THE EFFECT OF ADDING COLD-STANDBY REDUNDANCY ON THE INCREASE OF SYSTEM RELIABILITY

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Abstract

In this research, we study the effect of adding cold-standby redundancy in increasing system reliability. Cold-standby redundancy is essential in the study of maintenance polices.

تاثير اضافة المجانبة الباردة على زيادة معولية النظام

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الخلاصة

في هذا البحث سوف ندرس تاثير اضافة وحدات اضافية(المجانبة الباردة) للنظام على زيادة معولية النظام . ا ن المجانبة الباردة ضرورية في دراسة الصيانة.

Introduction

System reliability can be increased by adding active redundancy or cold-standby redundancy. Active redundancy is used for electronic components but with mechanical designs it is difficult to implement. So, we often use cold-standby redundancy which is much easier to implement [1]. In maintenance polices they focus on the operation of adding lifetimes [2,3]. K.Shen and M.Xie [4] study the increase of system reliability through active redundancy. In this research work, we give bounds on the increase of system reliability through cold-standby redundancy.

Notation

$\bar{F}_i$ reliability of component i

$\bar{F}$ reliability of system

F_i life CDF of component i

F life CDF of system

Δ_i the increase of system reliability through cold-standby redundancy

$\bar{F}^{(i)}$ reliability of system after adding cold-standby redundancy to component i

$\bar{F}_i^{'}$ reliability of component i through cold-standby redundancy

p_s the probability of successful operation of the switching device.

Assumptions

The components of the system are independent.

1. The standby redundant component neither degrades nor fails while in standby state.
2. The switching device which we need to replace the failed component by the standby one may be perfect or imperfect.

Main Results

We consider a series(parallel) system of n components with lifetimes $T_1, T_2, \ldots\ldots, T_n$ and component reliabilities $\bar{F}_1, \bar{F}_2, \ldots\ldots, \bar{F}_n$, respectively; standby redundant component(s) is(are) added to the existing component in the position i of the system (the standby component may be identical or not to the ith component of the system).

In our work, we focus on the question "what upper and lower bounds may be given on the increase of system reliability through standby redundancy?" In the next results we answer this question; our bounds can be written in terms of Birnbaum importance measure of component i, $I_B^{(i)}$. The Birnbaum importance measure is one of the early measures [5] and it is time dependent.

The system reliability function $\overline{F}(t)$ is defined in [2] as :

$$\overline{F}(t) = \overline{F}_i(t)h(1_i, \overline{F}(t)) + (1-\overline{F}_i(t))h(O_i, \overline{F}(t))$$

$$I_B^{(i)}(t) = h(1_i, \overline{F}(t)) - h(O_i, \overline{F}(t))$$

It is obvious that

$$I_B^{(i)} = \frac{\partial \overline{F}}{\partial \overline{F}_i} = \frac{\partial F}{\partial F_i}$$

To insert the standby component in the place of the failed one, we need a mechanism called a switching device, in the next part we assume that the switching device is perfect (has reliability equals to one).

Results with perfect switching

If we consider a system of n components with lifetimes $T_1, T_2, \ldots\ldots, T_n$ and component reliabilities $\overline{F}_1, \overline{F}_2, \ldots\ldots, \overline{F}_n$, respectively. The increase of system reliability through standby redundancy of component i is:

$$\Delta_i = \overline{F}^{(i)} - \overline{F}$$

Where

$$\overline{F}^{(i)} = \overline{F}_i^{'} h(1_i, \overline{F}) + (1-\overline{F}_i^{'})h(O_i, \overline{F})$$

And

$$\overline{F}(t) = \overline{F}_i(t)h(1_i, \overline{F}(t)) + (1-\overline{F}_i(t))h(Oi, \overline{F}(t))$$

So,

$$\Delta_i = (\overline{F}_i^{'} - \overline{F}_i).I_B^{(i)} \quad \textbf{............ (1)}$$

In the next results, we assume that the i th component of the system may be enhanced with one spare component.

