

# INELASTIC LONGITUDINAL C6 ELECTRON SCATTERING FORM FACTORS IN Ti-50, (RESIDUAL INTERACTION CONSIDERATION) 

Firas Z. Majeed, Nadia M. A. Albana'a, Ala'a A. Qassim abosajjad_altameme@yahoo.com<br>Department of physics, College of Science, University of Baghdad. Baghdad-Iraq


#### Abstract

The single orbit $1 \mathrm{f}_{7 / 2}$ has been adopted as a model space. The use of modern realistic M3Y effective nucleon- nucleon interaction with two sets of fitting parameters (Ried fitting (M3Y-P1), and Paris fitting (M3Y-P0)) beside the use of MSDI has been done as a residual interactions within the calculation of core polarization effects in Inelastic longitudinal electron scattering C6 form factor in Ti50 within the framework of first order perturbation theory (microscopic theory) with $2 \hbar \omega$ excitation energy coupling the core orbits to the higher configurations one across the model space at normal transition. Harmonic oscillator wave functions (H.O) has been adopted as a single particle wave functions in $1 \mathrm{f}_{7 / 2}$ and with the aid of F 7 MBZ model space $1 \mathrm{f}_{7 / 2}$ effective interaction to generate the model space wave functions. The present results have been compared with the experimental data.


PACS:-21.60 Cs; 23.20-g;24.10 Cu;25.30 Dh;27.40-Z.
Keywords: ${ }^{\mathrm{Ti}-50}$, Inelastic electron scattering C 6 form factor calculated with core polarization, M3Y as a residual interaction.

> عوامل التشكل للاستطاره الالكترونيه الطوليه غير المرنـه متعدده الاقطاب C6 لنواةTi-50 ( أعتبارات تفاعل البقيه)

الخلاصه
تم أعتماد المدار الدنفرد IFY/r كـأنموذج فضـاء وأستخدام التفاعل الواقعي المؤثر بين النيوكليونين من
الفصيله M3Y و بواقع مجموعتين مـن عوامـل الضبط الاولـى مـن نوع Paris و الاخرى مـن نـوع Ried
بالاضافه الى أستخدام التفاعل اللضبط MSDI ، كتفاعلات بقيه من خلال حسابات تأثيرات أستقطاب القلب

الاطار العام لنظريه الاضطراب ذي المرتبه الاولى و بطاقه تهيج مقدارها $2 \hbar \omega$ و التي تربط أغلفه القلب
الخامل مع التشككيلات العليا عبر الفضاء الانموذجي . كمـا و تم أعتماد دوال المتذبذب النتوافقي كداله للجسيم

الموجيه الخاصه بأنموذج الفضاء. تمت مقارنه النتائج الحالية مع المعطيات العمليه.

## Introduction

With electron scattering, one can immediately relate the cross section to the transition matrix elements of the local charge and current density operators and thus directly to the structure of the target itself. Of course, the same considerations apply to processes involving real photons, but electrons have the second great advantage that for a fixed energy loss of the electron, one can vary the threemomentum transferred $q$ to the nucleus, the only restriction being that the four momentum transfer be space-like[1]
Theoretical solutions to the nuclear many-body problem are partly phenomenological, and thus theory and experiment are closely tied together. Theory takes its inspiration from experiment in guiding the structure of the models and their parameters; the nuclear shell model is the primary example [2]. Nuclear experiment takes its inspiration from theory in helping to choose which experiments are most important to prove or disprove model assumptions.
Through microscopic theory, the discarded space has been included as a first order correction through the first order perturbation theory that is particle hole state ( $\mathrm{p}-\mathrm{h}$ ), and using mixing interaction in order to calculate these effects as a residual interaction, some of the most widely used mixing interaction to calculate this effect are modifed-surface delta function interaction (MSDI)[3], Michigan sum of threerange Yukawa potential (M3Y)[4], Skyrme-type hamiltonian (SKX)[5], Skyrme-type hamiltonian (MSK7)[6], Dirac-Hartree hamiltonian (NL3)[7], Gogny interactions D1S[8],.etc and the process is called Core polarization (CP).
Differential cross sections for inelastic electron scattering from ${ }^{50} \mathrm{Ti}$ had been measured and fitted using two representations for the transition charge, (i) the hydrodynamic liquid drop, and (ii) a phenomenological model. [9]. The cross sections for the strongly excited, $2^{+}, 3^{-}, 4^{+}$and $5^{-}$levels had been measured over a range of momentum transfer q of $0.4-2.6 \mathrm{fm}^{-1}$. This Experimental data are listed in ref. [10].
The inelastic Coulomb form factor for electroexcitation of the yrast $6^{+}$state in ${ }^{50} \mathrm{Ti}$ had been measured [11]. The value for the root mean square (r.m.s) charge radius of the $1 \mathrm{f}_{7 / 2}$ proton orbit deduced from these data is appreciably larger than the one extracted from the M7 form factor for elastic magnetic scattering from ${ }^{51} V$.

