Solving of fuzzy Multiobjective Optimization Problem Using The Weighting Method

Osama H.Mohammed*,Fadhel S.Fadel, Iraq Tariq Abbas*****
(*) and (**) Al-Nahrain University,College of Science , Dept. Math. & Comptr. Appl.,Baghdad ,Iraq.
(***)Baghdad University, College of Science,Dept. Math., Baghdad ,Iraq.
iraqtariq74@yahoo.com.

Abstract:
In this paper fuzzy multiobjective optimization problem with or without constraints is presented. The weighted method is considered to formulate the fuzzy multiobjective problem as a fuzzy singleobjective function and then Iskandars' approach is used to solve the resulting fuzzy optimization problem.

Keyword: Fuzzy sets, Multiobjective linear programming.

حلول المسائل الامثلية المتعددة الاهداف الضبابية بأستخدام طريقة الاوزان

اسامة حميد،فاضل صبحي،عراق طارق
جامعة النهرين- كلية العلوم،جامعة النهرين-كلية العلوم،جامعة بغداد –كلية العلوم-قسم الرياضيات،بغداد- العراق.

الخلاصة
في هذا البحث تم تقديم المسائل الامثلية المتعددة الاهداف الضبابية مع القيود.وقد تم اعتبار طريقة الاوزان لصياغة مسائل الامثلية المتعددة الاهداف الضبابية كدالة هدف ضبابية منفردة وتم استخدام اسلوب اسكندرس لتحويل مسائل النمذجة الخطية الضبابية الى ما كافئها في الحالة الاعتيادية ومن ثم تم حل المسائل الناتجة بأستخدام طريقة ملائمة من طرق النمذجة الخطية

Introduction

Optimization is an important activity in many fields of science and engineering. A lot of modeling ,design ,control and decision making problems can be formulated in terms of mathematical optimization .The classical frame work for the optimization is the minimization (or maximization)of the objectives ,given the constraints for the problems to be solved .

Many design problems however ,are characterized by multi-objectives .The first note on multi-criterion optimization was given by Pareto ; since ,then the determination of the compromise set of a multi-objective problem is called Pareto optimization [1,2].The engineer often encounters a problem in develop-ment of a precise mathematical model of the systems.

Vagueness and impreciseness often arise due to poorly defined data, system boundaries, and unsatisfactory formulation of design objective and in ability in evaluating the relative importance between the objectives. As the complexity of the systems increases, more assumption are made about its behavior and hence, the ability of the engineering to exactly model the systems in the precise mathematical terms is severely hampered [3].

To model vagu and imprecise nature of the design problem ,one has to use the fuzzy set theory [3,4].The fuzzy set theory was originally developed by professor Zadeh and is a beautiful way of describing a natural condition in optimization mathematically .Since ,the appearance of the fuzzy theory ,some researches and applications have accomplished ,such as [3,5].

In one formulation of fuzzy optimization due to Zimmerman[6],concepts from Bellman and Zadeh model of decision making[7]are used for formulating the fuzzy optimization problem .The linear fuzzy membership function[8,9]and non linear fuzzy membership function[10,11]for structural optimization are used to represent the fuzzy nature of failure .Rao presented a formulation for the fuzzy optimization of engineering systems involving multiple objective[3].

In this paper, we shall use the weighting method in order to transform the muti-objective linear programming problem to singleobjective ones and then the transform problem will be solved by the technique given by Iskander's [12].

Weighting Method:-

In the weighting method the multiobjective linear programming problems will be transformed to an equivalent singleobjective linear programming ones [5], in which the problem:

$$\text{Maximize } Z(\underline{X}) = [\,Z_1(\underline{X}), Z_2(\underline{X}), \ldots, Z_k(\underline{X})\,]$$

$$\text{Subject to: } g_i(\underline{X}) \le 0 \qquad i=1, 2\ldots.I$$

$$\underline{X} = (x_1, x_2, \ldots. x_n)$$

Will transformed to:

$$\text{Maximize } z^* = \sum_{i=1}^{k} w_i Z_i(\underline{X})$$

Subject to:

$$g_i(\underline{X}) \le 0 \qquad i=1, 2\ldots.I$$

Where w_i are called the rational weighed for the objective function z_i, and the set of all feasible solutions will be obtained by changing the coefficients w_i.

