



STUDYING INELASTIC TRANSVERSE ELECTRON SCATTERING FORM FACTORS OF THE ISOSCALAR TRANSITIONS IN ^{10}B NUCLEUS

Ban S. H. Al-qazaz

banalqazaz@yahoo.com

Department of Physics, College of Science for Women, University of Baghdad. Baghdad-IRAQ.

Abstract

The inelastic transverse electron scattering form factors are studied for the four isoscalar transitions for the states of ^{10}B nucleus. These states are specified by $J^{\pi}T = 1^+0$ (0.718 MeV), 1^+0 (2.154 MeV), 2^+0 (3.587 MeV) and 3^+0 (4.774 MeV). These form factors are analyzed in the framework of the harmonic oscillator shell model.

The transverse form factors have been calculated in the framework of the multi-nucleon configuration mixing shell model using the two-body interaction of Cohen and Kurath (C-K) to generate the 1p-shell wave function. The core polarization (CP) effects are included in the calculations through effective g-factors. A higher configuration effect outside the 1p-shell model space enhances the form factors for q-values and reasonably reproduces the data. The value of the size parameter (b) is chosen to reproduce the root mean square charge radius. The present results are compared with available experimental data and with that of other models.

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Keywords:- electron scattering and inelastic, electron scattering Nuclear structure electron scattering shell model, Elastic and inelastic.

دراسة عوامل التشكل غير المرنة للأستطارة الألكترونية المستعرضة للانتقالات غير المتجهة للنواة ^{10}B

بان صباح حميد القزاز

قسم الفيزياء، كلية العلوم للبنات، جامعة بغداد. بغداد- العراق

banalqazaz@yahoo.com

الخلاصة

تم دراسة عوامل التشكل غير المرنة للأستطارة الألكترونية المستعرضة للحالات الأربعة غير المتجهة للنواة ^{10}B . عينت الحالات باعداد الكم $J^{\pi}T = 1^+0$ (0.718 MeV)، 1^+0 (2.154 MeV)، 2^+0 (3.587 MeV) و 3^+0 (4.774 MeV). تم تحليل عوامل التشكل في إطار أنموذج القشرة للمتذبذب التوافقي.

تم حساب عوامل التشكل المستعرضة في إطار أنموذج القشرة ذي التشكيلات المختلطة للنويات المتعددة باستعمال تفاعل الجسيمين لـ Cohen –Kurath لتشكيل الدالة الموجية للقشرة -1p. أدخل تأثير استقطاب القلب من خلال عوامل- g الفعالة. تأثير توزيع المدارات العالية خارج فضاء أنموذج القشرة - 1p حسنت

عوامل التشكل لقيم q وأعادت ترتيب البيانات الى حد مقبول. أختيرت قيم معلم الحجم b لنسخ قيم الجذر التربيعي لمعدل نصف قطر الشحنة. قورنت النتائج الحالية مع البيانات العملية وكذلك مع نتائج نماذج أخرى.

1. Introduction

Inelastic scattering of medium energy electrons provides a well-understood probe of charge, current, and magnetization densities which characterize nuclear excitations. In light nuclei the most extensive microscopic calculation of the transition densities can be performed and tested [1].

Inelastic electron scattering was best documented by the work on p-shell nuclei by Rand et. al. [2]. The magnetic form factors of the 1p-shell nuclei (${}^6\text{Li}$, ${}^7\text{Li}$, ${}^9\text{Be}$, ${}^{10}\text{B}$, ${}^{11}\text{B}$ and ${}^{14}\text{N}$) have been measured by high energy electron scattering at higher momentum transfers. Fagg et. al. (1976) [3] have studied the electron scattering of excited transitions in ${}^{10}\text{B}$ of 5.11, 6.02, 7.48, 8.90, 10.79 and 11.56 MeV. Ansaldo et. al. (1979) [4] measured the electron scattering form factors for six transitions in ${}^{10}\text{B}$ in the momentum transfer range ($0.61 \leq q \leq 1.81 \text{ fm}^{-1}$). The transverse form factors for the 1.74 and 5.17 MeV levels agree well with predictions based on Cohen-Kurath wave functions [5].