Inelastic Electron Scattering from fp Shell Nuclei had been studied [12]. The calculated form factors for ${ }^{50} \mathrm{Ti}$ had been performed by the use of Hartree Fock theory, results are in a good agreement with the experimental data.
Shell-model and core-polarization calculations for the yrast $\mathrm{J}^{\pi}=2^{+}, 4^{+}, 6^{+}$levels in ${ }^{50} \mathrm{Ti}$, (Charge form factors) had been deduced and compared with experimental data [13], emphasizing the radial shape of the model wave function in comparison with the $q$-dependence of the data. The radial effect is explained in detail by configuration mixing and corepolarization effects.
In ${ }^{50} \mathrm{Ti}, 1 \mathrm{f}_{7 / 2}$ neutron orbit radius had been measured by the use of elastic electron scattering experiment and the result was (r=3.96 $\pm 0.05$ ) [14].
In the framework of the Hartree-Fock model the form factors for the inelastic electron scattering to $2^{+}, 4^{+}$, and $6^{+}$states in ${ }^{46,48,50} \mathrm{Ti},{ }^{50,52,54} \mathrm{Cr}$ and ${ }^{54,56} \mathrm{Fe}$ were studied [15, 16]. The calculations are performed in the $1 \mathrm{f}_{7 / 2}, 2 \mathrm{p}_{3 / 2}, 1 \mathrm{f}_{5 / 2}, 2 \mathrm{p}_{1 / 2}$ model space using a modified Kuo-Brown effective interaction. Inelastic longitudinal C6 electron scattering form factors had been studied for ${ }^{50} \mathrm{Ti}$ [17], by the use of shell model wave functions in $1 \mathrm{f}_{7 / 2}$ and $1 \mathrm{f}_{7 / 2} 1 \mathrm{~d}_{3 / 2}$ model spaces with the inclusion of core polarization in the calculation of the form factor with M3Y- E fitting [19] as a residual interaction, the core polarization contributions are in positive sign with respect to model space parts for all the three cases.

## Theory

For a selected operator $T_{J T}^{\eta}$ the reduced matrix elements are written as the sum of the product of the one-body transition density matrix elements (OBDM) times the single-particle transition matrix elements [3]:

$$
\begin{equation*}
\left\langle\Gamma_{f}\left\|\hat{T}_{\Lambda}^{n}\right\| \Gamma_{i}\right\rangle=\sum_{\alpha, \beta} \operatorname{OBDM}\left(\Gamma_{i}, \Gamma_{f}, \alpha, \beta\right)\left\langle\alpha\left\|\hat{T}_{\Lambda}^{\eta}\right\| \beta\right\rangle \tag{1}
\end{equation*}
$$

where $\Lambda=J T$ is the multipolarity and the states $\Gamma_{i} \equiv J_{i} T_{i} \quad$ and $\quad \Gamma_{f} \equiv J_{f} T_{f}$ are initial and final states of the nucleus respectively. While $\alpha$ and $\beta$ denote the final and initial single-particle states,
respectively (isospin is included). The OBDM in the present work are calculated by generating the wave functions of a given transition in the known nuclei from the modified version of the shell model code (OXBASH V.2005) [18] which contains a complete library of shell model effective interaction. All the informations about transitions of given multipolarities are contained in the OBDM which represents the combination of the model space wave functions.
The reduced matrix elements of the electron scattering operator $\hat{T}_{\Lambda}^{\eta}$ consist of two parts, one is for the "Model space" matrix elements, and the other is for the "Core-polarization" matrix elements [3].
$\left\langle\Gamma_{f}\left\|\hat{T}_{\Lambda}^{\eta}\right\| \mid \Gamma_{i}\right\rangle=\left\langle\Gamma_{f}\left\|\hat{T}_{\Lambda}^{\eta}\right\| \Gamma_{i}\right\rangle_{M S}+\left\langle\Gamma_{f}\left\|\delta \hat{T}_{\Lambda}^{\eta}\right\| \| \Gamma_{i}\right\rangle_{C P}$

Where, $\left\langle\Gamma_{f}\left\|\hat{T}_{\Lambda}^{\eta}\right\| \Gamma_{i}\right\rangle_{M S}$ are the model-space
matrix elements, and, $\left\langle\Gamma_{f}\left\|\delta T_{\Lambda}^{\eta}\right\| \Gamma_{i}\right\rangle_{C P}$ are the core-polarization matrix elements.
$\left|\Gamma_{i}\right\rangle$ and $\left|\Gamma_{f}\right\rangle$ are described by the model-space wave functions.
The core-polarization matrix elements in equation (2) can be written as [21]:
$\left\langle\left.\Gamma_{f}\left\|\delta \hat{T}_{\Lambda}^{\eta}\right\|\right|_{i}\right\rangle_{c p}=\sum_{\alpha, \beta} O B D M(\alpha, \beta)\left\langle\alpha\left\|\delta \hat{T}_{\Lambda}^{\eta}\right\| \mid \beta\right\rangle_{c p}$

