RIGHT (σ,τ)-DERIVATIONS ON LEFT IDEALS

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Abstract

Let R be a prime ring and I a nonzero left Ideal of R which is a semi prime as a ring. For a right (σ,τ) *– derivations* $\delta: R \rightarrow R$*, we prove the following results:*

(1) *If* δ *acts as a homomorphism on I, then* $\delta = 0$ *on R.*

(2) *If* δ *acts as an anti- homomorphism on I, then either* $\delta = 0$ *on R or* $I \subseteq Z(R)$*.*

Keywords:derivation,right derivation, (σ,τ)- derivation, right (σ,τ)-derivation

المشتقات(σ,τ) اليمنى على المثاليات اليسرى

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الخلاصة

لتكن R حلقة اولية و I مثالي ايسر غير صفري ل R شبه مثالي كحلقة، برهنا للاشتقاق (σ,τ) الايمن $\delta: R \rightarrow R$ النتائج الاتية :

(١) اذا كان δ هو تشاكل على I ، فأن $\delta = ٠$ على R .

(٢) اذا كان δ هو تشاكل مضاد على I ، فأن اما $\delta = ٠$ على R او $I \subseteq Z(R)$.

1. Introduction:

Throughout the present paper R will be denote an associative ring with center $Z(R)$. Recall that R is prime if $aRb=\{0\}$ implies that either $a=0$ or $b=0$. Let $x, y \in R$, the commutator $[x,y]$ will denoted $xy-yx$. An additive mapping $d:R \rightarrow R$ is called a derivation (resp.,Jordan derivation) on R if $d(xy)=d(x)y+xd(y)$ (resp., $d(x^2)=d(x)x+xd(x)$) holds, for all $x, y \in R$. Let σ,τ are two mappings of R. An additive mapping $d:R \rightarrow R$ is called a (σ,τ)- derivation (resp., Jordan (σ,τ)- derivation) on R if $d(xy)=d(x)\sigma(y)+\tau(x)d(y)$(resp.,$d(x^2)=d(x)\sigma(x)+\tau(x)d(x)$) holds, for all $x, y \in R$. Clearly every $(1,1)$- derivation (resp., Jordan $(1,1)$-derivation), where 1 is the identity mapping on R is derivation (resp., Jordan derivation) on R. An additive mapping δ: $R \rightarrow R$ is called a left derivation (resp., Jordan left derivation) if $\delta(xy)= x\,\delta(y)+y\,\delta(x)$ (resp.,$\delta(x^2)=2x\delta(x)$) holds, for all $x, y \in R$. In view of the definition of a (σ,τ)- derivation the notion of left (σ,τ)- derivation can be extended as follows:

An additive mapping δ: $R \rightarrow R$ is called a left (σ,τ)- derivation (resp., Jordan left (σ,τ)-derivation) on R if $\delta(xy)=\sigma(x)\delta(y)+\tau(y)\delta(x)$ (resp., $\delta(x^2)=\sigma(x)\delta(x)+\tau(x)\delta(x)$) holds, for all $x, y \in R$. Clearly every left $(1,1)$- derivation (resp., Jordan left $(1,1)$-derivation) is a left derivation (resp., Jordan left derivation) on R. An additive mapping δ: $R \rightarrow R$ is called a right derivation (resp., Jordan right derivation) on R if $\delta(xy)=\delta(y)\,x+\delta(x)y$ (resp.,$\delta(x^2)= 2\delta(x)x$) holds, for all $x, y \in R$.

An additive mapping δ: $R \rightarrow R$ is called a right (σ,τ)- derivation (resp., Jordan right (σ,τ)-derivation) on R if $\delta(xy)= \delta(y)\sigma(x)+\delta(x)\,\tau(y)$

(resp.,$\delta(x^2)= \delta(x)\sigma(x)+\delta(x)\tau(x)$) holds, for all $x, y \in R$. Clearly, every right (*1,1*)- derivation (resp., Jordan right (*1,1*)-derivation) on *R* is a right derivation (resp.,Jordan right derivation) on *R*.

