumption, hence $S._{semi}\, cl_\delta(\{w\}) \cap A \neq \phi$.
Now, consider $S._{semi}\, cl_\delta(\{w\}) \cap A \neq \phi$ and assume $w \notin S._{semi}\, ker_\delta(A)$, so there is an $S.\delta$semi. open set O containing A with $w \notin O$. Let $z \in S._{semi}\, cl_\delta(\{w\}) \cap A$ hence $z \in S._{semi}\, cl_\delta(\{w\})$, but O is $S.\delta$semi. neighborhood of z which does not contain w a contradiction (Definition 2.4) implies $w \in S._{semi}\, ker_\delta(A)$.
Definition 4.3: A supra space (C,τ_C^S) is called supra temperate δsemi ρ_0-space, symbolized by $S.t\delta.semi\ \rho_0$ if $\cap_{w\in C} S._{semi}\, cl_\delta(\{w\}) = \phi$.
Example 4.4: In Example 3.20. We see that $\cap_{m\in M} S._{semi}\, cl_\delta(\{m\}) = \phi$, so (M,τ_M^S) is an $S.t\delta.semi\ \rho_0$.
Theorem 4.5: A supra space (C,τ_C^S) is an $S.t\delta.semi\ \rho_0$ if and only if $S._{semi}\, ker_\delta(\{w\}) \neq C$ for each $w \in C$.
Proof: Let (C,τ_C^S) be an $S.t\delta.semi\ \rho_0$, and let $z \in C$ with $S._{semi}\, ker_\delta(\{z\}) = C$. Since C is an $S.t\delta.semi\ \rho_0$, so there is an $S.\delta$semi. open set O of C such that $z \notin O$ implies $z \in \cap S._{semi}\, cl_\delta(\{w\})$ (Theorem 4.2) but this is a contradiction.
Conversely: Let $S._{semi}\, ker_\delta(\{z\}) \neq C$ for each $w \in C$. If there is an element $z \in C$ with $z \in \cap_{w\in C} S._{semi}\, cl_\delta(\{w\})$. So, any $S.\delta$semi. open set containing z must contain each elements of C and since C is an $S.\delta$semi. open, hence $S._{semi}\, ker_\delta(\{z\}) = C$ which is a contradiction, and hence (C,τ_C^S) is an $S.t\delta.semi\ \rho_0$.
Theorem 4.6: Let (C,τ_C^S) be an $S.t\delta.semi\ \rho_0$ and (Z,τ_Z^S) is a supra space, then $C \times Z$ is an $S.t\delta.semi\ \rho_0$.
Proof: Since $\cap_{(a,b)\in C\times Z} S._{semi}\, cl_\delta(\{(a,b)\}) \nsubseteq \cap_{(a,b)\in C\times Z}(S._{semi}\, cl_\delta(\{a\}) \times S._{semi}\, cl_\delta(\{b\}))$ [26], which is equal to $\cap_{a\in C} S._{semi}\, cl_\delta(\{a\}) \times \cap_{b\in Z} S._{semi}\, cl_\delta(\{b\}) \nsubseteq \phi \times Z = \phi$, [27]. Thus $C \times Z$ is an $S.t\delta.semi\ \rho_0$.