The first order perturbation theory says that the single-particle matrix element for the higherenergy configurations can be expressed as [3]:
$\langle\alpha| \delta \hat{T}_{J}^{\eta}|\beta\rangle=\langle\alpha| V_{\text {res }} \frac{Q}{E-H^{(0)}} \hat{T}_{J}^{\eta}|\beta\rangle+\langle\alpha| \hat{T}_{J}^{\eta} \frac{Q}{E-H^{(0)}} V_{\text {res }}|\beta\rangle$
where $V_{\text {res }}$ represents a residual nucleon-nucleon interaction, and might be simplified as fallows [3, 21]:
$\langle\alpha| \hat{T}_{J}^{\eta} \frac{Q}{E-H^{(0)}} V_{r e s}|\beta\rangle=\sum_{p, h}\langle h| \hat{T}_{J}^{\eta}|p\rangle \times \frac{1}{e_{\beta}-e_{\alpha}-e_{p}+e_{h}}$
$\times\langle\alpha p| V_{\text {res }}|\beta h\rangle$
$\langle\alpha| V_{\text {res }} \frac{Q}{E-H^{(0)}} \hat{T}_{J}^{\eta}|\beta\rangle=\sum_{p, h}\langle p| \hat{T}_{J}^{\eta}|h\rangle \times \frac{1}{e_{\alpha}-e_{\beta}-e_{p}+e_{h}}$
$\times\langle\alpha h| V_{r e s}|\beta p\rangle$
the summation includes all possible particlehole states and $e_{k}$ is the single-particle energy $(\mathrm{k} \equiv \alpha, \beta, \mathrm{p}, \mathrm{h})$.
In equations 5 and 6 , the energy denominators are the excitation energies of the intermediate states with respect to the unperturbed valance states $|\alpha\rangle$ and $|\beta\rangle$.
In order to reduce the single-particle matrix elements, Wigner-Eckart theorem [22] is used with taking care of the proper normalization of the angular momentum coupled two-particle states one can obtain from equation 5 the expression[3]:-

$$
\begin{align*}
& \left\langle j_{\alpha} m_{\alpha}\right| T_{J, M}^{\eta} \frac{Q}{E-H^{(0)} V_{r e s}\left|j_{\beta} m_{\beta}\right\rangle=} \sum_{\substack{j_{p}, m_{p}, j_{h}, m_{h}, J, M}} \frac{\left\langle j_{p} m_{p} J M \mid j_{h} m_{h}\right\rangle}{\sqrt{2 j_{h}+1}} \\
& \times\left\langle j_{h}\right|\left|\hat{T}_{J}^{\eta} \| j_{p}\right\rangle \times \frac{1}{e_{\beta}-e_{\alpha}-e_{p}+e_{h}} \times\left\langle j_{\alpha} m_{\alpha} j_{p} m_{p} \mid J M\right\rangle \\
& \times\left\langle j_{\beta} m_{\beta} j_{h} m_{h} \mid J M\right\rangle \times\left\langle j_{\alpha} j_{p}\right| V_{r e s}\left|j_{\beta} j_{h}\right\rangle J \\
& \quad \times \sqrt{\left(1+\delta_{\alpha p}\right)\left(1+\delta_{\beta h}\right)} \tag{7}
\end{align*}
$$

The summation over the three Clebsch-Gorden coefficients can be performed to yield a 3 j symbol and 6 j -symbol, which is given as [3]:

$$
\begin{align*}
& \left\langle{ }_{\alpha}{ }_{\alpha}{ }_{\alpha}\right| \hat{T}_{J, M}^{\eta} \frac{Q}{E-H^{(0)}} V_{r e s}\left|{ }^{j^{m} m^{m}}{ }_{\beta}\right\rangle=(-1)^{j^{\prime}-m_{\alpha}}\left(\begin{array}{ccc}
j_{\alpha} & J & j_{\beta} \\
-m_{\alpha} & M & m_{\beta}
\end{array}\right) \\
& \times \sum_{j_{p}, j_{h}, J^{\prime}}\left\langle j_{h}\right| \hat{\Gamma}_{J}^{\eta}\left|\|_{p}\right\rangle \times \frac{\left(2 J^{\prime}+1\right)}{e_{\beta}-e_{\alpha}-e_{p}+e_{h}}(-1) j_{p}+j_{h}+J^{\prime}\left\{\begin{array}{lll}
j_{\alpha} & j_{\beta} & J \\
j_{h} & j_{p} & J
\end{array}\right\} \\
& \times\left\langle j_{\alpha} j_{p}\right| V_{r e s}\left|j_{\beta} j_{h}\right\rangle_{J^{\prime}} \times \sqrt{\left(1+\delta_{\alpha p}\right)\left(1+\delta_{\beta h}\right)} \\
& \text { where }\left\{\begin{array}{l}
\ldots \\
\ldots
\end{array}\right\} \text { is the } 6 \mathrm{j} \text {-symbol. } \tag{8}
\end{align*}
$$

