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A Study of Wear Articular Cartilage of Synovial Human Knee Joint Using Non-Newtonian Elastic Mathematical Model

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Abstract
In this paper, the wear in layers of articular cartilage was calculated, parameters effective on elastic deformation were studied in normal and diseased knee joints, and relations between elastic deformation and squeeze film characteristics under lubrication condition were discussed with using a mathematical model. Conferring to the results obtained, elastic deformation effects on the performance of synovial human knee joint were analyzed from medical and dynamics perspectives. Relationships between elastic deformation and wear of layers were also discussed.

Keywords: synovial fluid, elastic deformation, eccentricity ratio

دراسة تأكل الغضروف المفصلي لمفصل الركبه الزليلي باستخدام نموذج مطاطي غير نيوتوني

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الخلاصة
في هذه البحث ، تم حساب التآكل في طبقات الغضروف المفصلي ،و دراسة المعلمات المؤثرة على التشوه المرن في مفصل الركبة الطبيعي و المصاب .علاقة التشويه المرن بخصائص انضغاط الغشاء تحت شروط التزييت تم مناقشتها باستخدام موديل رياضي. أستناد إلى النتائج التي تم الحصول عليها فأن تشوه المرن يؤثر على أداء مفصل الركبة الزليلي البشري من الجانب الطبي وحركي. العلاقة بين التشوه المرن تأكل طبقات الغضروف تم مناقشتها .

1. Introduction

The knee joint is one of the most significant joints in the human body. It allows us to walk, run, and jump. The joint has the ability to bear body weight and perform a number of movements . Yet the knee joint is also one of our most flexible joints and allows a greater range of motion than all other joints in the body, except for the shoulder [1]. The primary function of the articular cartilage is to serve as the bearing material in the diarthrodial joint, transmitting loads while minimizing friction and wear .The friction coefficient of cartilage has been characterized extensively in the literature ,using standard measurements of normal and tangential forces action across a sliding interface . As deformation occurs, internal intermolecular forces arise consistent with applied force If the applied force is not too strong ,these forces may be sufficient to completely combat the applied force and allow the object to

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assume a new equilibrium state and to return to its original state when the load is removed .A larger applied force may lead to a constant deformation of the object or even to its constitutional failure [2]. The elastic action do not succumb any material deformation when shear stress is removed and is called purely viscous fluid .The shear stress depends only on the rate of deformation and not on the scope of deformation (strain). Properties viscous and elastic are called viscoelastic [3]. Normal joint surface is covered with a smooth layer of cartilage . When the surface of the cartilage is thin then this case is called osteoarthritis, which is caused by more wearing away (degradation) and less repair of the cartilage surface . There is a mechanical (wearing away) part of osteoarthritis and a biological (abnormal function) part of the disease[4]. Wear occurs when asperities (microscopic surface roughness) from opposing surfaces come into contact and deform ,resulting in removal of material [5] . The wear of the layer of the articular cartilage varies by the type of lubrication and load carrying capacity where hydrodynamic lubrication wear of superficial zone. In squeeze lubrication, reach wear to middle zone in late arthritis the damage reaches to deep zone. Increasing elastic deformation forms to protect the cartilage layers and reduce damage and early injury arthritis [6]. Gait cycle plays an important role in elastic deformation of articular cartilage, which is increased in the stance phase while reduced in the swing phase[7]. Previous studies indicated that corrosion is a result of the increase of friction and the reduction of elasticity. With time, it turns to wear damage the articular cartilage, where the rate of wear increases after 45 years of age. Several factors affect the increase of wear, including age, sex and work.

2. Basic Equation

Consider the flow of synovial fluid in the gap – shaped film in human knee joint with elastic deformation under different loads . The journal of radius R is rotating with the sliding motion U on the basis of the preceding assumption for the quasi- static squeeze thin film. The continuity equation and Navier- Stokes equations reduce to :

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu}\frac{\partial p}{\partial r}\delta - a^3 v \tag{2}$$

$$\frac{\partial^2 v}{\partial z^2} = \frac{1}{\mu}\frac{\partial p}{\partial r} \tag{3}$$

By integrating Navier- Stokes equation (3) twice with respect to z and using the boundary conditions of the tangential component of the fluid velocity in the film region, we have:

$$v = 0 \quad at \quad z = 0$$
$$v = 0 \quad at \quad z = h \tag{4}$$

The tangential component of the fluid velocity in the film region satisfies

$$v = \frac{1}{2\mu}\frac{\partial p}{\partial r}(z^2 - hz) \tag{5}$$

Substituting the tangential component of the fluid velocity in the film region equation (1.5) into the Navier- Stokes equations (2) gives

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{2\mu}\frac{\partial p}{\partial r}(2\delta - a^3 z^2 + a^3 hz) \tag{6}$$

By applying the no- slip condition on both surfaces radially, we have:

$$u = 0 \quad at \quad z = 0$$
$$u = 0 \quad at \quad z = h \tag{7}$$

By integrating (6) with the above condition , the component of the fluid velocity in the region becomes