Possibility Programming Method in Fuzzy SingleObjective Functions [12]:

Consider the formulation of the fuzzy singleobjective linear programming

$$\text{Maximize } Z = \sum_{j=1}^{n} \tilde{c}_j x_j \qquad \ldots.. \qquad (1)$$

Subject to:

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i, \quad i=1, 2\ldots m \quad \ldots. \qquad (2)$$

$$x_j \ge 0, \qquad j=1, 2 \ldots n \quad \ldots. \qquad (3)$$

Where x_j, j=1,2,….n, are non-negative decision variable $\tilde{c}_j$ is the fuzzy coefficient of the jth decision variable ,$\tilde{a}_{ij}$ represents the fuzzy coefficient of the jth decision variable in the ith constraint, while $\tilde{b}_i$ is the fuzzy right-hand side in the ith constraint .Hence for simplicity $\tilde{c}_j$, $\tilde{a}_{ij}$ and $\tilde{b}_i$ are considered to be of triangular fuzzy numbers i.e., $\tilde{c}_j = c_j - \sqrt{1-\alpha}$, c_{j0}, $c_j + \sqrt{1-\alpha}$], $\tilde{a}_j = [a_{ij} - \sqrt{1-\alpha}, a_{ij0}, a_{ij} + \sqrt{1-\alpha}]$, $\tilde{b}_j = [b_j - \sqrt{1-\alpha}, b_{j0}, b_j + \sqrt{1-\alpha}]$

Thus, according to the triangular fuzzy numbers, the equivalent crisp model for the fuzzy model (1)-(3), is given below:

$$\text{Maximize } Z = \sum_{j=1}^{n} [(1-\beta)\overline{c}_j + \beta c_{j0}] x_j$$

Subject to:

$$\sum_{j=1}^{n} [(1-\beta)\underline{a}_{ij} + \beta a_{ijo}] x_j \le (1-\beta)\overline{b}_i + \beta b_{i0}$$

Where α is a predetermined value of the minimum required possibility, $\beta \in (0, 1]$,and upon using a suitable method for solving the linear programming problem, we get our desired solution for the above problem.

Fuzzy Multiobjective Linear Programming in terms of the weighting Method:

`In this section the fuzzy multiobjective linear programming problem will be transformed to a singleobjective linear programming ones by using the weighting method and before that let us give two fuzzy operations which are the addition between two fuzzy number and multiplication of fuzzy number by a real constant as follows:

1-Fuzzy Operation:

Consider $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers in triangular form and $c \in R$ then:

1) $\tilde{a} + \tilde{b} = [a - \sqrt{1-\alpha}, a, a + \sqrt{1-\alpha}] + [b - \sqrt{1-\alpha}, b, b + \sqrt{1-\alpha}]$
$= (a+b-2\sqrt{1-\alpha}, a+b, a+b+2\sqrt{1-\alpha})$.

2) $c\,\tilde{a} = c\,[a - \sqrt{1-\alpha}, a, a + \sqrt{1-\alpha}]$
$= (ca - c\sqrt{1-\alpha}, c\,a, ca + c\sqrt{1-\alpha})$.

2-The Weighting Method for the Multiobjective Linear Programming Problem.

Consider the fuzzy multiobjective linear programming:

Maximize $\tilde{Z} = [\,\tilde{Z}_1(\underline{X}),\ \tilde{Z}_2(\underline{X}), \ldots .. \tilde{Z}_k(\underline{X})\,]$

Subject to:

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i, i=1,2\ldots m \quad (4)$$

$$x_j \geq 0, \qquad j=1, 2 \ldots n$$

And by using the weighting method which illustrated by section (2), equation (4) will takes the form

$$\text{Maximize } \tilde{Z}^* = \sum_{i}^{k} w_i \tilde{Z}_i(\underline{X})$$

Subject to:

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i, \quad i=1, 2\ldots m$$

$$x_j \geq 0, \qquad j=1, 2 \ldots n$$

And the resulting fuzzy singleobjective linear programming will be solved by Islander's approach given by [12] for different value of w_i,in order to get our feasible solution.