In space and isospace coordinates, equation (8) becomes [3]:
$\left\langle j_{\alpha} t_{\alpha}\left\|\hat{T}_{J, T}^{\eta} \frac{Q}{E-H^{(0)}} V_{r e s}\right\| j_{\beta} t_{\beta}\right\rangle=\sum_{\substack{j_{p}, j_{h}, J \\ t_{p}, t_{h}, T}} \frac{\left(2 J^{\prime}+1\right)\left(2 T^{\prime}+1\right)}{e_{\beta}-e_{\alpha}-e_{p}+e_{h}} \times(-1)^{j_{\beta}+j_{h}+J^{\prime}}$ $\times(-1)^{t^{t}+t_{h}+T^{\prime}} \times\left\{\begin{array}{ccc}j_{\alpha} & j_{\beta} & J \\ j_{h} & j_{p} & J^{\prime}\end{array}\right\} \times\left\{\begin{array}{ccc}t_{\alpha} & t_{\beta} & T \\ t_{h} & t_{p} & T^{\prime}\end{array}\right\} \times\left\langle j_{\alpha} j_{p}\right| V_{\text {res }}\left|j_{\beta} j_{h}\right\rangle_{J^{\prime}, T^{\prime}}$

$$
\begin{equation*}
\times\left\langle j_{h} t_{h}\left\|\hat{T}_{J, T}^{\eta}\right\| j_{p} t_{p}\right) \times \sqrt{\left(1+\delta_{\alpha p}\right)\left(1+\delta_{\beta h}\right)} \tag{9}
\end{equation*}
$$

The Greek-symbols is used to denote quantum numbers in space and isospace coordinates, equation 9 becomes [3]:

$$
\begin{align*}
& \left\langle\alpha\left\|\hat{T}_{\Lambda}^{\eta} \frac{Q}{E-H^{(0)}} V_{r e s}\right\| \beta\right\rangle=\sum_{\alpha_{1}, \alpha_{2}, \Gamma} \frac{(-1)^{\beta+\alpha_{2}}+\Gamma}{e_{\beta}-e_{\alpha}-e_{\alpha_{1}}+e_{\alpha_{2}}} \\
& (2 \Gamma+1) \times\left\{\begin{array}{ccc}
\alpha & \beta & \Lambda \\
\alpha_{2} & \alpha_{1} & \Gamma
\end{array}\right\} \\
& \times\left\langle\alpha \alpha_{1}\right| V_{r e s}\left|\beta \alpha_{2}\right\rangle_{\Gamma} \times\left\langle\alpha _ { 2 } \left\|\left|\hat{T}_{\Lambda}^{\eta} \| \alpha_{1}\right\rangle \times \sqrt{\left(1+\delta_{\alpha_{1} \alpha}\right)\left(1+\delta_{\alpha_{2} \beta}\right)}\right.\right. \tag{10}
\end{align*}
$$

where the index $\alpha_{1} \equiv j_{p} t_{p}$ runs over particle states and $\alpha_{2} \equiv j_{h} t_{h}$ runs over hole states, $\alpha \equiv j_{\alpha} t_{\alpha}$ and $\beta \equiv j_{\beta} t_{\beta}$ are the final and initial single-particle states respectively, $\Gamma \equiv J^{\prime} T^{\prime}$ correspond to the total spin in both spaces, , $\hat{T}_{\Lambda}^{\eta}$ denotes the single-particle transition operator of rank J in space coordinate and rank T in isospace $(\Lambda \equiv J T), T=0$ or 1 are denoting the isoscalar or isovector contribution.
Similarly one can find the contribution of the second term of equation 6 as [3]:

$$
\begin{align*}
& \left\langle\alpha\left\|\left\|_{\text {res }} \frac{Q}{E-H^{(0)}} \hat{T}_{\Lambda}^{n}\right\| \beta\right\rangle=\sum_{\alpha_{1}, \alpha_{2}, \Gamma} \frac{\left(-1 e^{\beta+e_{\alpha}}\right)^{\beta+e_{\alpha_{1}}+e_{\alpha_{2}}}}{}(2 \Gamma+1)\right. \\
& \times\left\{\begin{array}{lll}
\alpha & \beta & \Lambda \\
\alpha_{1} & \alpha_{2} & \Gamma
\end{array}\right] \\
& \times\left\langle\alpha \alpha_{2}\right| V_{\text {res }}\left|\beta \alpha_{1}\right\rangle_{\Gamma} \times\left\langle\alpha_{1}\right|\left|\hat{T}_{\Lambda}^{n} \| \alpha_{2}\right\rangle \times \sqrt{\left(1+\delta_{\alpha_{2} \alpha}\right)\left(1+\delta_{\alpha_{1} \beta}\right)} \tag{11}
\end{align*}
$$