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial r}\left[\delta z^2 - a^3\frac{z^4}{12} + a^3\frac{hz^3}{6} - \frac{\delta h}{2} - a^3\frac{h^3}{12}z\right] \tag{8}$$

Now we substitute the radial velocity of equation (8) into the continuity equation (1) and integrate across the film thickness .The boundary condition for velocity component $w(r,z)$ at the surfaces of the plate and sphere is :

$$w(r,0) = 0 \qquad and \qquad w(r,h) = \frac{\partial h}{\partial t} \tag{9}$$

Then the modified Reynolds equation governing the film pressure at film thickness is as follows:

$$\frac{\partial h}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\,.r\frac{1}{2\mu}\frac{\partial p}{\partial r}\,[g(\delta,h)] \tag{10}$$

$$\text{where } g(\delta,h) = \frac{\delta h^3}{3} - \ a^3\frac{h^5}{60} - \frac{\delta h^3}{4} \tag{11}$$

By introducing the non-dimensional variables and parameters [8], we have

$$\overline{p} = -\frac{p h_0^2}{\mu R \frac{\partial h}{\partial t}} \qquad \overline{\delta} = \frac{\delta}{r^2} \qquad \bar{h} = \frac{h}{r} \qquad \bar{r} = \frac{r}{R}$$

The modified Reynolds equation can now be written in a non-dimensional form as:

$$\frac{2}{\beta^2 R^2} = \frac{\partial}{\partial \bar{r}}\left[g\left(\delta,\bar{h}\right)\frac{\partial \bar{p}}{\partial \bar{r}}\right] \tag{12}$$

$$\text{where } g\left(\delta,\bar{h}\right) = R^5[\frac{\bar{\delta}}{12}\bar{h}^3 - \frac{a^3}{60}\bar{h}^5 \tag{13}$$

The boundary condition for the pressures in the film and bearing is

$\bar{p} = 0 \qquad at \qquad \bar{r} = 1$

$$\frac{\partial \bar{p}}{\partial \bar{r}} = 0 \qquad at \qquad \bar{r} = 0 \tag{14}$$

3. Squeeze – film characteristics

There are three types of squeeze film characteristics which are pressure distribution , load carrying capacity and friction force.

3.1 Pressure distribution

Integrating modified Reynolds equation with boundary condition film pressure is obtained as follows :

$$\overline{p} = \int_{\bar{r}}^{1} \frac{2\,\bar{r}}{g\left(\delta,\bar{h}\right)\beta^2 R^2} d\bar{r} = \frac{(1-\bar{r}^2)}{g\left(\delta,\bar{h}\right)\beta^2 R^2} \tag{15}$$

3.2 Load carrying capacity

Considering the pressure distribution on the journal during gait cycle (swing phase – stance phase) for normal and diseased humans , we can compute the load - carrying capacity by integrating pressure, as follows

$$w = 2\pi \int_0^1 a^3\, PR\, dr \tag{16}$$

Introducing the dimensionless quantity gives:

$$\overline{W} = -\frac{w\delta}{\mu u c R^2} \tag{17}$$

The dimensionless load carrying capacity is given by

$$\overline{W} = 2\pi \int_0^1 \bar{P}\, a^3 d\bar{r} \tag{18}$$

$$\overline{W} = \frac{240\,\pi a^3}{3\beta^2 \bar{h}^2 R^2(-\,\bar{\delta} + a^3\bar{h}^2\,)} \tag{19}$$

3.3. Friction force

Consider now friction $J = \mu \frac{\partial u}{\partial z}$, hence total friction F is

$$F = \int J\, dr = L \int \mu \frac{\partial u}{\partial z}\, dr \tag{20}$$

$$\frac{\partial u}{\partial r} = \frac{\partial p}{\partial r}\left(\frac{z}{\mu} - \frac{L}{2\mu}\right) + \frac{u}{e} \tag{21}$$

The frictional forces needed on two articular cartilage surfaces $z = h$ and $z = 0$ are

$$\frac{F}{L} = \int_0^2 (\frac{\partial p}{\partial r}\frac{L}{2} + \frac{u}{e}\mu)dr \tag{22}$$

$$\frac{F}{L} = \int_0^2 \frac{\partial p}{\partial \bar{r}}\frac{L}{2}\, d\bar{r} + \frac{u\mu\, R}{e}\, d\bar{r} \tag{23}$$

The dimensionless friction force is given by :

$$\bar{F} = \frac{Fc}{\mu uRL} \tag{24}$$

$$\bar{F} = \int_0^2 \frac{\partial}{\partial \bar{r}}\left(\frac{phc}{\mu uR}\,\frac{\bar{L}}{2}\right) d\bar{r} + \frac{1}{\epsilon}\, d\bar{r} \tag{25}$$

$$\bar{F} = \frac{0.025L^4\bar{r}}{R^2\beta^2\left(-2.5\times 10^{-19}\, h^3 + 0.0416667a\bar{h}^5\right)} + \frac{0.025L^4\bar{r}\,(-0.1\bar{r} + \frac{r^2}{2})}{R^2\beta^2(-2.5\times 10^{-19} + 0.0416667a\bar{h}^2)h^3} + \frac{\bar{r}}{\varepsilon}$$

