UNSTEADY MHD FLOW OF A VISCOELASTIC FLUID WITH THE FRACTIONAL BURGERS' MODEL

Ahmad M.Abdul Hadi
Departement of Mathematics , College of Science, University of Baghdad .Baghdad- Iraq

abstract

The aim of this paper is to study the effects of a magnetic field on unsteady flow of a viscoelastic fluid with the fractional Burgers' model between two parallel plates .The fractional calculus approach is introduced to establish the constitutive relationship of the viscoelastic fluid. Closed form solutions for velocity and shear stress are obtained by using the finite Fourier sine transform and discrete Laplace transform of the sequential fractional derivatives. For $\alpha=\beta=1$ the solution that are obtained are going corresponding to ordinary Oldroyd-B fluid. Finally, the effect of the material parameters on the velocity profile and shear stress profile spotlighted by means of the graphical illustrations.

الجريان اللامستقر في حقل مغناطيسي لمائع لزج من النمط بيركر ذو المشتقات الكسرية

أحمد مولود عبد الهادي
قسم الرياضيات، كلية العلوم، جامعة بغداد. بغداد – العراق

الخلاصة

إنّ هدفَ هذا البحث هو دراسة تأثيراتَ حقل مغناطيسي على التدفقِ اللا مستقرِ بين صفيحتين متوازية لمائع لزج بنموذجِ بيركر ذو المشتقات الكسريه. حساب التفاضل والتكاملِ الكسرى قد استخدم لكتابة معادلات الحركة . باستخدام تحويلات فورية ولابلاس حصلنا على الحل المضبوط لكل من حقل السرعةَ و إجهادِ القصِّ . عندما $\alpha=\beta=1$ نحصل على الحَلّ لمائع أولدرويد من النمط B . أخيراً، تأثير المعلمات على كل من حقل السرعةَ و إجهادِ القصِّ قد درست من خلال الإيضاحات التخطيطية .

1. Introduction

In recent years, the flow of non-Newtonian fluid(paints, grease, oils liquid polymers,…) has received much attention for their increasing industrial and technological applications such as extrusion polymers fluid, exotic lubricants, colloidal and suspension solutions, food stuff and many other applications. Mathematically the non-Newtonian fluids have non-linear relationship between the stress and the rate of strain, because of this there is no model which can alone describe the behavior of all non-Newtonian fluid. Therefore, several constitutive equations for the non-Newtonian fluid models have been proposed. The Oldroyd-B fluid is one of them which cannot describe by a typical relation between shear stress and the rate of strain, for this reason many models of constitutive equations have been proposed for these fluids [1,2,3]. The subject of fractional calculus [4] has been successfully used to describe the viscoelasticity.In general, the constitutive equations for generalized non-Newtonian fluids are modified from the well known models by replacing the time derivative of an integer order with the so called Riemann-Liouville fractional calculus operators. Qi and Xu[1] discussed the Stokes' first problem for a viscoelastic fluid with generalized Oldroyd-B model. Fetecau et al.[5,6] discussed some accelerated flows of generalized Oldroyd-B fluid. Khan[7] investigated the magnaticohydrodynamic (MHD) flow of generalized Oldroyd-B fluid in a circular pipe. Liancun et al.[8] considered the MHD flow of an incompressible generalized Oldroyd-B fluid due to an infinite accelerating plate. Later on [9],the same authors, investigated the slip effects on MHD flow of a generalized Oldroyd-B fluid with fractional derivative. Hyder Ali[10] , discussed some unidirectional flow of a viscoelastic fluid between two parallel plates with fractional Burgers' fluid model.

In the present work we studied the effects of MHD on the unsteady flow of a viscoelastic fluid between two parallel plates with fractional Burgers' fluid model that given by Hyder [10], also some especial cases are recovered. The exact solutions for velocity distribution and shear stress are established by using the finite Fourier sine transform and discrete Laplace transform of the sequential fractional derivatives.

2. Governing equations

The fundamental equations governing the unsteady motion of an incompressible viscoelastic fluid include the continuity equation and the momentum equations are

$$\operatorname{div} \mathbf{V} = 0 \tag{1}$$

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}.\nabla\mathbf{V})\right] = -\nabla p + \nabla.\mathbf{S} \tag{2}$$

Where $\mathbf{V}$ is the velocity field, ρ is the fluid density, p the pressure and $\mathbf{S}$ the extra stress tensor. The extra stress tensor $\mathbf{S}$ for a fractional Burgers' fluid satisfies the following constitutive equation

$$\left(1 + \lambda_1^{\alpha}\tilde{D}_t^{\alpha} + \lambda_2^{\alpha}\tilde{D}_t^{2\alpha}\right)\mathbf{S} = \mu\left(1 + \lambda_3^{\beta}\tilde{D}_t^{\beta}\right)\mathbf{A} \tag{3}$$

