Γ-CENTRALIZING MAPPINGS OF SEMIPRIME Γ-RINGS

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Abstract

Let M be a Γ-Ring with center Z(M) and S a non-empty subset of M. A mapping F from M to M is called Γ-centralizing on S if $[x,F(x)]_\alpha$= xαF(x)-F(x)αx∈Z(M) for all x∈S,α∈Γ . we show that a semi-prime Γ-ring M must have a non-trivial central ideal if it admits an endomorphism which is Γ-centralizing on some non-trivial one –sided ideal.

Key word: Γ-Ring,Derevitions, Γ-centralizing, prim Γ-ring , semi-prim Γ-ring.

التطبيقات المركزية من النمط –Γ على الحلقات شبه الاولية من النمط–Γ

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الخلاصة

لتكن M حلقة من النمط–Γ ذات مركز Z(M) و S مجموعة غير خالية من M. الدالة F من M إلى M تدعى تمركز من النمط–Γ اذا كان $[x,F(x)]_\alpha$= xαF(x)-F(x)αx∈Z(M) لكل x∈S,α∈Γ . سنبين أن الحلقة من النمط– Γ شبه الأولية M يجب ان تحتوي مثالي مركزي غير صفري اذا كانت F تشاكل تقابلي متمركز من النمط–Γ على مثالي (ايمن او ايسر) غير صفري.

الكلمات المفتاحية: حلقات كاما،المشتقات، تمركزات كاما،حلقات اولية من النمط–Γ ،حلقات شبه اولية من النمط–Γ.

1.Introduction

The purpose of introducing the concept of a Γ-ring is to generalize that of a classical ring. In the last few decades, a number of modern algebraists have determined a lot of fundamental properties of Γ-rings and extended numerous significant results in classical ring theory to gamma ring theory. Note that the notion of a Γ-ring was first introduced by N. Nobusawa[1] and then generalized by W. E. Barnes[2]. They obtained many important fundamental properties of Γ-rings, and also S.Kyuno[3], J.Luh[4], G.L.Booth[5] and some other prominent mathematicians characterized much more significant results in the theory. let R denote a ring with center Z, and let S be a nonempty subset of R. A mapping F from R to R is called centralizing on S if [x,F(x)] ∈Z for all x∈S ;in the special case where [x,F(x)]=0 for all x∈S, the mapping F is described as commuting on S. in [6] Mayne prove that if a prime ring R admits either a nonidentity automorphism or a nonzero derivation which is centralizing on some nonzero ideal U of R , then R is commutative in this paper we will extend the results of H.E.Bell and W.S. Martindale[7].

2.Some basic definitines and exmpel

Definition 2.1[2]

Let *M* and Γ be two additive abelian groups. If there exists a mapping(*a*,α ,*b*)→*a*α*b* of *M* × Γ ×*M* →*M* which satisfies the conditions:

(a) $(a + b)\alpha c = a\alpha c + b\alpha c$, $a(\alpha + \beta)b = a\alpha b + a\beta b$, $a\alpha (b + c) = a\alpha b + a\alpha c$ and
(b) $(a\alpha b)\beta c = a\alpha (b\beta c)$ for all $a,b, c\in M$ and $\alpha,\beta \in\Gamma$, then M is called a **Γ-ring** in the sense of Barnes[2], or simply, a Γ-ring.

Example 2.2

suppose that R is a ring with identity 1 and $M_{m,n}(R)$ is the set of all $m\times n$ matrices over R. Then M is a Γ-ring with respect to the usual addition and multiplication of matrices if we choose $M = M_{m,n}(R)$ and $\Gamma = M_{n,m}(R)$. In particular, if we let $M = M_{1,2}(R)$ and $\Gamma=\{({}^{n.1}{}_{0})$(;n is an integer} , then M is a Γ-ring.