Hicks et. al. (1988) [6] measured the transverse form factors for elastic electron scattering from ${}^{10}\text{B}$ and ${}^{11}\text{B}$, as well as for the electro-excitation of the 1.74 MeV ($J^\pi T = 0^+1$) and 5.17 MeV (2^+) level of ${}^{10}\text{B}$. The results were compared with the results of shell-model calculations that employ the 1p-shell amplitudes of Cohen-Kurath wave functions. An agreement exists between the experimental data and the theory, even for highest momentum transfers. Cichocki et. al. (1995) [7] measured the longitudinal and transverse form factors for low-energy transition in ${}^{10}\text{B}$. The measurements span the momentum transfer range $q = 0.48 - 2.58 \text{ fm}^{-1}$. Longitudinal and transverse form factors were compared with theoretical results of conventional 1p-shell models, models with $1\hbar\omega$ and $2\hbar\omega$ configurations, and model that included core-polarization (CP) effects. The inclusion of CP effects was essential to reproduce the experimental data.

Millener (2004) [8] fitted the inelastic electron scattering form factors with polynomial times Gaussian expressions in the variable

$y = (bq/2)^2$ to extract electromagnetic transition strengths at the photon point for ${}^{10}\text{B}$ nucleus.

Karataglidis et. al (2006)[9] studied the elastic electron scattering form factors, longitudinal and transverse, from the He and Li isotopes and from ${}^8\text{B}$. Large space shell model functions have been assumed. The precise distribution of the neutron excess has little effect on the form factors of the isotopes though there is a mass dependence in the charge densities. However, the form factors of the proton-rich nucleus, ${}^8\text{B}$, are significantly changed by the presence of the proton halo. Roca-Maza et. al (2008)[10] calculated the results for elastic electron scattering by nuclei with charge densities of Skyrme forces and covariant effective Lagrangians that accurately describe nuclear ground states are compared against experiment in stable isotopes. Hassan et. al (2011)[11] studied inelastic longitudinal and transverse electron scattering form factors of low-lying $T=0, T=1$ particle-hole states of ${}^{12}\text{C}$ and ${}^{16}\text{O}$ in the framework of the Tamm-Dancoff approximation (TDA). The Hamiltonian with the Michigan-three-Yukawa (M3Y) potential is diagonalized

The aim of the present work is to use the multi-nucleon configuration mixing shell model to analyze the inelastic electromagnetic form factors of ${}^{10}\text{B}$. The two-body interaction of Cohen-Kurath (C-K) [5] is used to generate the 1p-shell wave functions. The calculations incorporate the single-particle wave functions of the harmonic oscillator potential with the size parameter (b) chosen to reproduce the root mean square (rms) charge radius. Effective operators for the different multipoles are used to account the core-polarization effects. The higher configurations are included by mixing some percentage of the 2p-shell wave functions.

2. Theory

The transverse form factors involving the angular momentum J, isospin T and momentum transfer q, between the initial \mathbf{i} and final \mathbf{f} nuclear shell model states of spin $J_{i,f}$ and isospin $T_{i,f}$ is [12],

$$|F_f^\lambda(q)|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left| \sum_{T=0,1} \begin{pmatrix} T_f & T & T_i \\ -T_z & 0 & T_z \end{pmatrix} \times \langle J_f T_f \| \hat{T}_T^\lambda \| J_i T_i \rangle \right|^2 \times F_{c.m.}(q) \times F_{f.s.}(q)$$

(2.1)

Where T_z is the z-component of the isospin for the initial and final states and is given by $T_z = (Z-N)/2$. The bracket (\dots) is the 3j-symbol and λ –stands the transverse magnetic or electric. The reduced matrix elements of the transverse electron scattering operator \hat{T}_{JT}^λ is expressed as the sum of the one body density matrix (OBDM) $\chi_{J_i J_f}^{JT}(\alpha, \beta)$ times the single-particle elements and is given by:

$$\langle f || \hat{T}_{JT}^\lambda || i \rangle = \sum_{\alpha, \beta} \chi_{J_i J_f}^{JT}(\alpha, \beta) \langle \alpha || \hat{T}_{JT}^\lambda || \beta \rangle \quad (2.2)$$

Where α and β label single particle states (isospin is included) for the model space. For 1p-shell nuclei, the orbits $1p_{3/2}$ and $1p_{1/2}$ define the model space. The states $|f\rangle$ and $|i\rangle$ are described by the model space wave functions. The single-particle matrix element reduced in both spin and isospin.