Where $\mathbf{A} = \mathbf{L} + \mathbf{L}^{\mathbf{T}}$ is the first Rivlin-Eriksen tensor with $\mathbf{L}$ the velocity gradient, μ the dynamic viscosity, $\lambda_1$and λ_3 ($\lambda_3 < \lambda_1$) are the relaxation and retardation times, respectively, λ_2 is a material constant. α and β are fractional calculus parameters such that $0 \leq \alpha \leq \beta$ and $\tilde{D}_t$ is the upper convected time derivative defined as[11],

$$\tilde{D}_t^{\alpha}\mathbf{S} = D_t^{\alpha} + (\mathbf{V}.\nabla)\mathbf{S} - \mathbf{LS} - \mathbf{SL}^{\mathrm{T}}$$

and $$\tilde{D}_t^{2\alpha}\mathbf{S} = \tilde{D}_t^{\alpha}(\tilde{D}_t^{2\alpha}\mathbf{S}) \tag{4}$$

$$\tilde{D}_t^{\beta}\mathbf{S} = D_t^{\beta} + (\mathbf{V}.\nabla)\mathbf{S} - \mathbf{LS} - \mathbf{SL}^{\mathrm{T}}$$

Where D_t^{α} is the fractional differential operator, which is defined as[4]

$$D_t^{\alpha}[f(t)] = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-\tau)^{-\alpha} f(\tau)\,d\tau,$$
$$0 \leq \alpha \leq 1$$

Where $\Gamma(.)$ is the Gamma function.For unidirectional flow , we shall consider unsteady flows wherein velocity and stress filed are of the form

$\mathbf{V}=\mathbf{V}(y,t)=u(y,t)\mathbf{i},\qquad \mathbf{S}=\mathbf{S}(y,t),$ (5)

Where **i** is the unit vector in the x-coordinate direction of the Cartesian coordinate system. For such flow the constraint of incompressibility is automatically satisfied. Substituting Eq.(5) into Eqs.(1) and (3) and taking into account the initial condition

$$\mathbf{S}(y,0)=\frac{\partial \mathbf{S}(y,0)}{\partial t}=\mathbf{0} \qquad (6)$$

i.e., the fluid being at rest up to time t=0. For the components of stress field S we have $S_{yy}=S_{zz}=S_{xz}=S_{yz}=0$ and $S_{xy}=S_{yx}$, we obtain the relevant equations

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{\partial S_{xy}}{\partial y}-\sigma B_0^2 u \qquad (7)$$

$$\left(1+\lambda_1^\alpha \tilde{D}_t^\alpha+\lambda_2^\alpha \tilde{D}_t^{2\alpha}\right)S_{xy}=\mu\left(1+\lambda_3^\beta \tilde{D}_t^\beta\right)\frac{\partial u}{\partial y} \qquad (8)$$

Eliminating S_{xy} between Eqs. (7) and (8) , we arrive at the following fractional differential equation

$$\left(1+\lambda_1^\alpha D_t^\alpha+\lambda_2^\alpha D_t^{2\alpha}\right)\frac{\partial u(y,t)}{\partial t}=-\frac{1}{\rho}\left(1+\lambda_1^\alpha D_t^\alpha+\lambda_2^\alpha D_t^{2\alpha}\right)\frac{\partial p}{\partial x}+\nu\left(1+\lambda_3^\beta D_t^\beta\right)\frac{\partial^2 u(y,t)}{\partial y^2}-M\left(1+\lambda_1^\alpha D_t^\alpha+\lambda_2^\alpha D_t^{2\alpha}\right)u(y,t) \qquad (9)$$

Where $\nu=\mu/\rho$ is the kinematic viscosity of the fluid and $M=\sigma B_0^2/\rho$.

3.Plane Poiseuille flow

Let us consider the flow problem of an incompressible fractional Burgers' fluid between two infinitely parallel plates. Suppose that the fluid is bounded y two parallel plates in the (x,z) plane, these plates are placed at y= 0 and y = h. Initially, the fluid is at rest and at time $t=0^+$ and the motion starts due to a constant pressure gradient $A=-\frac{1}{\rho}\frac{\partial p}{\partial x}$. Under these assumptions, the governing equation in the present of the pressure gradient in the flow direction is given by

$$\left(1+\lambda_1^\alpha D_t^\alpha+\lambda_2^\alpha D_t^{2\alpha}\right)\frac{\partial u(y,t)}{\partial t}=\left(1+\frac{\lambda_1^\alpha t^{-\alpha}}{\Gamma(1-\alpha)}+\frac{\lambda_2^\alpha t^{-2\alpha}}{\Gamma(1-2\alpha)}\right)A+\nu\left(1+\lambda_3^\beta D_t^\beta\right)\frac{\partial^2 u(y,t)}{\partial y^2}-M\left(1+\lambda_1^\alpha D_t^\alpha+\lambda_2^\alpha D_t^{2\alpha}\right)u(y,t) \qquad (10)$$