Definition 2.3[3]

An additive subgroup U of M is said to be a left (or right) ideal of M if MΓU ⊂U(or ,UΓM ⊂U) ,whereas U is called a two – sided ideal , or simply , an ideal of M if U is a left as well as a right ideal of M.

Definition 2.4[3]: If M is a Γ-ring then M is called prime if aΓMΓb=0 (with a,b∈M) Implies either a=0 or b=o Note that this concept of prime Γ-ring were introduced by J. Luh[4], and some analogous results corresponding to the prime rings were obtained by him as well as by S. Kyuno[3].For a,b∈M and α∈Γ, then $[a,b]_\alpha$=aαb-bαa is called the commutator of a and b with respect to α. The set Z(M)={a∈M;aαm=mαa for all α∈ Γ and m ∈M} is called the center of Γ-ring M.

Definition2.5[3]

A subset S of a Γ-ring M is called strongly nilpotent if there exists a positive integer n such that $(S\Gamma)^nS=(0)$.

Definition 2.6[3]

An ideal P of a Γ-ring M is prime if for any ideals A,B⊂M,AΓB⊂P implies A⊂P or B⊂P. and an ideal Q of M is semi-prime if for any ideal U,UΓU⊂Q implies U⊂Q Also a Γ-ring M is semi-prime if the zero ideal is semi-prime ideal. And we can prove that a semi-prime Γ-ring contains no strongly nilpotent one sided – ideal. $(a,\alpha ,b)\to a\alpha b$ of $M \times \Gamma \times M \to M$ which satisfies the conditions
(a) $(a + b)\alpha c = a\alpha c + b\alpha c$, $a(\alpha + \beta)b = a\alpha b + a\beta b$, $a\alpha (b + c) = a\alpha b + a\alpha c$ **(b)** $(a\alpha b)\beta c = a\alpha (b\beta c)$ for all $a,b, c\in M$ and $\alpha ,\beta \in\Gamma$ then M is called a **Γ-ring** in the sense of Barnes[2], or simply, a Γ-ring.

***R*emark2.7[8]:**T.K. Mukherjee and M.K.Sen give equivalent definition of prime ideal ,if P is an ideal of Γ-ring M ,then the following are equivalent
P is prime ideal of M.
if a,b∈M and aΓMΓb⊂P implies a∈P or b∈P

Definition 2.8[9]: An additive subgroup S of a Γ-ring M is called subring if SΓS⊂S.

Definition 2.9[3]: let M and N be two Γ-rings let T be a map from M to N then T is called Γ-ring homomorphism iff T(xαy)=T(x)αT(y) and T(x+y)=T(x)+T(y) , for all x,y ∈M.
In the following we will define Γ-centralizing mapping on Γ-rings.

Definition 2.10: Let M be a Γ-ring with center Z(M) and S be a non-empty subset of M. A mapping F:M→M is called Γ-centralizing on S if $[x,F(x)]_\alpha\in Z(M)$ for all x∈S and α∈Γ ;in the special case where $[x,F(x)]_\alpha=0$,for all x∈S and α∈Γ,the mapping F is described as Γ-commuting on S.

Example2.11: Let M_1 be Γ_1-ring ,put $M=M_1\oplus M_1$ and $\Gamma= \Gamma_1\oplus \Gamma_1$ then M is a Γ-ring. Define a mapping d:M⟶M by d((x,y))=(y,x) for all x,y∈ M_1, and let S={(x,0)|x∈ M_1}be a subset of M .
Then
$[(x,0),d((x,0)]_\alpha=(x,0)\alpha(0,x)(0,x)\alpha(x,0)$
$=(x\alpha0,0\alpha x)-(0\alpha x,x\alpha0)=(0,0)$.
That is mean d is Γ-centralizing on S.

3.Γ-Centralizing mappings of semiprime Γ-rings:

For proving our main result, we have need some important results which we have proved here as lemmas. So, we start as follows:

Lemma3.1

The center of a semi-prime Γ-ring M contains no non-zero strongly nilpotent elements.

