IMAGING OF BINARY STARS IN THE PRESENCE OF DEFOCUSING

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Abstract

In this paper a simulation for a mathematical two dimensional model was conducted to assess the presence of defocusing due to telescope aperture quality. The point spread function (psf), Modulation transfer function (MTF), and Fourier frequencies of binary star are computed in the absence and presence of defocusing. The results show that the total frequency components of MTF decreases very sharply as defocusing values goes to 6 and after this value the total frequency changes very slightly afterward. The relative heights of the second to third peaks decrease rapidly as defocusing values goes to 3 and slightly changes after that.

تصوير النجوم الثنائية بوجود دالة التبأير

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الخلاصة

في هذه الدراسة، جرت عملية محاكاة لنموذج رياضي ثنائي الابعاد لتخمين وجود تأثير دالة التبأير بسبب جودة فتحة التلسكوب. تم دراسة دالة الانتشار النقطية ودالة التضمين لمصدر نقطي وكذلك المركبات الترددية في اطياف النجوم الثنائية بوجود وعدم وجود دالة التبأير.
دلت الدراسة على ان مجموع مركبات الترددات في دالة التضمين تتناقص بصورة اسية عندما يكون معامل دالة التبأير تقترب من ٦ وهذا السلوك يصبح ناعما بعد هذه القيمة.

Introduction

All telescopes have limitations to their diffraction limiting resolution because of the diffraction of light at the telescope aperture. This diffraction make the optical system to acts as a low-pass filter in the formation of the observed image by the telescope (the images become blurred). Each aperture has a certain diffraction limited cut-off frequency that depends on the shape and size of the aperture.

The theoretical resolving power of a 5 meters optical telescope using visible light of 450 nm is 0.023 arc sec. This value becomes much lower in the presence of defocusing.

There are many criteria that describing the performance of the quality of any optical imaging system. These criteria depend on the wavelength of the incident wave front and the diameter of the aperture of the imaging system . In addition to these criteria there are many others performances criteria such as the MTF measurements, *strehl* ratio that are a very well known in assessing the quality of the optical systems [1,2,3,4].

There are many papers in the literatures that tackle the problem, that concerning imaging with obstructing aperture [5, 6, 7, 8].

The shape and size of the aperture play an important role in any imaging system. Any error in this aperture yields severe limitations in the corresponding psf and MTF of a reference star. The aim of this paper is to assess the quality of psf and MTF of an imaging system and the frequency components of a binary star in the absence and presence of defocusing.

Theory

The essential equation for the formation of an image by any ideal optical imaging system is described by the following equations [10]:

$$g(x,y)=\iint_{-\infty}^{\infty} O(x',y')psf(x-x',y-y')dx'dy' \quad .....(1)$$

or it my be written in the form[10]:

$$g(x,y)= O(x,y)\otimes psf(x, y) \quad ....(2)$$

Where: g(x,y) is the two dimensional intensity distribution of the observed image, o(x,y) is the object intensity, and ⊗ denotes Convolution. Equation (2) describes the imaging system in Cartesian Coordinates. In Fourier Coordinates this equation becomes[10]:

$$G(u,v)=O(u,v).T(u,v) \quad ....(3)$$

where capital letters denotes Fourier transformation and G(u,v),O(u,v),and T(u,v) are complex Fourier transformation of the corresponding function given in equation(2) respectively. MTF is taken to be |T(u,v)| .

In general, the resolution of any imaging system is limited only by the luck of large optical elements that are free from inherent distortions. The wave aberrations in an optical system with circular pupil could be described by a weighted sum of Zernike polynomials [9]

$$Z_n^k(\rho,\theta)=N_n^k\, R_n^{|k|}(\rho)\cos(k\theta) \quad \text{for } k\geq 0, 0\leq\rho\leq 1, 0\leq\theta\leq 2\Pi$$
$$=N_n^k\, R_n^{|k|}(\rho)\sin(k\theta) \quad \text{for } k<0, 0\leq\rho\leq 1, 0\leq\theta\leq 2\Pi \quad ....(4)$$

For a given n, k takes values –n,-n+2,-n+4 ,…, n

N_n^k is the normalization factor

$$N_n^k=\sqrt{\frac{2(n+1)}{1+\delta mo}} \quad \delta_{ko}=1 \text{ for } k=0, \delta_{ko}=0 \text{ for } k\neq 0 \quad ....(5)$$

$R_n^{|k|}(\rho)$ is the radial polynomial

Where k and n denote the degree and order of the polynomial.

The solution of Zernik polynomial for field curvature defocus is given by [9] : $(2\rho^2-1)$.