Illustrative Examples:-

Example (1): Maximize $\tilde{Z}_1(X) = \tilde{5}\, x_1 + 2\tilde{0}\, x_2$

Maximize $\tilde{Z}_2(X) = 2\tilde{3}\, x_1 + 3\tilde{2}\, x_2$

Subject to:

$$1\tilde{0}\, x_1 + \tilde{6}\, x_2 \leq 250\tilde{0}$$

$$-\tilde{5}\, x_1 - 1\tilde{0}\, x_2 \leq -200\tilde{0}, \quad x_1, x_2 \geq 0$$

Hence,by using the weighting method the problem will becomes:

$$\text{Maximize } \tilde{Z}^* = \tilde{\omega}_1\, \tilde{Z}_1(x) + \tilde{\omega}_2\, \tilde{Z}_2\ (x)$$

Subject to:

$$1\tilde{0}\, x_1 + \tilde{6}\, x_2 \leq 250\tilde{0}$$

$$-\tilde{5}\, x_1 - 1\tilde{0}\, x_2 \leq -200\tilde{0}$$

$$x_1, x_2 \geq 0$$

1-Choose $\overline{W} = (\tilde{\omega}_1, \tilde{\omega}_2) = (1, 0)$, thus we have

$$\therefore \text{ Maximize } Z^* = \tilde{5}\, x_1 + 2\tilde{0}\, x_2$$

Subject to:

$$1\tilde{0}\, x_1 + \tilde{6}\, x_2 \leq 250\tilde{0}$$

$$-\tilde{5}\, x_1 - 1\tilde{0}\, x_2 \leq -200\tilde{0}$$

$$x_1, x_2 \geq 0$$

The fuzzy singleobjective linear programming in triangular form takes the form:

Maximize $(5-\sqrt{1-\alpha},\ 5,\ 5+\sqrt{1-\alpha})\ x_1 + (20-\sqrt{1-\alpha},\ 20,\ 20+\sqrt{1-\alpha})x_2$

Subject to:

$(10-\sqrt{1-\alpha},\ 10,\ 10+\sqrt{1-\alpha})x_1 + (6-\sqrt{1-\alpha},\ 6,\ 6+\sqrt{1-\alpha})\ x_2 \leq (2500-\sqrt{1-\alpha}, 2500,\ 2500+\sqrt{1-\alpha})$

$-\ (5-\sqrt{1-\alpha},\ 5,\ 5+\sqrt{1-\alpha})\ x_1 - (10-\sqrt{1-\alpha}, 10, 10+\sqrt{1-\alpha})x_2 \leq -(2000-\sqrt{1-\alpha},\ 2000,\ 2000+\sqrt{1-\alpha})$

Thus, the equivalent crisp objective linear programming in the case of possibility programming is which is given in section three stated as:

Maximize $[(1-\beta)\ (5+\sqrt{1-\alpha})\ +5\beta]\ x_1 + [(1-\beta)(20-\sqrt{1-\alpha}) +20\beta)]\ x_2$

Subject to:

$[(1-\beta)\ (10-\sqrt{1-\alpha}+10\beta)]\ x_1 + [(1-\beta)\ (6-\sqrt{1-\alpha}+6\beta)]\ x_2 \leq (1-\beta)\ (2500+\sqrt{1-\alpha}+2500\beta)$

$-[(1-\beta)(5-\sqrt{1-\alpha})+5\beta]x_1 - [(1-\beta)(10-\sqrt{1-\alpha}+10\beta]x_2 \leq -(1-\beta)(2000+\sqrt{1-\alpha}+2000\beta)$

$x_1, x_2 \geq 0.$

Which can be solved easily by using a suitable method for solving linear programming problems.

