Nucleon Momentum Distributions and Elastic Electron Scattering Form Factors for some sd-shell Nuclei

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Abstract

The nucleon momentum distributions (NMD) for the ground state and elastic electron scattering form factors have been calculated in the framework of the coherent fluctuation model and expressed in terms of the weight function (fluctuation function). The weight function has been related to the nucleon density distributions of nuclei and determined from theory and experiment. The nucleon density distributions(NDD) is derived from a simple method based on the use of the single particle wave functions of the harmonic oscillator potential and the occupation numbers of the states. The feature of long-tail behavior at high momentum region of the NMD has been obtained using both the theoretical and experimental weight functions. The observed electron scattering form factors for ^{35}Cl ,^{37}Cl and ^{39}K nuclei are very well reproduced by the present calculations throughout all values of momentum transfer q.

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توزيعات زخم النيكليون وعوامل التشكل للاستطاره الالكترونيه المرنه لبعض نوى القشره sd

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الخلاصة

تم أستخدام أنموذج التموج المتشاكه في حساب كل من توزيعات زخم النيكليون النووي للحالة الأرضية وعوامل التشكل للأستطارة الألكترونية المرنة ، حيث تم التعبير عنهما بدلالة دالة تسمى دالة التموج. لقد تم التعبير عن دالة التموج بواسطة توزيعات كثافة النيكليونات وتم حسابها من النتائج النظرية والعملية لتوزيعات كثافة النيوكليونات. ان حساب توزيعات كثافة النيكليونات يعتمد بالأساس على كل من اعداد اشغال الحالات النووية وعلى الدوال الموجية للجسيمة المنفردة المتواجدة في الجهد التوافقي.

تميزت نتائج توزيعات زخوم النيكليونات(المستندة على دالة التموج النظرية والعملية)بصفة الذيل الطويل عند منطقةالزخم العالي.أظهرت هذه الدراسة بأن عوامل التشكل النظرية تتفق مع النتائج العملية للنوى(^{37}Cl, ^{35}Cl (^{39}K,

Introduction

The nucleon momentum distribution (*NMD*) is of interest in many research subjects of modern physics. In the last three decades, there has been significant effort for the determination of the *NMD* in nuclear matter and finite nucleon systems [1]. *NMD* is related to the cross sections of various kinds of nuclear reactions. The experimental evidence obtained from inclusive and exclusive electron scattering on nuclei established the existence of a high-momentum component for momenta k >2 fm [2,3]. It has been shown that, in principle, mean field theories cannot describe correctly *NMD* and density distribution simultaneously [4] and the main features of *NMD* depend a little on the effective mean field considered [5]. The reason is that *NMD* is sensitive to short-range and tensor nucleon-nucleon correlations which are not included in the mean field theories. Thus, theoretical approaches, which take into account short range correlations (SRC) due to the character of the nucleon-nucleon forces at small distances, are necessary to be developed.

There are several theoretical studies of elastic electron-nucleus scattering, Bertulani [6] and Karataglidis *et al.* [7] have performed theoretical studies on electron scattering from exotic nuclei. Roca-Maza et al. [8] systematically investigated elastic electron scattering off both stable and exotic nuclei with the phase-shift analysis method. Chu et al. [9] have studied the elastic electron scattering along *O* and *S* isotopic chains and shown that the phase-shift analysis method can reproduce the experimental data very well for both light and heavy nuclei. Recently, the nucleon momentum distributions and elastic electron scattering form factors for some even and odd p-shell nuclei have been studied by Hamoudi et al. [10]

In the coherent fluctuation model (CFM), which is exemplified by the work of Antonov et al. [11,12], the local NDD and the NMD are simply related and expressed in terms of an experimentally obtainable fluctuation function (weight function) $|f(x)|^2$. They [11, 12] investigated the NMD of (^4He and ^{16}O), ^{12}C and (^{39}K, ^{40}Ca and ^{48}Ca) nuclei using weight functions $|f(x)|^2$ expressed in terms of, respectively, the experimental two parameter Fermi ($2PF$) NDD [13], the experimental data of Ref. [14] .