<u>Result (1)</u>

Consider a series system of n components with lifetimes $T_1, T_2, \ldots\ldots, T_n$ and component reliabilities $\overline{F}_1, \overline{F}_2, \ldots\ldots, \overline{F}_n$, respectively. The increase of system reliability through standby redundancy of component i is bounded as below:

$$\overline{F}F_i \le \Delta_i \le I_B^{(i)} F_i \quad \textbf{............... (2)}$$

Proof:

The reliability of the series system is given by:

$$\overline{F}(t) = \overline{F}_1(t).\overline{F}_2(t).......\overline{F}_i(t).......\overline{F}_n(t)$$

And the system reliability after adding standby redundancy at component i is:

$$\overline{F}^{(i)}(t) = \overline{F}_1(t).\overline{F}_2(t).......\overline{F}_i^{'}(t).......\overline{F}_n(t)$$

Where

$$\overline{F}_i^{'} = 1 - P(T_i + T_i \le t)$$

$$= 1 - \int_0^t F_i(t-x)dF_i(x)$$

$$= \overline{F}_i(t) + \int_0^t \overline{F}_i(t-x) f_i(x)dx$$

But

$$\overline{F}_i(t) \le \overline{F}_i(t-x)$$

So,

$$\overline{F}_i(t) + \overline{F}_i(t).F_i(t) \le \overline{F}_i(t) + \int_0^t \overline{F}_i(t-x).f_i(x)dx$$

….. (1)

And

$$\overline{F}_i(t-x) f_i(x) \le f_i(x)$$

Then

$$\overline{F}_i(t) + \int_0^t \overline{F}_i(t-x).f_i(x)dx \le \overline{F}_i(t) + F_i(t)$$

………………… (2)

Now,

$$\Delta_i = (\overline{F}_i^{'} - \overline{F}_i).I_B^{(i)}$$

From (1) and (2) we can give the following bounds on Δ_i

$$\overline{F}.F_i \le \Delta_i \le I_B^{(i)}.F_i$$

(where $I_B^{(i)} = \frac{\partial \overline{F}}{\partial \overline{F}_i} = \prod_{\substack{k=1 \\ k \ne i}}^{n} \overline{F}_k$ **).**

<u>Result (2)</u>

Consider a parallel system of n components with lifetimes $T_1, T_2, \ldots\ldots, T_n$ and component reliabilities $\overline{F}_1, \overline{F}_2, \ldots\ldots, \overline{F}_n$, respectively. The increase of system reliability through standby redundancy of component i is bounded as below:

$$F\overline{F}_i \le \Delta_i \le I_B^{(i)} F_i \qquad \text{……………….. (3)}$$

Proof:

The reliability of the parallel system is given by:

$$\overline{F}(t) = 1 - F_1(t).F_2(t)......F_i(t).........F_n(t)$$

And the system reliability after adding standby redundancy at component i is:

$$\overline{F}^{(i)}(t) = 1 - F_1(t).F_2(t)......F_i^{'}(t).........F_n(t)$$

$$\Delta_i = (\overline{F}_i^{'} - \overline{F}_i).I_B^{(i)}$$

But,

$$\overline{F}_i(t) + \overline{F}_i(t).F_i(t) \le \overline{F}_i^{'} \le \overline{F}_i(t) + F_i(t)$$

So,

$$F\overline{F}_i \le \Delta_i \le I_B^{(i)} F_i$$

(where $I_B^{(i)} = \dfrac{\partial F}{\partial F_i} = \Pi_{\substack{k=1\\k\ne i}}^{n} F_k$).

Now, assume that the i th component of the system may be immediately replaced upon failure and all the n components of the system are exponentially distributed with parameters $\alpha_1, \alpha_2, ..., \alpha_n$, respectively.

As known [6], if a component having a constant failure rate α is immediately replaced upon failing, the number of failures observed over a time period t has a poisson distribution. The probability of observing m failures in time t is given by the probability mass function $p_m(t)$:

$$p_m(t) = \frac{e^{-\alpha t}(\alpha t)^m}{m!} \qquad \text{m=0,1,2,…….}$$

If s spare components are available to support a continuous operation over a time period t, then $\sum_{m=0}^{s} p_m(t)$ is the cumulative probability of s or fewer failures occurring in time t, also it represents the probability of satisfying all demands for spare components during time t, therefore $\sum_{m=0}^{s} p_m(t)$ is the component reliability if there are s spares available for immediate replacement when a failure occurs.

