The effect of slip condition on heat transfer of MHD oscillatory third order fluid flow in a channel filled with porous medium

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Abstract

The combined effect of a transverse magnetic field and radiative heat transfer to unsteady flow of a conducting optically third order fluid through a channel filled with saturated porous medium and non-uniform walls temperature is investigated.It is assumed that the no-slip condition between the wall and the fluid remains no longer valid. The third order fluid equations of continuity momentum and energy are obtained. Analytical solutions for problem are established.The effect of wall slip on velocity field is presented by figures. The basic properties of the flow are studied.

Keywords: Third order fluid; MHD; heat transfer; porous medium.

التأثير المشترك لحقل مغناطيسي وإنتقال الحرارة بالاشعاع في الجريان اللامستقر لمائع من الرتبة الثالثة في قناة مملوءة بوسط مثقب

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الخلاصة

في هذا البحث تمت دراسة التأثير المشترك لحقل مغناطيسي وانتقال الحرارة بالاشعاع في الجريان اللامستقر لمائع من الرتبة الثالثة في قناة مملوءة بوسط بوريس وذات جدران مختلفة الحرارة وقد اعتبر أن شرط عدم الانزلاق بين الجدران والمائع غير متحقق .أوجدنا معادلات الاستمرارية والحركة والطاقة للمائع من الرتبة الثالثة وقد حصلنا على حلول تحليلية للمعادلات أعلاه وكذلك وضحنا الصفات الرئيسية للمسألة من خلال الرسوم .

INTRODUCTION

The magnetofluiddynamics is the study of electrically conducting fluids in electric and magnetic fields. It unifies in a common framework the electromagnetic and fluid dynamic theories to yield a description of the concurrent effects of magnetic field on the flow and the flow on the magnetic field. Magneto hydrodynamics (MHD) is specifically concerned with electrically conducting liquids and ionized compressible gases.

There are many natural phenomenon and engineering problems susceptible to magnetofluiddynamics analysis. It is useful in engineering problems such as magnetohyrodnamics (MHD) generators and many applications [8]. In the last few decades, several simple flow problems associated with classical hydrodynamics have received considerable attention within the more general context of magnetohydrodynamics (MHD), an important field of application is electromagnetic propulsion. In recent years, the flow of fluids through porous media has become an important topic because of the recovery of crude oil from the pores of the reservoir rocks [6]. In this paper we study the effect of slip condition on unsteady MHD of third grade fluid. Exact analytic solution is presented. This paper is organized into five sections. Section two concern with third order fluid. Section three describe the mathematical model of the problem, and the last two sections give the solution and results, which we want to investigate in it the combined effects of a transverse magnetic field and radiative heat transfer on unsteady flow of conducting optically thin third order fluid through a channel filled with saturated porous medium and non uniform walls temperature. In the following section, the problem of the governing equation formulated, solved and results with discussion.

Third order fluid

An incompressible simple fluid is defined as a material whose state of present stress is determined by the history of the deformation gradient without preferred reference configuration [3]. Its constitutive equation can be written in the form of a functional of the form

$$T(t) = -p_1 I + \sum_{s=0}^{\infty} F_1^t(s) \qquad (1)$$

Where $p_1 I$ is the undetermined part of the stress tensor and F is the deformation gradient.
Coleman and Noll [2] defined the incompressible fluid of differential type of grade n as the simple fluid obeying the constitutive equation

$$T(t) = -p_1 I + \sum_{j=0}^{\infty} S_j \qquad (2)$$

Obtained by asymptotic expansion of the functional in (1) through a retardation parameter x. If $n = 3$, the first three tensors S_j are given by

$$S_1 = \mu A_1,$$

$$S2 = \alpha_1 A2 + \alpha_2 A_1^2,$$

$$S3 = \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (tr A_1^2) A_1,$$

is the coefficient of viscosity and μ where are the material β_3 and $\alpha_1, \alpha_2, \beta_1, \beta_2$ are kinematical tensors A_3 and A_1, A_2 modules, defined by