Bell and Kappe [1] proved that if *d* is a derivation of a prime ring *R* which acts as a homomorphism or as an anti- homomorphism on a nonzero right ideal *I* of *R*, then *d=0* on *R*, further Yenigul and Arac [2] obtained the above result for α-derivation in prime rings. Recently Ashraf, *et al.* [3] extended the result for (σ,τ)-derivation in prime and semiprime ring. Also in [4] Ö.Glbasi and N. Aydin proved that if *d* is a (σ,τ)-derivation which acts homomorphism or as an anti-homomorphism on a prime ring *R*, then *d=0* on *R*. In [5] Majeed and Hamdi Asawer extended the above results for (σ, σ)-derivation which acts as a homomorphism or as an anti- homomorphism on a nonzero Jordan ideal and a subring *J* of a 2-torsion -free prime ring *R*, then they generalized the above extension for generalized (σ, σ)-derivation. Also they proved that if $d:R \to R$ is a (σ,τ)-derivation which acts as a homomorphism on a nonzero Jordan ideal and a subring *J* of a 2-torsion-free prime ring *R*, then either *d=0* on *R* or $J \subseteq Z(R)$.

In [6] Zaidi, *et al.* proved that if *R* is a 2-torsion-free prime ring, *J* a nonzero Jordan ideal and a subring of *R* and *d* is a left (σ, σ)-derivation of *R*, which acts as a homomorphism or as an anti-homomorphism on *R*, then *d=0* on *R*. Hamdi Asawer in[7] extended this result to a left (σ,τ)-derivation which acts as a homomorphism or as an anti- homomorphism on a nonzero Jordan ideal and a subring *J* of *R*.

As for more details and fundamental results used in this paper without mention we refer to [1,3,4,8,9,10 ,12].

The aim in this paper is to extend the above results and the theorem of Ö.Glbasi and N. Aydin [4] which state that if *d* is a nonzero (σ,τ)- derivation which acts as a homomorphism or as an anti-homomorphism on a nonzero left ideal *I* of prime ring *R* which is a semiprime as a ring, then *d=0* on *R* to a right (σ,τ)- derivation on *R* which acts as a homomorphism or as an anti- homomorphism on a nonzero left ideal *I* of prime ring *R* which is a semiprime as a ring, then either $\delta = 0$ on *R* or $I \subseteq Z(R)$.

2. Right (σ,τ)-derivation as a homomorphism or as an anti-homomorphism:

Let *R* be a ring and *d* is a derivation of *R*. If $d(xy)=d(x)d(y)$(resp., $d(xy)=d(y)\, d(x)$) holds, for all $x, y \in R$, then we say that *d* acts as a homomorphism (resp., anti- homomorphism) on *R*.

To prove the main result the following lemma is needed.

<u>Lemma(2.1):</u>[4]

Let *R* be a prime ring, *I* a nonzero left ideal of *R* which is semiprime as a ring. If *Ia=0* (*aI=0*), for $a \in R$, then *a=0*.

We are now well- equipped to prove the main theorem:

<u>Theorem (2.2):</u>

Let *R* be a prime ring, *I* a nonzero left ideal of *R* which is a semiprime as a ring. Suppose σ,τ are automorphisms of *R* and $\delta: R \to R$ is a right (σ,τ)-derivation of *R*. Then the following are holds:

(i) If δ acts as a homomorphism on *I*, then $\delta = 0$ on *R*.

(ii) If δ acts as an anti- homomorphism on *I*, then either $\delta = 0$ on *R* or $I \subseteq Z(R)$.

<u>Proof:</u>

(i) If δ acts as a homomorphism on *I*, then we have $\delta(uv)= \delta(v)\sigma(u)+ \delta(u)\tau(v)$, for all $u, v \in I$ .. . . (2.1)

Replacing *u* by *ut* , $t \in I$ in (2.1),we get $\delta(v)\sigma(ut)+ \delta(ut)\tau(v)= \delta(ut)\, \delta(v)$ Since δ is a homomorphism on *R* and σ ,τ are automorphisms of *R*, we have

$\delta(v)\sigma(u)\sigma(t)+\delta(u)\delta(t)\tau(v)=\delta(u)\delta(t)\delta(v)=$
$\delta(u)\delta(tv)=\delta(u)[\delta(v)\sigma(t)+\delta(t)\tau(v)]$, for all $u,v,t \in I$.

Or equivalently

$\delta(v)\sigma(u)\sigma(t)=\delta(u)\delta(v)\sigma(t)$,for all $u,v,t \in I$ ..(2.2)

This implies that $[\delta(v)\sigma(u) - \delta(u)\delta(v)]\sigma(t) = 0$, for all $u,v,t \in I$.