So, the reliability of the i th component with s standby redundant components to replace it can be defined as:

$$\overline{F}_i^{'}(t) = \sum_{m=0}^{s} \frac{e^{-\alpha_i t}(\alpha_i t)^m}{m!}$$

<u>Result (3)</u>

Consider a series system of n components with component lifetimes $T_1, T_2, ..., T_n$ are exponentially distributed with parameters $\alpha_1, \alpha_2, ..., \alpha_n$, respectively. The increase of system reliability through standby redundancy of component i is bounded as below (assume that s spares are available at the i th position):

$$\sum_{m=0}^{s-1} \frac{(\alpha_i t)^m}{(m+1)!}.e^{-\sum_{k=1}^{n}\alpha_k t}.(1 - e^{\alpha_i t}) \le \Delta_i \le$$
$$\sum_{m=0}^{s-1} \frac{(\alpha_i t)^m}{(m+1)!}.I_B^{(i)}.(1 - e^{-\alpha_i t})$$

….. (4)

Proof:

The reliability of the series system is given by:

$$\overline{F}(t) = e^{-\sum_{k=1}^{n}\alpha_k t}$$

And the system reliability after adding standby redundancy at component i is:

$$\overline{F}^{(i)}(t) = e^{-\sum_{\substack{k=1\\k\ne i}}^{n}\alpha_k t}.\sum_{m=0}^{s} \frac{e^{-\alpha_i t}(\alpha_i t)^m}{m!}$$

$$\Delta_i = (\overline{F}_i^{'} - \overline{F}_i).I_B^{(i)} = \sum_{m=1}^{s} \frac{(\alpha_i t)^m}{m!}.e^{-\alpha_i t}.I_B^{(i)}$$

If we take m=1 (the case of one spare component available), the increase in reliability will be:

$$\Delta_i = \alpha_i.t.e^{-\alpha_i t}.I_B^{(i)}$$

We use inequality (2) to give bounds on Δ_i as:

$$(1 - e^{-\alpha_i t}).e^{-\sum_{k=1}^{n}\alpha_k t} \le \Delta_i \le (1 - e^{-\alpha_i t}).I_B^{(i)}$$

Now let us consider that s spares available at the i th position

$$\Delta_i = \sum_{m=1}^{s} \frac{(\alpha_i t)^m}{m!}.e^{-\alpha_i t}.I_B^{(i)}$$
$$= \sum_{m=0}^{s-1} \frac{(\alpha_i t)^m}{(m+1)!}.\alpha_i t.e^{-\alpha_i t}.I_B^{(i)}$$

Δ_i is bounded as:

$$\sum_{m=0}^{s-1}\frac{(\alpha_i t)^m}{(m+1)!}.e^{-\sum_{k=1}^{n}\alpha_k t}.(1-e^{\alpha_i t}) \le \Delta_i \le \sum_{m=0}^{s-1}\frac{(\alpha_i t)^m}{(m+1)!}.I_B^{(i)}.(1-e^{-\alpha_i t})$$

(where $I_B^{(i)} = e^{-\sum_{k=1, k\neq i}^{n}\alpha_k t}$)

Result (4)

Consider a parallel system of n components with component lifetimes $T_1, T_2, ..., T_n$ are exponentially distributed with parameters $\alpha_1, \alpha_2, ..., \alpha_n$, respectively. The increase of system reliability through standby redundancy of component i is bounded as below (assume that s spares are available at the i th position) :

$$\sum_{m=0}^{s-1}\frac{(\alpha_i t)^m}{(m+1)!}.\Pi_{k=1}^{n}(1-e^{-\alpha_k t}).e^{-\alpha_i t} \le \Delta_i \le \sum_{m=0}^{s-1}\frac{(\alpha_i t)^m}{(m+1)!}.I_B^{(i)}.(1-e^{-\alpha_i t})$$