$$A_1 = grad V + (\operatorname{grad} V)^T,$$

$$A_n = \frac{d}{dt} A_{n-1} + A_{n-1}(\operatorname{grad} V) + \qquad (3)$$

$$(\operatorname{grad} V)^T A_{n-1}, \text{ with } n = 2,3,\cdots,$$

Where V denotes the velocity field, grad is the gradient operator and d/dt is the material time derivative which is defined by

$$\frac{d}{dt}(.) = \frac{\partial}{\partial t}(.) + V.\operatorname{grad}(.), \qquad (4)$$

Where $\partial/\partial t$ is the partial derivative with respect to time. A detailed thermodynamic analysis of the model, represented by (2.2) is given in [7]. It was shown that if all the motions in the sense that these motions meet the Clausius – Duhem inequality and if it is assumed that the specific Helmholtz free energy is minimum , when the fluid is locally at rest, then:

$$\mu \geq 0, \quad \alpha_1 \geq 0, |\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta_3},$$

$$\beta_1 = \beta_2 = 0, \quad \beta_3 \geq 0 \qquad (5)$$

Therefore, the constitutive relation for a thermodynamically compatible fluid of third grade becomes

$$T = -p_1 I + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_3 (tr A_1^2) A_1. \qquad (6)$$

In this paper we consider a fluid of third order whose state equation is of the form (6).

Mathematical model of the problem

In the present paper, consideration is given to unsteady, incompressible, viscous electrically conducting fluid of third order saturated porous medium with constant temperature and the radiation effect is also taken to account.
The fluid disjoint by two parallel plates by a space as shown in Fig.1.
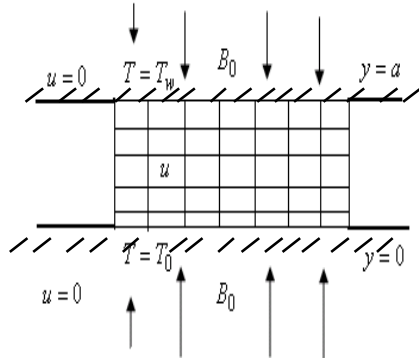
(Figure-1)Geometry of the proble

In this problem, the following assumptions have been made:-

1.A uniform magnetic (μ_e) field of β_0 strength $\beta_0 = (\mu_0 H_0)$ is applied perpendicular to the plates.

2.The electromagnetic induction is small,and the electromagnetic force produced is very small.

3.It is assumed that both walls temperature T_0, T_W arehigh enough to induce radiative heat transfer.

4. It is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux [5] is given by :

$$\frac{\partial q}{\partial y} = 4\alpha^2 (T_0 - T) \qquad (7)$$

Where q is the radiative heat flux, α the mean radiation absorption coefficient, T_0 the temperature at $y = 0$, T the temperature at $y = \alpha$

.

The Governing equations

Under the above assumptions, the governing equations can be written as:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial \chi} + v\frac{\partial^2 u}{\partial y^2} - \frac{v}{K}u - \frac{\sigma_e B_0^2}{\rho}u + g\beta(T - T_0) \qquad (8)$$

$$+\alpha_1 \frac{\partial^3 u}{\partial y^2 \partial t} + 6\beta_3 \text{ Re}\left(\frac{\partial u}{\partial y}\right)^2 \left(\frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial T}{\partial t} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q}{\partial y} \quad .. \qquad . (9)$$

With

$$u - \gamma \frac{\partial u}{\partial y} = 0, \ \theta = 0 \ , \ \text{on y} = 0\ldots\ldots \qquad (10)$$

$$u = 0 \ , \theta = 1, \ \text{on y} = 1 \qquad (11)$$

where u is the axial velocity, t the time, ρ the fluid density, P the pressure, x the axial distance , v the kinematic viscositycoefficient, y transverse distance, $\underset{v}{K}$ the porous medium permeability coefficient, k the conductivity of the fluid, g the gravitational force,β the coefficient of volume expansion due to temperature, the coecostic parameter, is the material modal, k the thermal conductivity,

the specific heat at constant pressure.

