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Numerical study of soliton generation in microresonators

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Abstract

Soliton generation in microresonator is studied numerically using the Lugiato-Lefever equation (LLE). Optically induced effects, duration, and shape are studied via the adaptive split-step Fourier method (SSFM). It investigates how the microresonator affects the evolution of the optical field in cases of nonlinearity, dispersion, slow time, and fast time. A study is done on the optical frequency comb produced and the frequency domain dynamics of the microresonator. The generated soliton wave was a dispersive soliton. The study shows that the soliton generation and its properties depend strongly on the parameters included in this study.

Keywords: Microresonators, Cavity solitons, Lugiato–Lefever equation, Dispersion, Dissipative Kerr solitons, nonlinear optics, SSFM.

دراسة عددية لتوليد السوليتون في المرنانات الدقيقة

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الخلاصة

تمت دراسة توليد الموجات المنعزلة (soliton) في المرنانات الميكروية عدديًا باستخدام معادلة Lugiato–Lefever (LLE). تتم دراسة التأثيرات المستحثة بصريًا والمدة والشكل من خلال طريقة فورييه ذات الخطوة المقسمة (SSFM). نبحث في كيفية تأثير المرنان الميكروي على تطور المجال البصري لحالات اللاخطية والتشتت والزمن البطيء والزمن السريع. تم إجراء دراسة على مشط التردد البصري المتولد وديناميكيات مجال التردد للمرنان الميكروي. وكانت موجة سوليتون المتولدة عبارة عن سوليتون مشتت. أظهرت الدراسة أن توليد السليتون وخصائصه يعتمد بشكل كبير على العوامل المتضمنة في هذه الدراسة.

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1. Introduction

Many topics like nonlinear optics and hydrodynamics [1], plasma physics [2], and biology, have conducted extensive theoretical and experimental research on solitons, also known as solitary waves. In optics, the natural tendency for an optical wave packet (a pulse or a beam) to spread as it moves across a medium is due to chromatic dispersion or spatial diffraction. The novel use of optical solitons is being investigated for the purpose of creating optical frequency combs in microresonators [3]. Soliton wave generation in basic and photonic crystal optical fibers was studied [4,5], also optical frequency comb generation was studied by several researchers [6]. The last ten years have seen the discovery of rich nonlinear dynamics in microresonators[7-14].

The microresonator used to create frequency combs consists of a small circular piece of glass-like optical component made of silicon nitride, magnesium fluoride, calcium fluoride, or fused silica [15]. They can range in size from the width of hair to a few millimeters in diameter. Light is trapped in the microresonators in a way that forces it to travel around the resonator's circumference. If the resonators are of sufficiently high quality, a quality factor (Q) of 10^6 to 10^{10}, then large amounts of light can be stored inside. At high enough intensities, nonlinear interactions occur between the light and the microresonator. This can result in a single color of light being turned into a rainbow of light containing many colors (four-wave mixing) [16], creating an optical frequency comb, as shown in Figure (1).

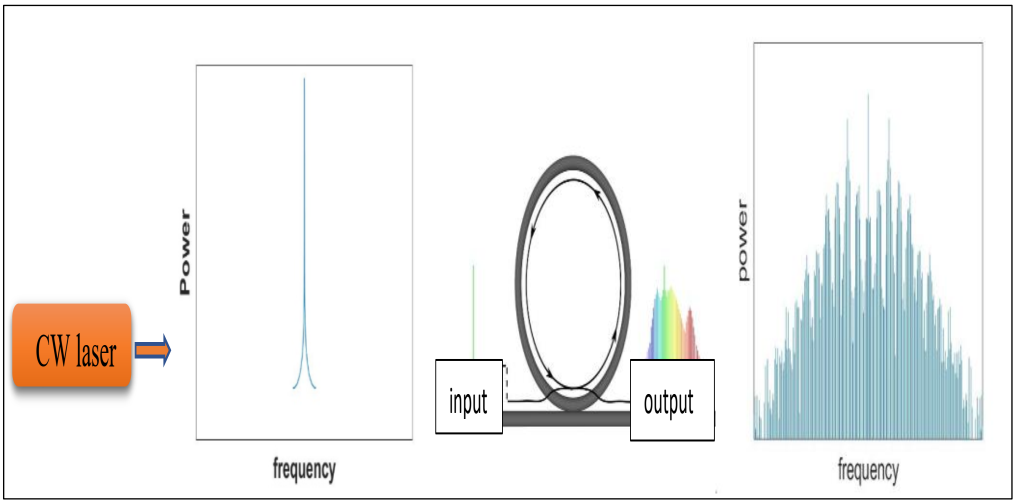

Figure 1: The process of optical frequency comb and soliton generation using microresonator.

