ON GENERALIZED(θ, ϕ)- REVERSE DERIVATIONS OF PRIME RINGS

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Abstract

In this paper we will introduce the concept of (θ, ϕ) reverse derivation, generalized (θ, ϕ) reverse derivation and generalized left (θ, ϕ) derivation. Let R be a 2-torsion free prime ring and U a Lie ideal of R such that $u^2 \in U$, for all $u \in U$. The main result of this paper states that if F is a generalized (θ, θ) reverse derivation on R which also acts as a homomorphism or as anti- homomorphism on U, then either $d=0$ or $U \subseteq Z(R)$ Further, as an application of this result it is shown that, if every generalized left (θ, θ) derivation on R which also acts as a homomorphism or as anti- homomorphism on U, then either $d=0$ or $U \subseteq Z(R)$.

Key Words: Lie ideals, prime rings, left derivation, reverse derivation, left (θ, ϕ) derivation.

حول تعميم مشتقات- (θ, ϕ) المعكوسه للحلقات الاوليه

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الخلاصه

في هذا البحث سنقدم مفهوم كل من مشتقة (θ, ϕ) المعكوسه، تعميم مشتقة (θ, ϕ) المعكوسه وكذلك تعميم مشتقة (θ, ϕ) اليسرى.

لتكن R حلقة اوليه طليقة الالتواء من النمط ٢ و U مثالي لي بحيث $u^2 \in U$ لكل $u \in U$.

الهدف الرئيسي لبحثنا هو اذاكانت F هي تعميم مشتقة (θ, ϕ) المعكوسه على R بحيث تكون تشاكل او تشاكل ضد على U فاما $d=0$ او $U \subseteq Z(R)$.بالاضافه الى ذلك، كتطبيق على هذه النتيجه برهنا كل تعميم مشتقة (θ, ϕ) على R بحيث تكون تشاكل او تشاكل ضد فاما $d=0$ او $U \subseteq Z(R)$.

1. Introduction

Throughout the present paper, R will denote an associative ring with center $Z(R)$.For any $x, y \in R$, the symbol [x, y], will represent the commutator $xy-yx$. A ring R is said to be prime if for any $x, y \in R$, $xRy=0$ implies that either $x=0$ or $y=0$, [1]. A ring R is said to be 2-torsion free if whenever $2x=0$ with $x \in R$, then $x=0$, [1]. An additive subgroup U of R is said to be a Lie ideal of R if $[u, r] \in U$, for all $u \in U$, $r \in R$, [1]. The Lie ideal U verifies $u^2 \in U$ for all $u \in U$ such that $U \not\subset Z(R)$ is called admissible Lie ideal of R, [2]. An additive mapping $d: R \rightarrow R$ is called derivation if $d(xy)=d(x)y+xd(y)$ holds for all $x, y \in R$, [3]. Let θ, ϕ be endomorphisms of R. An additive mapping $d: R \rightarrow R$ is called (θ, ϕ) derivation if $d(xy)=d(x)\theta(y)+\phi(x)d(y)$, for all $x, y \in R$, [4]. Of course a (1,1)-derivation is a derivation on R, where 1 is the identity mapping on R. Inspired by definition of (θ, ϕ) derivation the notation of generalized (θ, ϕ) derivation was extended as follows: Let θ, ϕ be endomorphisms of R and S be a non-empty subset of R. An additive mapping F: R→R is called generalized (θ, ϕ) derivation on S if there exist a (θ, ϕ) derivation d: R→R such that F(xy)=F(x)θ(y)+ϕ(x)d(y), for all x, y∈S, [5]. An additive mapping d: R→R is called left derivation if

d(xy)=xd(y)+yd(x), for all x, y∈R, [6]. An additive mapping d: R→R is called left (θ, ϕ) derivation if

d(xy)=θ(x)d(y)+ϕ(y)d(x),

for all x, y∈R, [4]. Clearly, every left (1, 1) - derivation is a left derivation on R. Shaheen [6], introduced the concept of generalized left derivation as an additive mapping F: R→R, if there exist a left derivation d: R→R such that F(xy)=xF(y)+yd(x), for all x, y∈R.

Breŝar and Vukman [7], have introduced the notion of a reverse derivation as an additive mapping d from a ring R into itself satisfying d(xy)=d(y)x+yd(x), for all x, y∈R.

Obviously, if R is commutative, then both derivation and reverse derivation are the same.