The following result is useful for the remainder of our work.
Properties 4.7: Let (C,τ_C^S) be a supra space, and $w,z \in C$. Then, $z \in S._{semi}\, ker_\delta(\{w\})$ if and only if $w \in S._{semi}\, ker_\delta(\{z\})$.
Proof: Assume $z \notin S._{semi}\, ker_\delta(\{w\})$ implies there is an $S.\delta$semi. open sets O containing w with $z \notin O$, so $w \notin S._{semi}\, ker_\delta(\{z\})$. Proof of the converse is smaller.
Theorem 4.8: For any two elements w,z in a supra space (C,τ_C^S), the following statements are equivalent.
1. $S._{semi}\, ker_\delta(\{w\}) \neq S._{semi}\, ker_\delta(\{z\})$;
2. $S._{semi}\, cl_\delta(\{w\}) \neq S._{semi}\, cl_\delta(\{z\})$.
Proof: Let $S._{semi}\, ker_\delta(\{w\}) \neq S._{semi}\, ker_\delta(\{z\})$ implies there is an element $y \in C$ such that $y \in S._{semi}\, ker_\delta(\{w\})$ and $y \notin S._{semi}\, ker_\delta(\{z\})$. Hence, $w \in S._{semi}\, ker_\delta(\{y\})$, so $S._{semi}\, ker_\delta(\{w\}) \subseteq S._{semi}\, ker_\delta(S._{semi}\, ker_\delta(\{y\}))$ (Theorem 2.12) implies $S._{semi}\, ker_\delta(\{w\}) \subseteq S._{semi}\, ker_\delta(\{y\})$ (Corollary 2.11), and since $z \notin S._{semi}\, ker_\delta(\{y\})$ hence $z \notin S._{semi}\, ker_\delta(\{w\})$ therefore $S._{semi}\, cl_\delta(\{w\}) \neq S._{semi}\, cl_\delta(\{z\})$.
Conversely: Let $S._{semi}\, cl_\delta(\{w\}) \neq S._{semi}\, cl_\delta(\{z\})$, hence there is an element $y \in C$ with $y \in S._{semi}\, cl_\delta(\{w\})$ and $y \notin S._{semi}\, cl_\delta(\{z\})$. Thus, there is an $S.\delta$semi. open set containing w but not z, hence $z \notin S._{semi}\, ker_\delta(\{w\})$ which complete the proof.

5. Supra $\boldsymbol{\delta}$. semi $\boldsymbol{\rho_0}$ and supra $\boldsymbol{\delta}$. semi $\boldsymbol{\rho_1}$-spaces

In this section, we introduce two new concepts of supra spaces, namely supra δsemi ρ_0, ρ_1. Furthermore, study the relationship between these two concepts and also, we demonstrate a collection of results and conclusions associated with them, as well discuss the connection and properties between these concepts and the others which we have been presented in previous sections.

Definition 5.1: A supra space (C,τ_C^S) is said to be supra δsemi ρ_0 , symbolized by $S.\delta.semi\ \rho_0$ if for each $w\in V$, then $S._{semi}\, cl_\delta(\{w\})\subseteq V$, where $V\in S.\delta semi.O(C,\tau_C^S)$.

Definition 5.2: A supra space (C,τ_C^S) is termed supra δsemi ρ_1 , symbolized by $S.\delta.semi\ \rho_1$ if for any $w,z\in C$ with $S._{semi}\, cl_\delta(\{w\})\neq S._{semi}\, cl_\delta(\{z\})$, there are disjoint $O,V\in S.\delta semi.O(C,\tau_C^S)$ that contain $S._{semi}\, cl_\delta(\{w\})$ and $S._{semi}\, cl_\delta(\{z\})$, respectively.

Examples 5.3: In Example 3.20, we can see that (M,τ_M^S) is an $S.\delta.semi\ \rho_0$ and an $S.\delta.semi\ \rho_1$.

Theorem 5.4: Let (C,τ_C^S) be a supra space, if C is an $S.\delta.semi\ \rho_1$, then it is an $S.\delta.semi\ \rho_0$.

Proof: Assume (C,τ_C^S) is an $S.\delta.semi\ \rho_1$, $w\in C$ and O is an $S.\delta$semi. open set containing w. If there is no $p\notin O$ implies $O=C$, so $S._{semi}\, cl_\delta(\{w\})\subseteq O$. If there is $z\notin O$, then $w\notin S._{semi}\, cl_\delta(\{z\})$ hence $S._{semi}\, cl_\delta(\{w\})\neq S._{semi}\, cl_\delta(\{z\})$. By assumption there is a $S.\delta$semi. open set V such that $S._{semi}\, cl_\delta(\{z\})\subseteq V$ and $z\notin V$ implies $z\in S._{semi}\, cl_\delta(\{w\})$. Hence, $S._{semi}\, cl_\delta(\{w\})\subseteq O$, and we are the done.