Proof: Let a∈Z(M) be a strongly nilpotent element then there exits smallest positive integer n such that

$(a\Gamma)^na=(0)$. … (1)

Then from (1) we have

$(a\Gamma)^{n-1}a\Gamma a=(0)$. … (2)

Since M is a Γ-ring ,we get

$(a\Gamma)^{n-1}a\Gamma a\Gamma M=(0)$. … (3)

Now from (3) and since $(a\Gamma)^{n-2}a\in M$, therefore

$(0)=(a\Gamma)^{n-1}a\Gamma M\Gamma a\Gamma(a\Gamma)^{n-2}a$

$=(a\Gamma)^{n-1}a\Gamma M\Gamma(a\Gamma)^{n-1}a$. But M is a semi-prime Γ-ring we have from above relation , $(0)=(a\Gamma)^{n-1}a$
But n is smallest positive integer such that $(a\Gamma)^{n}a=(0)$, then a=0.

Lemma3.2

Let M be a Γ-ring then for all a,b,c∈M, α,β ∈ Γ
(1)$[a,b+c]_\alpha=[a,b]_\alpha+[a,c]_\alpha$
(2)$[a+b,c]_\alpha=[a,b]_\alpha+[a,c]_\alpha$
(3)$[a\beta b,c]_\alpha=a\beta[b,c]_\alpha+[a,c]_\alpha+a\beta(c\alpha b)-a\alpha(c\beta b)$.
Proof: Obvious.

Throughout this paper ,the condition aβcαb=aαcβb,for all a,b,c∈M and α,β∈Γ will represent by (*).

Lemma3.3

Let M be a semi-prime Γ-ring satisfying (*) and let a∈M such that $a\beta[a,m]_\alpha=0$ (or $[a,m]_\alpha\beta a=0$) , for all m∈ M and α, β ∈ Γ. Then a∈ Z(M).

Proof

For all m_1∈M and δ∈Γ , then
$0=a\beta[a,m\delta m_1]_\alpha=a\beta(m\delta[a,m_1]_\alpha+[a,m]_\alpha\delta m_1)$
$a\beta(m\delta[a,m_1]_\alpha$ (1)
By assumption and (*) ,we get
$0=m\delta a\beta[a,m_1]_\alpha=m\beta a\delta[a,m_1]_\alpha$. …(2)
Now from (1) and (2), we have
$[a,m]_\beta\delta[a,m_1]_\alpha=0$. …(3)
In (3) replace m_1 by $m_1\gamma m$,for all γ∈Γ,we have
$0=[a,m]_\beta\delta[a,m_1\gamma m]_\alpha=[a,m]_\beta\delta(m_1\gamma[a,m]_\alpha+[a,m_1]_\alpha\gamma m)=[a,m]_\beta\delta m_1\gamma[a,m]_\alpha$.
Now for all β ∈ Γ take β = α and since M is a semi-prime Γ-ring therefore
$[a,m]_\alpha=0$, for all m∈ M and α∈ Γ, thus a∈Z(M). Similarly we can prove the lemma ,when $[a,m]_\alpha\beta a=0$.

Lemma3.4

let M be a semi-prime Γ-ring ,U be a left ideal of M and A,B:M×M→M ,be two bi-additive maps ,if A(x,y)ΓUΓB(x,y)=0, then A(x,y)ΓUΓB(u,v)=0 , for all x,y,u,v∈U.