Hence $\sigma^{-1}([\delta(v)\sigma(u) - \delta(u)\delta(v)])\, I = \{0\}$, for all $u,v \in I$ and then we have $\sigma^{-1}([\delta(v)\sigma(u) - \delta(u)\delta(v)])\, RI = \{0\}$,for all $u,v \in I$, since *R* is a prime ring and *I* is a nonzero left ideal of *R*, we have $\delta(v)\sigma(u) - \delta(u)\delta(v) = 0$, for all $u,v \in I$, since δ is a homomorphism on *R*, we get

$0 = \delta(v)\sigma(u) - \delta(uv)$
$= \delta(v)\sigma(u) - \delta(v)\sigma(u) - \delta(u)\tau(v)$

$= -\delta(u)\,\tau(v)$, for all $u, v \in I$.

This implies that $\delta(u)\ \tau(v) = 0$, for all $u, v \in I$ .Replacing v by rv , $r \in R$, we get

$0 = \delta(u)\tau(rv) = \delta(u)\tau(r)\ \tau(v)$, for all $u, v \in I$, $r \in R$. Since R is a prime ring and I is a nonzero left ideal of R, we have $\delta(u) = 0$, for all $u \in I$. Now, replacing replacing u by ru , $r \in R$, we find

$0 = \delta(ru)$

$= \delta(u)\ \sigma\ (r) + \delta(r)\ \tau(u) = \delta(r)\ \tau(u)$, for all $u \in I$, $r \in R$. Since R is a prime ring, I a nonzero left ideal of R and τ is an automorphism of R, we have $\delta = 0$ on R.

(ii) If δ acts as an anti- homomorphism on I, then we have $\delta(uv) = \delta(v)\sigma(u) + \delta(u)\tau(v) = \delta(v)\ \delta(u)$, for all $u, v \in I$ . ... (2.3)

Replacing u by uv in (2.3), we get $\delta(v)\sigma(uv) + \delta(uv)\tau(v) = \delta(v)\ \delta(uv)$ for all $u, v \in I$.

Since δ is a homomorphism on R and σ, τ are automorphisms of R, we have $\delta(v)\sigma(u)\ \sigma(v) + \delta(v)\ \delta(u)\tau(v) = \delta(v)\ \delta(v)\ \sigma(u) + \delta(v)\ \delta(u)\ \tau(v)$, for all $u, v \in I$.

This implies that $\delta(v)\sigma(u)\sigma(v) = \delta(v)\delta(v)\ \sigma(u)$, for all $u, v \in I$ . . . . (2.4)

Replacing u by ut , $t \in I$ in (2.4), we get $\delta(v)\sigma(u)\sigma(t)\sigma(v) = \delta(v)\ \delta(v)\ \sigma(u)\sigma(t)$, for all $u, v, t \in I$ . . (2.5)

In view of (2.4), the relation (2.5) yields that $\delta(v)\sigma(u)\sigma(t)\sigma(v) = \delta(v)\ \sigma(u)\ \sigma(v)\sigma(t)$, for all $u, v, t \in I$, this implies that $\delta(v)\ \sigma\ (u)\ [\sigma\ (v)\ ,\ \sigma\ (t)] = 0$, for all $u, v, t \in I$ and hence $\sigma^{-1}(\delta(v))\ I\ [v,t] = \{0\}$, for all $v, t \in I$.

Since R is a prime ring, we have either $\delta(v) = 0$ or $I\,[v,t] = \{0\}$, for all $v, t \in I$.

If $\delta(v) = 0$, for all $v \in I$, replacing v by rv, where $r \in R$, to get $\delta(rv) = \delta(v)\sigma(r) + \delta(r)\tau(v)$, this implies that $\delta(r)\tau(v) = 0$, for all $v \in I$, $r \in R$.

Since R is a prime ring, I a nonzero ideal of R and τ is an automorphism of R, we have $\delta = 0$ on R.

If $I\,[v,t] = \{0\}$ thus by Lemma (2.1), we find that $[v,t] = 0$, for all $v, t \in I$.

Now, replacing v by rv, where $v \in I$ and $r \in R$,we get

$0 = [rv,t]$

$= r[v,t] + [r,t]v$

$= [r,t]\ v$, for all $v, t \in I$ and $r \in R$ and hence we have $[R,I]I = \{0\}$

Since R is a prime ring, I a nonzero left ideal of R, we have $[R,I] = \{0\}$,therefore we have $I \subseteq Z(R)$.

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