And its corresponding initial and boundary conditions are

$u(y,0)=0;\quad 0<y<h$ (11)

$u(0,t)=u(h,t)=0;\quad \text{for } t\geq 0$ (12)

4. Calcution of the velocity field

In order to solve the above problem, we shall use the finite Fourier sine transform and fractional Laplace transform. Consequently, multiplying Eq.(10) by $(2/\pi)\sin(n\pi y/h)$,integration with respect to y from 0 to h and keeping in mind the conditions(12) we get

$$\left(1+\lambda_1^\alpha D_t^\alpha+\lambda_2^\alpha D_t^{2\alpha}\right)\frac{\partial u_s(n,t)}{\partial t}=\frac{A(1-(-1)^n)}{\epsilon_n}\left(1+\frac{\lambda_1^\alpha t^{-\alpha}}{\Gamma(1-\alpha)}+\frac{\lambda_2^\alpha t^{-2\alpha}}{\Gamma(1-2\alpha)}\right)-\nu\epsilon_n^{\,2}\left(1+\lambda_3^\beta D_t^\beta\right)u_s(n,t)-M\left(1+\lambda_1^\alpha D_t^\alpha+\lambda_2^\alpha D_t^{2\alpha}\right)u_s(n,t) \qquad (13)$$

Where $u_s(\xi,t)$ is the finite Fourier sine transform of $u(y,t)$ and $\epsilon_n=n\pi/h$. Next, applying Laplace transform for sequential fractional derivative to Eq.(13) and using initial condition(10), we get

$$\bar{U}(n,s)=\frac{\bar{u}(n,s)}{\frac{Ah^2}{8\nu}}=\frac{8\nu h^{-2}}{\epsilon_n}\left(1-(-1)^n\right)\times\left(1+\lambda_1^\alpha s^\alpha+\lambda_2^\alpha s^{2\alpha}\right)/s\left(s+\lambda_1^\alpha s^{\alpha+1}+\lambda_2^\alpha s^{2\alpha+1}+\nu\,\epsilon_n^{\,2}\left(1+\lambda_3^\beta s^\beta\right)+M\left(1+\lambda_1^\alpha+\lambda_2^\alpha s^{2\alpha}\right)\right) \qquad (14)$$

In order to obtain an analytic solution for this problem and to avoid lengthy calculations of residues and contour integrals, we apply the discrete inverse Laplace transform method [4]. However, for more suitable presentation of final result, we rewrite Eq.(14) in series form

$$\bar{U}(n,s) = \frac{8\nu\,(1-(-1)^n)}{h^2\epsilon_n} \times \sum_{k=0}^{\infty}(-1)^k \sum_{a+b+c+d+e+f=k}^{a,b,c,d,e,f\geq 0} \frac{k!\,M^{d+e+f}\,(\nu\,\epsilon_n{}^2)^{b+c}}{a!\,b!\,c!\,!\,d!\,e!\,f!} \times \frac{\lambda_1^{-\alpha(k+1)+\alpha e}\lambda_2^{\alpha(a+f)}\lambda_3^{\beta c}\,s^{\delta}\,(1+\lambda_1^{\alpha}s^{\alpha}+\lambda_2^{\alpha}s^{2\alpha})}{(s^{\alpha}+\frac{1}{\lambda_1^{\alpha}})^{k+1}}$$

Where

$\delta = -(k+2) + \alpha e + (2\alpha+1)a + 2\alpha f + \beta c$

.Taking the inverse Laplace transform of the last equation, we obtain

$$u_s(n,t) = \frac{8\nu\,(1-(-1)^n)}{n^3\pi^3/h} \sum_{k=0}^{\infty}(-1)^k \times \sum_{a+b+c+d+e+f=k}^{a,b,c,d,e,f\geq 0} \frac{k!\,M^{d+e+f}\,(\nu\,\epsilon_n{}^2)^{b+c}}{a!\,b!\,c!\,!\,d!\,e!\,f!} \times \lambda_1^{-\alpha(k+1)+\alpha e}\lambda_2^{\alpha(a+f)}\lambda_3^{\beta c}\,t^{\alpha k+\alpha-\delta-1} \left[E^{(k)}_{\alpha,\,\alpha-\delta}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right) + \frac{\lambda_1^{\alpha}}{t^{\alpha}} E^{(k)}_{\alpha,\,-\delta}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right) + \frac{\lambda_2^{\alpha}}{t^{2\alpha}} E^{(k)}_{\alpha,-\alpha-\delta}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right) \right] \quad (15)$$