The following dimensionless variables and parameters are introduced:-

$$R_e = \frac{Ua}{v}, \bar{x} = \frac{x}{a}, \bar{y} = \frac{y}{a}, \bar{u} = \frac{u}{U}, \theta = \frac{T - T_0}{T_w - T_0},$$

$$H^2 = \frac{a^2 \sigma_e B_0^2}{\rho v}, \bar{t} = \frac{tU}{a}, \bar{P} = \frac{aP}{\rho v U}, Da = \frac{K}{a^2}, \qquad (12)$$

$$Gr = \frac{g\beta(T_w - T_0)a^2}{vU}, P_e = \frac{Ua\rho c_p}{k}, N^2 = \frac{4\alpha^2 a^2}{k},$$

$$\alpha_1 = \frac{\alpha_1'}{a^2}, \beta_3 = \frac{\beta_3' U}{a^3}$$

where U is the flow mean velocity,θ the non dimensional temperature R_e,H , D_a ,Gr , P_e,N are Reynolds number, Hartmann number, Darcy number, Grashoff number, **Pe'cle** number, Radiation parameter respectively.

The governing equations for this flow geometry together with the appropriate boundary conditions, in dimensionless form can be written as

$$\text{Re}\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial \chi} + \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho a}u - H^2 u$$

$$+G_r\theta + \alpha_1 \frac{\partial^3 u}{\partial y^2 \partial t} + 6\beta_3 \text{ Re}^2\left(\frac{\partial u}{\partial y}\right)^2\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad (13)$$

$$P_e \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - N^2 \theta \qquad (14)$$

With associated boundary conditions are

u-γ=0, θ =0 on y=0,   u = 0,θ=1,on y =1   **(15)**

It is clear that if we set $\alpha l = \beta_3 = 0$ in equation (13), we obtain the corresponding equations in the case of second order fluid as obtained by [1].

Solution of the problem

For purely an oscillatory flow we take

$$-\frac{\partial P}{\partial x} = \lambda e^{i\omega t} \qquad (16)$$

Where λ is a constant and ω is the frequency of the scillation.

Due to the selection from of pressure gradient we assume the solution of the equations (13), (14) of the form:

$u(y, t) = u_0 + \beta u_1$, $\theta(y, t) = \theta_0(y) e^{i\omega t}$ **(17)**

Substituting these expressions into equation (13) and equating the coefficients of equal power in β_3 we obtain:

Zero order in β_3:

$$R_e \frac{\partial u_0}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u_0}{\partial y^2} - \frac{1}{\rho a} u_0 -$$

$$H^2 u_0 + G_r \theta + \alpha_1 \frac{\partial^3 u_0}{\partial y^2 \partial t} \qquad (18)$$

With boundary conditions:

$$u_0 - \gamma \frac{\partial u_0}{\partial y} = 0, \qquad \text{on } y = 0$$

$u_0 = 0$,   on y = 1

First order in β_3 is:
wher

$$R_e \frac{\partial u_1}{\partial t} = \frac{\partial^2 u_1}{\partial y^2} - -\frac{1}{\rho a} u_1 - H^2 u_1 + \alpha_1 \frac{\partial^3 u_2}{\partial y^2 \partial t}$$

$$+ 6 \text{Re}^2 \left(\frac{\partial u_0}{\partial y} \right)^2 \left(\frac{\partial^2 u_0}{\partial y^2} \right) \qquad (19)$$

where:

$$u_1 - \gamma \frac{\partial u_1}{\partial y} = 0, \text{on } y = 0$$

$u_1 = 0$,   on y = 1

Putting the second part of equation (17)into energy equation (14), we get:

$$\frac{\partial^2 \theta_0}{\partial y^2} + m_1^2 \theta_0 = 0 \dots \qquad (20)$$