The finite size (f.s.) nucleon form factor is $F_{f.s.}(q) = \exp(-0.43q^2/4)$, and

$F_{c.m.}(q) = \exp(q^2 b^2 / 4A)$ is the correction for the lack of translation invariance in the shell model. A is the mass number [13, 14]. The total transverse form factor is given by:

$$|F^\lambda(q)|^2 = \sum_{J \geq 0} \left\{ |F_J^{mag}(q)|^2 + |F_J^{ele}(q)|^2 \right\} \quad (2.4)$$

When the 1p-shell model space is extended to include the 2p-shell model space, the wave functions of the initial (i) and final (f) states will be written as [15]:

$$|i\rangle = \alpha |i(1p)\rangle + \sqrt{1-\alpha^2} |i(2p)\rangle \quad (2.5)$$

$$|f\rangle = \gamma |f(1p)\rangle + \sqrt{1-\gamma^2} |f(2p)\rangle \quad (2.6)$$

Where α and γ are mixing parameters. Since the C-K interaction depends on the angular parts only, the same OBDM are used for both 1p and 2p shells.

3. Results and Discussion

The transverse electron scattering form factors have been studied for some low lying excited isoscalar transitions for states of ^{10}B .

The transverse form factors are calculated with size parameter $b_{\text{rms}}=1.61\text{fm}$ [16] chosen to reproduce the root mean square charge radius. In the calculations of Sato et al. [16] the optimal value of b is between $b=1.5\text{ fm}$ and $b=1.6\text{ fm}$, while Cichocki et al. [7] carried out their calculations with $b=1.7\text{ fm}$. In present work, the results of 1p-shell only will be denoted by dashed curves, while that results (1p+2p)-shell with CP effects including will be denoted by solid curves.

3.1 The 0.718 MeV (1^+0) state

The isoscalar transverse form factor to the 1^+0 state is associated with two multipoles, namely E2(dotted curve) and M3(cross symbol curve). The results of 1p-shell with free g-factors and $b_{\text{rms}}=1.61\text{ fm}$ (dashed curve) are in accordant with E2 (dotted curve) component and presented in Figure 1. The 1p-shell results reproduce the experimental data of Cichocki et al. [7] (circles) at $q > 1.7\text{ fm}^{-1}$ and underestimate the data at other region. In Figure 2, the solid curve represents the present results including the core-polarization effects through effective g-factors ($g_{s\text{-eff}}^{p/n} = 1.19 g_{s\text{-free}}^{p/n}$) reproduce the experimental data for region of momentum transfer up to $q \sim 1.9\text{ fm}^{-1}$. The present results (solid curve) are compared with that of Cichocki et al. results [7](dotted curve). The form factor of the present work and the ($2\hbar\omega + \text{CP}$)-result of Cichocki et al. (dotted curve) are close to each other in shape and magnitude at $q < 2.1\text{ fm}^{-1}$, and they are different in magnitude at higher momentum transfer.

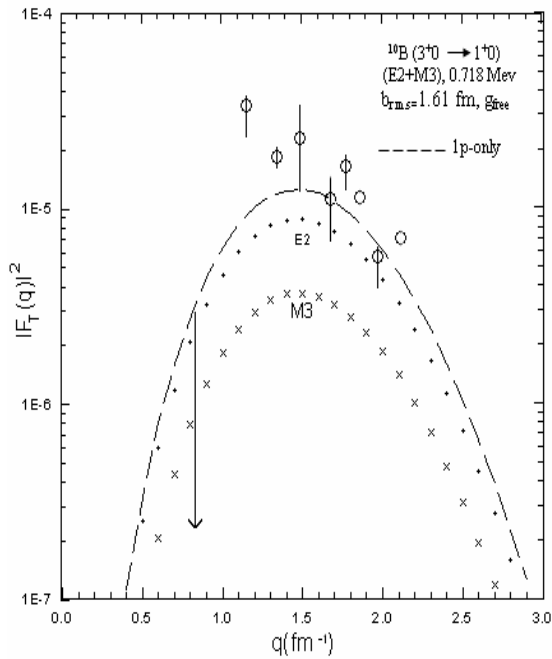


Fig. 1 :The transverse form factors for 1^+0 state in ^{10}B calculated in 1p-shell model space only (dashed curve).The individual multipole contributions are shown. The experimental data are taken from Ref.[7] (circles).