In which $E_{\alpha,\beta}(z)$ is the generalized Mittage-Leffler function [4] and

$$E^{m}_{\alpha,\beta}(z) \equiv \frac{d^m}{dx^m} E^{m}_{\alpha,\beta}(z) = \sum_{j=0}^{\infty} \frac{(j+m)!\,z^j}{j!\,\Gamma(\alpha j+\alpha m+\beta)}$$

In obtaining Eq.(15), the following property of the generalized Mittage-Leffler function is used[4];

$$L^{-1}\left\{\frac{m!\,s^{\lambda-\mu}}{(s^{\lambda}\pm c)}\right\} = t^{\lambda m+\mu-1} E^{m}_{\lambda,\mu}(\mp c t^{\lambda})$$

Finally, the inverse finite Fourier sine transform gives the analytic solution of velocity distribution

$$u(y,t) = 16 \sum_{n=1}^{\infty} \frac{(1-(-1)^n)}{n^3\pi^3} \sum_{k=0}^{\infty}(-1)^k \times \sum_{k=0}^{\infty}(-1)^k \sum_{a+b+c+d+e+f=k}^{a,b,c,d,e,f\geq 0} \frac{k!\,M^{d+e+f}\,(\nu\,\epsilon_n{}^2)^{b+c+1}}{a!\,b!\,c!\,!\,d!\,e!\,f!} \times \lambda_1^{-\alpha(k+1)+\alpha e}\lambda_2^{\alpha(a+f)}\lambda_3^{\beta c}\,t^{\alpha k+\alpha-\delta-1} \left[E^{(k)}_{\alpha,\,\alpha-\delta}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right) + \frac{\lambda_1^{\alpha}}{t^{\alpha}} E^{(k)}_{\alpha,\,-\delta}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right) + \frac{\lambda_2^{\alpha}}{t^{2\alpha}} E^{(k)}_{\alpha,-\alpha-\delta}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right) \right] \sin\left(\frac{n\pi}{h}y\right) \quad (16)$$

5.Calcution of the shear stress

Applying the Laplace transform to Eq.(8) and using the initial condition(6), we obtain

$$(y,s) = \mu \frac{1+\lambda_3^{\beta}s^{\beta}}{(1+\lambda_1^{\alpha}s^{\alpha}+\lambda_2^{\alpha}s^{2\alpha})} \frac{\partial\bar{u}\,(y,s)}{\partial y} \quad (17)$$

The image function $\bar{u}(y,s)$ of u(y, t) can be obtained in terms of (16). Substituting the result into Eq.(17),yields

$$\tau = 16\mu \sum_{n=1}^{\infty} \frac{(1-(-1)^n)}{n^2\pi^2 h} \times \sum_{k=0}^{\infty}(-1)^k \sum_{a+b+c+d+e+f=k}^{a,b,c,d,e,f\geq 0} \frac{k!\,M^{d+e+f}\,(\nu\,\epsilon_n{}^2)^{b+c+1}}{a!\,b!\,c!\,!\,d!\,e!\,f!} \times \lambda_1^{-\alpha(k+1)+\alpha e}\lambda_2^{\alpha(a+f)}\lambda_3^{\beta c}\,t^{\alpha k+\alpha-\delta-1} \left[E^{(k)}_{\alpha,\,\alpha-\delta}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right) + \frac{\lambda_3^{\beta}}{t^{\beta}} E^{(k)}_{\alpha,\,\alpha-\beta-\delta}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right) \right] \cos\left(\frac{n\pi}{h}y\right) \quad (18)$$

6.The limiting cases

1- Making the limit of Eqs.(16) and (18) when $\alpha \neq 0$ $\lambda_2 \to 0$ (a=0) and $M \to 0$ (d=e=f=0),we can get the velocity distribution for a generalized Oldroyd-B fluid , as obtained in Ref[12].Thus the velocity and the stress fields reduce to

$$u(y,t)=16\sum_{n=1}^{\infty}\frac{(1-(-1)^n)}{n^3\pi^3}$$
$$\times\sum_{k=0}^{\infty}(-1)^k\sum_{c=0}^{k}\binom{k}{c}\frac{\nu^{k+1}\,\epsilon_n^{\;2k+2}}{\lambda_1^{\alpha k}\,k!}t^{\alpha k+k+1-\beta c}$$
$$\left[\begin{matrix}\frac{\lambda_1^{\alpha}}{t^{\alpha}}E^{(k)}_{\alpha,\,\alpha+k+2-\beta c}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right)\\ +E^{(k)}_{\alpha,\,k+2-\beta c}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right)\end{matrix}\right]\sin\left(\frac{n\pi}{h}y\right)\qquad(19)$$