Proof: By assumption
A(x,y)ΓUΓB(x,y)=(0) , for all x,y∈U…(1)
In (1) replace x by x+u for all u∈U we get
(0)=A(x+u,y)ΓUΓB(x+u,y)
=(A(x,y)ΓUΓA(u,y))ΓUΓ(B(x,y)+ B(u,y))
A(x,y)ΓUΓB(u,y)+A(u,y)ΓUΓB(x,y)=(0). (2)
Now from (2) and semi-primness of M we can prove
(A(x,y)ΓUΓB(u,y))ΓMΓ(A(x,y)ΓUΓB(u,y)
=-A(u,y)ΓUΓB(x,y)ΓMΓA(x,y)ΓUΓB(u,y)
but UΓB(x,y)ΓMΓ A(x,y)ΓU⊆U , (U be a left ideal), therefore
(A(x,y)ΓUΓB(u,y))ΓMΓ(A(x,y)ΓUΓB(u,y)=0.
Then
A(x,y)ΓUΓB(u,y)=(0). … (3)
In (3) replace y with y+v , for all v∈U we get
(0)=A(x,y+v)ΓUΓB(u,y+v)
=(A(x,y)+A(x,v))ΓUΓ(B(u, y)+ B(u,v))
A(x,y)ΓUΓB(u,v)+A(x,v)ΓUΓB(u,v)=0. … (4)
Also we can prove that
(A(x,y)ΓUΓB(u,v))ΓMΓ(A(x,y)ΓUΓB(u,v) =-A(x,v)ΓUΓB(u,v)ΓMΓA(x,y)ΓUΓB(u,v).
From the above relation ,since U be a left ideal, then UΓB(u,v)ΓMΓA(x,y)ΓU⊂U , therefore
A(x,y)ΓUΓb(u,v)ΓMΓA(x,y)ΓUΓB(u,v)=0, but M be a semi-prime Γ-ring then A(x,y)ΓUΓB(u,v)=0 ,for all x,y,u,v∈U.

Lemma3.5

Let M be semiprime Γ-ring satisfying (*) and let U be left ideal of M then Z(U)⊂Z(M).

Proof

Let a∈Z(U) then for all α∈Γ and x∈M ,xαa∈U and $[a,x\alpha a]_\beta=0$ for all β∈Γ, then by lemma 3.3 , a∈Z(M).

Lemma3.6

Let U be a nonzero left ideal of the semi-prime Γ-ring M satisfying (*) if T is an endomorphism of M which is Γ-centralizing on U .Then T is Γ-commuting on U

Proof: By assumption $[x,T(x)]_\alpha$∈ Z(M),for all x∈U and α∈Γ. Polarizing the above relation we have
$[x,T(y)]_\alpha+[y,T(x)]_\alpha$∈Z(M) , for all x,y∈U and α∈Γ. …(1)
In (1) replacing y by xβx , then we get
$[x,T(x\beta x)]_\alpha+[x\beta x,T(x)]_\alpha=[x,T(x)\beta T(x)]_\alpha+x\beta[x,T(x)]_\alpha+[x,T(x)]_\alpha\beta x$
$=2T(x)\beta[x,T(x)]_\alpha+2x\beta[x,T(x)]_\alpha$∈ Z(M).
Now since $[x,T(x)]_\alpha$∈ Z(M) , then
$[2x\beta[x,T(x)]_\alpha,x]_\alpha=0$,for all x∈U.
Therefore $2x\beta[x,T(x)]_\alpha$∈ Z(U)⊂ Z(M) (by Lemma 3.5) ,so $2x\beta[x,T(x)]_\alpha$∈ Z(M) , by additive subgroup of Z(M) we have $2T(x)\beta[x,T(x)]_\alpha$ Therefore
$0=2[T(x)\beta[x,T(x)]_\alpha,x]_\alpha$
$=2[x,T(x)]_\alpha\beta[x,T(x)]_\alpha$, for all x∈U and α∈Γ. Which means that

$(2[x,T(x)]_\alpha\beta)^3(2[x,T(x)]_\alpha)$
$=2^3(2[x,T(x)]_\alpha\beta[x,T(x)]_\alpha)\beta[x,T(x)]_\alpha\beta[x,T(x)]_\alpha$
$=0.$