Depending on the dimensionless friction force Coefficient of the friction, we can obtain :

$$C_f = \frac{\bar{F}}{N} \tag{26}$$

4. Results and discussion

On the basis of quasi-static squeeze film equations, this paper discusses the effectiveness of elastic deformation of articular cartilage on squeeze film characteristics in synovial human knee joint in the cases of normal and disease joint. We also determine wear layers of articular cartilage (superficial zone – middle zone – deep zone) in young and elderly.

4.1 Squeeze film Pressure

Figure-1 illustrates the dimensionless of the pressure distribution on layers of articular cartilage generated by squeeze film action during the normal walk, with different values of film thickness for (hydrodynamic – squeeze- elastohydrodynamic) lubrication. It was found that the dimensionless pressure distribution (p^*) increases with decreasing the value of film thickness parameters ($h_0^* = 3$). Inversely , increasing film thickness ($h_0^* = 5$) leads to decreasing the dimensionless pressure distribution. The percentage rate of increase in dimensionless pressure in the swing phase was approximately 5% at (a = 0.044,$\delta = 1.5\times 10^{-18}$), while it was found that the percentage rate of increase in dimensionless pressure (p^*) in the stance phase was approximately 80% at (a = 0.044).

The different dimensionless pressure distribution as a function of radius of sphere for different values of elastic deformation is seen in Figure -2. It is observed that when elastic deformation is small , it implies that the hydrodynamic pressure between the articular cartilages is very high. This is protect for layers of articular cartilage (superficial zone – middle zone – deep zone), when low hydrodynamic pressure inside the gap leads to increasing elastic deformation of the superficial zone in the healthy knee joint. Table -1 shows the relationship between pressure distribution and elastic deformation.

In Figure-3, it is shown that the normal articular cartilage of knee joint has higher water content. This provides an increased stiffness and lower hydraulic permeability to knee cartilage. The result is a higher pressure distribution that could protect knee cartilage from continuous different movements. It was determined that the percentage rate of increase in dimensionless pressure was approximately

(80% at δ= 1.5$\times 10^{-18}$ in knee joint's synovial fluid of patients suffering from OA. The apparent synovial permeability (SP) to each protein was calculated and the results showed that the normal synovia were significantly more permeable (δ = 1.5$\times 10^{-18}$) than the osteoarthritic synovia. The relation between size pore of articular cartilage and pressure is shown in Figure-4 .When pore size was smaller, the comb pressure at the cartilage edge was high and, therefore, the synovial fluid in the synovial cells flows freely. The percentage rate of increase in dimensionless pressure was approximately 80% at $\beta = 0.01$ in the stance phase, while the swing phase showed a decreased dimensionless pressure which was approximately 16% at $\beta = 0.05$.

Table 1-Relationship between elastic deformation and pressure distribution in normal walk.

Hydrodynamic lubrication		Squeeze lubrication	
Elastic	Pressure	Elastic	Pressure
0.032	1.5	0.032	285.5
0.046	1.04	0.046	198.6
0.055	0.87	0.055	166.1
0.058	0.82	0.058	157.5
0.061	0.78	0.061	149.8
0.064	0.75	0.064	142.7
0.067	0.71	0.067	136.4
0.069	0.69	0.069	132.4

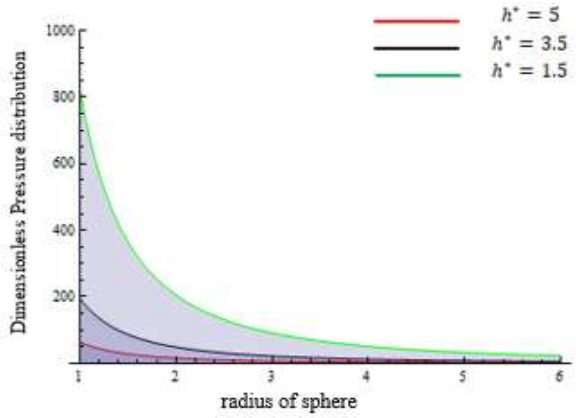

Figure 1-Variation of dimensionless pressure distribution with radius of sphere for different values of film thickness