Bell and Kappe [8], proved that if R is a semiprime ring and d is a derivation of R which is either an endomorphism or an anti-endomorphism, then d=0 Also, they are in the mentiond paper showed that if R is a prime ring and d is a derivation of R which acts as a homomorphism or an anti- homomorphism on a nonzero right ideal U of R, then d=0 on R. Further, Yenigul and Argac [9], obtained the above result for α-derivation on prime rings. Recently, Ashraf , Nadeem and Quadri [10], extended the result for (θ, ϕ) derivation in pime and semiprime rings. Ali and Kumar [5], extended the result for (θ, ϕ) derivations in prime rings.

We extend the above result by introducing the concepts of (θ, ϕ) reverse derivation, generalized (θ, ϕ) reverse derivation and generalized left (θ, ϕ) reverse derivation. We will make use of the following basic commutator identities without any speeific mention :

$[xy,z]=x[y,z]+[x,z]y,$

$[x,yz]=y[x,z]+[x,y]y.$

2. Prelimineries

Now we will introduce the following new definitions which are (θ, ϕ) reverse derivation, generalized (θ, ϕ) reverse derivation and generalized left (θ, ϕ) reverse derivation as follows

2.1 Definition:

Let S be a non-empty subset of R. An additive mapping d: R→R is called (θ, ϕ) - reverse derivation on S if

d(xy)=d(y)θ(x)+ϕ(y)d(x), for all x, y∈S. Clearly a (1, 1)-reverse derivation is a reverse derivation on R, where 1 is the identity mapping on R.

2.2 Definition:

Let S be a non-empty subset of R.An additive mapping F: R→R is called generalized (θ, ϕ) reverse derivation on S if there exist a (θ, ϕ) revese derivation d: R→R such that

F(xy)=F(y)θ(x)+ϕ(y)d(x), for all x, y∈S.

2.3 Example

Consider the ring $R=\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a,b\in Z\right\}$,

where Z denotes the set of integer numbers .

Define d: R→R by $d\left(\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\right)=\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$

Then d is (θ, ϕ) reverse derivation with the endomorphisms θ and ϕ of R which are defined by

$\theta\left(\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\right)=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ and

$\phi\left(\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\right)=\begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix}$

2.4 Example:

Consider the ring R as in Example (2.3). Define F: R→R by

$$F\left(\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\right)=\begin{bmatrix} -a & 0 \\ 0 & 0 \end{bmatrix}.$$

Then there exist (θ, ϕ) -reverse derivation d as in Example (2.3) .Thus, F is generalized (θ, ϕ) reverse derivation.

2.5 Definition:

Let S be a non-empty subset of R. An additive mapping F: R→R is called generalized left (θ, ϕ) derivation on S if there exist a left (θ, ϕ) derivation d: R→R such that

F(xy)=θ(x)F(y)+ϕ(y)d(x), for all x, y∈S.

2.6 Example:

In Example (2.4), it is easy to check that F is generalized left (θ, ϕ) derivation since there exists a left (θ, ϕ) derivation d which is defined as in Example (2.3).

We begin with following lemmas which are essential in developing the proof of our main results.

2.7 Lemma: [2]

If U$\not\subset$Z(R) is a Lie ideal of a 2-torsion free prime ring R and a, b$\in$R such that aUb=0, then a=0 or b=0.

2.8 Lemma: [11]

If R is a semiprime ring and U is a Lie ideal of R with u^2=0, for all u$\in$U, then U=0.

2.9 Lemma: [12]

Let R be a 2-torsion free prime ring and U be nonzero admissible Lie ideal of R. Then U contains a nonzero ideal of R.

3. Main results

3.1 Lemma:

Let R be a 2-torsion free prime ring and U a nonzero Lie ideal of R. Let θ, ϕ be automorphisms of R. If R admits a(θ, ϕ) reverse (i) derivation d such that d(u)=0, then d=0 or U$\subseteq$Z(R). (ii)

Proof:

We have d(u)=0, for all u$\in$U. This yields d[u, r]=0, for all u$\in$U, r$\in$R. Now using that fact d(u)=0, the above expression yields that

$$0 = d(r)\theta(u) + \phi(r)d(u) - d(u)\theta(r) - \phi(u)d(r)$$

$$= d(r)\theta(u) - \phi(u)d(r) \text{, for all } u\in U, r\in R. \quad (1)$$

Now, for any s$\in$R, replace r by rs in equation (1) and use equation (1) , to get

$$0 = d(rs)\theta(u) - \phi(u)d(rs)$$
$$= (d(s)\theta(r) + \phi(s)d(r))\theta(u) - \phi(u)(d(s)\theta(r) + \phi(s)d(r))$$
$$= d(s)\theta(r)\theta(u) + \phi(s)\phi(u)d(r) - d(s)\theta(u)\theta(r) - \phi(u)\phi(s)d(r)$$
$$= d(s)[\theta(r),\theta(u)] + [\phi(s),\phi(u)]d(r) \text{, for all } u\to U, r, s\in R. \quad (2)$$

Again replacing s by vs in equation (2), our hypothesis yield that

$$0 = d(vs)[\theta(r),\theta(u)] + [\phi(vs),\phi(u)]d(r)$$
$$= (d(s)\theta(v) + \phi(s)d(v))[\theta(r),\theta(u)] + \phi(v)[\phi(s),\phi(u)]d(r) + [\phi(v),\phi(u)]\phi(s)d(r)$$
$$= \phi(v)d(s)[\theta(r),\theta(u)] - \phi(v)d(s)[\theta(r),\theta(u)] + [\phi(v),\phi(u)]\phi(s)d(r)$$
$$= [\phi(v),\phi(u)]\phi(s)d(r) \text{, for all u, v}\in U, r, s\in R.$$

This implies that [v, u]Rθ^{-1}(d(r))=0. Thus the primeness of R implies that either [u, v]=0 or d(r)=0.

If [u, v]=0, for all u, v$\in$U, then it follows that [u, [u, rs]]=0. Since R is 2-torsion free , the above relation yields [u, r][u, s]=0, for all u$\in$U, r, s$\in$R.

Thus, [u, r]x[u, s]=[u, rx][u, s]=0, for all u$\in$U, r, s, x$\in$R, and hence [u, r]=0, for all u$\in$U, r$\in$R i.e, U$\subseteq$Z(R).

We are now well-equipped to prove our main results .

3.2 Theorem:

Let R be a 2-torsion free prime ring and U be a nonzero Lie ideal of R with $u^2\in$U, for all u$\in$U. Suppose that θ and ϕ are automorphisms of R and d: R→R is a (θ, ϕ) reverse derivation .

If d acts as a homomorphism on U, then either d=0 on R or U$\subseteq$Z(R).

If d acts as anti-homomorphism on U, then either d=0 on R or U$\subseteq$Z(R).

Proof:

Suppose that U$\not\subset$Z(R).

(i) If d acts as a homomorphism on U, then we have

$$d(uv) = d(u)d(v) = d(v)\theta(u) + \phi(v)d(u) \text{, for all u, v}\in U. \quad (1)$$

Replacing v by 2uv in equation (1) and using that fact R is 2-torsion free, we get

$$d(u)(d(v)\theta(u) + \phi(v)d(u)) = (d(v)\theta(u) + \phi(v)d(u))\theta(u) + \phi(u)\phi(v)d(u)$$

Using equation (1), the above relation yields that $(d(u) - \phi(u))\phi(v)d(u) = 0$, for all u, v$\in$U. and hence

$\phi^{-1}(d(u) - \phi(u))U\phi^{-1}(d(u)) = 0$, for all u$\in$U.

Hence by Lemma (2.7), either d(u)−ϕ(u)=0 or d(u)=0.

If d(u)−ϕ(u)=0, for all u$\in$U, then by replacing v by u in equation (1), we get d(u)θ(u)=0, for all u$\in$U.

Now, replace u by u+v, to get

$d(u)\theta(v) + d(v)\theta(u) = 0$,forall $u, v \in U$.

Replace u by 2vw and since R is 2-torsion free, we get

$$0 = d(w)\theta(v)\theta(v) + \phi(w)d(v)\theta(v) + d(v)\theta(v)\theta(w)$$

$= d(w)\theta(v)\theta(v)$, for all v, w∈U.

Replace w by 2wx in the last equation and since R is 2-torsion free, then

$0 = (d(x)\theta(w) + \phi(x)d(w))\theta(v)\theta(v)$

$= d(x)\theta(w)\theta(v^2)$, for all v, w, x∈U.

This implies that, $\theta^{-1}(d(x))Uv^2=0$, for all v, x∈U By Lemma (2.7), either $v^2=0$ or d(w)=0. If $v^2=0$, for all v∈U, then by Lemma (2.8), U=0 and this contradication. On the other hand if d(w)=0, for all w∈U, then by Lemma (3.1),we get the required result .