The aim of the present work is to derive an analytical form for the NDD of nuclei presented at the end of the *sd*-shell based on the use of the single particle harmonic oscillator wave functions and the occupation numbers of the states. The derived NDD is employed in determining the theoretical weight function $|f(x)|^2$ which is used in the CFM to study the NMD and elastic form factors for some odd nuclei lying at the end of the *sd*-shell, (such as $^{35}Cl, ^{37}Cl$ and ^{39}K nuclei). We shall see later that the theoretical $|f(x)|^2$, based on the derived *NDD*, is capable to give information about the NMD and elastic electron scattering form factors .

Theory

The nucleon density distribution (NDD) of one body operator can be written as [15]

$$\rho(r) = \frac{1}{4\pi}\sum_{nl} \zeta_{nl}\, 4(2l+1)\phi^*_{nl}(r)\phi_{nl}(r) ...(1)$$

Here ζ_{nl} is the nucleon occupation probability of the state nl ($\zeta_{nl}=0$ or 1 for closed shell nuclei and $0<\zeta_{nl}<1$ for open shell nuclei) and $\phi_{nl}(r)$ is the radial part of the single particle harmonic oscillator wave function.

The *NDD* of nuclei under study ($^{35}Cl, ^{37}Cl$ and ^{39}K) presented at the end of the *sd*-shell is derived on the assumption that there is a core of filled $1s$ and $1p$ shells and the occupation numbers of nucleons in $2S$ and $1d$ shells are equal to, respectively, $4-\delta$ and $A-32+\delta$ and not to 4 and $A-32$ as in the simple shell model. Using this assumption with the help of eq.(1), an analytical form for $\rho(r)$ is obtained as:

$$\rho(r)=\frac{e^{-r^2/b^2}}{2\pi^{3/2}b^3}\left\{8+16\left(\frac{r}{b}\right)^2+\frac{96}{15}\left(\frac{r}{b}\right)^4+(4-\delta)\left(3-4\left(\frac{r}{b}\right)^2+\frac{4}{3}\left(\frac{r}{b}\right)^4\right)+(A-32+\delta)(8/15)\left(\frac{r}{b}\right)^4\right\} \quad ...(2)$$

Here, the parameter δ characterizes the deviation of the nucleon occupation numbers from the prediction of the simple shell model (δ=0), A is the nuclear mass number and b is the harmonic oscillator size parameter.

The normalization condition of the *NDD* is given by [11]:

$$A=4\pi\int_0^\infty \rho(r)r^2dr \quad ....(3)$$

The central *NDD* is obtained from eq.(2)

$$\rho(0)=\frac{1}{2\pi^{3/2}b^3}\{8+12-3\delta\}............ \quad (4)$$

Therefore, δ can be obtained from eq.(4) as

$$\delta=\frac{(20-2\rho(0)\pi^{3/2}b^3)}{3} \quad ......(5)$$

The *NMD* of *sd*-shell nuclei is also determined by the shell model using the single particle harmonic oscillator wave function in momentum representation and is given by [16]

$$n(k)=\frac{b^3}{\pi^{3/2}}e^{-b^2k^2}\times\left[10+(8/3)(bk)^4+4\frac{(A-20)}{15}(bk)^4\right]..(6)$$

In the *CFM* [11,12], the mixed density is given by:

$$\rho(r,r')=\int_0^\infty |f(x)|^2\rho_x(r,r')dx \quad ....(7)$$

Where:

$$\rho_x(r,r')=3\rho_0(x)\frac{j_1(k_F(x)|\vec{r}-\vec{r}'|)}{k_F(x)|\vec{r}-\vec{r}'|}\times\theta\left(x-\frac{|\vec{r}+\vec{r}'|}{2}\right) \quad ...(8)$$

is the density matrix for A nucleons uniformly distributed in the sphere with radius x and density $\rho_0(x)=3A/4\pi x^3$. The Fermi momentum is defined as[11,12]

$$k_F(x)=\left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3}=\left(\frac{9\pi A}{8}\right)^{1/3}\frac{1}{x}$$
$$=\frac{\alpha}{x}; \quad \alpha=\left(\frac{9\pi A}{8}\right)^{1/3} \quad .... (9)$$

and the step function θ is defined by

$$\theta(y)=\begin{cases}1, & y\geq 0\\ 0, & y<0\end{cases} \quad .... \quad (10)$$