Following,**(Table -1)**which represents the solution of the above example.

(Table -1)

β	0.1	0.5	0.75	0.9	1
Z	8190.3249	7241	6862.0452	6715.3204	6666.667
x_1	0	0	0	0	0
x_2	485.9626	442.8189	425.5531	418.8769	416.6666

2-Choose $\tilde{w} = (\omega_1, \omega_2) = (0.5, 0.5)$

$$\text{Maximize } \tilde{Z}^* = 0.5\, \tilde{Z}_1(x)\ + 0.5\, \tilde{Z}_2(x)$$

Subject to:

$$1\tilde{0}\, x_1 + \tilde{6}\, x_2 \leq 250\tilde{0}$$

$$-\tilde{5}\, x_1 - 1\tilde{0}\, x_2 \leq -200\tilde{0},$$

$$x_1, x_2 \geq 0$$

Thus, the problem becomes:

Maximize $\tilde{Z}^* = 0.5[(5-\sqrt{1-\alpha},\ 5,\ 5+\sqrt{1-\alpha})]\ x_1 + 0.5\ [(20-\sqrt{1-\alpha},\ 20,\ 20+\sqrt{1-\alpha})]\ x2$ $+0.5\ [(23-\sqrt{1-\alpha},\ 23,\ 23+\sqrt{1-\alpha})]\ x2 + 0.5[(32-\sqrt{1-\alpha},\ 32,\ 32+\sqrt{1-\alpha})]\ x2$

Subject to:

$(10-\sqrt{1-\alpha},\ 10,\ 10+\sqrt{1-\alpha})x_1 + (6-\sqrt{1-\alpha},\ 6,\ 6+\sqrt{1-\alpha})\ x_2 \leq (2500-\sqrt{1-\alpha}, 2500,\ 2500+\sqrt{1-\alpha})$

$-(5-\sqrt{1-\alpha}, 5, 5+\sqrt{1-\alpha})x_1 - (10-\sqrt{1-\alpha}, 10, 10+\sqrt{1-\alpha})x_2 \leq -(2000-\sqrt{1-\alpha},\ 2000,\ 2000+\sqrt{1-\alpha})$

$x_1, x_2 \geq 0$.

Hence, the fuzzy singleobjective linear programming in triangular form will takes the form:

Maximize $\tilde{Z}^* = [(14-\sqrt{1-\alpha}, 14, 14+\sqrt{1-\alpha})]\ x_1 + [(26-\sqrt{1-\alpha}, 26, 26+\sqrt{1-\alpha})]$ x2

Subject to:

$(10-\sqrt{1-\alpha}, 10, 10+\sqrt{1-\alpha})x_1 + (6-\sqrt{1-\alpha}, 6, 6+\sqrt{1-\alpha})\ x_2 \leq (2500-\sqrt{1-\alpha}, 2500, 2500+\sqrt{1-\alpha})$

$-(5-\sqrt{1-\alpha}, 5, 5+\sqrt{1-\alpha})x_1 - (10-\sqrt{1-\alpha}, 10, 10+\sqrt{1-\alpha})x_2 \leq -(2000-\sqrt{1-\alpha}, 2000, 2000+\sqrt{1-\alpha})$

$x_1, x_2 \geq 0$.

Thus, the equivalent crisp singleobjective linear programming in the case of possibility programming given in section three is stated as:

Maximize $\tilde{Z}^* = [(1-\alpha)(14+\sqrt{1-\alpha}) + 14\alpha]\ x_1 + [(1-\alpha)(26-\sqrt{1-\alpha}) + 26\alpha)]\ x_2$

Subject to:

$[(1-\beta)(10-\sqrt{1-\alpha}+10\beta)]\ x_1 + [(1-\beta)(6-\sqrt{1-\alpha}+6\beta)]\ x_2 \leq (1-\beta)(2500+\sqrt{1-\alpha}+2500\beta)$

$-[(1-\beta)(5-\sqrt{1-\alpha})+5\beta]x_1 - [(1-\beta)(10-\sqrt{1-\alpha}+10\beta]x_2 \leq -(1-\beta)(2000+\sqrt{1-\alpha}+2000\beta)$

Which also can be solved easily by using a suitable method for solving linear programming problems.