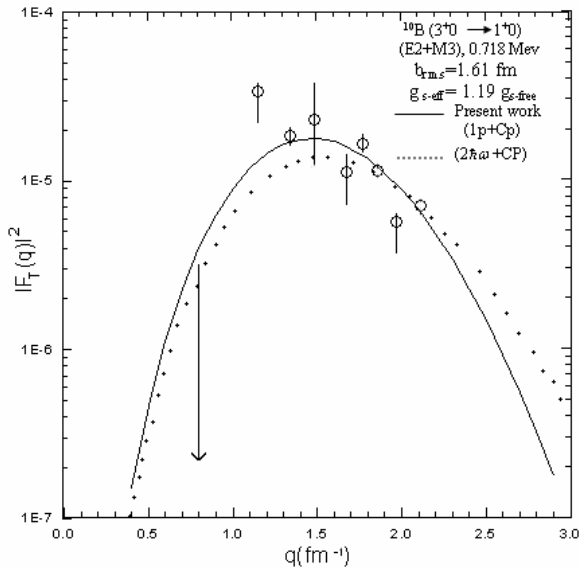


Fig. 2 :The transverse form factors for 1^+0 state in ^{10}B calculated with 1p+CP (solid curve). The present results are compared to that of Ref.[7] (dotted curve).

3.2 The 2.154 MeV (1_2^+0) state

The calculated form factors for 2.154 MeV (1_2^+0) state with the size parameter $b_{rms}=1.61\text{fm}$ and with free g-factors are presented in Figure 3

(dashed curve). The results are compared with the experimental data of Cichocki et al. [7] (circles). The individual multipoles contributions E2 and M3 are denoted by dotted curve and cross symbol curves respectively. The M3 multipole is dominant over the E2 multipole. The experimental data are reproduced at higher momentum transverse in 1p-shell model space (dashed curve), but they are overestimated at low q-value. The inclusion of core-polarization effects using the effective g-factors ($g_{s-eff}^{p/n} = 1.05 g_{s-free}^{p/n}$) are shown in Figure 4 (solid curve). With these parameters the experimental data are reproduced for region of momentum transfer up to $q \sim 1.3\text{fm}^{-1}$. The present results are compared with that of Cichocki et al. results (dotted curve), this comparison is shown in Figure 4. The total form factors of the present results and ($2\hbar\omega + \text{CP}$)-results [7] give the same behavior and close to each other in shape and the magnitude at the $q < 2.0 \text{ fm}^{-1}$ and slightly different at the higher momentum transfer.

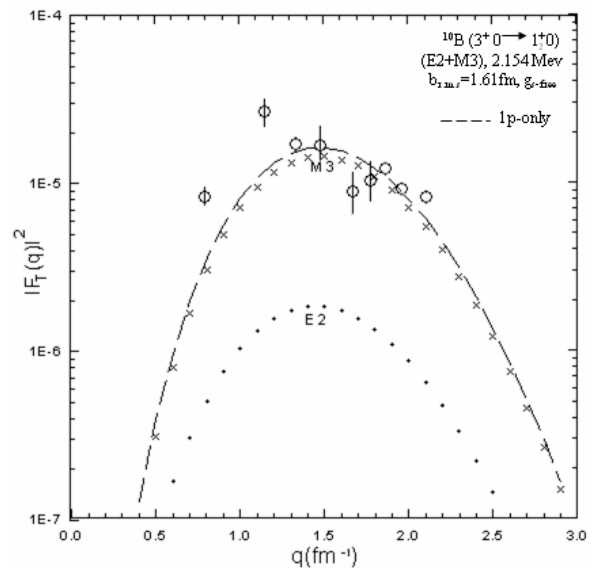


Fig. 3 :The transverse form factors for 1_2^+0 state in ^{10}B calculated in 1p-shell model space only (dashed curve).The individual multipole contributions are shown. The experimental data are taken from Ref.[7] (circles).

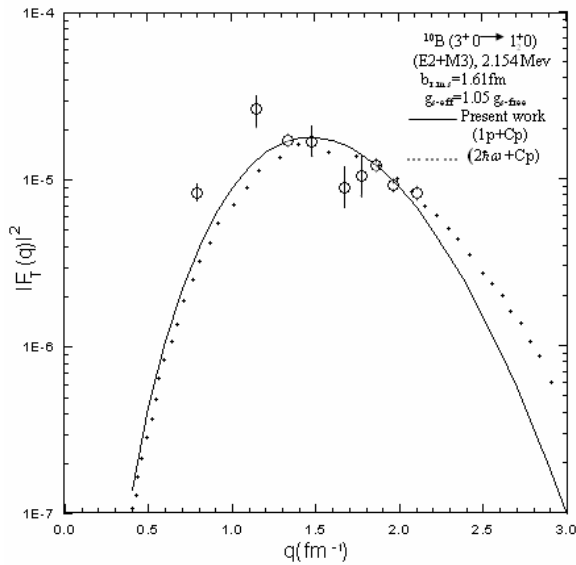


Fig. 4 The transverse form factors for 1_2^+ state in ^{10}B calculated with 1p+CP (solid curve). The present results are compared to that of Ref.[7] (dotted curve).