Corollary 5.5: Let (C,τ_C^S) be a supra space, then C is an $S.\delta.semi\ \rho_1$ if and only if for any $w,z\in C$, $S._{semi}\, ker_\delta(\{w\})\neq S._{semi}\, ker_\delta(\{z\})$, there are $O,V\in S.\delta semi.O(C,\tau_C^S)$ such that $O\cap V=\phi$ and containing $S._{semi}\, cl_\delta(\{w\})$, $S._{semi}\, cl_\delta(\{z\})$, respectively.

Proof: Follows from Theorem 4.8.

Theorem 5.6: Let (C,τ_C^S) be a supra space, then C is an $S.\delta.semi\ \rho_0$ if and only if for any $w,z\in C$, $S._{semi}\, cl_\delta(\{w\})\neq S._{semi}\, cl_\delta(\{z\})$ implies $S._{semi}\, cl_\delta(\{w\})\cap S._{semi}\, cl_\delta(\{z\})=\phi$.

Proof: Let (C,τ_C^S) be an $S.\delta.semi\ \rho_0$, and let $w,z\in C$ with , $S._{semi}\, cl_\delta(\{w\})\neq S._{semi}\, cl_\delta(\{z\})$, hence there is $y\in C$ such that $y\in S._{semi}\, cl_\delta(\{w\})$ and $y\notin S._{semi}\, cl_\delta(\{z\})$ or vice versa. Thus, there is $O\in S.\delta semi.O(C,\tau_C^S)$ such that $z\notin O$ and $y\in O$, hence $w\in O$, and thus, $w\notin S._{semi}\, cl_\delta(\{z\})$. Thus, $w\in C/S._{semi}\, cl_\delta(\{z\})\in S.\delta semi.O(C,\tau_C^S)$ implies $S._{semi}\, cl_\delta(\{w\})\subseteq C/S._{semi}\, cl_\delta(\{z\})$, that means the intersection is empty and we are done.

Conversely: Let $w\in O\in S.\delta semi.O(C,\tau_C^S)$ and assume $z\notin O$, hence $z\notin S._{semi}\, cl_\delta(\{w\})$ implies $S._{semi}\, cl_\delta(\{w\})\neq S._{semi}\, cl_\delta(\{z\})$. By assumption we have $S._{semi}\, cl_\delta(\{w\})\cap S._{semi}\, cl_\delta(\{z\})=\phi$, hence $z\notin S._{semi}\, cl_\delta(\{w\})$ and hence $S._{semi}\, cl_\delta(\{w\})\subseteq O$.

Theorem 5.7: Let (C,τ_C^S) be a supra space, then C is an $S.\delta.semi\ \rho_0$ if and only if for any $w,z\in C$, $S._{semi}\, ker_\delta(\{w\})\neq S._{semi}\, ker_\delta(\{z\})$ implies $S._{semi}\, ker_\delta(\{w\})\cap S._{semi}\, ker_\delta(\{z\})=\phi$.

Proof: Let (C,τ_C^S) be an $S.\delta.semi\ \rho_0$ and $w,z\in C$ such that $S._{semi}\, ker_\delta(\{w\})\neq S._{semi}\, ker_\delta(\{z\})$.

Now, by Theorem 4.8 we have $S._{semi}\, cl_\delta(\{w\})\neq S._{semi}\, cl_\delta(\{z\})$, hence $S._{semi}\, cl_\delta(\{w\})\cap S._{semi}\, cl_\delta(\{z\})=\phi$ (Theorem 5.6). Assume there is $y\in S._{semi}\, ker_\delta(\{w\})\cap S._{semi}\, ker_\delta(\{z\})=\phi$, so $w\in S._{semi}\, ker_\delta(\{y\})$ (Definition 2.4) and by Theorem 5.6 we have $S._{semi}\, cl_\delta(\{w\})=S._{semi}\, cl_\delta(\{y\})$. Similarly, $S._{semi}\, cl_\delta(\{z\})=S._{semi}\, cl_\delta(\{y\})$ a contradiction, hence $S._{semi}\, ker_\delta(\{w\})\cap S._{semi}\, ker_\delta(\{z\})=\phi$.