The core-polarization form factor can be evaluated by replacing the single-particle matrix elements in equation 3 with the two matrixelements which are given in equations 10 and 11.

The total form factors are given by:

$$
\begin{gather*}
\left\langle\alpha\left\|\delta \hat{r}_{\Lambda}^{\eta}\right\| \beta\right\rangle_{C P}=\left\langle\alpha\left\|\hat{T}_{\Lambda}^{\eta} \frac{Q}{E-H^{(0)}} V_{\text {res }}\right\| \beta\right\rangle_{C P} \\
+\left\langle\alpha\left\|V_{\text {res }} \frac{Q}{E-H^{(0)}} \hat{T}_{\Lambda}^{\eta}\right\| \|_{C P}\right. \tag{12}
\end{gather*}
$$

The single-particle energies $\mathrm{e}_{\mathrm{nj}}$ are calculated according to [3]:

$$
e_{n l j}=\left(2 n+l-\frac{1}{2}\right) \hbar \omega+\left\{\begin{array}{rr}
-\frac{1}{2}(l+1)\langle f(r)\rangle_{n l} \text { for }{ }^{j=l-\frac{1}{2}}  \tag{13}\\
\frac{1}{2} l\langle f(r)\rangle_{n l} & \text { for }{ }_{j=l+\frac{1}{2}}
\end{array}\right.
$$

with:

$$
\begin{align*}
& \langle f(r)\rangle_{n l} \approx-20 A^{-2 / 3} \mathrm{MeV} \\
& \hbar \omega=45 A^{-1 / 3}-25 A^{-2 / 3} \tag{14}
\end{align*}
$$

For the two-body matrix elements of the residual interaction $\left\langle\alpha \alpha_{2}\right| V_{\text {res }}\left|\beta \alpha_{1}\right\rangle_{\Gamma}$, which appear in equations 10 and 11, the Michigan sum of three range Yukawa potential (M3Y)
interaction of Berstch et. al [4] is adopted. This interaction containing terms like those were given in LS-coupling and tensor effects. A transformation of the wave function from jj to LS coupling must be done to get the relation between the two-body shell model matrix elements and the relative and centre of mass coordinates, using the harmonic oscillator radial wave functions with Talmi-Moshinsky transformation.
The realistic M3Y effective NN interaction, which is used in electron scattering ( $\mathrm{V}_{\text {res }}=v 12$ ) is expressed as a sum of the central potential part $v_{12}^{(C)}$, spin-orbit potential part $v_{12}^{(L S)}$, and long range tensor part $v_{12}^{(T N)}$, as follows [4, 23]:
$V_{12}^{(M 3 Y-P 0)}=\nu_{12}^{(c)}+V_{12}^{(L S)}+\nu_{12}^{(T N)}$
$\nu_{12}^{(M 3 Y-P 1)}=\nu_{12}^{(c)}+\nu_{12}^{(L S)}+\nu_{12}^{(T N)}+\nu_{12}^{(D D)}$

The four potentials are expressed as [23]:-
$v_{12}^{(c)}=\sum_{n}\left(t_{n}^{(S E)} P_{S E}+t_{n}^{T E} P_{T E}+t_{n}^{(S O)} P_{S o}+t_{n}^{(T O)} P_{T o}\right) f_{n}^{(C)}\left(r_{12}\right)$
$v_{12}^{(L S)}=\sum_{n}\left(t_{n}^{(L S E)} P_{T E}+t_{n}^{(L S O)} P_{T o}\right) f_{n}^{(L S)}\left(r_{12}\right) L_{12} \cdot\left(\overrightarrow{\boldsymbol{S}}_{1}+\vec{S}_{2}\right)$
$\nu_{12}^{(T N)}=\sum_{n}\left(t_{n}^{(T N E)} P_{T E}+t_{n}^{(T N O)} P_{T O}\right) f_{n}^{(T N)}\left(r_{12}\right) r_{12}^{2} S_{12}$
$v_{12}^{(D D)}=\left(t_{d d}^{S E} P_{S E}+t_{d d}^{T E} P_{T E}\right) \delta\left(r_{12}\right)$
(16)