Since the center of a semi-prime Γ-ring contains no nonzero strongly nilpotent elements we conclude that

$2[x,T(x)]_\alpha=0$ …(2)

and hence

$2([x,T(y)]_\alpha+[y,T(x)]_\alpha)=0$ for all $x,y\in U$ and $\alpha\in\Gamma$. …(3)

Now by use (1) and (2) we can proved that
$[x\beta y+y\beta x,xT(x)]_\alpha+[x\beta x,T(y)]_\alpha=2y\beta[x,T(x)]_\alpha+2x\beta([x,T(y)]_\alpha+[y,T(x)]_\alpha)=0$, for all $x,y\in U$ and $\alpha,\beta\in\Gamma$.

Therefore $[x\beta y+y\beta x,xT(x)]_\alpha+[x\beta x,T(y)]_\alpha=0$, for all $x,y\in U$ and $\alpha,\beta\in\Gamma$. …(4)

Now in (4) let $T(x)=z$ and take $y=z\delta x\mu x$, for all $\delta,\mu\in\Gamma$, then

$[x\beta z\delta x\mu x+z\delta x\mu x\beta x,z]_\alpha+[x\beta x,T(z\delta x\mu x)]_\alpha$
$=[x\beta z\delta x\mu x,z]_\alpha+[z\delta x\mu x\beta x,z]_\alpha+[x\beta x,T(z)\delta z\mu z]_\alpha$
$=x\beta z\delta[x\mu x,z]_\alpha+[x\beta z,z]_\alpha\delta x\mu x+z\delta x\mu x\beta[x,z]_\alpha+[z\delta x\mu x,z]_\alpha\beta x+[x\beta x,T(z)\delta z\mu z]_\alpha$
$=x\beta z\delta(x\mu[x,z]_\alpha+[x,z]_\alpha\mu x)+(x\beta[z,z]_\alpha+[x,z]_\alpha\beta z)\delta x\mu x+z\delta x\mu x\beta[x,z]_\alpha+(z\delta x\mu[x,z]_\alpha+[z\delta x,z]_\alpha\mu x)\beta x+[x\beta x,T(z)\delta z\mu z]_\alpha$
$=x\beta z\delta(2x\mu[x,z]_\alpha)+2[x,z]_\alpha\beta z\delta x\mu x+z\delta x\mu[x,z]_\alpha\beta x+z\delta[x,z]_\alpha\mu x\beta x+[x\beta x,T(z)\delta z\mu z]_\alpha$
$=2[x,z]_\alpha\beta z\delta x\mu x+[x\beta x,T(z)\delta z\mu z]_\alpha$
$=[x\beta x,T(z)\delta z\mu z]_\alpha$.

Therefore

$0=[x\beta x,T(z)\delta z\mu z]_\alpha$
$=T(z)\delta[x\beta x,z\mu z]_\alpha+[x\beta x,T(z)]_\alpha\delta z\mu z$
$=T(z)\delta(x\beta(2[x,z]_\alpha\mu z))+[x\beta x,T(z)]_\alpha\delta z\mu z$
$[x\beta x,T(z)]_\alpha\delta z\mu z=0$ …(5)

On the other hand taking $y=z\delta x$ in (4) for all $\delta\in\Gamma$, we have

$[x\beta z\delta x+z\delta x\beta x,x,z]_\alpha+[x\beta x,T(z\delta x)]_\alpha$
$=[x\beta z\delta x+z\delta x\beta x,x,z]_\alpha+[x\beta x,T(z)\delta z]_\alpha$
$=[x\delta z\beta x+z\delta x\beta x,x,z]_\alpha+[x\beta x,T(z)\delta z]_\alpha$
$=[(x\delta z+z\delta x)\beta x,x,z]_\alpha+T(z)\delta[x\beta x,z)]_\alpha+[x\beta x,T(z)]_\alpha\delta z$
$=[([x,z]_\delta+2z\delta x)\beta x,z]_\alpha+[x\beta x,T(z)]_\alpha\delta z$
$=[[x,z]_\delta\beta x,z]_\alpha+2[z\delta x\beta x,z]_\alpha+[x\beta x,T(z)]_\alpha\delta z$
$=[x,z]_\delta\beta[x,z]_\alpha+[[x,z]_\delta,z]_\alpha\beta x+[x\beta x,T(z)]_\alpha\delta z.$
$0=[x,z]_\delta\beta[x,z]_\alpha+[x\beta x,T(z)]_\alpha\delta z\in Z(M)$. …(6)