The line spacing of a frequency comb, light's inverted roundtrip time in a microresonator is known as the Free Spectral Range (FSR). Depending on a microresonator's free spectral range, this frequency may be in the GHz or THz range. Figure (2) depicts a simulation of the evolution of a basic soliton without higher-order effects. The fundamental soliton maintains its form (time duration, amplitude, and intensity distribution) during the propagation distance.

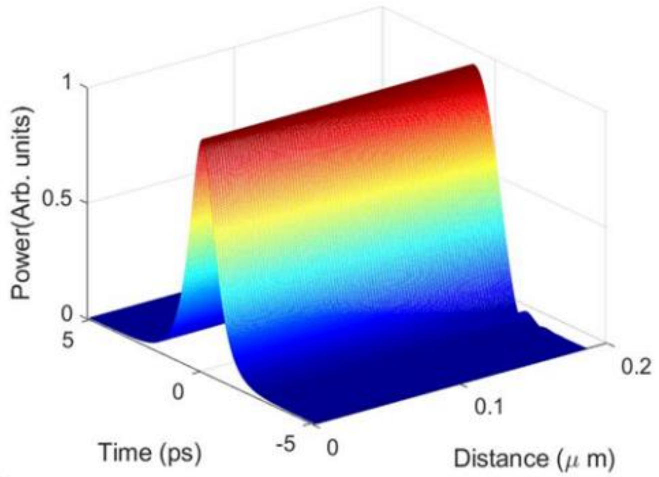

Figure 2: Sample of the spatiotemporal evolution of solitons of order N = 1, input pulse width τ_s = 100 ps, and peak power P = 4 W.

For four-wave mixing processes that result in sharp spectral peaks, which in the temporal domain corresponds to oscillatory soliton tails, wavelength matching is very useful. If the soliton's duration is brief enough to allow its bandwidth to reach the standard dispersion regime, it is dispersive [17], approaching the GVD's zero-dispersion regime (zero-dispersion wavelength) [18].

The Lugiato-Lefever Equation (LLE) with higher order dispersion components sometimes referred to as the damped driven nonlinear Schrodinger equation [19], explains the temporal dissipative cavity Kerr solitons' behavior in microresonators. The optical field envelope in a coordinate frame spinning in a microresonator with the pump field phase velocity is represented by this equation.

This work presents a theoretical analysis of the production of temporal dissipative cavity solitons in SiN microresonators. Silicon nitride utilized to create this microresonator is a common dielectric used in both linear and nonlinear optical applications. The combination of its moderate optical field confinement in waveguides, low intrinsic loss, and extremely high clear window make it an ideal semiconductor. Due to its nonlinear Kerr coefficient, silicon nitride with high silicon content waveguides is desirable for nonlinear optics. In this scenario, the detuning frequency and the noise strength term are disregarded [20].

1. Theory

The LLE describes how light moves through a high-finesse resonator. To examine the development of microresonator frequency combs in detail, the LLE was numerically simulated [21], particularly, considering the various comb operating regimes, such as temporal cavity solitons. When utilized to find an equilibrium spectrum solution, the LLE model has been shown to agree highly with experimental findings [22]. This allows for the description of the temporal profile of the field envelope moving at this rate in the cavity. To link the intracavity field at the end of one round trip with the field at the beginning of the next, it is necessary to include boundary conditions that consider the field's evolution over each round trip. According to mathematics, this relationship is provided by Eq. (1), which provides a relationship between the intracavity fields at the beginning and end of consecutive round trips in the optical resonator [23]:

$$E_{m+1}(L,\tau) = \sqrt{\theta}E_0 + \sqrt{1-\theta}\,E_m(L,\tau)e^{i\varphi_0} \quad (1)$$