Equation (7) corresponds to the general statement of the *CFM* in which the *NDD* of the nuclear matter fluctuates around the average distribution, keeping spherical symmetry and uniformity. The diagonal element of eq. (7) gives the one-particle density as

$$\rho(r)=\rho(r,r'=r)$$
$$=\int_0^\infty |f(x)|^2\rho_x(r)dx \quad .... (11)$$

In eq. (11), $\rho_x(r)$ and $|f(x)|^2$ have the following forms [12]:

$$\rho_x(r)=\rho_0(x)\theta(x-|\vec{r}|)......... .... (12)$$

$$|f(x)|^2=-\frac{1}{\rho_0(x)}\frac{d\rho(r)}{dr}\Big|_{r=x} \quad .... \quad (13)$$

The weight function $|f(x)|^2$ of eq.(11), determined in terms of the *NDD* $\rho(r)$, satisfies the normalization condition

$$\int_0^\infty |f(x)|^2 dx = 1 \quad (14)$$

and holds only for monotonically decreasing *NDD*, i.e. $\frac{d\rho(r)}{dr} < 0$.

On the basis of eq.(11), the *NMD*, $n(k)$, is expressed as [14]

$$n(k) = \int_0^\infty |f(x)|^2 n_x(k) dx \quad (15)$$

where

$$n_x(k) = \frac{4}{3}\pi x^3 \theta\left(k_F(x) - |\vec{k}|\right) \quad (16)$$

is the Fermi-momentum distribution of the system with density $\rho_0(x)$. By means of eqs.(13), (15) and (16), an explicit form for $n(k)$ is expressed in terms of $\rho(r)$ as

$$n(k) = \left(\frac{4\pi}{3}\right)^2 \frac{4}{A} \times \left[6\int_0^{\alpha/k} \rho(x) x^5 dx - \left(\frac{\alpha}{k}\right)^6 \rho\left(\frac{\alpha}{k}\right)\right] \quad (17)$$

with normalization condition

$$\int n(k) \frac{d^3k}{(2\pi)^3} = A \quad (18)$$

The form factor $F(q)$ of the nucleus is also expressed in the *CFM* and is given by [12]

$$F(q) = \frac{1}{A}\int |f(x)|^2 F(x,q) dx \quad (19)$$

where $F(x,q)$ is the form factor of uniform charge density distribution given by:

$$F(x,q) = \frac{3A}{(qx)^2}\left[\frac{\sin(qx)}{qx} - \cos(qx)\right] \quad (20)$$

Equation (20) reflects the physical scattering picture, inherent by the *CFM*, in which the scattering amplitude is a superposition of different uniform charge distributions. The nucleon finite size (fs) form factor is defined by $F_{fs}(q) = Exp(-0.43q^2/A)$ [17]and $F_{cm}(q) = Exp(q^2b^2/4A)$ is the correction for the lack of translational invariance in the shell model (center of mass correction) [17]. Inclusion of $F_{fs}(q)$ and $F_{cm}(q)$ in the calculations requires multiplying the form factor of eq. (19) by these corrections.

It is important to point out that all physical quantities studied above in the framework of the *CFM* such as $n(k)$ and $F(q)$ are expressed in terms of the weight function $|f(x)|^2$. Therefore, it is worthwhile trying to obtain the weight function firstly from the *NDD* of $3PF$ model extracted from the analysis of elastic electron-nuclei scattering experiments and secondly from theoretical considerations. The *NDD* of $3PF$ models is given by [13]

$$\rho_{3PF}(r) = \rho_0\left(1 + \frac{wr^2}{c^2}\right)\Big/\left(1 + e^{\frac{r-c}{z}}\right); \rho_0 = \frac{A}{4\pi}\frac{1}{\int_0^\infty \left(1 + \frac{wr^2}{c^2}\right)\left(1 + e^{\frac{r-c}{z}}\right)^{-1} r^2 dr} \quad (21)$$