Conversely: If $S._{semi}\, cl_\delta(\{w\})\neq S._{semi}\, cl_\delta(\{z\})$, then $S._{semi}\, ker_\delta(\{w\})\neq S._{semi}\, ker_\delta(\{z\})$ (Theorem 4.8) as a result it will be $S._{semi}\, cl_\delta(\{w\})=S._{semi}\, cl_\delta(\{z\})=\phi$. If $y\in S._{semi}\, cl_\delta(\{w\})\cap S._{semi}\, cl_\delta(\{z\})$, hence $w\in S._{semi}\, ker_\delta(\{y\})$ (Proposition 4.7) and hence $S._{semi}\, ker_\delta(\{w\})\cap S._{semi}\, ker_\delta(\{y\})\neq\phi$. Thus, $S._{semi}\, ker_\delta(\{w\})=S._{semi}\, ker_\delta(\{y\})$ and in the same manner $S._{semi}\, ker_\delta(\{y\})=S._{semi}\, ker_\delta(\{z\})$ a contradiction which leads to $S._{semi}\, cl_\delta(\{w\})\cap S._{semi}\, cl_\delta(\{z\})=\phi$ and by Theorem 5.6, (C,τ_C^S) is an $S.\delta.semi\ \rho_0$.

Theorem 5.8: For a supra space (C,τ_C^S), the following properties are equivalent:

1. (C,τ_C^S) is an $S.\delta.semi\ \rho_0$ space;

2. For any non-empty set K and $L \in \mathrm{S}.\delta semi.O(C,\tau_C^S)$ such that $K \cap L \neq \phi$, there is $M \in \mathrm{S}.\delta semi.O(C,\tau_C^S)$ such that $K \cap M \neq \phi$ and $M \subseteq L$;
3. Any $L \in \mathrm{S}.\delta semi.O(C,\tau_C^S)$, $L = \bigcup\{M \in \mathrm{S}.\delta semi.C(C,\tau_C^S)/M \subseteq L\}$;
4. Any $M \in \mathrm{S}.\delta semi.C(C,\tau_C^S)$, $M = \bigcap\{L \in \mathrm{S}.\delta semi.O(C,\tau_C^S)/M \subseteq L\}$;
5. For any $w \in C$, $\mathrm{S}._{semi}\, cl_\delta(\{w\}) \subseteq \mathrm{S}._{semi}\, ker_\delta(\{w\})$.

Proof: (1→2): Let K be a non-empty set of C and $L \in \mathrm{S}.\delta semi.O(C,\tau_C^S)$ such that $K \cap L \neq \phi$, so there is $w \in K \cap L$. Now, $w \in L \in \mathrm{S}.\delta semi.O(C,\tau_C^S)$, hence $\mathrm{S}._{semi}\, cl_\delta(\{w\}) \subseteq L$. And since $\mathrm{S}._{semi}\, cl_\delta(\{w\})$ is S. δsemi. closed (Theorem 2.12) then $M = \mathrm{S}._{semi}\, cl_\delta(\{w\})$ is the required set.

(2→3): Let $L \in \mathrm{S}.\delta semi.O(C,\tau_C^S)$, then clear that $\bigcup\{M \in \mathrm{S}.\delta semi.C(C,\tau_C^S): M \subseteq L\} \subseteq L$. Let w be any point in L, so (2) guarantees the existence of $M \in \mathrm{S}.\delta semi.C(C,\tau_C^S)$ such that $w \in M$ and $M \subseteq L$, therefore $L \subseteq \bigcup\{M \in \mathrm{S}.\delta semi.C(C,\tau_C^S)|M \subseteq L\}$. Thus, $L = \bigcup\{M \in \mathrm{S}.\delta semi.C(C,\tau_C^S)|M \subseteq L\}$.