$$\tau=16\mu\sum_{n=1}^{\infty}\frac{(1-(-1)^n)}{n^2\pi^2h}$$
$$\times\sum_{k=0}^{\infty}(-1)^k\sum_{c=0}^{k}\binom{k}{c}\frac{\nu^{k+1}\,\epsilon_n^{\;2k+2}}{\lambda_1^{\alpha(k+1)}\,k!}$$
$$\times$$
$$t^{\alpha k+\alpha+k-\beta c+1}\left[E^{(k)}_{\alpha,\,\alpha+k+2-\beta c}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right)+\frac{\lambda_3^{\beta}}{t^{\beta}}E^{(k)}_{\alpha,\,\alpha-\beta+(k+2)-\beta c}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right)\right]\cos\left(\frac{n\pi}{h}y\right)\qquad(20)$$

The consequence of this step is that, all the limiting cases that are covered in Ref[5],such as generalized second grade fluid, generalized Maxwell fluid and the Oldroyd-B fluid, can be covered also through our work.

2- Making the limit of Eqs.(16) and (18) when $\alpha\neq0$ and $M\rightarrow0$ (d=e=f=0),we can get similar solutions velocity distribution for unsteady flows of a viscoelastic fluid with the fractional Burgers' model, as obtained in Ref[10].Thus the velocity and the stress fields reduce to

$$u(y,t)=16\sum_{n=1}^{\infty}\frac{(1-(-1)^n)}{n^3\pi^3}$$
$$\sum_{k=0}^{\infty}(-1)^k\sum_{a+b+c=k}^{a,b,c\geq0}\frac{k!\;(\nu\,\epsilon_n^{\;2})^{b+c+1}}{a!\,b!\,c!}t^{\alpha k+\alpha-\delta-1}$$
$$\left[\begin{matrix}E^{(k)}_{\alpha,\,\alpha-\delta}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right)+\frac{\lambda_2^{\alpha}}{t^{\alpha}}E^{(k)}_{\alpha,\,-\delta}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right)+\\ \frac{\lambda_2^{\alpha}}{t^{2\alpha}}E^{(k)}_{\alpha,\,-\alpha-\delta}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right)\end{matrix}\right]\sin\left(\frac{n\pi}{h}y\right)\qquad(21)$$

$$\tau=16\mu\sum_{n=1}^{\infty}\frac{(1-(-1)^n)}{n^2\pi^2h}$$
$$\sum_{k=0}^{\infty}(-1)^k\sum_{a+b+c=k}^{a,b,c\geq0}\frac{k!\;(\nu\,\epsilon_n^{\;2})^{b+c+1}}{a!\,b!\,c!}\lambda_1^{-\alpha(k+1)}$$
$$\lambda_2^{\alpha a}\lambda_3^{\beta c}\,t^{\alpha k+\alpha-\delta-1}\left[E^{(k)}_{\alpha,\,\alpha-\delta}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right)+\frac{\lambda_3^{\beta}}{t^{\beta}}E^{(k)}_{\alpha,\,\alpha-\beta-\delta}\left(\frac{-t^{\alpha}}{\lambda_1^{\alpha}}\right)\right]\cos\left(\frac{n\pi}{h}y\right)\qquad(22)$$

In Eqs.(21) and (22) we have

$$\delta=-(k+2)+(2\alpha+1)a+\beta c\,.$$

7. Results and discussion

Due to the increase in the importance of the viscoelastic fluid the exact analytic solution to unsteady flow of a viscoelastic fluid with the fractional Burgers' model is obtained in terms of Mittage-Leffler function by using the finite Fourier sine and the Laplace transform for fractional derivative. In the limiting cases $\alpha\neq0$ $\lambda_2\rightarrow0$, $M\rightarrow0$ and $\alpha\neq0$ and $M\rightarrow0$,our solutions reduce to those corresponding to a generalized Oldroyd-B fluid(and the consequences cases) and for unsteady flows of a viscoelastic fluid with the fractional Burgers' model.

In this section numerical results are given to illustrate the velocity and stress fields corresponding to Eqs (16) and (18) respectively. The results are interpret with respect to the variation of emerging parameters of interest. In all figures we take ν=0.002.