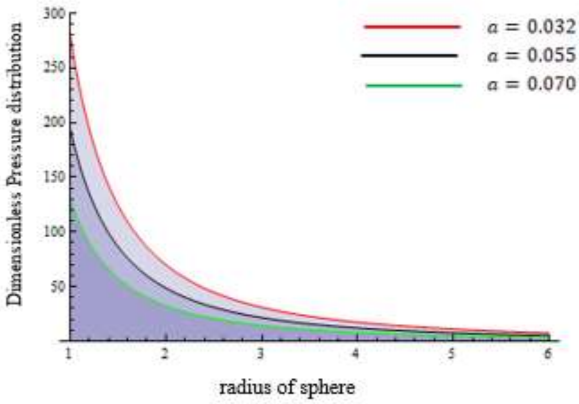

Figure 2-Variation of dimensionless pressure distribution with radius of sphere for different values of elastic deformation

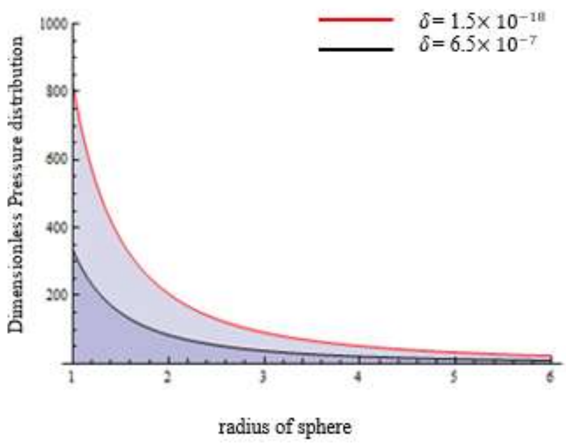

Figure 3 Variation of dimensionless pressure distribution with radius of sphere for different values of permeability deformation

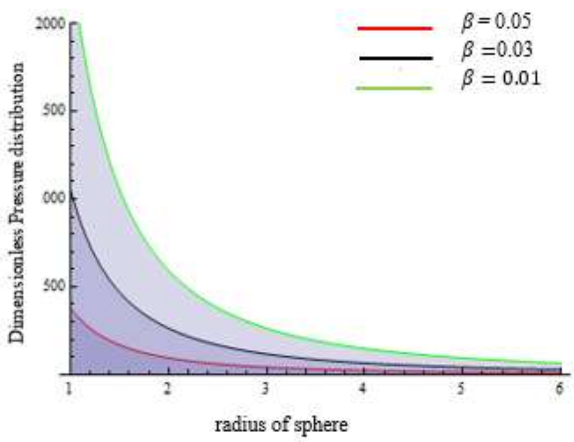

Figure 4-Variation of dimensionless pressure distribution with radius of sphere for different values of pore size

4.2 Load-carrying capacity

Figure-5 shows the effects of film thickness on the load carrying capacity for the radius of the sphere .The effect of film thickness parameter h^* signifies a growing load carrying capacity especially for small values of h^* =1.5 ,$t = 1.4s$ compared with that of the film thickness of the hydrodynamic which was h^* =5,$t = 1.4\ s$. The effects of pore size and porosity on the ability of the articular cartilage to bear different weights is shown in Figure-6. Size pore of the articular cartilage in the swing phase is large since the load on the joint is low. Therefore, the load carrying capacity was approximately 5% at a = 0.055, when the foot touch the earth start stance phase and increased load on lower joint that leads to pore diameters become smaller where reach to ($\beta = 0.01$ in toe off and the load caring capacity was approximately 60% at a = 0.055 . The relationship between elastic deformation and load through cycle time is shown in Figure-7. The loads originates from the main body weight which doubles during movement (normal walk and run), which leads increase the elasticity of the layer of the articular in the stance Phase where strain is high, while it is decreased in the swing phase where strain is 10^{-3} , indicating a positive relationship between the load and elasticity.

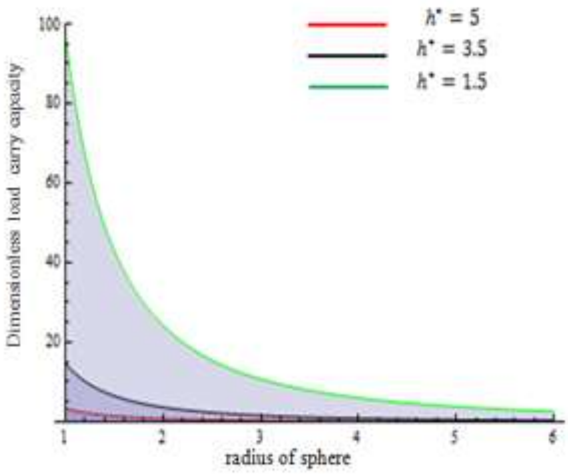

Figure 5 -Variation of dimensionless load carrying capacity with radius of sphere for different values of film thickness