...............................(5)

Proof:

The reliability of the parallel system is given by:

$$\bar{F}(t) = 1 - \Pi_{k=1}^{n}(1-e^{-\alpha_k t})$$

And the system reliability after adding standby redundancy at component i is:

$$\bar{F}^{(i)}(t) = 1 - \Pi_{k=1, k\neq i}^{n}(1-e^{-\alpha_k t}).(1-\sum_{m=0}^{s}\frac{e^{-\alpha_i t}(\alpha_i t)^m}{m!})$$

$$\Delta_i = (\bar{F}_i^{'} - \bar{F}_i).I_B^{(i)} = \sum_{m=1}^{s}\frac{(\alpha_i t)^m}{m!}.e^{-\alpha_i t}.I_B^{(i)}$$

If we take m=1 (the case of one spare component available), the increase in reliability will be:

$$\Delta_i = \alpha_i.t.e^{-\alpha_i t}.I_B^{(i)}$$

We use inequality (3) to give bounds on Δ_i as

$$\Pi_{k=1}^{n}(1-e^{-\alpha_k t}).e^{-\alpha_i t} \le \Delta_i \le I_B^{(i)}.(1-e^{-\alpha_i t})$$

Now let us consider that s spares available at the i th position

$$\Delta_i = \sum_{m=0}^{s-1}\frac{(\alpha_i t)^m}{(m+1)!}.\alpha_i t.e^{-\alpha_i t}.I_B^{(i)}$$

Δ_i is bounded as:

$$\sum_{m=0}^{s-1}\frac{(\alpha_i t)^m}{(m+1)!}.\Pi_{k=1}^{n}(1-e^{-\alpha_k t}).e^{-\alpha_i t} \le \Delta_i \le \sum_{m=0}^{s-1}\frac{(\alpha_i t)^m}{(m+1)!}.I_B^{(i)}.(1-e^{-\alpha_i t})$$

(where $I_B^{(i)} = \frac{\partial \bar{F}}{\partial \bar{F}_i} = \prod_{k=1, k\neq i}^{n}(1-e^{-\alpha_k t})$)

In all of the previous results, we assume that iid component(s) is (are) added to the component in the i th position of the system.

In the next results, we assume that different component may be added through

(where $I_B^{(i)} = \frac{\partial \bar{F}}{\partial \bar{F}_i} = \prod_{k=1, k\neq i}^{n}(1-e^{-\alpha_k t})$)

In all of the previous results, we assume that iid component(s) is (are) added to the component in the i th position of the system.

In the next results, we assume that different component may be added through standby redundancy at ˋıe i th position of the system.

Result (5)

Consider a series system of n components with lifetimes $T_1, T_2, ..., T_n$ and component reliabilities $\bar{F}_1, \bar{F}_2, ..., \bar{F}_n$, respectively. The increase of system reliability through standby redundancy of component i is bounded as below:

$$I_B^{(i)}.\bar{F}_i^{*}F_i \le \Delta_i \le I_B^{(i)}.F_i \qquad \text{............(6)}$$

(where $\bar{F}_i^{*}$ is the reliability of the standby component)

Proof:

The reliability of the series system is given by:

$$\bar{F}(t) = \bar{F}_1(t).\bar{F}_2(t).......\bar{F}_i(t).......\bar{F}_n(t)$$

And the system reliability after adding standby redundancy at component i is:

$$\bar{F}^{(i)}(t) = \bar{F}_1(t).\bar{F}_2(t).......\bar{F}_i^{'}(t).......\bar{F}_n(t)$$

Where

$$\overline{F}_i^{'} = 1 - P(T_i + T_i^{*} \le t)$$
$$= 1 - \int_0^t F_i^{*}(t-x)\, dF_i(x)$$
$$= \overline{F}_i(t) + \int_0^t \overline{F}_i^{*}(t-x) f_i(x) dx$$