Following, **,(Table-2)**which represents the solution of the above example.

(Table-2)

β	0.1	0.5	0.75	0.9	1
Z	1304	1166	1111	1090	7086
x_1	0	0	0	0	5061
x_2	485.9624	442.7262	425.5236	418.8796	416.6635

<u>Example (2):</u> Maximize $\tilde{Z}_1(X) = 0.\tilde{4}\ x_1 + 0.\tilde{3}\ x_2$

Maximize $\tilde{Z}_2(X) = \tilde{1}\ x_1$

Subject to:

$\tilde{1}\ x_1 + \tilde{1}\ x_2 \leq 40\tilde{0}$

$\tilde{2}\ x_1 + \tilde{1}\ x_2 \leq 50\tilde{0}$,

$x_1, x_2 \geq 0$

Hence, by using the weighting method the problem will becomes:

Maximize $\tilde{Z}^* = \tilde{\omega}_1\ \tilde{Z}_1(x) + \tilde{\omega}_2\ \tilde{Z}_2(x)$

Subject to:

$\tilde{1}\ x_1 + \tilde{1}\ x_2 \leq 40\tilde{0}$

$\tilde{2}\ x_1 + \tilde{1}\ x_2 \leq 50\tilde{0}$,

$x_1, x_2 \geq 0$

1-Choose $\overline{W} = (\tilde{\omega}_1, \tilde{\omega}_2) = (1, 0)$, thus we have

Maximize $\tilde{Z}^* = 0.\tilde{4}\ x_1 + 0.\tilde{3}\ x_2$

Subject to:

$\tilde{1}\ x_1 + \tilde{1}\ x_2 \leq 40\tilde{0}$

$\tilde{2}\ x_1 + \tilde{1}\ x_2 \leq 50\tilde{0}$,

$x_1, x_2 \geq 0$

Hence, the fuzzy singleobjective linear programming in triangular form will takes the form:

Maximize $\tilde{Z}^* = [(0.4-\sqrt{1-\alpha}, 0.4, 0.4+\sqrt{1-\alpha})]\ x_1 + [(0.3-\sqrt{1-\alpha}, 0.3, 0.3+\sqrt{1-\alpha})]$ x2

Subject to:

$[(1-\sqrt{1-\alpha}, 1, 1+\sqrt{1-\alpha})]\ x_1 + [(1-\sqrt{1-\alpha}, 1, 1+\sqrt{1-\alpha})]\ x_2 \leq (400-\sqrt{1-\alpha}, 400, 400+\sqrt{1-\alpha})$

$[(2-\sqrt{1-\alpha}, 2, 2+\sqrt{1-\alpha})]$ x1 $+ [(1-\sqrt{1-\alpha}, 1, 1+\sqrt{1-\alpha})]\ x_2 \leq (500-\sqrt{1-\alpha}, 500, 500+\sqrt{1-\alpha})$

$x_1, x_2 \geq 0$.

Thus, the equivalent crisp singleobjective linear programming in the case of possibility programming is stated as:

Maximize $[(1-\alpha)(0.4+\sqrt{1-\alpha}) + 0.4\alpha]\ x_1 + [(1-\alpha)(0.3-\sqrt{1-\alpha}) + 0.3\alpha)]\ x_2$

Subject to:

$[(1-\alpha)(1-\sqrt{1-\alpha}+\alpha)]\ x_1 + [(1-\alpha)(1-\sqrt{1-\alpha}+\alpha)]\ x_2 \leq (1-\alpha)(400+\sqrt{1-\alpha}+400\alpha)$

$[(1-\alpha)(2-\sqrt{1-\alpha})+2\alpha]x_1 + [(1-\alpha)(1-\sqrt{1-\alpha}+\alpha]x_2 \leq (1-\alpha)(500+\sqrt{1-\alpha}+500\alpha)$

$x_1, x_2 \geq 0$.