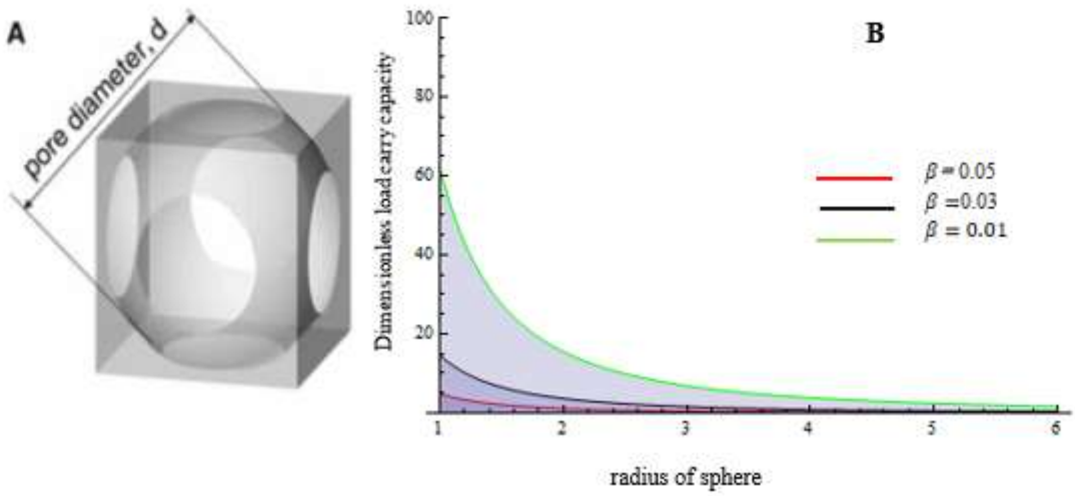

Figure 6 (-) pore diameter (B) Variation of dimensionless load carrying capacity with radius of sphere for different values of pore size

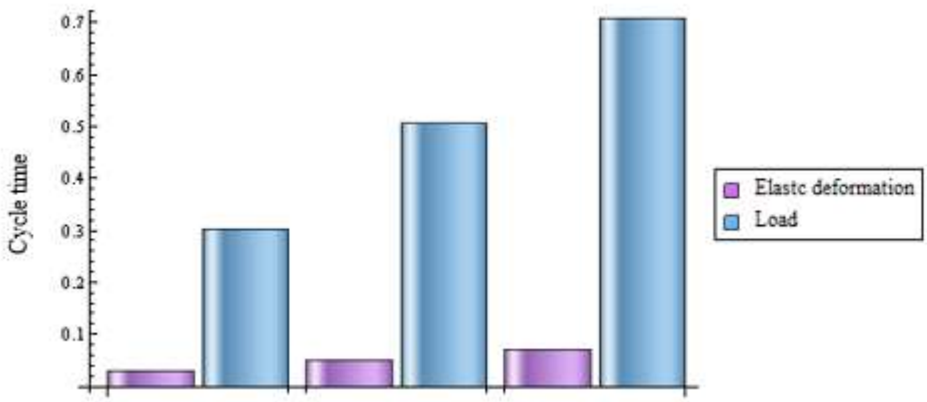

Figure 7-Effective of cycle time on elastic deformation and load

Parameters effective on elastic deformation during motion (walk , run, jump) with different loads are showen in Figure-8. One parameter is radius that changes in stages of human life. accompanied with an increaisng elastic deformation. This increase starts as low in childhood and youth stages , while, when a person reaches to a higher aging stage, the curvature increases and deformation is in its highest level. Elastic modules are two parameter that are effective on the deformation. When the elastic modules value is $E = 10^7$ then the percentage of deformation is 35%, while when the elastic modules $is\ E = 10^9$ then the percentage of deformation is72%.