$E_{(m+1)}(L,\tau)$ represents the intracavity field at the start of the $(m + 1)^{th}$ round trip. It indicates the strength and characteristics of the field at the beginning of the next cycle. The linear phase buildup of the intracavity field with respect to the pump field throughout a single roundtrip is φ_0; the coefficient of transmission on the coupler is given by θ. The intracavity field envelope can be thought of as varying somewhat between successive round trips in the limit of low loss. In these circumstances, averaging the prior infinite-dimensional map can provide the externally driven nonlinear Schrodinger equation (NLSE) [24]:

$$t_r \frac{\partial E(t,\tau)}{\partial t} = \left[-(\alpha + i\delta_o) + iL\sum_{k\geq 2}\frac{\beta_k}{k!}\left(i\frac{\partial}{\partial\tau}\right)^k + iL\gamma|E(t,\tau)|^2\right]E(t,\tau) + \sqrt{\theta}E_{in} \quad (2)$$

Where: E (t,τ) is the intracavity field, t_r (t_r=2Ln_0/c) is the roundtrip duration [25], t is the slow time scale for this profile's progression over consecutive roundtrips, it is presumed that the field follows the cavity roundtrip time, which is E(t + 2,τ) = E(t,τ) and which determines the field's temporal profile in ordinary (fast) time [26], β_k is the dispersion coefficients, $\alpha = (\alpha_i + \theta)/2$ is the overall cavity losses, $\delta_0 = 2k\pi - \varphi_0 \ll 1$ the cavity detuning from the nearest resonance, with the order of the cavity resonance closest to the driving field, k is the power transmission coefficient, and γ is interaction nonlinearity coefficient. The spatial dissipative features in optical systems were the inspiration for the development of the LLE [27], which has recently had a significant influence in the field of integrated photonics. It is utilized to grasp and anticipate Kerr-mediated nonlinear optical phenomena such as parametric frequency comb generation [28]within microresonators [29]. This equation has become the basic model for studying frequency combs based on microresonators and leads to the analysis of various structures inside the cavity, such as luminous and dark soliton, spatial and temporal, etc.

2. Results and discussion

The soliton dynamics within the LLE were numerically simulated using the Split-Step Fourier Method (SSFM) [30]. Table (1) displays the simulation parameters used in this study.

Table 1: The simulation parameters used in this work are the typical parameters for a silicon nitride (SiN) ring resonator [25], [31], [32]].

Symbol	Description	Value Unit	Unit
λ_0	Lasing Wavelength	1.55	µm
n_2	The nonlinear refractive index	2.4×10^{-19}	m^2/W
ω_0	the frequency of the optical cw pump	193.5	THz
n_0	the refractive index	1.99	
A_{eff}	the effective model area of the resonator mode	2.5×10^{-12}	m^2
Q	Quality factor	1.5×10^6	
θ	the external coupling coefficient	0.03	
P_{in}	input cw pump power	0.5, 1, 1.5, 2	Watt
γ	nonlinearity coefficient	1.2	$W^{-1}m^{-1}$
α	total cavity losses inside the resonator	0.00161	
β_2	dispersion coefficient	-4.7×10^{-26}	s^2m^{-1}
L	Cavity length	428.6	µm
t_r	the roundtrip time	14.28	ns
a	radius	100	µm
τ	Fast time	2	ps

2.2 . The effect of the input field on the production of solitons

Increasing the strength of the optical pulse as a result of a change in the refractive index of the nonlinear medium causes modification in the phase, which in turn affects the spread of the pulse inside the microresonator. As a result, the speed of transmission of the pulse within the resonator is affected. At the same time, the medium has dispersion and affects the phase of that pulse; dispersion leads to distortion of this pulse (reducing its height and expanding it in time). The dominant process (nonlinearity or dispersion) determines the output shape. The greatest effect of the power of the pulse is the stability of the rest of the parameters in Eq. (2), which means an increase in the intensity of the input pulse (third term in Eq. (2). Depending on previous studies [4, 5, 6, 33], the intensity of the pulse leads to a change in the effect of the dispersion phenomenon. It is also clear that the intensity of the pulse changes the refractive index of the medium according to the relationship [$n=n_0+n_2$ (I)]. To study the effect of input power on the time and spectral evolution of the microresonator, the input pulse power, P_{in}, was varied (1, 1.2, 1.4, 1.8, 2, 2.5) W, while, keeping the parameters of the microresonator, shown in Table (1), fixed. The peaks in Figure (3-a) mean that frequencies (modes) are growing inside the microresonator at the expense of other modes.