Table 1: The values of the best fit to the potential parameters belongs to Ried fitting (M3Y-P1) [23], for $\mathrm{TN}, \mathrm{R}_{\mathbf{2}}=\mathbf{0 . 7} \mathrm{fm}$.

|  | $\mathrm{R}_{1}=0.25 \mathrm{fm}$ | $\mathrm{R}_{2}=0.40 \mathrm{fm}$ | $\mathrm{R}_{3}=1.414 \mathrm{fm}$ |
| :--- | :--- | :--- | :--- |
| Oscillator matrix elements (Channel) | $\mathrm{t}_{1}$ | $\mathrm{t}_{2}$ | $\mathrm{t}_{3}$ |
| Centeral Singlet-Even (SE) | 8599.5 | -3556 | -10.463 |
| Central Triplet-Even (TE) | 10475.25 | -4594 | -10.463 |
| Central Singlet-Odd (SO) | -1418 | 950 | 31.389 |
| Central Triplet-Odd (TO) | 11345 | -1900 | 3.488 |
| Tensor-Even (TNE) | -131.52 | -3.708 | 0.0 |
| Tensor-Odd (TNO) | 29.28 | 1.872 | 0.0 |
| Spin-Orbit Even (LSE) | -9181.8 | -606.6 | 0.0 |
| Spin-Orbit Odd (LSO) | -3414.6 | -1137.6 | 0.0 |
| Density dependent Singlet-Even (SE) <br> $t_{d d}^{(S E)}$ <br> $=1090$ |  |  |  |
| Density dependent triplet-Even (TE) <br> $t_{d d}^{(T E)}=1332$ |  |  |  |

Table 2:The values of the best fit to the potential parameters belong to Paris [23].

|  | $\mathrm{R}_{1}=0.25 \mathrm{fm}$ | $\mathrm{R}_{2}=0.40 \mathrm{fm}$ | $\mathrm{R}_{3}=1.414 \mathrm{fm}$ |
| :--- | :--- | :--- | :--- |
| Oscillator matrix elements (Channel) | $\mathrm{t}_{1}$ | $\mathrm{t}_{2}$ | $\mathrm{t}_{3}$ |
| Centeral Singlet-Even (SE) | 11466 | -3556 | -10.463 |
| Central Triplet-Even (TE) | 13967 | -4594 | -10.463 |
| Central Singlet-Odd (SO) | -1418 | 950 | 31.389 |
| Central Triplet-Odd (TO) | 11345 | -1900 | 3.488 |
| Tensor-Even (TNE) | 0.0 | -171.7 | -78.03 |
| Tensor-Odd (TNO) | 0.0 | 283.0 | 13.62 |
| Spin-Orbit Even (LSE) | 0.0 | -813.0 | 0.0 |
| Spin-Orbit Odd (LSO) | -2672 | -620.0 | 0.0 |

with the exception of TN , where $\mathrm{R}_{2}=0.7 \mathrm{fm}$.
MSDI as a residual interaction is the touchstone in this paper because this type of interactions (adjustable) will give us the general behavior of the core polarization part with respect to the model space one and expose the reasons behind the negative sign in the addition process (core polarization + model space) as we had seen and without a prolonged analysis the matrix element of this type of interaction might be expressed as follows [24]

$+[(2 T(T+1)-3) B+C] \delta_{12} \delta_{34}$
Where,
$\mathrm{A} 0=\mathrm{A} 1=\mathrm{B}=25 / \mathrm{A}, \mathrm{C} \approx 0$.

## Results, Discussion and Conclusions

The core-polarization effects has been included in order to account the contribution of configurations from outside of the model space in the transition. The $\mathbf{1 f}_{7 / 2}$ subshell orbit adopted in this work as a model space for ${ }^{50} \mathrm{Ti}$. Core-
polarization effects are taken into account through first order perturbation theory, which allows particle-hole excitation from shell core orbits $\mathbf{1 s}_{1 / 2}, \mathbf{1 p}_{3 / 2}, \mathbf{1 p}_{1 / 2}, \quad \mathbf{1 d}_{5 / 2}, \mathbf{1 d}_{3 / 2}$ and $\mathbf{2 s}_{1 / 2}$ (shell model space having ${ }^{40} \mathrm{Ca}$ as an inert core ). Mc Cullen-Bayman-Zamick(F7MBZ) [25] effective interaction has been adopted for the model space $1 \mathrm{f}_{7 / 2}$ to generate the $\mathbf{1 f}_{7 / 2}$ shell model wave functions for ${ }^{50} \mathrm{Ti}$.

The cp effects are calculated with the realistic effective interactions M3Y-P1 and M3Y-P0 [18] and MSDI [3] as a residual interaction. In this interaction, the Ried and Paris fitting have been used to calculate the radial integral.