But $[x,z]_\delta\beta[x,z]_\alpha\in Z(M)$, therefore from (6) $[x\beta x,T(z)]_\alpha\delta z\in Z(M)$.

Now from (5)
$0=[x\beta x,T(z)]_\alpha\delta z\mu z=z\mu[x\beta x,T(z)]_\alpha\delta z$
$0=[x\beta x,T(z)]_\alpha\delta z\mu[x\beta x,T(z)]_\alpha\delta z$

but the center of a semi-prime Γ-ring contains no nonzero strongly nilpotent elements we conclude that. $[x\beta x,T(z)]_\alpha\delta z=0$. Therefore from (6) we have $[x,z]_\delta\beta[x,z]_\alpha=0$, for all $\delta\in\Gamma$, thus $[x,z]_\alpha\beta[x,z]_\alpha=0$, therefore $[x,z]_\alpha=[x,T(x)]_\alpha=0$.

4.Main result

Theorem 4.1:

Let M be a semi-prime Γ-ring satisfying (*) and U be a non zero left ideal of M, suppose that M admits an endomorphism T which is one-to-one on U, Γ-centralizing on U and not the identity on U, if $T(U)\subseteq U$. Then M contains a non zero central ideal.

Proof:

Let x^T be the image of element x under the mapping T.

Now, by Lemma 3.6, we have
$[x,x^T]_\alpha=0$, for all $x\in U$, $\alpha\in\Gamma$.

Polarizing the above relation we have
$[x,y^T]_\alpha=[x^T,y]_\alpha$ for all $x,y\in U$, $\alpha\in\Gamma$. …(1)

Substituting $x\beta y$ for y and applying (1), we then get
$(x-x^T)\beta[x^T,y]_\alpha=0$, for all $x,y\in U$ $\beta,\alpha\in\Gamma$. …(2)

Replacing y by $u\gamma y$ in (2) for all $u\in U$ and $\gamma\in\Gamma$ yields
$(x-x^T)\beta U\gamma[x^T,y]_\alpha=(0)$
therefore $(x-x^T)\Gamma U\Gamma[x^T,y]_\alpha=(0)$
now by Lemma3.4 then
$(x-x^T)\Gamma U\Gamma[z^T,y]_\alpha=(0)$, for all $x,y,z\in U$. … (3)

Let $P=\{P_i \mid P_i$ is prime ideal with $\cap P_i=(0)$; $i\in I\}$, therefore from (3), we get
$(x-x^T)\Gamma M\Gamma U\Gamma[z^T,y]_\alpha\subset P_i$.

Therefore by Remark2.6 either
(a) $(x-x^T)\subset P_i$ or (b) $U\Gamma[z^T,y]\subset P_i$.