Also can be solved easily by using a suitable method for solving linear programming problems.

Following, **,(Table-3)** which represents the solution of the above problem with different value of realβ.

(Table-3)

β	0.1	0.5	0.75	0.9	1
Z	3173.8661	414.7548	204.3464	146.9916	130
x_1	100	100	100	100	100
x_2	2642.0909	519.3141	357.2757	313.0948	300

2-Choose $\tilde{w} = (\omega_1, \omega_2) = (0.5, 0.5)$

Maximize $\tilde{Z}^* = 0.5\ \tilde{Z}_1(x) + 0.5\ \tilde{Z}_2(x)$

Subject to:

$\tilde{1}\ x_1 + \tilde{1}\ x_2 \leq 40\tilde{0}$

$\tilde{2}\ x_1 + \tilde{1}\ x_2 \leq 50\tilde{0}$,

$x_1, x_2 \geq 0$

Hence, the fuzzy singleobjective linear

programming in triangular form will takes the form:

Maximize $\tilde{Z}^*$=o.5 [(0.4- $\sqrt{1-\alpha}$, 0.4, 0.4+$\sqrt{1-\alpha}$)] x_1+o.5[(0.3-$\sqrt{1-\alpha}$, 0.3, 0.3+$\sqrt{1-\alpha}$)] x2+0.5[1-$\sqrt{1-\alpha}$,1,1+$\sqrt{1-\alpha}$]x_1

= (0.7-$\sqrt{1-\alpha}$, o.7, 0.7+$\sqrt{1-\alpha}$) x_1+ (0.15+0.5$\sqrt{1-\alpha}$, 0.15, 0.15+0.5$\sqrt{1-\alpha}$) x_2

Subject to:

[(1-$\sqrt{1-\alpha}$, 1, 1+$\sqrt{1-\alpha}$)] x_1+ [(1-$\sqrt{1-\alpha}$, 1, 1+$\sqrt{1-\alpha}$)] $x_2 \leq$ (400-$\sqrt{1-\alpha}$, 400,400+$\sqrt{1-\alpha}$)

[(2-$\sqrt{1-\alpha}$, 2, 2+$\sqrt{1-\alpha}$)] x1+ [(1-$\sqrt{1-\alpha}$, 1, 1+$\sqrt{1-\alpha}$)] $x_2 \leq$ (500-$\sqrt{1-\alpha}$, 500,500+$\sqrt{1-\alpha}$)

x_1, $x_2 \geq 0$.

Thus, the equivalent crisp singleobjective linear programming in the case of possibility programming is stated as:

Maximize $\tilde{Z}^*$=(1α)(0.7+$\sqrt{1-\alpha}$)+0.7α]x_1+[(1-α)(0.15+0.5$\sqrt{1-\alpha}$)+0.15α] x_2

Subject to:

[(1- α) (1-$\sqrt{1-\alpha}$+α)] x_1+ [(1- α) (1-$\sqrt{1-\alpha}$+α)] $x_2 \leq$ (1- α) (400+$\sqrt{1-\alpha}$+400α)

[(1-α)(2-$\sqrt{1-\alpha}$)+2α]x_1+[(1-α)(1-$\sqrt{1-\alpha}$+α]$x_2 \leq$(1-α)(500+$\sqrt{1-\alpha}$+500α)

x_1, $x_2 \geq 0$.

Also can be solved easily by using a suitable method for solving linear programming problems.

Following,**(Table-4)** which represents the solution of the above problem with different value of realβ.

(Table-4)

β	0.1	0.5	0.75	0.9	1
Z	1669.6281	320.1738	220.0550	185.8559	175
x_1	100	303.8990	266.7333	254.0324	250
x_2	2642.0989	0	0	0	0

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