## Inelastic longitudinal C6 Electron scattering

 Form Factor in ${ }^{50} \mathrm{Ti}$ ( $\mathbf{1 f}_{7 / 2}$ Model Space)The nucleus ${ }^{50} \mathrm{Ti}$ is really ${ }^{48} \mathrm{Ca}$ +two protons distributed in subshell $\mathbf{1 f}_{7 / 2}$ the space that be chosen to study the behavior of this system under excitatioin by the use of electron scattering and calculating multipole form factor. ${ }^{40} \mathrm{Ca}$ as an inert core and the model space effective interaction F7MBZ [25] has been adopted to generate the model space wave functions. Table 3 shows the values of OBDM for the C 6 form factors for the model space $\left(\mathbf{1 f}_{7 / 2}\right.$ ) and F7MBZ as a model space effective interaction calculation in ${ }^{50} \mathrm{Ti}$.

Table 3: The values of (OBDM) for the C6
form factors in ${ }^{50} \mathrm{Ti}$.

| $J_{i}$ | $J_{f}$ | OBDM <br> $(\Delta \mathbf{T}=\mathbf{0})$ | OBDM <br> $(\Delta \mathbf{T}=\mathbf{1})$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{7 / 2}$ | $\mathbf{7 / 2}$ | $\mathbf{- 1 . 3 2 2 8 8}$ | 0.88192 |

Figure 1 shows the C6 form factor calculated with the use of M3Y-P0 as a residual interaction and this results are consistent with the experimental one and it is clear that the model space form factor is in a good agreement with the experimental one and the results show that the core contribution has a minor effect and in opposite sign with respect to model space so that the total C6 form factors are quenched. Protons in model space will be dominantly responsible to this coulomb form factor ( $\left.e_{p}=1 e\right)$.
Return to the figure 1, it is clear that the core contribution, model space and total form factors are in phase. The experimental data are taken from ref. [9].


Fig. 1: Inelastic longitudinal C6 form factors in
${ }^{50} \mathrm{Ti}$ nucleus, using M3Y-P0 residual interaction. The data are taken from ref. [9].
Figure 2 shows the C6 form factor calculated with the use of MSDI as a residual interaction and this results are consistent with the experimental data and it is clear that the model space form factor is in a good agreement with the experimental data and the results show that the core contribution is weak in some extent but has the property of constructive interference so that the total C6 form factors are enlarged and overestimated and are in phase.


Fig. 2: Inelastic longitudinal C6 form factors in ${ }^{50} \mathrm{Ti}$ nucleus. using MSDI residual interaction. The data are taken from ref. [9].
figure. 3 shows Inelastic longitudinal C6 form factors in ${ }^{50} \mathrm{Ti}$ nucleus, for the $1 \mathbf{f}_{7 / 2}$ model space calculation, and F7MBZ effective interaction, (M3Y-P1) as a residual interaction, where the core polarization effect has a constructive and enhancement properties, a good agreement have been obtained, with the experimental data.


Fig. 3: Inelastic longitudinal C6 form factors in
${ }^{50} \mathrm{Ti}$ nucleus. using M3Y-P1 residual interaction. The data are taken from ref. [9].

## Conclusions

1. In fp shell model space with many body problem, the model space contribution is found dominant with respect to the core one for the three cases.
2. Core polarization effect has a constructive contribution when we use (MSDI) and (M3Y-P1) as a residual interactions in comparison with M3Y-P0 because of the density dependent term.
3. Use of two versions of M3Y with two different sets of fitting parameters do change the behavior of the core polarization contribution especially in amplitudes and phases where the density dependence has the dominant contribution.

Acknolegement: Authers would like to express their thanks to Prof. Radhi for his providing of the original computer program codes written in FORTRAN language which are used to producing the computed results