(Fig 1) is plotted to illustrate the effect of the variation of λ_1 .It can be seen that an increase in λ_1 is to slow down the flow and this is true for small and large time. In contrary, the increase in λ_2 has leads to increase in the velocity value, as shown in

(Fig 2). Figures (3 and 4) are depicted to show the velocity changes with the parameters λ_3 and the fractional parameter β .From them we can see that they have opposite effect on the velocity distribution. Figures (5 and 6) are plotted to illustrate the effect of the fractional parameter α and the MHD parameter M, we observed that they have the same effect with exception that they have different effect on the velocity value .(Fig 7),shows the effect of the time t, in which as t increases there is increase in velocity.

Figs.(8 and 14) show the effect of different parameters upon the shear stress. The parameters λ_1 and β as they increase they have the same effect upon the shear stress ,see figures(8 and 11).For small value of λ_2 leads to decrease in shear stress value ,but for large value of λ_2 we observed opposite effect, see(fig 9). As λ_3 increases there is small increasing in the shear stress as shown in (fig 10).Finally, the variation in α, the MHD parameter M and the time t, we noted that there is a variation in shear stress about the origin as shown in figures (12,13 and 14).

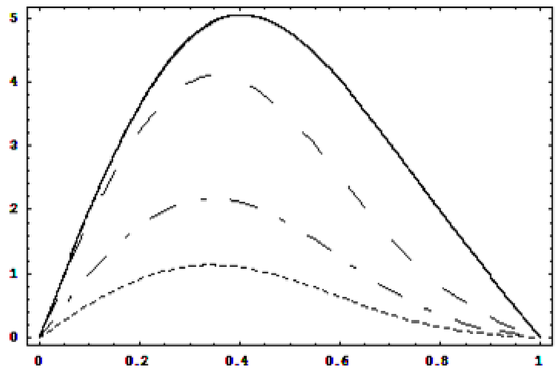

Fig 1 : Velocity profile for λ_1=0.8, 1, 1.2,1.4 (Solid line, dash, dot-dash, dot) λ_2=0.1,λ_3=0.5,β=0.6,α=0.4,M=1,t=2.

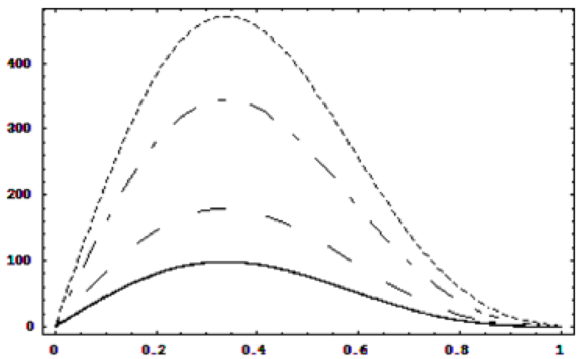

Fig 2 : Velocity profile for λ_2=2, 3, 5 , (Solid line, dash, dot-dash, dot) λ_1=0.9,λ_3=0.2,β=0.6,α=0.4,M=1,t=0.05.

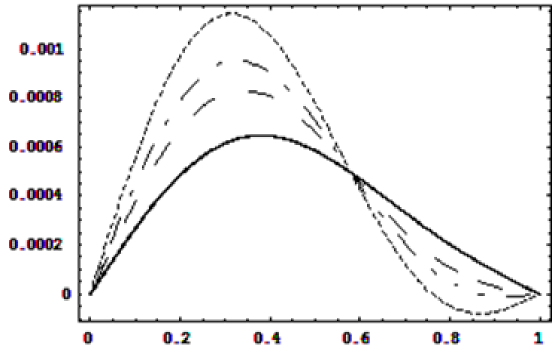

Fig 3 : Velocity profile for λ_3=0.05, 1, 2,3 (Solid line, dash, dot-dash, dot) λ_1=4.2,λ_2=1,β=0.6,α=0.4,M=1,t=1.

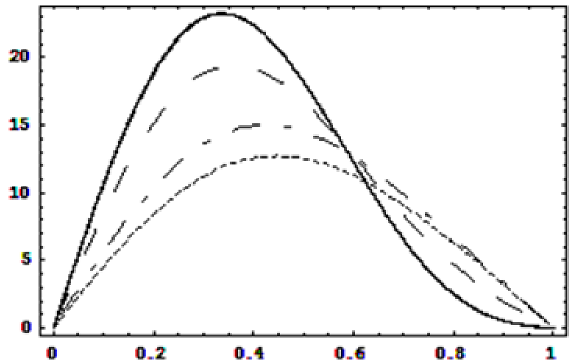

Fig 4 : Velocity profile for β =0.5 , 2 , 2.5 , 3 (Solid line, dash, dot-dash, dot) λ_1=1,λ_2=0.4,λ_3=0.9,α=0.4,M=5,t=0.5.