But
$\overline{F}_i^{*}(t) \le \overline{F}_i^{*}(t-x)$
So,
$$\overline{F}_i(t) + \overline{F}_i^{*}(t).F_i(t) \le \overline{F}_i(t) + \int_0^t \overline{F}_i^{*}(t-x) f_i(x) dx$$
…………… (1)

And
$\overline{F}_i^{*}(t-x) f_i(x) \le f_i(x)$
Then
$$\overline{F}_i(t) + \int_0^t \overline{F}_i^{*}(t-x).f_i(x)dx \le \overline{F}_i(t) + F_i(t)$$
……………… … (2)

Now,
$\Delta_i = (\overline{F}_i^{'} - \overline{F}_i).I_B^{(i)}$

From (1) and (2) we can give the following bounds on Δ_i
$I_B^{(i)}.\overline{F}_i^{*}.F_i \le \Delta_i \le I_B^{(i)}.F_i$

(where $I_B^{(i)} = \dfrac{\partial \overline{F}}{\partial \overline{F}_i} = \prod_{k=1, k\ne i}^{n} \overline{F}_k$ **).**

Result (6)
Consider a parallel system of n components with lifetimes $T_1, T_2, \ldots, T_n$ and component reliabilities $\overline{F}_1, \overline{F}_2, \ldots, \overline{F}_n$, respectively. The increase of system reliability through standby redundancy of component i is bounded as below:

$$F\overline{F}_i^{*} \le \Delta_i \le I_B^{(i)} F_i \quad \text{………. (7)}$$

(where $I_B^{(i)} = \dfrac{\partial F}{\partial F_i} = \prod_{k=1, k\ne i}^{n} F_k$).

Proof:
The reliability of parallel system is given by:
$\overline{F}(t) = 1 - F_1(t).F_2(t)......F_i(t).........F_n(t)$

And the system reliability after adding standby redundancy at component i is:

$\overline{F}^{(i)}(t) = 1 - F_1(t).F_2(t)......F_i^{'}(t).........F_n(t)$
$\Delta_i = (\overline{F}_i^{'} - \overline{F}_i).I_B^{(i)}$

But,
$\overline{F}_i(t) + \overline{F}_i^{*}(t).F_i(t) \le \overline{F}_i^{'} \le \overline{F}_i(t) + F_i(t)$
So,
$F\overline{F}_i^{*} \le \Delta_i \le I_B^{(i)} F_i$
(where $I_B^{(i)} = \dfrac{\partial F}{\partial F_i} = \prod_{k=1, k\ne i}^{n} F_k$).

Results with imperfect switching
Now, we consider that the switching device we use to insert the standby component in the place of the failed one is imperfect (has reliability less than one). We try the previous results in the case of imperfect switching device. We assume that an iid standby component is added to the i th component of the system. To calculate $\overline{F}_i^{'}$, let A_1 denotes the event that the switching device fails and A_2 denotes the event that the switching device operates successfully and let P_s be the probability of successful operation of the switching device. Denote by Q_1 the unreliability of the original component with successfully operating switching and by Q_2 the unreliability of the two components standby system with successfully operating switching. Then from the formula for total probabilities we have:

$$F_i^{'}(t) = p(A_1).Q_1(t) + p(A_2).Q_2(t)$$
$$= (1-p_s).(1-\overline{F}_i(t)) + P_s.(1 - \overline{F}_i(t) - \int_0^t \overline{F}_i(t-x) f_i(x) dx)$$
$$\overline{F}_i^{'}(t) = 1 - F_i^{'}(t)$$
$$= \overline{F}_i(t) + P_s.\int_0^t \overline{F}_i(t-x) f_i(x) dx \quad \text{(8)}$$

Result (7)
Consider a series system of n components with lifetimes $T_1, T_2, \ldots, T_n$ and component reliabilities $\overline{F}\ \overline{F}_2, \ldots, \overline{F}_n$, respectively. The increase of system reliability through standby redundancy of component i is bounded as below:

$$\overline{F}.F_i.P_s \le \Delta_i \le I_B^{(i)}.F_i.P_s \quad \text{………(9)}$$

Result (8)