(3→4): This is obvious.

(4→5): Let w be any point of C and $z \notin \mathrm{S}._{semi}\, ker_\delta(\{w\})$. There is $O \in \mathrm{S}.\delta semi.O(C,\tau_C^S)$ such that $w \in O$ and $z \notin O$, hence $\mathrm{S}._{semi}\, cl_\delta(\{z\}) \cap O = \phi$ (Proposition 4.7), and since $\mathrm{S}._{semi}\, cl_\delta(\{z\})$ is S. δsemi. closed (Theorem 2.12) then by (4) we have $\mathrm{S}._{semi}\, cl_\delta(\{z\}) = \bigcap\{L \in \mathrm{S}.\delta semi.O(C,\tau_C^S)|\mathrm{S}._{semi}\, cl_\delta(\{z\}) \subseteq L\}$, so there is $L \in \mathrm{S}.\delta semi.O(C,\tau_C^S)$ such that $w \notin L$ and since $\mathrm{S}._{semi}\, cl_\delta(\{z\}) \subseteq L$ consequently $\mathrm{S}._{semi}\, cl_\delta(\{w\})L = \phi$, hence $z \notin \mathrm{S}._{semi}\, cl_\delta(\{w\})$ and we are done.

(5→1): Let $L \in \mathrm{S}.\delta semi.O(C,\tau_C^S)$ and $w \in L$. Let $z \in \mathrm{S}._{semi}\, ker_\delta(\{w\})$, hence $w \in \mathrm{S}._{semi}\, cl_\delta(\{z\})$ (Proposition 4.7) and hence $z \in L$ which leads to $\mathrm{S}._{semi}\, ker_\delta(\{w\}) \subseteq L$. Thus, we have $w \in \mathrm{S}._{semi}\, cl_\delta(\{w\}) \subseteq \mathrm{S}._{semi}\, ker_\delta(\{w\}) \subseteq L$ implies (C,τ_C^S) is $\mathrm{S}.\delta.semi\ \rho_0$ space.

Corollary 5.9: For a supra space (C,τ_C^S), the following properties are equivalent:
1. (C,τ_C^S) is a $\mathrm{S}.\delta.semi\ \rho_0$ space;
2. $\mathrm{S}._{semi}\, cl_\delta(\{w\}) = \mathrm{S}._{semi}\, ker_\delta(\{w\})$ for all $w \in C$.

Proof: (1→2): By Theorem 5.8 we have $\mathrm{S}._{semi}\, cl_\delta(\{w\}) \subseteq \mathrm{S}._{semi}\, ker_\delta(\{w\})$ for each $w \in C$. Now, consider $z \in \mathrm{S}._{semi}\, ker_\delta(\{w\})$, then by Proposition 4.7 $w \in \mathrm{S}._{semi}\, cl_\delta(\{z\})$ and by Theorem 5.6 $\mathrm{S}._{semi}\, cl_\delta(\{w\}) = \mathrm{S}._{semi}\, cl_\delta(\{z\})$, hence $z \in \mathrm{S}._{semi}\, cl_\delta(\{w\})$ and hence $\mathrm{S}._{semi}\, ker_\delta(\{w\}) \subseteq \mathrm{S}._{semi}\, cl_\delta(\{w\})$.

(2→1): Follows directly from Theorem 5.8.

Theorem 5.10: For a supra space (C,τ_C^S), the following properties are equivalent.
1. (C,τ_C^S) is a $\mathrm{S}.\delta.semi\ \rho_0$ space;
2. $w \in \mathrm{S}._{semi}\, cl_\delta(\{z\})$ if and only if $z \in \mathrm{S}._{semi}\, cl_\delta(\{w\})$.