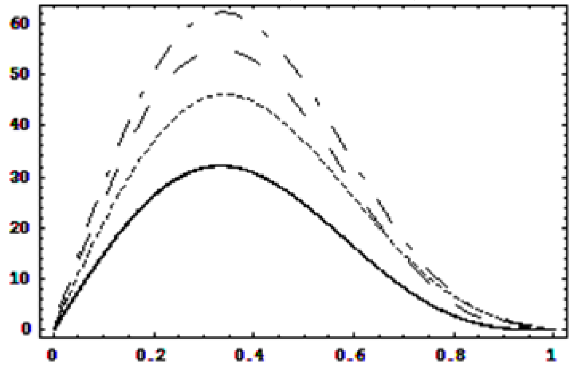

Fig 5 : Velocity profile for α =0.1,0.3,0.5,0.7 (Solid line, dash, dot-dash, dot) λ_1=0.9,λ_2=1,λ_3=0.8, β=0.8,M=1,t=0.5.

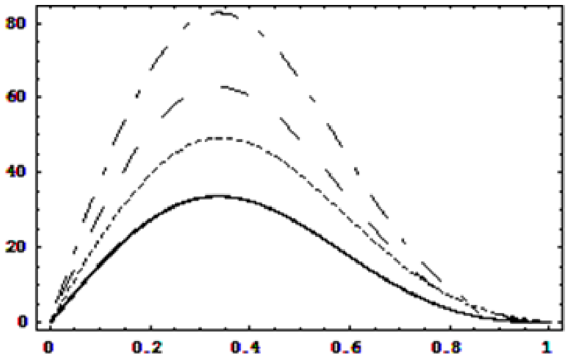

Fig 6 : Velocity profile for M =0.001,1,2,4 (Solid line, dash, dot-dash, dot) λ_1=0.9,λ_2=1,λ_3=0.8, β=0.4, α =0.4,t=0.5.

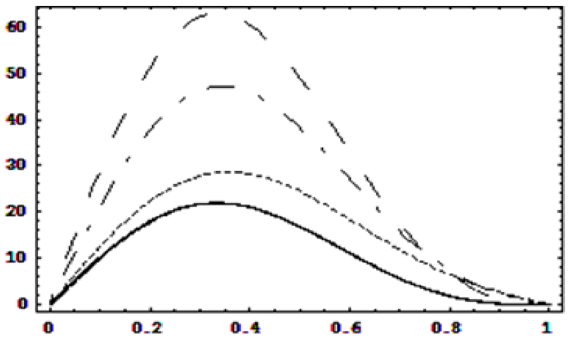

Fig 7: Velocity profile for t =0.3,0.5,1,1.2 (Solid line, dash, dot-dash, dot λ_1=0.9,λ_2=1,λ_3=0.8, β=0.5, α =0.4,M=1.

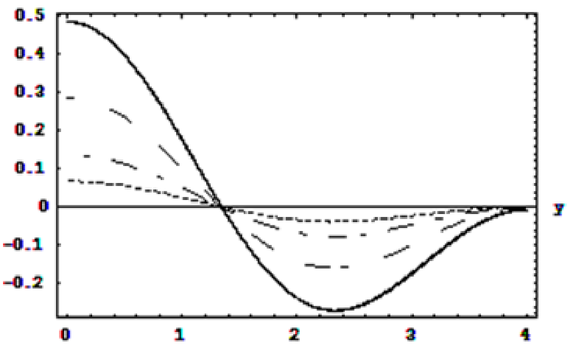

Fig 8 :Stress profile for λ_1=0.8, 1, 1.2,1.4 (Solid line, dash, dot-dash, dot) λ_2=0.1,λ_3=0.7,β=0.6,α=0.4,M=1,t=2.

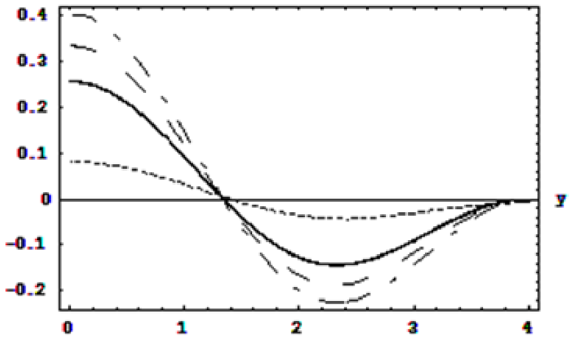

Fig 9 : Stress profile for λ_2=0.001,0.01,0.1,0.3 (Solid line, dash, dot-dash, dot) λ_1=0.9,λ_3=0.8,β=0.6,α=0.4,M=1,t=2.