And the associated boundary conditions are:

$\theta_0 = 1$ on   y = 0

$\theta_0 = 1$ on   y = 1

The solution of (20) is given by

$\theta_0 = \dfrac{\sin m_1 y}{\sin m_1}$, where

$$m_1 = \sqrt{N^2 - i\omega \rho a} \qquad (21)$$

Putting the last expression of θ_0 into (17), we have

$$\theta = \frac{\sin m_1 y}{\sin m_1} e^{i\omega t} \qquad (22)$$

To solve equations (18)and(19), we assume that

$u_0 = u_{00}$, $u_1 = u_{11} e^{3i\omega t}$,   **(23)**

Substitute expressions (16, 22, and 23) into equation (18),(19) and eliminating $e^{i\omega t}$, $e^{3i\omega t}$ we obtain:

$$\frac{\partial^2 u_{00}}{\partial y^2} - K_1^2 u_{00} = -K_2 - K_3 \frac{\sin m_1 y}{\sin m_1} \dots \qquad (24)$$

$$u_{00} - \gamma \frac{\partial u_{00}}{\partial y} = 0, \qquad \text{on } y = 0$$

$u_{00} = 0$,   on y = 1

$$K_1 = \frac{R_e + \dfrac{1}{\rho a} + H a^2}{1 + i w \alpha_1}, K_2 = \frac{\lambda}{1 + i\omega \alpha_1}, K_3 = \frac{Gr}{1 + i\omega \alpha_1},$$

$$u_{11}'' - C_1 u_{11} + C_2 u_{00}'^2 u_{00}'' = 0 \qquad (25)$$

$$u_{11} - \gamma \frac{\partial u_{11}}{\partial y} = 0, \qquad \text{on } y = 0$$

$u_{11} = 0$,   on y = 1

where:

$$C_1 = \frac{\dfrac{1}{\rho a} + H a^2 - 3iw \text{Re}}{1 + 3iw \alpha_1}, C_2 = \frac{6iw \text{Re}^2}{1 + 3i\omega \alpha_1}$$

The solution of equations (24) and (25) is found to be:

$$u(y) = e^{i\omega t} D_1 \left(\begin{array}{c} \cosh K_1 y + D_2 \sinh K_1 y + \dfrac{\lambda}{K_1^2} + \\ \dfrac{Gr}{\sin m_1 (m_1^2 - K_1^2)} \sin m_1 y \end{array} \right)$$

$+ \beta \, e^{3i\omega t}$

$$(\frac{1}{4}e^{i3\alpha t}-(12i\text{Re}^2(Ha^2+i^w\text{Re}+s^2)^2 w^2 \cosh$$

$$[\sqrt{Ha^2+i^w\text{Re}+s^2}y](-5(Ha^2+i^w\text{Re}+s^2)+$$

$$\frac{\frac{1}{\rho a}+Ha^2+3iw\text{Re}}{1+3iw\alpha}+(Ha^2+i^w\text{Re}+s^2 \_$$

$$\frac{\frac{1}{\rho a}+Ha^2+3iw\text{Re}}{1+3iw\alpha})\cosh 2\sqrt{Ha^2+i^w\text{Re}+s^2}y])$$

$$(\frac{\lambda}{Ha^2+i^w\text{Re}+s^2}+\gamma(Gr\sqrt{N^2+ipaw} \qquad \textbf{(26)}$$

$$(-Ha^2+N^2-i^w\text{Re}-s^2+ipaw)Csc(\sqrt{N^2+ipaw}]+........).$$

$$D_1=-\frac{\lambda}{K_1^2}+\gamma\left(D_2 K_1+\frac{Grm_1}{\sin m_1(m_1^2-K_1^2)}\right)$$

$$D_2=\frac{1}{\sinh K_1+\gamma K_1 \cosh K_1}$$

$$\left((\cosh K_1-1)\frac{\lambda}{K_1^2}-\frac{Gr}{(m_1^2-K_1^2)}(1+\frac{\gamma m_1 \cosh K_1}{\sin m_1})\right)$$