Call a prime ideal in P a type –one prime if it satisfies (a); call all other members of P type-two primes. Now let $P_1=\cap P_i$ (type –one prime) and $P_2=\cap P_i$ (type-two prime). It is clear that $P_1\cap P_2=(0)$. Now from (a) and (b) and since $T(U)\subseteq U$ then for all x in U we have $x^T\in U$ and $x-x^T\in U$. From (a) and (b) we get $U^T\Gamma[(x-x^T),y^T]\subset P_1\cap P_2=(0)$, therefore $(U\Gamma[x-x^T,y]_\alpha)^T=0$, but T is one to one on U then

$U\Gamma[x-x^T,y]_\alpha=0$ …(4)

From (4) we have $U\Gamma[x,y]_\alpha-U\Gamma[x^T,y]_\alpha=0\in P_i$ but $U\Gamma[x^T,y]_\alpha\subset P_i$ (type-two prime), therefore
$U\Gamma[x,y]_\alpha\subset P_i$ (type-two prime). …(5)

Now returning to (1) and replacing x by $x\beta y$ for all $x,y\in U$ and $\beta\in\Gamma$ we get
$[x\beta y,y^T]_\alpha=[(x\beta y)^T,y]_\alpha=[x^T\beta y^T,y]_\alpha$
$x\beta[y,y^T]_\alpha+[x,y^T]_\alpha\beta y=x^T\beta[y^T,y]_\alpha+[x^T,y]_\alpha\beta y^T$
$[x,y^T]_\alpha\beta y=[x^T,y]_\alpha\beta y^T=[x^T,y]_\alpha\beta y^T$ (from (1))

Therefore
$[x^T,y]_\alpha\beta(y-y^T)=0$, now since $T(U)\subseteq U$, then in above relation replace y by y^T ,we have $[x^T,y^T]_\alpha\beta(y^Ty^{TT})=0$,but T is one to one on U therefore $[x,y]_\alpha\beta(y-y^T)=0$, replace x by $x\gamma u$, for all $u\in U$ and $\gamma\in\Gamma$, thus
$0=[x\gamma u,y]_\alpha\beta(y-y^T)=(x\gamma[u,y]_\alpha+[x,y]_\alpha\gamma u)\beta(y-y^T)$
$= [x,y]_\alpha\gamma u\beta(y-y^T)$.
Therefore $[x,y]_\alpha\Gamma U\Gamma(y-y^T)=(0)$. Now by Lemma 3.4, then $[x,y]_\alpha\Gamma U\Gamma(z-z^T)=(0)$, for all x,y, z∈U. By definition of P then either $[x,y]_\alpha\in P_i$ or $U\Gamma(z-z^T)\subset P_i$. But $T(U)\subset U$, for all $z_1\in U$, then $z_1-z_1^T\in U$, therefore
$[x,y]_\alpha\Gamma(z_1-z_1^T)\Gamma(z_2-z_2^T)=(0)$,
for all x,y, z_1, $z_2\in U$. (6)

Define V be the left ideal generated by all elements of form $u\beta(v-v^T)$ for u,v ∈ U and β∈ Γ. We will show that V is commutative as Γ-ring ,it will suffice to show that
$[u_1\beta(v_1-v_1^T),u_2\gamma(v_2-v_2^T)]_\alpha=0$,for all $u_1,u_2,v_1,v_2\in U$ and β,γ∈Γ ...(7)

We note that
$[u_1\beta(v_1-v_1^T),u_2\gamma(v_2-v_2^T)]_\alpha=u_1\beta[(v_1-v_1^T),u\gamma(v_2-v_2^T)]_\alpha$
$+[u_1,u_2\gamma(v_2-v_2^T)]_\alpha\beta(v_1-v_1^T)$
$=u_1\beta[v_1-v_1^T,u_2\gamma v_2]_\alpha-u_1\beta[v_1-v_1^T,u_2\gamma v_2^T]_\alpha$
$+u_2\gamma[u_1,v_2-v_2^T]_\alpha\beta(v_1-v_1^T)$
$+[u_1,u_2]_\alpha\gamma(v_2-v_2^T)\beta(v_1-v_1^T)$
$=0$
according to (4) and (6), V is commutative ideal so by Lemma 3.5 , V is central left ideal of M .
Now if V =(0) then $u\beta(v-v^T)=0$, for all u,v∈ U and β∈ Γ , there for
$U\Gamma(y-y^T)=0$,for all y∈ U. ...(8)