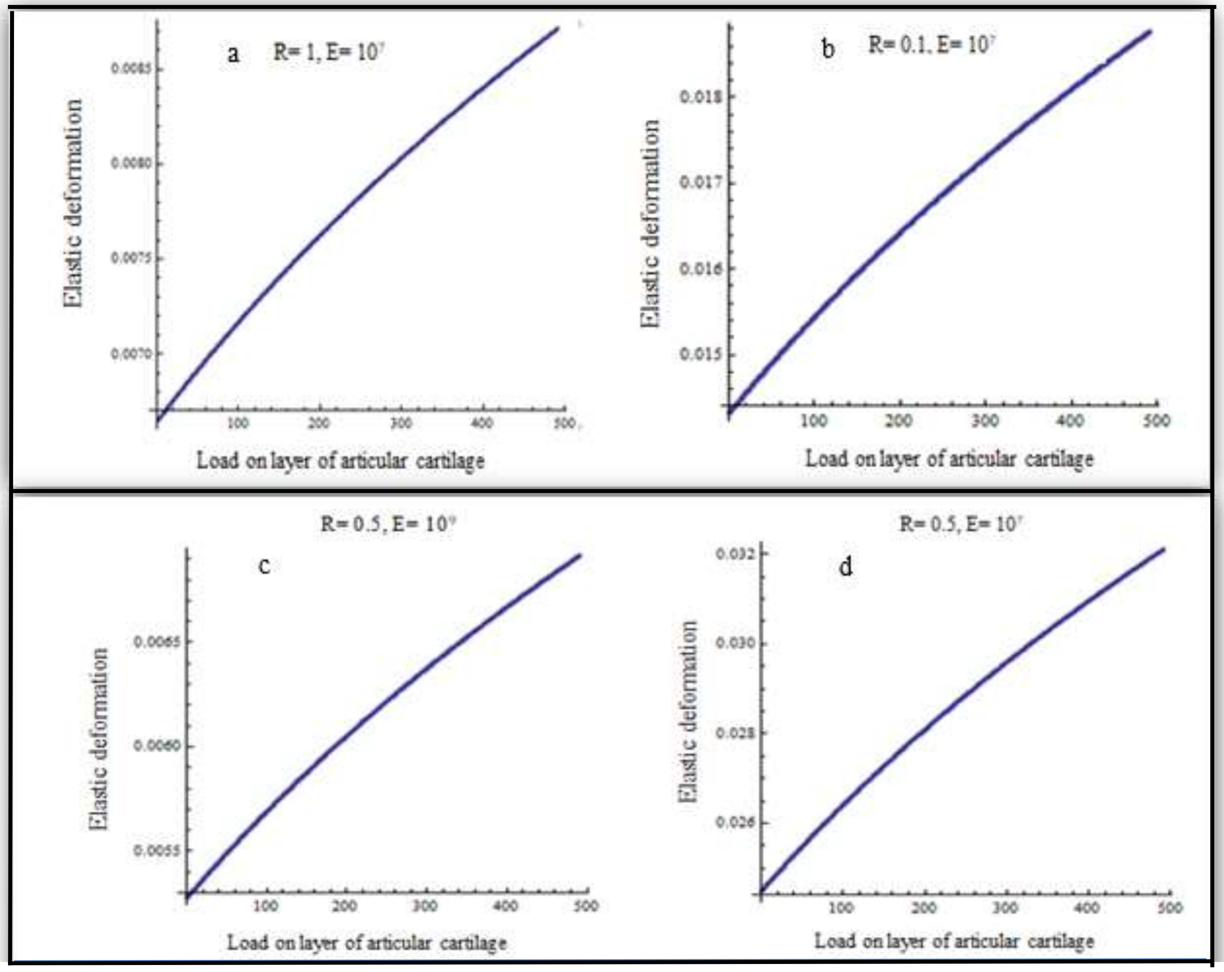

Figure 8-Parameters effective on elastic deformation with different loads . (a) radius of articular cartilage in elastic-hydrodynamic lubrication , (b) radius of articular cartilage in hydrodynamic, (c) elastic module in stance phase (d) elastic module in swing phase

4.3. Friction

Friction force and film thickness of the (hydrodynamic–squeeze- elastohydrodynamic) lubrication are shown in Figure-9. Parameters effective on elastic deformation with different loads . (a) radius of articular cartilage in elasto-hydrodynamic lubrication , (b) radius of articular cartilage in hydrodynamic, (c) elastic module in stance phase (d) elastic module in swing phase.

In Figure-10, When film thickness between the articular cartilages becomes sufficiently high, i. e $h > 2,$ friction tends towards being produced after 3 cycle time, i.e. 4.2 s. After 5 cycle time, it was observed that film thickness became lower, which resulted in a high friction force that reached 50.55% when h < 1.1.

In Figure-10, effective of lubricton and elastic deformation on friction force are shown. The elasto deforimation was different with respect to lubricton of knee jont in the hydrodynamic lubrication. It was observed that when the elastic deformation is small, then the value of friction force was low. Lubrication (hydredynamic- squeeze) transformed to elasto-hydrodynamic where velocity flow synovial fluid and load was played main in lubrication. Value of elasto deforimation been large compared to its previous values thus friction force be large.seen Table-(2, 3)

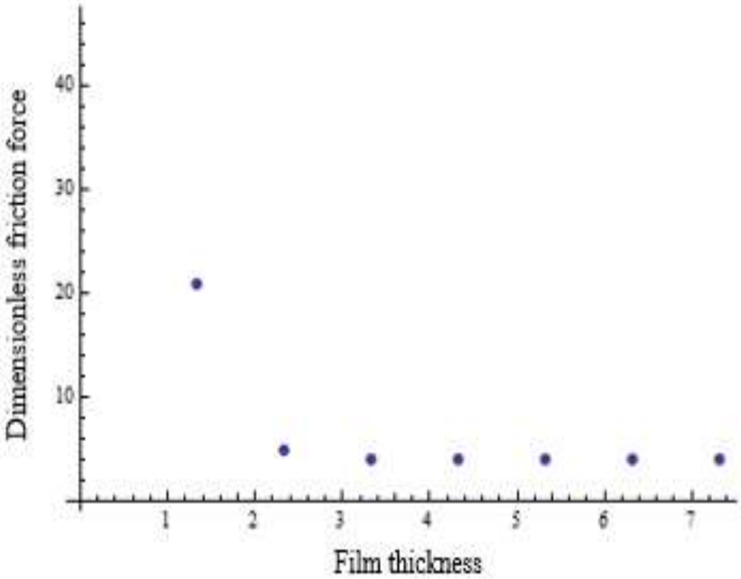

Figure 9 -Relationship between dimensionless friction force and film thickness of type lubrication