(ii) If d acts as an anti- homomorphism on U, then we have

$d(uv) = d(v)d(u) = d(v)\theta(u) + \phi(v)d(u)$,

for all u, v∈U. (2)

Replacing u by u^2 in equation (2), and by using equation (2), we get

$d(v)d(u^2) = d(v)\theta(u^2) + \phi(v)d(u^2)$. That is,

$d(v)(d(u)\theta(u) + \phi(u)d(u))$

$= d(v)\theta(u)\theta(u) + \phi(v)(d(u)\theta(u) + \phi(u)d(u))$

Then $(d(v) - \phi(v))\phi(u)d(u) = 0$, for all u, v∈U. (3)

Replacing v by u in equation (2) and using equation (3), we get

$0 = (d(v) - \phi(v))(d(u^2) - d(u)\theta(u))$

$= d(v)d(u^2) - d(v)d(u)\theta(u) - \phi(v)d(u^2) + \phi(v)d(u)\theta(u)$

$= d(u^2v) - d(uv)\theta(u) - \phi(v)d(u^2) + \phi(v)d(u)\theta(u)$

$= F(uv)F(u) - F(uv)\theta(u) - \theta(v)(F(u^2) - F(u)\theta(u))$

In view of equation (2), the last equation yields that

$0 = \phi(uv)d(u) - \phi(v)\phi(u)d(u)$

$= [\phi(u), \phi(v)]d(u)$, for all $u, v \in U$. (4)

By Lemma (2.9), U contains a nonzero ideal of R and hence

$0 = [\phi(u), \phi(I)]d(u) = [\phi(u), \phi(RI)]d(u)$

$= (\phi(R)[\phi(u), \phi(I)] + [\phi(u), \phi(R)]\phi(I))d(u)$

$= [\phi(u), R]R\phi(I)Rd(u)$

Since R is prime and U⊄Z(R), then the last equation implies that

$I\phi^{-1}(d(u)) = IR\phi^{-1}(d(u)) = 0$. Since R is prime and I≠0, then d(u)=0, for all u∈U and hence by Lemma(3.1) , we get the required result .

In the next Lemma, we explaine the relationship between (θ, ϕ) reverse derivation and left (θ, ϕ) derivation.

3.3 Lemma:

A mapping d on a 2-torsion free semiprime ring R is a (θ, ϕ) reverse derivation iff it is a left (θ, ϕ) derivation.

Proof:

Suppose that d is a (θ, ϕ) reverse derivation. Then

$d(xy^2) = d(y^2)\theta(x) + \phi(y^2)d(x)$

$= (d(y)\theta(y) + \phi(y)d(y))\theta(x)) + \phi(y)\phi(y)d(x)$;

that is

$d(xy^2) = d(y)\theta(y)\theta(x) + \phi(y)d(y)\theta(x) + \phi(y)\phi(y)d(x)$, for all x, y∈R. (1)

Also,

$d((xy)y) = d(y)\theta(xy) + \phi(y)d(xy)$

$= d(y)\theta(x)\theta(y) + \phi(y)(d(y)\theta(x) + \phi(y)d(x))$

that is

$d(xy^2) = d(y)\theta(x)\theta(y) + \phi(y)d(y)\theta(x) + \phi(y)\phi(y)d(x)$ (2)

From equation (1) and equation (2), we get $d(y)[\theta(x), \theta(y)]=0$, for all x, y∈R. (3)

Replacing x by z_1x in equation (3) and using equation (3) again, we get

$d(y)\theta(z_1)[\theta(x), \theta(y)] = 0$

That is

$d(y)z[\theta(x), \theta(y)]=0$, for all x, y, z∈R. (4)

A linearization of equation (3) on y leads to

$d(y)[\theta(x), \theta(u)] + d(u)[\theta(x), \theta(y)] = 0$

That is,

$d(y)[\theta(x), \theta(u)] = -d(u)[\theta(x), \theta(y)]$, for all x, y, u∈R. (5)

Replacing z by $[\theta(u), \theta(x)]zd(u)$ in equation (4) and using equation (5), we get

$0 = d(y)[\theta(u), \theta(x)]zd(u)[\theta(x), \theta(y)]$

$= -d(u)[\theta(x), \theta(y)]zd(u)[\theta(x), \theta(y)]$ (6)

Since R is semiprime, by (6), we get $d(u)[\theta(x), \theta(y)]=0$, that is, d(u)[r, s]=0, for all r, s, u∈R. By [3, Lemma 1.1.8], d(u)∈Z(R), for all

$u\in R$. Hence $d(xy)=d(y)\theta(x)+\phi(y)d(x)$

$$=\theta(x)d(y)+\phi(y)d(x)$$

This shows that d is left (θ, ϕ) derivation.