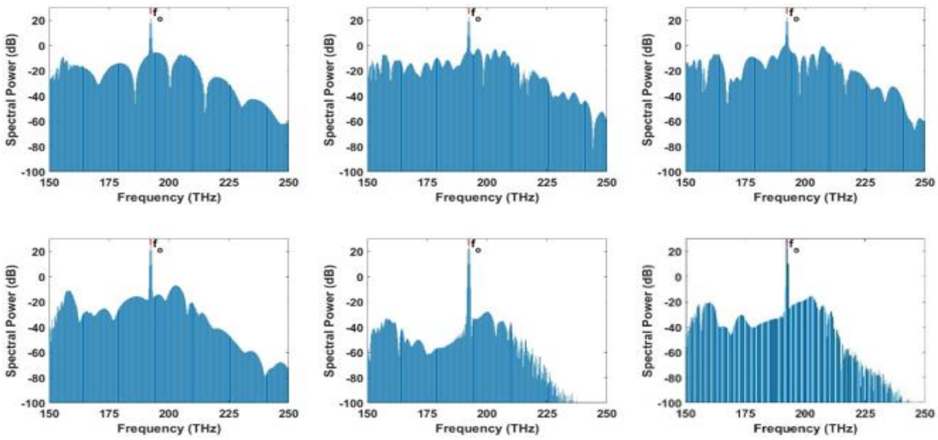

Figure 3a: The relationship between spectral power and the frequency comb of the microresonator field, for input power (1, 1.2, 1.4, 1.8, 2, 2.5) W.

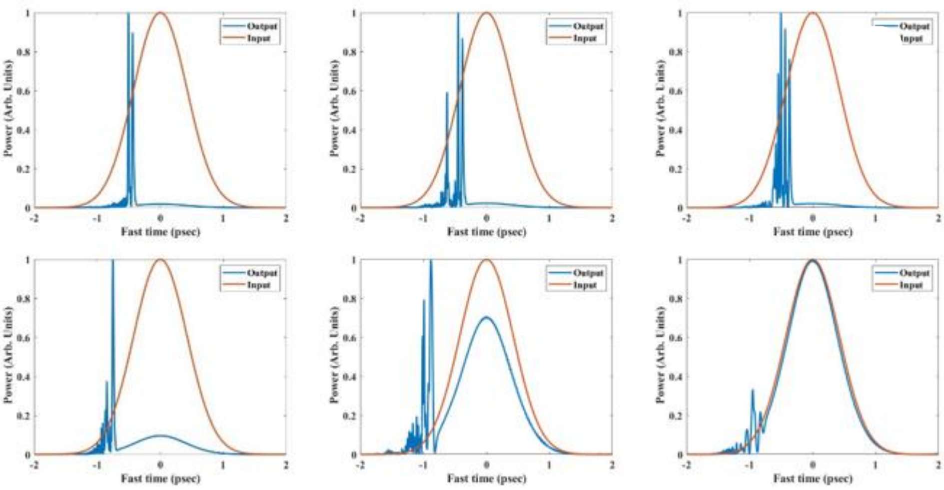

Figure 3:b: The input-output pulse corresponding to different input power levels of (1, 1.2, 1.4, 1.8, 2, 2.5) W

2.3. Effect of pulse width on soliton generation

The pulse width and its effect on soliton generation in a microresonator were studied, knowing that the effect of the pulse width cannot be predicted easily. The pulse width directly affects the interaction between the pulse and the medium. The induced nonlinear effects, the induced refractive index and dispersion depend on the pulse width. The effect depends on the shape and intensity of the pulse. In addition, changes from large cavities to the microcavities leads to changes in the physical properties i.e. properties in small dimensions differ from those in large dimensions