Proof: (1→2): Let $w \in \mathrm{S}._{semi}\, cl_\delta(\{z\})$ and O be any S. δsemi. open set that contain z, hence $\mathrm{S}._{semi}\, cl_\delta(\{z\}) \subseteq L$ and hence $w \in O$ implies $z \in \mathrm{S}._{semi}\, cl_\delta(\{w\})$.

(2→1): Consider $w \in O \in \mathrm{S}.\delta semi.O(C,\tau_C^S)$ and $z \notin O$, hence $w \notin \mathrm{S}._{semi}\, cl_\delta(\{z\})$, so by assumption $z \notin \mathrm{S}._{semi}\, cl_\delta(\{w\})$ implies $\mathrm{S}._{semi}\, cl_\delta(\{w\}) \subseteq O$. Thus, (C,τ_C^S) is $\mathrm{S}.\delta.semi\ \rho_0$.

Corollary 5.11: A supra space (C,τ_C^S) is $\mathrm{S}.\delta.semi\ \rho_0$ if and only if it's S. δsemi. symmetric.

Proof: Follows from Theorem 5.10 and Definition 3.11.

Theorem 5.12: Let (C,τ_C^S) be a supra space, the following properties are equivalent:
1. (C,τ_C^S) is $\mathrm{S}.\delta.semi\ \rho_0$;
2. For any S. δsemi. closed K, then $K = \mathrm{S}._{semi}\, ker_\delta(K)$;
3. For any S. δsemi. closed K and $w \in K$, then $\mathrm{S}._{semi}\, ker_\delta(\{w\}) \subseteq L$;
4. If $w \in C$, then $\mathrm{S}._{semi}\, ker_\delta(\{w\}) \subseteq \mathrm{S}._{semi}\, cl_\delta(\{w\})$.

Proof: (1→2): Follows from Theorem 5.8.

(2→3): Since $\{w\} \subseteq K$ implies $\mathrm{S}._{semi}\, ker_\delta(\{w\}) \subseteq \mathrm{S}._{semi}\, ker_\delta(K)$ and by (2) $\mathrm{S}._{semi}\, ker_\delta(K) = K$, hence $\mathrm{S}._{semi}\, ker_\delta(\{w\}) \subseteq K$.

(3→4): Since $S._{semi}\, cl_{\delta}(\{w\})$ is $S.\delta$semi. closed (Theorem 2.12) and $w \in S._{semi}\, cl_{\delta}(\{w\})$. Thus $S._{semi}\, ker_{\delta}(\{w\}) \subseteq S._{semi}\, cl_{\delta}(\{w\})$.
(4→1): Consider $w \in S._{semi}\, cl_{\delta}(\{z\})$, so by Proposition 4.7 $z \in S._{semi}\, ker_{\delta}(\{w\})$. But $S._{semi}\, cl_{\delta}(\{z\})$ is $S.\delta$semi. closed (Theorem 2.12) and by use (4) we have $S._{semi}\, ker_{\delta}(\{w\}) \subseteq S._{semi}\, cl_{\delta}(\{w\})$, hence $z \in S._{semi}\, cl_{\delta}(\{w\})$ implies (C, τ_C^S) is $S.\delta.semi\ \rho_0$ (Theorem 5.10).

6. Conclusions

The study of semi delta open sets of supra spaces has very interesting and useful for simulating many spaces that cannot meet all the conditions of topology. Therefore, in this work, we have introduced new concepts of supra topological spaces like $S.\delta$semi. open, $S.\delta$semi. symmetric and $S.\delta$semi. kernel which have actively contributed to achieving confirmed results foe several theorems, which in turn will serve as a starting point for forming new types of concepts carefully constructed and organized in patterns to simulate future applications. In this research, many results related to supra topology were obtained using sets $S.\delta$semi. open, including results about interior, closure and separation axioms. Two types of supra topological spaces were also presented, namely $S.t\delta.semi\ \rho_0$ and $S.t\delta.semi\ \rho_1$ and many equivalent properties were obtained in them. This will elevate the perspective on various applications, making them clearer than what has been previously studied in other fields.

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