Results and discussion

In this section we study the effect of each of the dimensionless parameter that appear in the above motioned equation, upon the velocity distribution considering the real part of the solution given by eq. (26),if we set $\gamma_1=0$ we cover all the results that obtained by[4],in addition to that. If we set $\beta_3=\alpha_1=0$, all the results obtained by [1] can be coverd.

Also,to see the effect of any parameter we keep all other parameters fixed. The following results are obtained

• For As Re   increase there is decreases in the velocity distribution, see figure (2).

• For As β increase we observe that there is oscillatory velocity distribution about $\beta=0.03$, see figure (3).

• For As Ha increase there is decreases in the velocity distribution, see figure (4).

• For As Gr increase there is decreases in the velocity distribution , see figure (5).

• For As S increase there is decreases in the velocity distribution, see figure (6).

• For As w increase there is decreases in the velocity distribution , see figure (7).

• For As Pa increase there is oscillatory in the velocity   distribution   about $Pa=0.002$,   see figure(8)

• For As N increase there is decreases in the velocity distribution , see figure (9).

• For As a increase there is decreases in the velocity distribution , see figure (10).

• For As λ increase there is decreases in the velocity distribution, see figure (11)

• For As γ increase there is decreases in the velocity distribution , see figure (12).

• For As t increase there is decreases in the velocity distribution , see figures (13).
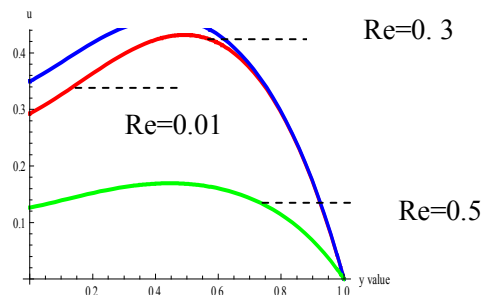
Fig.2 .Velocity curves for
Re= 0.01,0.3,0.5   Ha=1 γ =1 N=1 s=1
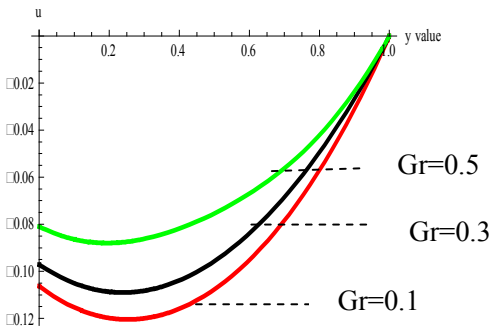**w=1   t=$\frac{\pi}{20}$ Pa=0.7 $\beta=1$**
$a=1$ λ=1   Gr=1
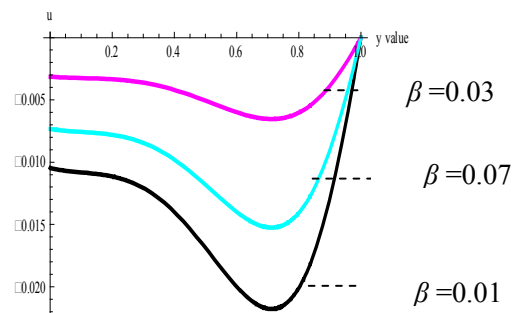
Fig.3. Velocity curves for
Re=1,   Ha=1 γ =1 N=1 s=1
w=1 $t=\frac{\pi}{20}$ Pa=0.7 $\beta=0.1,0.03,0.07$
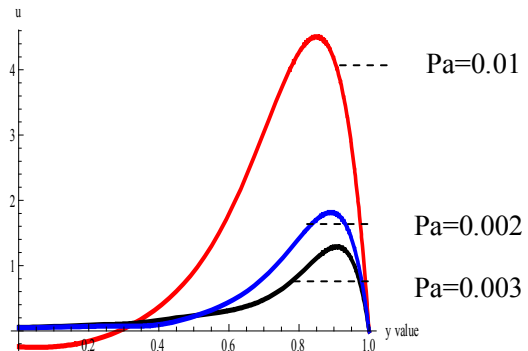λ=1   Gr=1 $a=1$
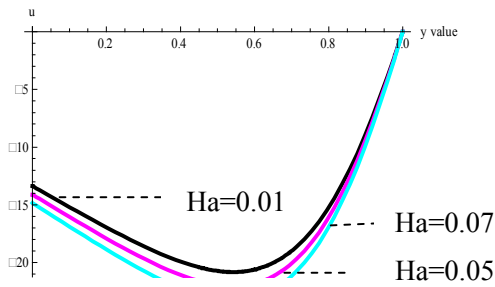
Fig.4. Velocity curves for
Re=1,   Ha=0.01,0.05,0.07γ =1 N=1 s=1
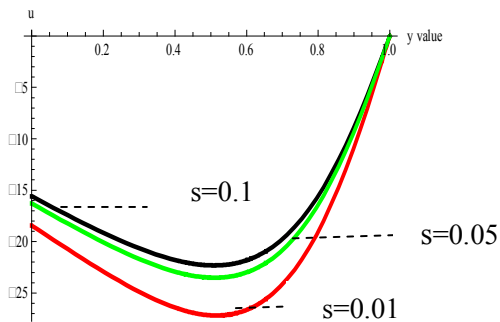**w=1   t =$\frac{\pi}{20}$ Pa=0.7 $\beta = 1$**
λ=1   Gr=1 $a = 1$
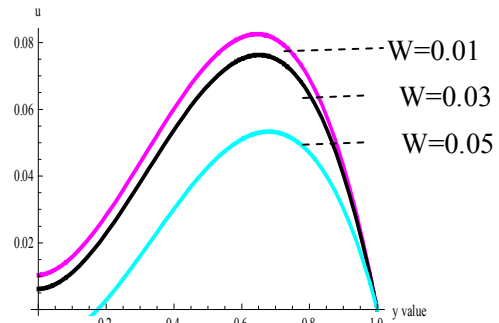
Fig.7. Velocity curves for
Re=1,   Ha=1 γ =1 N=1 s=1
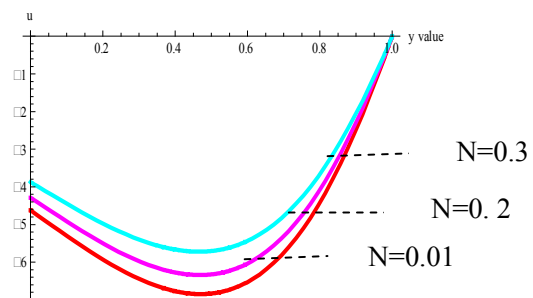**w=0.01,0.03,0.05   t =$\frac{\pi}{20}$ Pa=0.7 $\beta = 1$**
,λ=1   Gr=1 $a = 1$

Fig.5. Velocity curves for
Re=1,   Ha=1 γ =1 N=1 s=1
w=1   t =$\frac{\pi}{20}$ Pa=0.7 $\beta = 1$
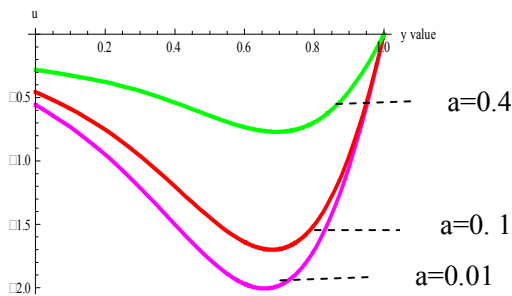**λ=1   Gr=0.1,0.3,0.5$a = 1$**