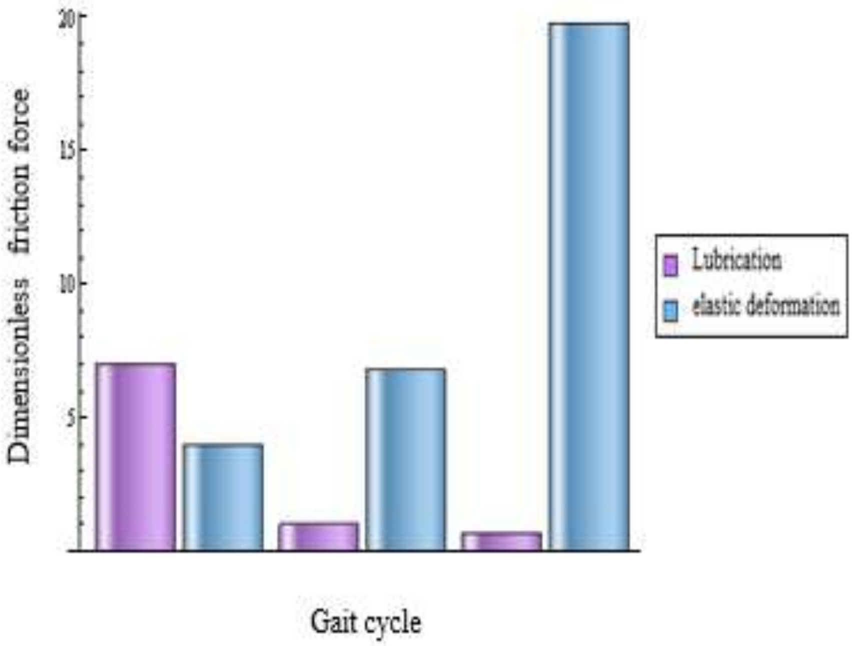

Figure 10 -Effective lubrication e & elastic defamation on dimensionless friction force

In Figure-11 influences of pore size and elastic deforimation are showen. The pore size of the articular cartlilage is one of the most important adsorbent parameters that determine the ability of the molecules of fluid to flow through tissues. When molecules of fluid are smaller than pore size, then pressure and load carrying capacity become higher, while friction force and elasto deforimation become lower. With changing patterns of movement and aging, pore size becomes bigger than volume of molecules. The result that friction force appeared higher in comparison with the elasticity

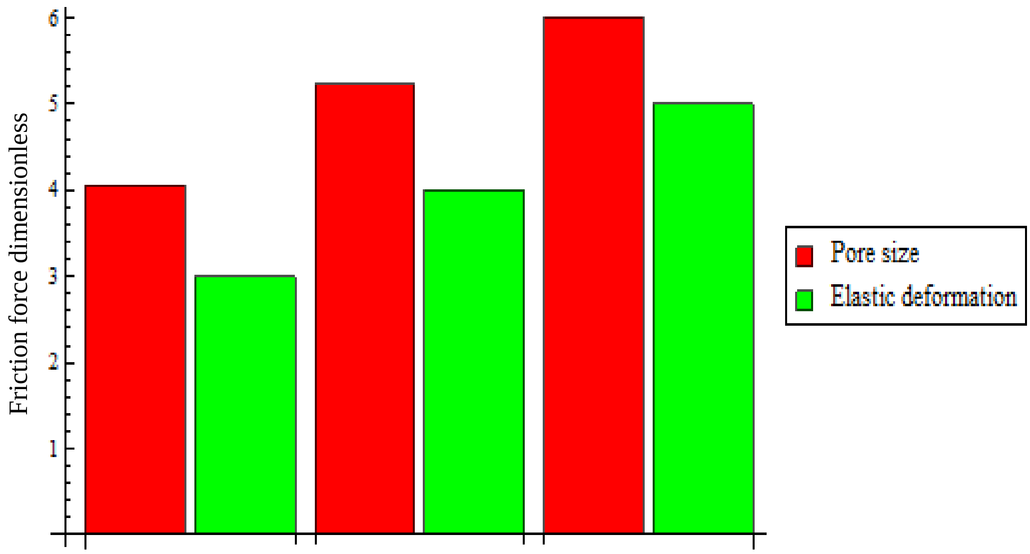

Figure 11 -Effective pore size & elastic defamation on dimensionless friction force

Table 2- Variation in friction force with different value of film thickness and elastic deformation

Film thickness	Friction force	Cycle time	Elastic deformation
Hydrodynamic lubrication			
7	4.00012		0.070
6	4.00027		0.060
5	4.00066		0.050
Squeeze lubrication			
3	4.00849		0.055
2	4.0644		0.040
Elasto hydrodynamic lubrication			
1	6.0625		0.044
0.7	16.2717		0.030

4.4. Coefficient of friction

The relationship between the weight of a person and the coefficient of friction with different age stages is shown in Figure-12. In the childhood phase, the weight range was between 10 and 33 with different coefficients of friction respect to lubricant we found rang increasing coefficient of friction [28%-42%]. Friction coefficients are affected by increased weight in the young age, but this effect is simple on the joint because of the presence of lubricants that reduce friction. This situation changes with progresses in age where weight increases and friction factors have a significant impact on cartilage.