Therefore the wave front of a point source at the aperture of optical imaging system takes the form[10]:

$$U(\eta,\gamma)=e^{k\phi(\eta,\gamma)} \quad ....(6)$$

Where: $\phi(\eta,\gamma)=(2\rho^2-1)$,η,γ are distances in the pupil function and ρ is given by:

$\rho=[(\eta-\eta_o)^2+(\gamma-\gamma_o)^2]^{1/2}$, η_o and γ_o are the coordinates of center of the aperture in a two-dimensional array.

The instantaneous psf of the entire imaging system is described by[10]:

$$psf(x,y)=|FT[H(\eta,\gamma)U(\eta,\gamma)]|^2 \quad ....(7)$$

Where $H(\eta,\gamma)$ represents the pupil function , FT denotes Fourier transform and MTF is given by:

$$MTF(u,v)=|FT[psf(x,y)]| \quad ....(8)$$

Simulations

Compute simulations are carried out by taking the pupil function H (η,γ) to be a two-dimensional circular aperture of radius R and of unity magnitude according to the following equation[10]:

$$H(\eta,\gamma)=\begin{cases} 1 & \text{if} \quad \rho\leq \mathbf{R} \\ 0 & \text{else where} \end{cases} \quad ....(9)$$

The size of this array is 512 by 512 pixels. This length is taken as large as possible in order to keep the theoretical diffraction limiting cut-off frequency vanishes to zero inside this array. The radius of the aperture is set to 100 pixels. This value is chosen to obtain as smoother as possible aperture.

Now consider an extremely small quasimono-chromatic point source located far away from the optical system.

In the absence of defocusing, the source would generate a plane wave falling on the aperture of the optical system, i.e. $U(\eta,\gamma)=1$.

In the presence of defocusing, $U(\eta,\gamma)$ takes the form that given in equation (6).

m is chosen to have values from 1 to 6. These values range from slight to severe defocusing.

Results and Discussion

Now after setting up all the functions and parameters that given in the theory. The psf and MTF are computed following equations (7) and (8) respectively. Fig.(1) demonstrate, the circular top hat function in the absence and presence of defocusing value (different values of m).

The intensity distribution, in the aperture, decreases very rapidly towards the center of the aperture. The half width of MTF becomes very narrow as m increases. This means a great loss in the Fourier frequency components.

The normalized frequency components of MTF could be calculated via:

$$N_f = \frac{1}{NM}\sum_{y=1}^{M}\sum_{x=1}^{N}\frac{MTF(u,v)}{MTF(0,0)} \quad \text{.......(10)}$$

Where MTF (0, 0) is The maximum value of the MTF (u,v) which is located at the middle of this array. Fig. (4) shows N_f as a function of m. It should be pointed out here that as m increases, the power of MTF falls very significantly as m approaches 6.

Now this study is extended towards assessing the quality of binary star in the presence of defocusing. This binary star is simulated by generating two dimensional Gaussian functions having the same magnitude. The width of each star is taken to be 3 pixels separated by a distance of 10 pixels. The size of array that having this binary star is 512 by 512 pixels. The center of the binary star is taken to be the center of this array.

Fig.(5) shows the Fourier Modulus of this binary star in the absence and presence of defocusing error. It is so clear that as m increases, the height of second and third peaks reduces significantly at m=4.

The normalized height of the second and third peaks beyond the central spike as a function of m is presented in Fig.(6).

Conclusions

The following conclusions could be drawn:

1- Introducing defocusing error on the aperture of any imaging system make the incoming wave front at exit pupil to have parabolic shape instead of a plane wave. The sharpness and smoothness of the parabolic function depend on the value of m as shown in Fig.(1)

2- The half width at half maximum of the distribution of the MTF that given in Fig.(4) is equal to 3. This means very clearly that the total frequency components of the MTF decreases very rapidly up to defocusing value m=3 and then its change becomes slightly smoother afterwards.

2- The first, second and third peaks beyond the center spike of the Fourier modulus of a binary star decrease very significantly as m approaches 3 and then slightly afterwards as shown in Fig.(6)