The experimental weight functions $|f(x)|^2_{3PF}$ are obtained by introducing eq.(21), into eq.(13) , Using eq.(2) in eq.(13), an analytical expression for theoretical weight function $|f(x)|^2$ is obtained as

$$|f(x)|^2 = \frac{8\pi}{3Ab^2}x^4\rho(x) - \frac{4}{6A\pi^{1/2}b^5}x^4$$

$$\left\{32 + 25.6\frac{x^2}{b^2} - 8(4-\delta) + \frac{16}{3}\frac{x^2}{b^2} + \frac{32}{15}(A - 32 + \delta)\frac{x^2}{b^2}\right\}e^{-x^2/b^2} \quad (22)$$

Results , discussion and conclusions

The nucleon momentum distributions $n(k)$ and elastic form factors for some *sd*- shell nuclei lying at the end of the shell are studied by means of the *CFM*. The distribution $n(k)$ of eq.(17) is calculated in terms of the weight function obtained firstly from the fit to the electron-nuclei scattering experiments, $|f(x)|^2_{3PF}$ and secondly from theory, as in eq.(22).

The harmonic oscillator size parameters b are chosen in such a way as to reproduce the measured root mean square radii (rms) of nuclei. The parameter δ are determined by introducing the chosen values of b and the experimental densities $\rho_{\exp}(0)$ into eq.(5). The values of b and δ together with the other parameters employed in the present calculations for $^{35}Cl, ^{37}Cl$ and ^{39}K nuclei are listed in table-1. The calculated *rms* $\langle r^2\rangle_{cal}^{1/2}$ and those of experimental data $\langle r^2\rangle_{\exp}^{1/2}$ [13] are displayed in this table as well for comparison. The comparison shows a remarkable agreement between $\langle r^2\rangle_{cal}^{1/2}$ and $\langle r^2\rangle_{\exp}^{1/2}$ for all considered nuclei.

The *NDD* for $^{35}Cl, ^{37}Cl$ and ^{39}K nuclei are shown in figure-1. The solid circles are the experimental nucleon density distributions of the three-parameter Fermi [13], the solid curves are the calculated nucleon density distributions of eq. (2), when $\delta \neq 0$, and the dotted symbols are the calculated nucleon density distributions of eq.(2) , when $\delta = 0$. This figure shows that the dotted curves are in poor agreement with experimental data, especially for small r (fm). Considering the parameter δ in the calculation leads to a satisfactory results with the experimental data. A small deviation between the calculated NDD and those of experimental data is seen at $0 \le r \le 2.5$ fm the region of small r , i.e., $0 \le r \le 2.5$ fm.

The dependence of the $n(k)$ (in fm^3) on k (in fm^{-1}) for $^{35}Cl, ^{37}Cl$ and ^{39}K nuclei is shown in figure-2 The dash-dotted distributions are the *NMD* of eq. (6) obtained by the shell model calculation using the single particle harmonic oscillator wave functions in the momentum representation. The solid circles and solid curves distributions are the *NMD*'s obtained by the *CFM* and expressed in terms of the experimental and theoretical weight functions, respectively. It is clear that the behavior of the dash-dotted distributions reproduced by the shell model calculations is in contrast with those of the solid circles and solid curves distributions reproduced by the *CFM*. The important feature of the dash-dotted distributions is the steep slope behavior when k increases. This behavior is in disagreement with other studies [11,15,18,19] and it is attributed to the fact that the ground state shell model wave functions given in terms of a Slater determinant does not take into account the important effect of the short range dynamical correlation functions. Hence, the short-range repulsive features of the nucleon-nucleon forces are responsible for the high momentum behavior of the *NMD* [15,18]. It is noted that the general structure of the solid circles and solid distributions at the region of high momentum components is almost the same for $^{35}Cl, ^{37}Cl$ and ^{39}K nuclei, where these distributions have the feature of long-tail behavior at momentum region $k \ge 2\,fm^{-1}$. In fact, the feature of long-tail behavior obtained by the *CFM*, which is in agreement with other studies [11, 15,20,19], is related to the existence of high densities $\rho_x(r)$ in the decomposition of eq.(11), though their weight functions $|f(x)|^2$ are small.