Conversely, if d is left (θ, ϕ) derivation .Then

$$d(xy^2)=\theta(x)d(y^2)+\phi(y^2)d(x)$$
$$=\theta(x)\theta(y)d(y)+\theta(x)\phi(y)d(y)+\phi(y)\phi(y)d(x) \quad (7)$$

On the other hand ,

$$d((xy)y)=\theta(x)\theta(y)d(y)+\phi(y)\theta(x)d(y)+\phi(y)\phi(y)d(x) \quad (8)$$

From equation (7) and equation (8), we get

$[\theta(x), \phi(y)]d(y)=0$, for all $x, y\in R$. (9)

Replacing x by xz_1 in equation (9) and using (9) again ,we get $[\theta(x), \phi(y)]\theta(z_1)d(y)=0$. That is $[\theta(x), \phi(y)]zd(y)=0$, for all $x, y, z\in R$. (10)

A linearization of equation (9) leads to $[\theta(x),\phi(y)]d(u)=-[\theta(x),\phi(u)]d(u)$, for all $x, y, u\in R$. (11)

Replacing z by $d(u)z[\theta(x), \phi(u)]$ in equation (10) and using equation (11) and since R is semiprime, we get

$[r, s]d(u)=0$, for all $r, s, u\in R$.

By [3,Lemma 1.1.8], $d(u)\in Z(R)$, for all $u\in R$. Hence

$$d(xy)=\theta(x)d(y)+\phi(y)d(x)$$
$$=d(y)\theta(x)+\phi(y)d(x)$$

This implies that d is (θ, ϕ) reverse derivation .

As an application of Theorem (3.2) and Lemma (3.3) we get the following result, which generalizes Theorem (4.2) in [4]:

3.4 Theorem:

Let R be a 2-torsion free prime ring and U be a nonzero Lie ideal of R with $u^2\in U$, for all $u\in U$. Suppose that θ and ϕ are automorphisms of R and $d: R\to R$ is a left (θ, ϕ) derivation .

(i) If d acts as a homomorphism on U, then either d=0 on R or $U\subseteq Z(R)$.

(ii) Ifd acts as anti-homomorphism on U, then either d=0 on R or $U\subseteq Z(R)$.

3.5 Theorem:

Let R be a 2-torsion free prime ring and U be a nonzero Lie ideal of R with $u^2\in U$, for all $u\in U$. Suppose that θ is an automorphism and $F: R\to R$ is a generalized (θ, θ) reverse derivation associated with (θ, θ) reverse derivation .

(i) If F acts as a homomorphism on U, then either d=0 on R or $U\subseteq Z(R)$.

(ii) If F acts as anti-homomorphism on U, then either d=0 on R or $U\subseteq Z(R)$.

Proof:

Suppose that $U\not\subset Z(R)$.

(i) If F acts as a homomorphism on U, then we have

$F(uv)=F(u)F(v)=F(v)\theta(u)+\theta(v)d(u)$, for all $u, v\in U$. (1)

Replacing v by 2uv in equation (1) and using that fact R is 2-torsion free , we get

$$F(u)(F(v)\theta(u)+\theta(v)d(u))$$
$$=(F(v)\theta(u)+\theta(v)d(u))\theta(u)+\theta(uv)d(u)$$

Using equation (1), the above relation yields that $(F(u)-\theta(u))\theta(v)d(u)=0$, for all $u, v\in U$, and hence

$\theta^{-1}(F(u)-\theta(u))U\theta^{-1}(d(u))=0$, for all $u\in U$. Hence by Lemma (2.7), either $F(u)-\theta(u)=0$ or $d(u)=0$.

If $F(u)-\theta(u)=0$, for all $u\in U$ then by replacing v by u in equation (1), we get $\theta(u)d(u)=0$, for all $u\in U$.