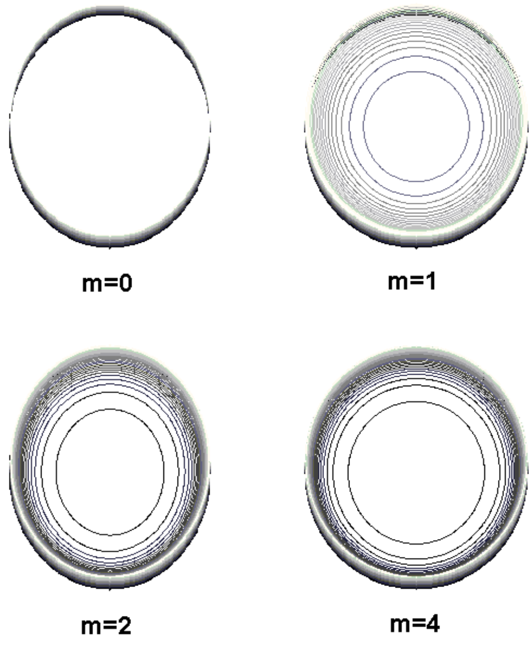

Figure 1: Circular top hat function at different values of m

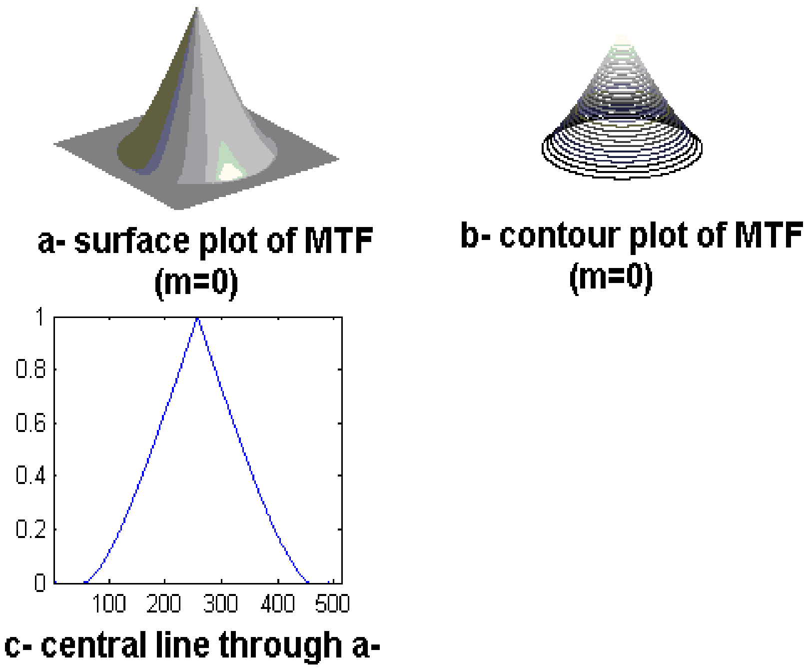

Figure 2: MTF in the absence of defocusing error (m=0).

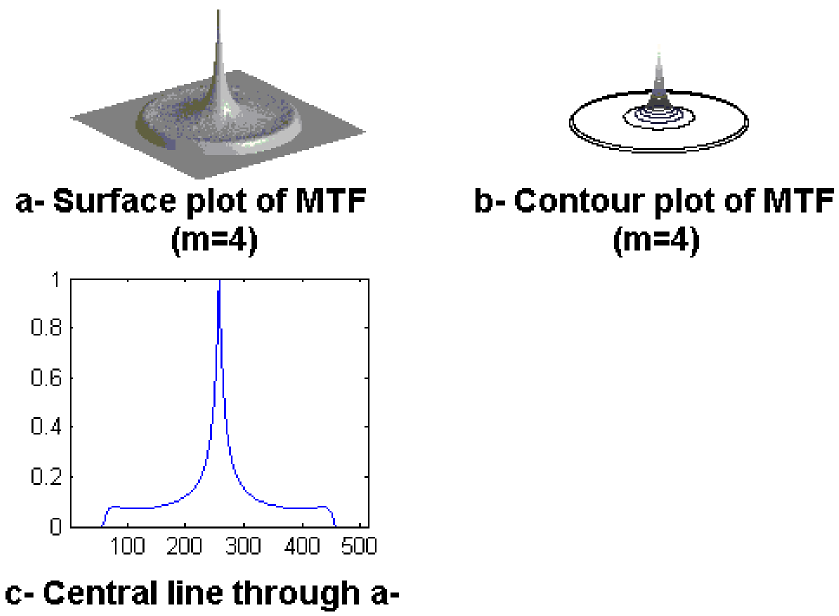

Figure 3: MTF in the presence of defocusing error (m=4).

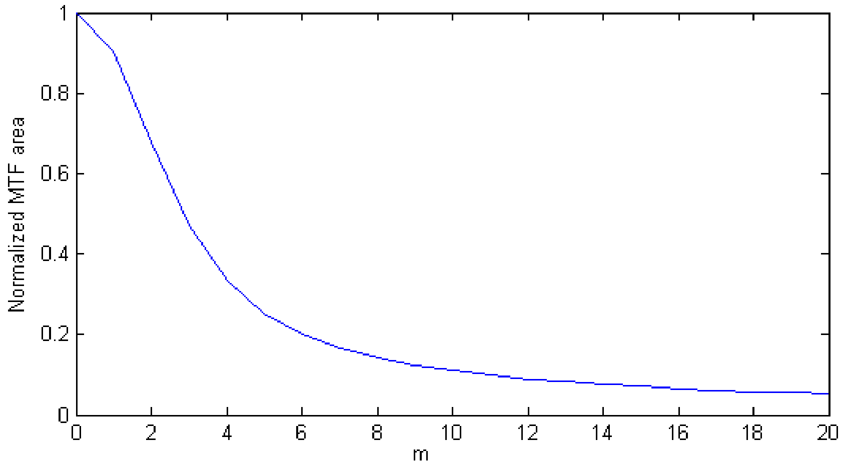

Figure 4: Normalized area under normalized two dimension MTF at different values of m.