The elastic electron scattering form factors from the considered spin zero nuclei are calculated in the frame work of the *CFM* as given by eq.(19). The present results for elastic form factors are plotted versus the momentum transfer q for $^{35}Cl, ^{37}Cl$ and ^{39}k nuclei as shown in figure-3. The dotted and solid curves are the calculated results obtained, respectively, without and with including the corrections of $F_{fs}(q)$ and $F_{cm}(q)$. The solid circles are the experimental data of elastic form factors for considered nuclei. It is clear that the experimental data [20,21] are in good agreement with both calculations of the solid and dotted curves throughout all values of q.

Including $F_{fs}(q)$ and $F_{cm}(q)$ corrections in the calculations of the $^{35}Cl, ^{37}Cl$ and ^{39}k nuclei leads to slight reduction in the calculated elastic form factors throughout all values of q. All the first and second diffraction minima are reproduced in the correct places.

Summary and Conclusions

It is concluded that the derived form of NDD of eq.(2) employed in the determination of theoretical weight function of eq. (22) is capable to reproduce information about the NMD and elastic form factors as do those of the experimental data.

Table-1: The values of various parameters employed in the present calculations together with $\langle r^2 \rangle_{cal}^{1/2}$ and $\langle r^2 \rangle_{\exp}^{1/2}$.

Nuclei	$3PF$ [16]			Experimental central NDD [16]	Calculated parameters and rms of the present work			Experimental rms [16]
	w (fm)	c (fm)	z (fm)	$\rho_{\exp}(0)$ (fm^{-3})	b (fm)	δ	$\langle r^2 \rangle_{cal}^{1/2}$ (fm)	$\langle r^2 \rangle_{\exp}^{1/2}$ (fm)
^{35}CL	-0.10	3.476	0.599	0.1710227	1.9704	1.806	3.371959	3.388
^{37}CL	-0.13	3.554	0.588	0.1773539	1.9574	1.726	3.367331	3.384
^{39}k	-0.201	3.743	0.585	0.1760695	1.98704	1.5356	3.434293	3.408

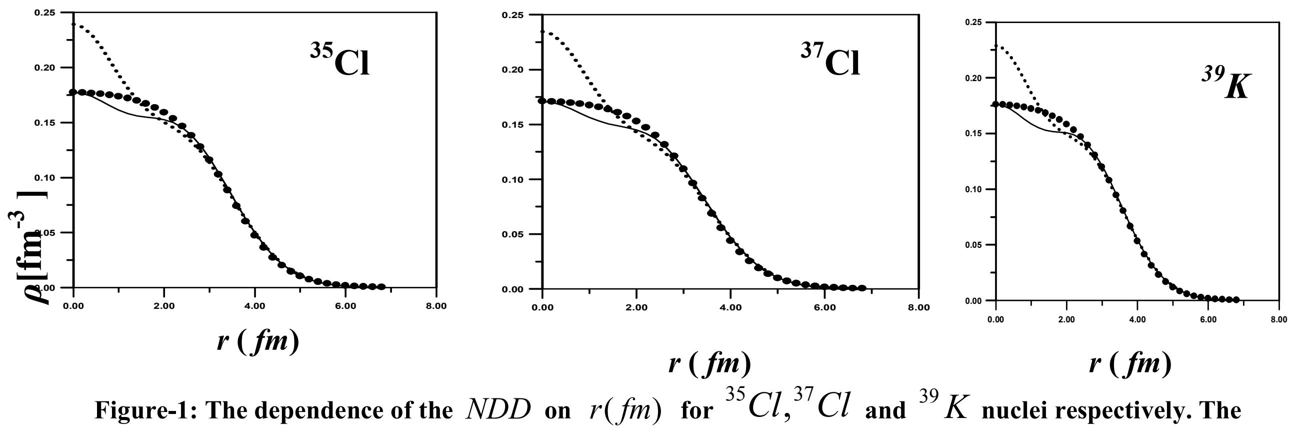

Figure-1: The dependence of the NDD on $r(fm)$ for $^{35}Cl, ^{37}Cl$ and ^{39}K nuclei respectively. The dotted and solid curves are the theoretical NDD of eq.(2) with δ =0 and $\delta \neq 0$, respectively .The solid circles curves are the experimental data [13]