Now, replace u by u+v, to get

$\theta(u)d(v)+\theta(v)d(u)=0$, for all $u, v\in U$. Replace u by 2wv and since R is 2-torsion free, we get

$$0=\theta(w)\theta(v)d(v)+\theta(v)(d(v)\theta(w)+\theta(v)d(w))$$
$$=\theta(v)\theta(v)d(w), \text{ for all } v, w\in U.$$

Replace w by 2xw in the last equation and since R is 2-torsion free, then

$$0=\theta(v^2)(d(w)\theta(x)+\theta(w)d(x))$$
$$=\theta(v^2)\theta(w)d(x)\,\theta, \text{ for all } x,v,w\in U.$$

This implies that, $v^2U\theta^{-1}(d(x))=0$, for all $x, v\in U$. By Lemma (2.7), either $v^2=0$ or $d(x)=0$. If $v^2=0$, for all $v\in U$, then by Lemma (2.6), U=0 and this contradication. On the other hand if $d(x)=0$, for all $x\in U$, then by Lemma (3.1), we get the required result.

(ii) If F acts as an anti- homomorphism on U, then we have

$F(uv)=F(v)F(u)=F(v)\theta(u)+\theta(v)d(u)$, for all $u, v\in U$. (2)

Replacing u by u^2 in equation (2), and by using equation (2), we get

$$F(v)F(u^2)=F(v)\theta(u^2)+\theta(v)d(u^2)$$

That is,

$$F(v)(F(v)\theta(u)+\theta(u)d(u))$$
$$=F(v)\theta(u)\theta(u)+\theta(v)(d(u)\theta(u)+\theta(u)d(u))$$

Then $(F(v)-\theta(v))\theta(u)d(u)=0$, for all $u, v\in U$. (3)

Replacing v by u in equation (2) and using equation (3), we get

$$0=(F(v)-\theta(v))(F(u^2)-F(u)\theta(u))$$

$$=F(v)F(u^2)-F(v)F(u)\theta(u)-\theta(v)F(u^2)-\theta(v)F(u)\theta(u)$$

$$=F(u^2v)-F(uv)\theta(u)-\theta(v)F(u^2)+\theta(v)F(u)\theta(u)$$

$$=F(uv)F(u)-F(uv)\theta(u)-\theta(v)(F(u^2)-F(u)\theta(u))$$

In view of equation (2), the last equation yields that

$$0=\theta(uv)d(u)-\theta(v)\theta(u)d(u)$$

=[θ(u), θ(v)]d(u), for all u, v∈U. (4)

By Lemma 2.9, U contains a nonzero ideal of R and hence

$$0=[\theta(u),\theta(I)]d(u)=[\theta(u),\theta(RI)]d(u)$$

$$=(\theta(R)[\theta(u),\theta(I)]+[\theta(u),\theta(R)]\theta(I))d(u)$$

$$=[\theta(u),R]R\theta(I)Rd(u)$$

Since R is prime and U⊄Z(R), then the last equation implies that

$$I\theta^{-1}(d(u))=IR\theta^{-1}(d(u))=0\ .$$

Since R is prime and I≠0, then d(u)=0, for all u∈U and hence by lemma (3.1), we get the required result .

3.6 Corollary:

Let R be a 2-torsion free prime ring and U be a nonzero Lie ideal of R with u^2∈U,for all u∈U.Suppose that θ is an automorphism and F: R→R is a generalized left (θ, θ) - derivation associated with left (θ, θ) - derivation d such that F(xy)=F(yx), for all x, y∈R.

(i) If F acts as a homomorphism on U, then either d=0 on R or U⊆Z(R).

(ii) If F acts as anti-homomorphism on U, then either d=0 on R or U⊆Z(R) .

Proof:

Since F is a generalized left (θ, θ) derivation, then

$$F(xy^2)=F((xy)y)$$

$$=\theta(xy)F(y)+\theta(y)d(xy)$$

$$=\theta(x)\theta(y)F(y)+\theta(y)\theta(x)d(y)+\theta(y)\theta(y)d(x) \quad (1)$$

On the other hand, since F(xy)=F(yx), then

$$F(xy^2)=F((yx)y)$$

$$=\theta(yx)F(y)+\theta(y)d(yx)$$

$$=\theta(y)\theta(x)F(y)+\theta(y)\theta(y)d(x)+\theta(y)\theta(x)d(y) \quad (2)$$

Compare equation (1) and (2), we get

[θ(x), θ(y)]F(y)=0, for all x, y∈R.

As proof of Lemma (3.3), we get F(u)∈Z(R), for all u∈R.

This implies that F is a generalized (θ, θ) - reverse derivation associated with (θ, θ) - reverse derivation. By applying Theorem (3.5), we get the required result.

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