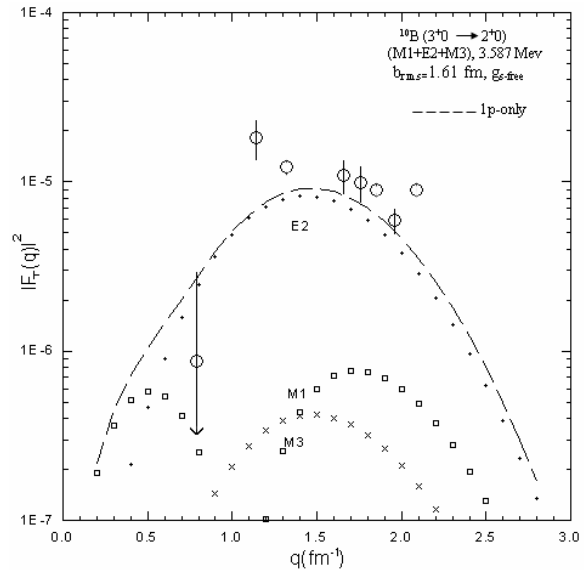


Fig. 5 :The transverse form factors for 2^+ state in ^{10}B calculated in 1p-shell model space only (dashed curve).The individual multipole contributions are shown. The experimental data are taken from Ref.[7] (circles).

3.3 The 3.587 MeV(2^+ 0) state

It would be more interesting to study the transition to 2^+ state where E2 multipole is dominant, because such study can essentially clarify the E2-transition operator in our case. The isoscalar transverse form factors with the M1, E2 and M3 multipoles are presented in Figure 5. The M1 (square symbol curve) and M3 (cross symbol curves) form factors are small throughout the entire range of momentum transverse. The results of 1p-shell with free g-factors and $b_{\text{rms}}=1.61\text{fm}$ (dashed curve) are underestimates the experimental data of Cichocki et al. [7] (circles), and in accordant with E2 (dotted curve) component. The present results are compared with that of $(2\hbar\omega + \text{CP})$ -results of Cichocki et al. This comparison is shown in Figure 6. The solid curve shows the present results that including the effective g-factors ($g_{s\text{-eff}}^{p/n} = 1.19 g_{s\text{-free}}^{p/n}$), while the results of Cichocki et al. [7] are indicated by dotted curve. Both models give same behavior in general. The present results give a reasonable agreement with the experimental data for large regions of q , and closely to the results of Cichocki et al. only at $q < 1.5 \text{ fm}^{-1}$, and they are different in magnitude in other regions of q .

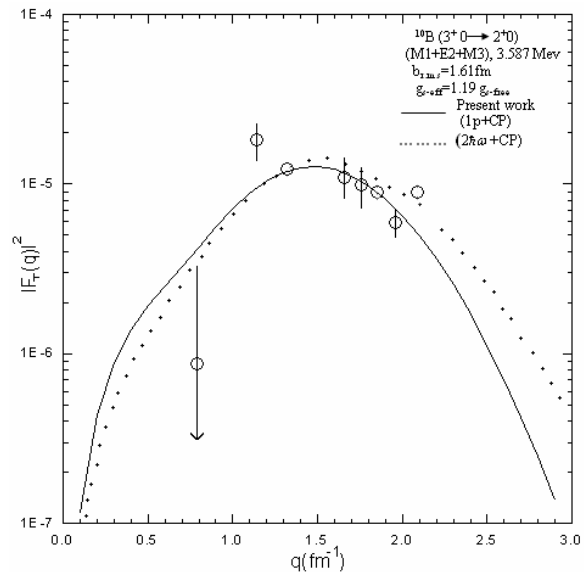


Fig. 6 :The transverse form factors for 2^+ state in ^{10}B calculated with 1p+CP (solid curve). The present results are compared to that of Ref.[7] (dotted curve).