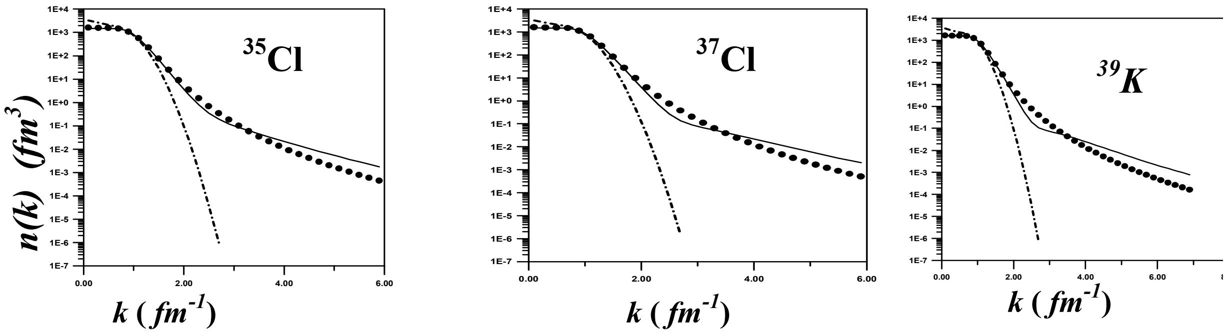

Figure-2:The NMD for $^{35}Cl, ^{37}Cl$ and ^{39}K nuclei, respectively. The dashed-dotted distributions are the results obtained by the shell model calculation of eq.(6) using the single particle harmonic oscillator wave functions in the momentum representation. The solid circles and solid curves distributions are the calculated results expressed by the CFM of eq.(17) using the experimental and theoretical weight functions, respectively .

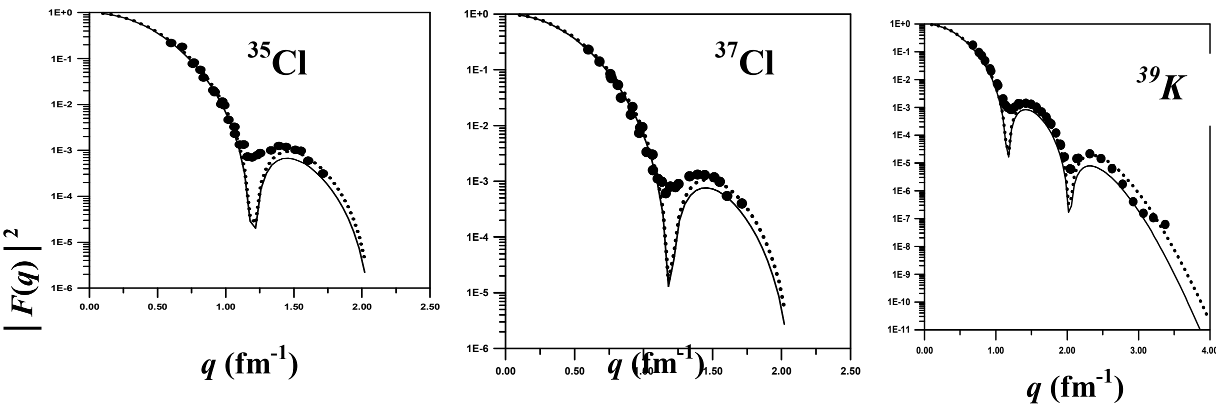

Figure-3;Elastic electron scattering is drawn as a function of momentum transfer for nuclei ^{35}Cl, $^{37}Cl$and ^{39}k respectively . The dotted and solid curve are the calculated results without and with including the corrections (finite size nucleon and center of mass corrections), respectively . The solid circles are the experimental data, taken from Refs. [20,21]

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