Fig.8. Velocity curves for
Re=1,   Ha=1 γ =1 N=1 s=1
w=1   t =$\frac{\pi}{20}$ Pa=0.01,0.002,0.003 $\beta = 1$
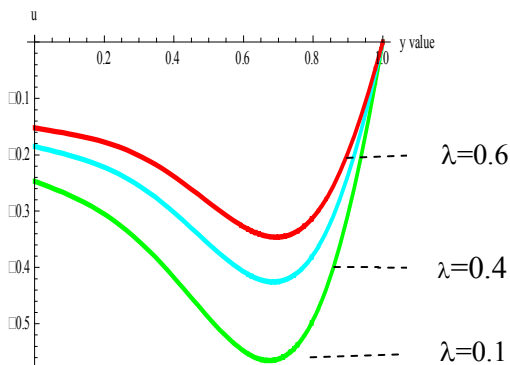**λ=1   Gr=1 $a = 1$**

Fig.6. Velocity curves for
Re=1,   Ha=1 γ =1 N=1 s=0.1,0.01,0.05
w=1   t =$\frac{\pi}{20}$ $\beta = 1$
λ=1   Gr=1 $a = 1$
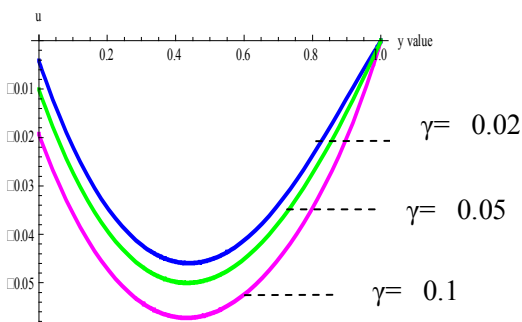

Fig.9. Velocity curves for
Re=1,   Ha=1 γ =1 N=0.01,0.2,0.3 s=1
w=1   t =$\frac{\pi}{20}$ Pa=0.7 $\beta = 1$
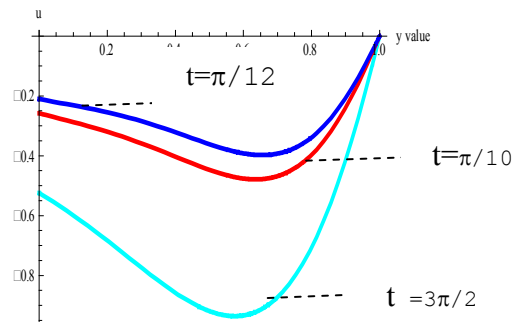**λ=1   Gr=1$a = 1$**

Fig.10. Velocity curves for
Re=1, Ha=1 γ =1 N=1 s=1
w=1 t =$\frac{\pi}{20}$ Pa=0.7 $\beta = 1$
λ=1   Gr=1   a= 0.01,0.1,0.4

Fig.11. Velocity curves for
Re=1, Ha=1 γ =1 N=1 s=1
w=1 t =$\frac{\pi}{20}$ Pa=0.7 $\beta = 1$
λ=0.1,0.4,0.6   Gr=1$a = 1$

Fig.12. Velocity curves for
Re=1, Ha=1 γ =0.1,0.02,0.05 N=1 s=1
w=1 t = $\frac{\pi}{20}$ Pa=0.7 $\beta = 1$
λ=1   Gr=1 $a = 1$

Fig.13. Velocity curves for
Re=1, Ha=1 γ =1 N=1 s=1
w=1 t = $\frac{3\pi}{20}, \frac{\pi}{10}, \frac{\pi}{12}$ Pa=0.7 $\beta = 1$
λ=1   Gr=1 $a = 1$

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