3.4 The 4.774 MeV (3_2^+ 0) state

The calculated form factors for 4.774 MeV(3_2^+ 0) state with the size parameter $b_{\text{rms}}=1.61\text{fm}$ and with free g-factors are presented in Figure 7 (dashed curve). The results are compared with the experimental data of Cichocki et al. [7] (circles). The individual multipoles contributions E2 and M3 are denoted

by dotted and cross symbol curves respectively, while the M1 multipole is disappeared because it has a negligible contribution. The 1p-shell model space (dashed curve) reproduced the experimental data at low momentum transverse ($1.1\text{fm}^{-1} < q < 1.4\text{fm}^{-1}$), and overestimated the experimental data at high q -value. The inclusion admixture of 2p-shell contribution with $\alpha = \gamma = -0.98$ and using the effective g -factors ($g_{s\text{-eff}}^{p/n} = 1.05 g_{s\text{-free}}^{p/n}$) are shown in Figure 8 (solid curve). With these parameters the experimental data are very well reproduced over all range of momentum transfers. The present results are compared with that of Ref. [7] (dotted curve), this comparison is shown in Figure 8. The total form factors of the present results (1p+2p+CP) and ($2\hbar\omega$ +CP)-results of Ref. [7] give the same behavior and close to each other in shape and magnitude at the $q < 1.5\text{fm}^{-1}$, but they are different in magnitude at higher momentum transfer.

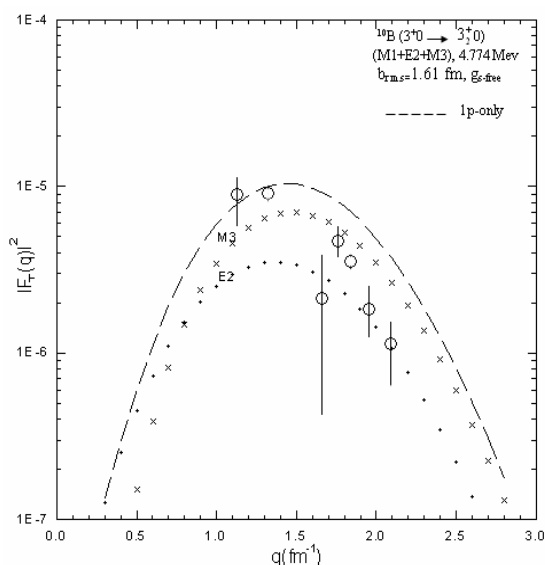


Fig. 7 The transverse form factors for 3_2^+ state in ^{10}B calculated in 1p-shell model space only (dashed curve). The individual multipole contributions are shown. The experimental data are taken from Ref.[7] (circles).

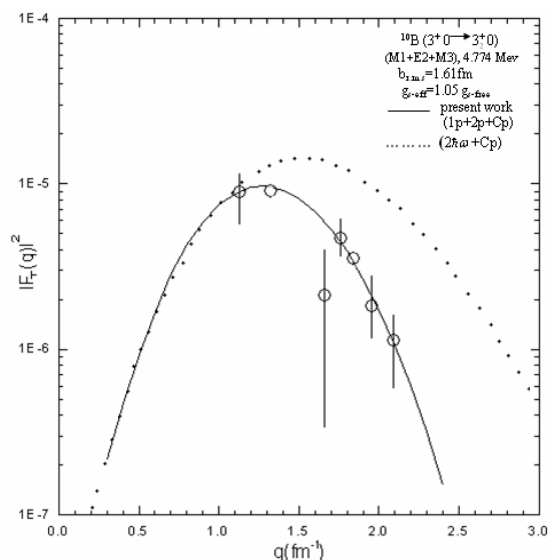


Fig. 8 The transverse form factors for 3_2^+ state in ^{10}B calculated with 1p+2p+CP (solid curve). The present results are compared to that of Ref.[7] (dotted curve).

4. Conclusions

The most important conclusions of the present work can be briefly summarized as follows:-

1. The transverse inelastic electron scattering form factors are fairly well predicted in the extended model with CP effects.
2. The transverse inelastic electron scattering form factors are influenced by the details of nuclear wave functions. The inclusion of the lowest-order configuration is essential to reproduce some of the q -data.
3. The inclusion of higher orbit contribution gives a remarkable improvement in the form factors at the 3_2^+ state.
4. The effective g -factors given in the text make a reduction to the form factors and sometimes reproduce the low q -data.

5. References

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