The spectral power against the frequency comb of the resonator field was plotted for different pulse widths in the range (1.2, 1.3, 1.35, 1.4, 1.46, 1.49) ps at intracavity pump (2 W), as shown in Figure (4). The output of the microresonator in the frequency space (wavelength) changed with the change of the pulse width. The processes that take place inside the microresonator depend on the time it takes for the round trip to go until the pulse leaves the microresonator, so depending on the dominant effect, and the change in the field distribution and intensity with time, there is a change in the amount of effects, for example, there may be an increase in the phenomenon of dispersion or an increase in nonlinear effects, and whenever the generated soliton wave is dispersed. It is noticed that the distribution of the pulses with time tends towards the left side, and the number of accompanying or embedded pulses (the accompanying pulses within the original wave) increases with the increase in the pulse width, except for the case when the pulse width is (1.46 ps), as in Figure (4-b). The reason for this may be the occurrence of a state of equilibrium or a momentary reaction, perhaps the cause of irregularity that made the medium inhomogeneous.

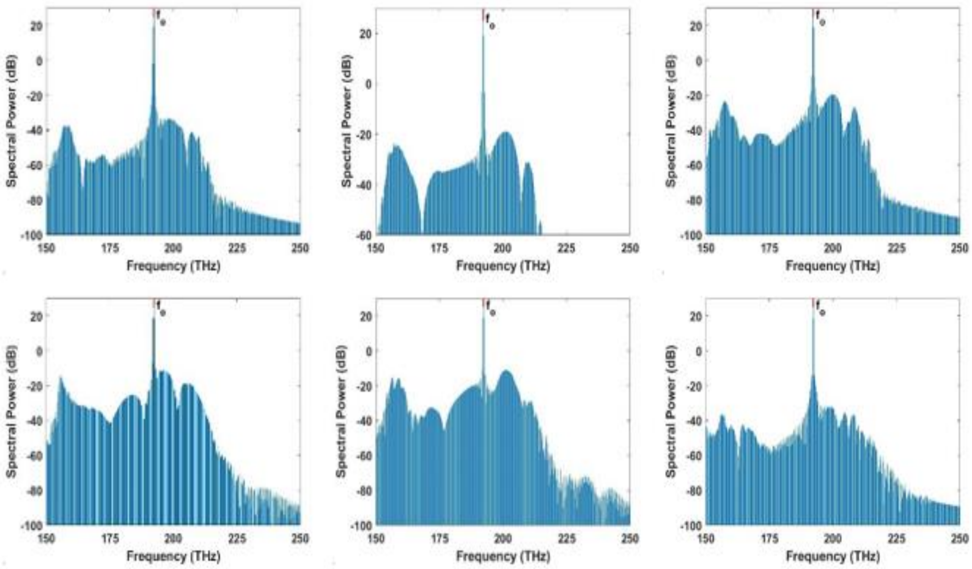

Figure 4a: The spectral power against the frequency comb of the resonator field for pulse widths in the range (1.2, 1.3, 1.35, 1.4, 1.46, 1.49) ps, with the intracavity pump (2 W).

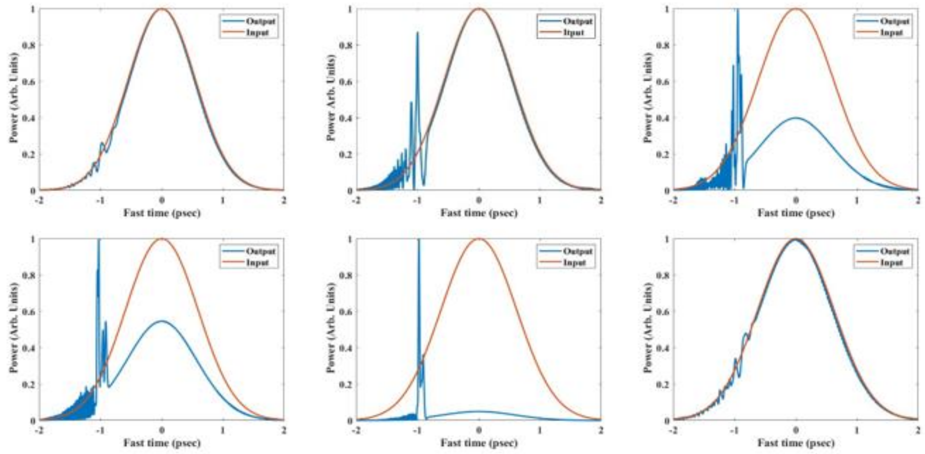

Figure 4b: The input-output pulse for different pulse widths within the range of (1.2, 1.3, 1.35, 1.4, 1.46, and 1.49) ps.