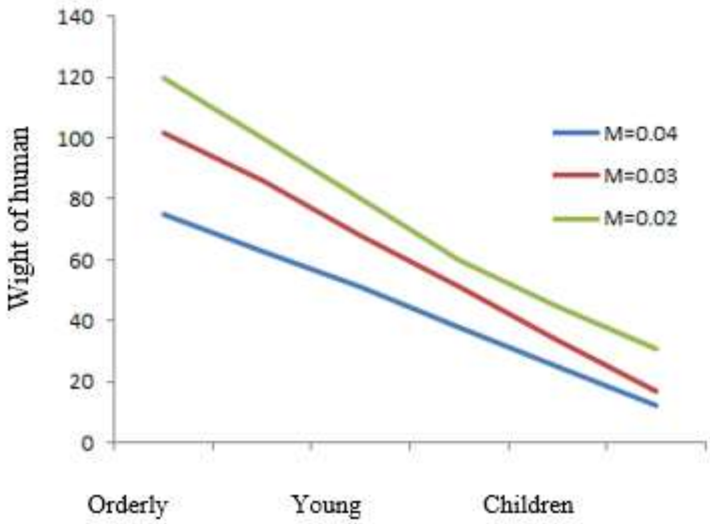

Figure 12-Variation of Friction force with age of human for different values of Coefficient of friction

4.5 Wear of the layer of articular cartilage

Classification of joints, i.e. normal or injured (Osteoarthritic), depends on the rate of wear. The parameters affecting wear are the load carrying capacity and friction. High load carrying capacity reduces the wear of the of layer articular cartilage (superficial zone, subsurface zone, middle zone)).Corrosion is transmitted from middle zone to deep zone when low load carrying capacity ,plays elastic deformation and friction force an important role in increasing wear articular cartilage for both sexes , in case Osteoarthritic.

Table 3-wear layer of articular cartilage

Hydrodynamic lubrication ,F=10 N, E=[0.070-0.050],superficial zone			
Load carrying capacity	thickness of layer cartilage	Case of joint	Wear of articular cartilage
100	10	Normal	Very low
80	8	Normal	Very low
60	6	Normal	Low
40	4	Normal	Low
20	2	Osteoarthritic	High
10	1	Osteoarthritic	Very high
Squeeze lubrication ,F=20 N, E=[0.050-0.040], subsurface zone			
Load carrying capacity	thickness of layer cartilage	Case of joint	Wear of articular cartilage
100	5	Normal	Very low
80	4	Normal	Very low
60	3	Normal	Low
40	2	Normal	Low
20	1	Osteoarthritic	High
10	0.5	Osteoarthritic	Very high
Elasto- Hydrodynamic lubrication ,F=30 N, E=[0.044-0.030], middle zone			
Load carrying capacity	thickness of layer cartilage	Case of joint	Wear of articular cartilage
100	3.33	Normal	Very low
80	2.66	Normal	Low
60	2	Osteoarthritic	Low
40	1.33	Osteoarthritic	Low
20	0.666	Osteoarthritic	Very high
10	0.333	Osteoarthritic	Very high

7. Conclusions

When film thickness is sufficiently high, pressure generated between articular cartilages will reduce surfaces contact, the labor of the film thickness of lubrication on improving the performance of the joint when permeably value is small. Under the conditions studied, the elastic deformation of the surface in a normal knee joint becomes low. This is related to high pressure and small size of pores through which the synovial fluid flows. Load carrying was divided to two phases (stance swing phase). It was found that film thickness and pore size in the stance phase become very low, to protect the joint from damage then increasing load carrying capacity marked swing phase. It was found that the radius of shape and elastic deformation have significant influences on the performance of knee joint. It was found that elastic deformation with high radius and lubricant reduces cartilage erosion. Friction force appears more important and effects on the safety of articular when decreased film thickness. With progress in age, an increase in body weight occurs, especially with females as compared to males as a result (sick, less move) that associate with increasing coefficient of friction. The wear of the layer of the articular cartilage varies by the type of lubrication and load carrying capacity. Where wear in hydrodynamic lubrication in superficial zone, in squeeze lubrication reach wear in middle zone in artitis the damage reaches to deep zone. Elastic deformation low protect the cartilage layers and reduces damage and early injury artitis

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