Consider a parallel system of n components with lifetimes $T_1, T_2, \ldots, T_n$ and component reliabilities $\bar{F}_1, \bar{F}_2, \ldots, \bar{F}_n$, respectively. The increase of system reliability through standby redundancy of component i is bounded as below:

$$F.\bar{F}_i.P_s \le \Delta_i \le I_B^{(i)}.F_i.P_s \;\ldots\ldots\ldots\ldots \textbf{(10)}$$

If the standby component, with reliability $\bar{F}_i^*$, is not similar to the existing one, then:

$$\bar{F}_i'(t) = \bar{F}_i(t) + P_s.\int_0^t \bar{F}_i^*(t-x) f_i(x)dx \;\ldots \textbf{(11)}$$

Result (9)

Consider a series system of n components with lifetimes $T_1, T_2, \ldots, T_n$ and component reliabilities $\bar{F}_1, \bar{F}_2, \ldots, \bar{F}_n$, respectively. The increase of system reliability through standby redundancy of component i is bounded as below:

$$I_B^{(i)}.\bar{F}_i^*.F_i.P_s \le \Delta_i \le I_B^{(i)}.F_i.P_s \;\ldots\ldots\ldots\ldots \textbf{(12)}$$

Result (10)

Consider a parallel system of n components with lifetimes $T_1, T_2, \ldots, T_n$ and component reliabilities $\bar{F}_1, \bar{F}_2, \ldots, \bar{F}_n$, respectively. The increase of system reliability through standby redundancy of component i is bounded as below:

$$F.\bar{F}_i^*.P_s \le \Delta_i \le I_B^{(i)}.F_i.P_s \;\ldots\ldots\ldots\ldots \textbf{(13)}$$

Examples

Example (1)

Consider a parallel-series system of three components (as in fig. (1) below). The component lifetimes are exponentially distributed with parameters 0.001, 0.002, 0.003, respectively. We try to add a similar standby component or a different one, which is exponentially distributed with parameter $\alpha = 0.005$, to each component in the system (let t= 10 yr time).

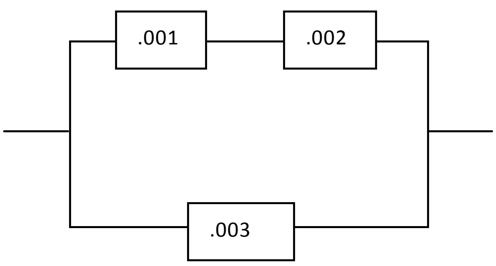

Figure 1:Parallel- series system

$$\Delta_i = (\bar{F}_i' - \bar{F}_i).I_B^{(i)}$$

Δ_1 =(9.9×10^{-3}).(0.02897)

$= 2.86803\times10^{-4}$

Δ_2 =(0.0196). (0.0293)

$= 5.7428\times10^{-4}$

Δ_3 = (0.03) (0.03)

$= 9\times10^{-4}$

So, the highest increase of system reliability will be when we duplicate component 3. System reliability increased from 0.99913 to 0.99999.

Now, we discuss the case when we add a standby component which is exponentially distributed with parameter .005 .

Δ_1 =(9.7×10^{-3}).(0.02897)

$= 2.81\times10^{-4}$

Δ_2 = (0.0192). (0.0293)

$= 5.626\times10^{-4}$

Δ_3 = (0.029). (0.03)

$= 8.7\times10^{-4}$

So, the highest increase of system reliability will be when we add this standby component at the third position of our system. System reliability increased from 0.99913 to 0.99998.

If we consider the case of imperfect switching device with probability P_s=0.5 of successful operation

Δ_3 = (0.03) (0.03). (0.5)

$= 4.5\times10^{-4}$ (similar standby component)

0.99913 $\longrightarrow$ 0.99958

Δ_3 = (0.029) (0.03). (0.5)

$= 4.35\times10^{-4}$ (different standby component)

0.99913 $\longrightarrow$ 0.99955

Example (2)

Consider a series system of four components. The component lifetimes are distributed according to weibull distribution. We try to add a similar standby component or a different one, which is distributed according to weibull distribution with scale parameter $\theta = 2000$, and shape parameter $\beta = 2.1$ to each component in the system (let t= 10 yr time).