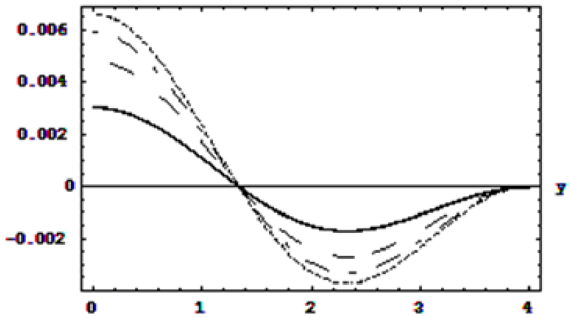

Fig 10 : Stress profile for λ_3=0.05, 1, 2,2.8 (Solid line, dash, dot-dash, dot) λ_1=3,λ_2=1,β=0.6,α=0.4,M=1,t=2.

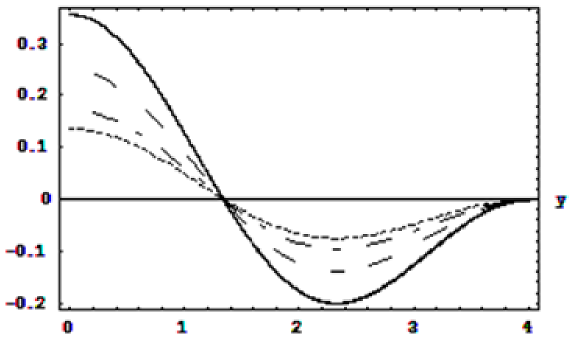

Fig 11: Stress profile for β =0.5,1,1.5,2 (Solid line, dash, dot-dash, dot) λ_1=0.9,λ_2=0.01,λ_3=0.5,α=0.4,M=1,t=2.

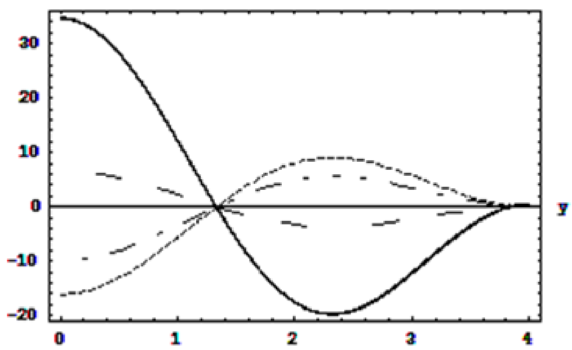

Fig 12 : Stress profile for α =0.01,0.1,0.5,0.7 (Solid line, dash, dot-dash, dot) λ_1=0.9,λ_2=1,λ_3=0.8, β=0.8,M=1.5,t=2.

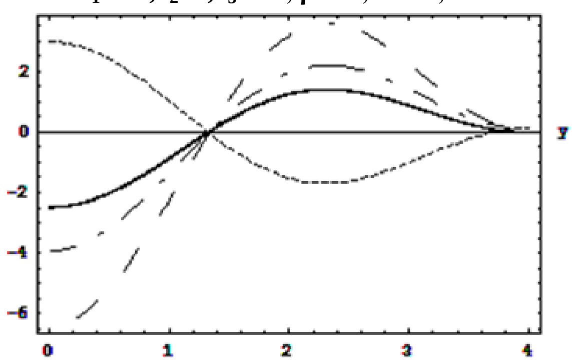

Fig 13: Stress profile for M =1,1.5,2.2,2.4 (Solid line, dash, dot-dash, dot) λ_1=0.9,λ_2=1,λ_3=0.8, β=0.6, α =0.4,t=2.

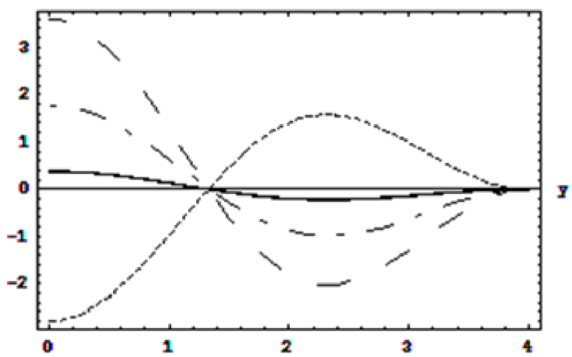

Fig 14 : Stress profile for t =0.01,0.5,1,1.5 (Solid line, dash, dot-dash, dot) , λ_1=0.9,λ_2=1,λ_3=0.8, β=0.6, α =0.4,M=1.5.

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