The effect of the fast time on the production of solitons

Equation (2) was formulated using the double-scale technique. In this approach, the variable "t" represents the slow time scale, which monitors the back-and-forth growth of the mean field and its effect on soliton formation. On the other hand, the fast time scale, represented by the second term in Eq. (2), corresponds to the time it takes the field to propagate inside the microresonator at the group velocity of light. The fast time scale is specifically related to the effects of Group Velocity Dispersion (GVD) and its effect on the scattering phenomenon. It is the time scale over which GVD affects the dynamics of the system. Therefore, the dispersion effect is directly affected by this fast time scale. It is worth noting that the input pulse width primarily determines the characteristics of the fast time scale. The behavior of the dispersion effect is closely related to this fast time scale, which in turn depends on the input pulse width. Figure 5 shows the spectral power against the frequency comb of the resonator field and the input-output pulse for a pulse width of (1 ps) and fast time of (1, 1.2, 1.4, 1.5, 1.6) ps with the intracavity pump 1.5 W

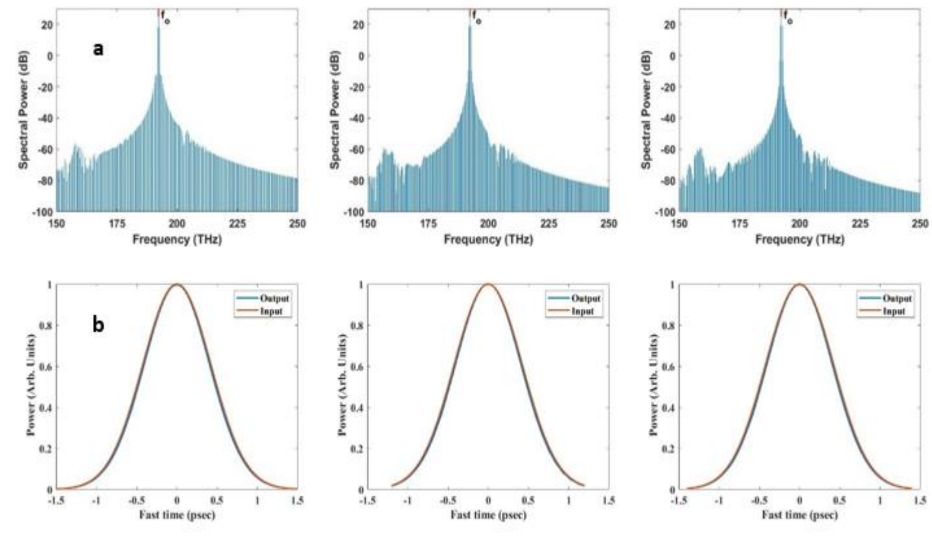

Figure 5: (a)The spectral power against the frequency comb of the resonator field (b) the input-output pulse for a pulse width of (1 ps) and fast time of (1, 1.2, 1.4, 1.5, 1.6) ps with the intracavity pump 1.5 W.

It is noted from Figure (5(b)) that the change in the input and output pulse remains the same; this means that the effect of fast time is through spectral power, and it is also a simple change that leads to the appearance of an increase or decrease some modes present at both ends of the central frequency. Specifically, the variations in the fast time scale result in an increase or decrease in the spectral power of specific modes located at the outer edges of the center frequency. This implies that the fast time scale affects the energy distribution within the resonator, leading to localized changes in the spectral power of certain modes.

Conclusion

The temporal and spectral dynamics of the microresonator field were studied numerically by solving the Lugiato-Lefever Equation (LLE). Several configurations of the microresonator were considered, including the slow time parameter. Input power pulse, duration, shape of the soliton, and optical frequency comb generation were investigated. The output pulses and optical frequency combs were generated depending on the parameters studied and investigated in the time domain. The power spectral density of the microresonator was studied. Many types of solitons are obtained with different dynamics depending on the microresonator configuration, the input field, power, and time duration.

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