## Refrences

1. Walecka, J. D. 2004 " Electron Scattering for Nuclear and Nucleon Structure "

Cambridge monographs on physics, Nuclear physics and cosmology. 12-42.
2. Haxel, O., Jensen J. H. D. ,and Suess, H. E. 1949. . On the "Magic Numbers" in Nuclear Structure. Phys. Rev. 75, 17661780.
3. Brussaard P.J., and Glademans, P. W. M. 1977 "Shell-model Application in Nuclear Spectroscopy", North-Holland Publishing Company, Amsterdam.
4. Berstch, G., Borysowicz, J. McManus, H., and Love, W. G. 1977. An effective interaction for inelastic scattering derived from the Paris potential. Nucl. Phys., A284, 399-418.
5. Goriely, S. Tondeur, F., and Pearson, J. M. 2001. Atomic and Nuclear Data sheet. At. Data \&Nucl. Data Sheets, 77, 311-420.
6. Lalazissis, G. A., Koonig, J., and Ring, P. 1997. Central and noncentral components of the effective sd-shell interaction. Phys. Rev., C55, 540-567.
7. Khoa, D. T., von Oertzen, W., and Ogloblin, A. A. 1996. Study of the equation of state for asymmetric nuclear matter and interaction potential between neutron-rich nuclei using the densitydependent M3Y interaction. Nucl. Phys., A602, 98-213.
8. Zalewski, M., Satuła, W., Nazarewicz, W., Stoitcheva, G., and Zdunczuk, H. 2007 Shell model and mean-field description of band termination in the $\mathrm{A}=44$ nuclei. Phys. Rev., C75, 054306-054318.
9. Hosoyama, K. 1975. Inelastic electron scattering form factors for the excitation of the $2^{+}$states in the $1 f_{7 / 2}$ nuclei. Reserch Reports in Nuclear Science (Tohoku University), 7279, 855-874.
10. Selig, M. Zacharov, I. E. De Witt Huberts, P. K. A. and Platchkov, S. K. 1985. Comparison of the size of the $1 \mathrm{f}_{7 / 2}$ orbit deduced from C6 and M7 form factors. Phys. Lett., B162, 251-270.
11. Sahu, R., I:\nsrsearch.cgiAhalpara,D. P. and Bhatt, K. H. 1986. Inelastic electron scattering form factors in some fp shell model space nuclei. Bull.Am.Phys.Soc. 31, No.10, 1764-1773, DC7.
12. Selig, M. de Vries, C. de Witt Huberts, P. K. A. and Zacharov, I. E. 1988. Effective electro-magnetic operators of ${ }^{50} \mathrm{Ti}$ investigated with the ( e , $\mathrm{e}^{\prime}$ ) reaction. Nucl. Phys., A476, 413-427.
13. Platchkov, S., Amroun, A. Bricault, P., Cavedon, J. M., de Witt Huberts, P. K. A., Dreux, P. Frois, B., Goodman, C. D., Goutte, D., Martino, J., Meot, V., Peterson, G. A., Phan, X. H., Raman, S., and Sick, I. 1988. Measurement of the $1 \mathrm{f}_{7 / 2}$-NeutronOrbit Radius in ${ }^{41} \mathrm{Ca}$. Phys. Rev. Lett., 61, 1465-1479.
14. Sahu, R., I:\nsrsearch.cgiAhalpara,D. P. and Bhatt, K. H., 1990. Inelastic electron scattering form factors in some fp shell model space nuclei. J. Phys. (London), G16, 733-746.
15. Sahu, R., I:\nsrsearch.cgiAhalpara,D. P. and Pandya, S. P. J. 1987. Inelastic electron scattering form factors in some fp shell model space nuclei. J. Phys. (London), G13, 603-621.
16. Yokoyama, A. 2005. Inelastic longitudinal C6 electron scattering form factors Ti-50. Jour. of Phys. Conf. Series, 20. 143-146.
17. Majeed, F. Z. 2009. Core Polarization effects on Electron scattering form factors for some fp-shell nuclei. Ph.D Thesis, Department of physics, College of Science, University of Baghdad, Baghdad, Iraq. 114-139.
18. Brown, B. A., Etchgoyen, A., Godwin, N. S., Rae, W. D. M., Richter, W. A., Ormand, W. E., Warburton, E. K., Winfield, J. S., Zhao, L. and Zimmerman, C. H. 2005 Nuclear Shell Model Code. MSU-NSCL report number1289. and private communication.
19. Elliott, P., and Skyrme, T.H.R. 1955. An effective interaction for inelastic scattering derived from the nucleon nucleon potential. Proc. Roy. Soc., A323, 561-576.
20. Heisenberg, J. McCarthy, J. S. and Sick, I. 1971. Inelastic electron scattering from several $\mathrm{Ca}, \mathrm{Ti}$ and Fe isotop. Nucl. Phys., A164, 353-374.
21. Radhi, R.A. Bouchebak, A. 2003. Microscopic calculations of C2 and C4 form factors in sd-shell nuclei. Nucl. Phys., A716, 87-97.
22. Edmonds, R. 1974. Angular Momentum in Quantum Mechanics, (Princeton University Press), Third ed. 70-113.
23. Nakada, H. and Sato, M. 2002. HartreeFock approach to nuclear matter and finite nuclei with M3Y-type nucleon-nucleon interactions. Nucl. Phys., A699, 511-525.
24. Yeslam, H. A. 2005. Core-polarization effects for electron scattering form factors
in some sd shell nuclei. Ph.D Thesis, Department of physics, College of Science, University of Baghdad, Baghdad, Iraq. 5670.
25. Mc Cullen, J. D. Bayman, B. F. and Zamick, L. 1963. Application to Electron Scattering of Center-of-Mass Effects in the Nuclear Shell Model Phys. Rev. 134, 515530