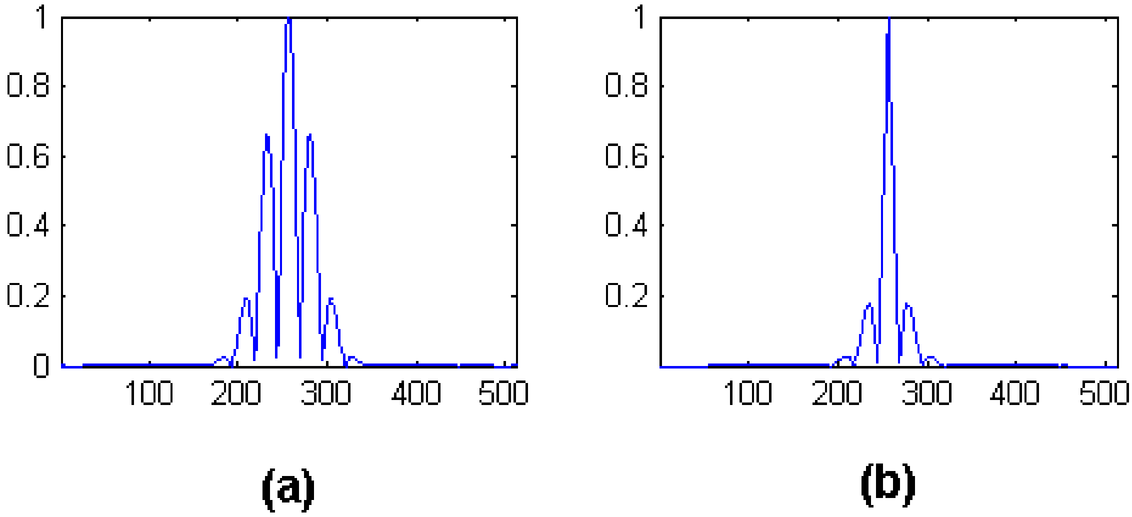

Figure 5: Binary star

a- Fourier modulus of binary star at m=0.

b- Fourier modulus of binary star at m=4.

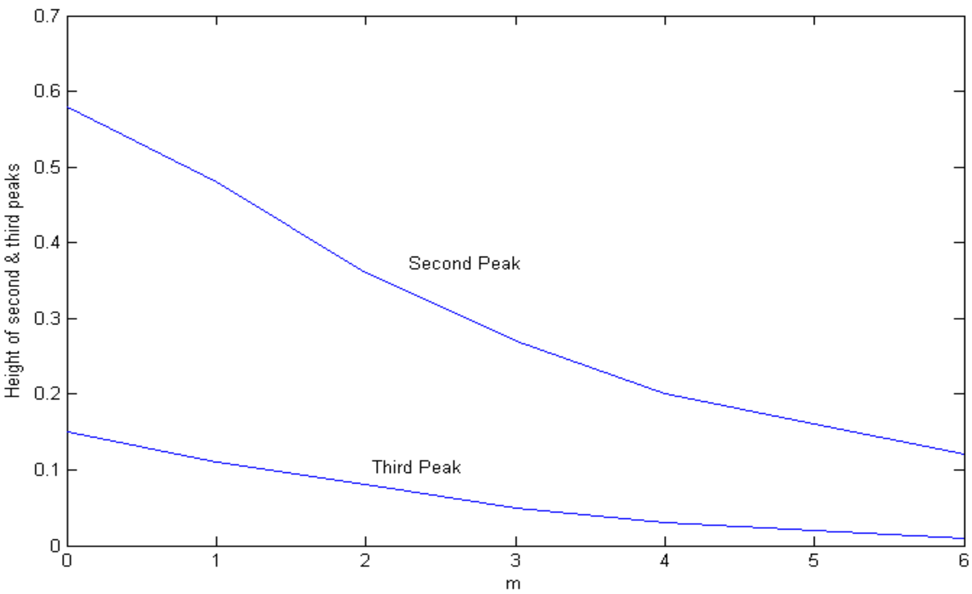

Figure 6: The height of the second and the third peaks of normalized MTF at different values of m.

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