Suppose that $F=\{u\in U \mid u^T=u\}$, then from (1) and (8) we can prove that $x\beta y+y\beta x\in F$ for all x, y∈U, β∈ Γ.
Since $U\Gamma(y-y^T)=0$, $U\Gamma(x-x^T)=0$ and $x,x^T,y,y^T\in U$, then
$x\beta y=x\beta y^T$ ...(a)
$y\beta x=y\beta x^T$...(b)
but from (1) , we have $[x,y^T]_\beta=[x^T,y]_\beta$, therefore from (a) and (b), we get
$x\beta y+y\beta x=x^T\beta y^T+y^T\beta x^T=(x\beta y+y\beta x)^T$.
Therefore
$x\beta y+y\beta x\in F$. ...(9)

Now from (1) if x∈ F then $[x,y^T]_\alpha=[x^T,y]_\alpha=[x,y]_\alpha$ for all y∈ U,α∈Γ therefore $[x,y-y^T]_\alpha=0$ and $(y-y^T)\alpha x=x\alpha(y-y^T)$, but by (8) then
$(y-y^T)\alpha x=0$,for all x∈F , y∈ U and α∈ Γ. ... (10)

Now from (9) then
$(y-y^T)\alpha(x\beta z+y\beta z)=0$ for all x,y,z∈ U ...(11)
therefore $(y-y^T)\alpha x\beta z+(y-y^T)\alpha y\beta z=0$.
Then
$2(y-y)\Gamma U\Gamma U=(0)$. ...(12)
But U be a left ideal then $2(y-y^T)\Gamma U\Gamma M\Gamma U=(0)$ and $2(y-y)\Gamma U\subset U$ then
$(2(y-y^T)\Gamma U)\Gamma M\Gamma(2(y-y^T)\Gamma U)=(0)$,but M is a semi-prime Γ-ring then
$2(y-y)\Gamma U=(0)$ for all y∈ U. ...(13)

Since $T(U)\subset U$ then $y-y^T\in U$ for all y∈U ,now from (8), we get
$(y-y^T)\beta(y-y^T)=0$ for all β∈Γ. ...(14)
Therefore $y^T\beta y^T=(y-(y-y^T))\beta(y-(y-y^T))$
$y^T\beta y^T=y\beta y$ (according to (8),(a)and (14)).
then $y\beta y\in F$ for all y∈ U and β∈ Γ.
In (11) replace x by xγx and z by mδx ,for all γ,δ∈ Γ and m∈ M ,we get
$0=(y-y^T)\alpha(x\gamma x)\beta m\delta x+(y-y^T)\alpha m\delta x\beta x\gamma x$
But if x∈ U then xγx ∈ F therefore from (10) we have
$(y-y^T)\alpha(x\gamma x)\beta m\delta x=0$
So that
$(y-y^T)\Gamma M\Gamma(x\Gamma)^2x=0$, for all x,y ∈U ...(15)

Now from definition of P then either (a′)
$(y-y^T)\in P_i$ for all y∈U or (b′)
$(x\Gamma)^2x\in P_i$ for all x∈ U.
We say P_i is of type three if satisfy (a′)
We say P_i is of type four if satisfy (b′)
and $P_4=\cap P_i$ (P_i is type four)
therefore $P_3\cap P_4=(0)$
Since T is not identity on U then there exists y∈ U such that $y-y^T\neq 0$
let $0\neq\tilde{U}=U\cap P_3$ that $\tilde{U}$ is left ideal .
For each x∈ Ũ then x∈ U there for
$(x\Gamma)^2x\subset P_4$ but $P_4\cap P_3\cap U=(0)$ then
For all x∈Ũ,then $(x\Gamma)^2x=0$ therefore $(\tilde{U}\Gamma)^2\tilde{U}=(0)$,but M has no such left ideal by Definition2.5 ,then V≠ 0 .

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