Components	scale parameter	shape parameter	reli. Per 10 yrs
Comp.1	7000	1.7	0.9999854343
Comp.2	510	1.8	0.9991562954
Comp.3	12000	2	0.9999993056
Comp.4	720	1	0.9862071167

$\overline{F}$ =0.9853600123

$\overline{F}F_i \le \Delta_i \le I_B^{(i)}F_i$ (add similar standby component)

$(0.98536001230).(1.45657\times10^{-5}) \le \Delta_1 \le (0.985374356).(1.45657\times10^{-5})$

The resulted reliability is $\overline{F}^{(1)}$ is bounded as:

$0.9853743648 \le \overline{F}^{(1)} \le 0.985374365 \longrightarrow$

$\overline{F}^{(1)} \in [0.9853743648, 0.985374365]$

$(0.9853600123).(8.437046\times10^{-4}) \le \Delta_2 \le (0.9861920671).(8.437046\times10^{-4})$

$0.9861913651 \le \overline{F}^{(2)} \le 0.9861920671 \longrightarrow$

$\overline{F}^{(2)} \in [0.9861913651, 0.9861920671]$

$(0.9853600123).(6.944\times10^{-7}) \le \Delta_3 \le (0.9853606965).(6.944\times10^{-7})$

$\overline{F}^{(3)} = 0.9853606965$

$(0.9853600123).(0.0137928833) \le \Delta_4 \le (0.9991410482).(0.0137928833)$

$0.998950968 \le \overline{F}^{(4)} \le 0.9991410482$

$\overline{F}^{(4)} \in [0.998950968, 0.9991410482]$

As seen, the highest increase of system reliability will be when we duplicate component 4.

Now, we add a standby component which is distributed according to weibull distribution with scale parameter $\theta = 2000$, and shape parameter $\beta = 2.1$ to each component in the system .

$I_B^{(i)}.\overline{F}_i^{*}F_i \le \Delta_i \le I_B^{(i)}.F_i$ (add different component in the system .

$I_B^{(i)}.\overline{F}_i^{*}F_i \le \Delta_i \le I_B^{(i)}.F_i$ (add different standby component)

$\overline{F}_i^{*} = 0.9999852825 \quad ,\forall i$

$I_B^{(1)}.\overline{F}_1^{*}F_1 \le \Delta_1 \le I_B^{(1)}.F_1$

$(0.985374365).(0.9999852825).(1.45657\times10^{-5} \le \Delta_1 \le (0.985374365).(1.45657\times10^{-5})$

The resulted reliability is bounded as:

$0.9853743648 \le \overline{F}^{(1)} \le 0.985374365 \longrightarrow$

$\overline{F}^{(1)} \in [0.9853743648, 0.985374365]$

$(0.9861920671).(0.9999852825).(8.437046\times10^{-4}) \le \Delta_2 \le (0.9861920671).(8.437046\times10^{-4})$

$0.9861920548 \le \overline{F}^{(2)} \le 0.9861920671 \longrightarrow$

$\overline{F}^{(2)} \in [0.9861920548, 0.9861920671]$

$(0.9853606965).(0.9999852825).(6.944\times10^{-7}) \le \Delta_3 \le (0.9853606965).(6.944\times10^{-7}) \longrightarrow$

$\overline{F}^{(3)} = 0.9853606965$

$(0.9991410482).(0.9999852825).(0.0137928833) \le \Delta_4 \le (0.9991410482).(0.0137928833)$

$0.9991408454 \le \overline{F}^{(4)} \le 0.9991410482 \longrightarrow$

$\overline{F}^{(4)} \in [0.9991408454, 0.9991410482]$

The highest increase of system reliability will be at component 4.

If we consider that the switching device is imperfect with $P_s = 0.5$. Then:

$\overline{F}.F_i.P_s \le \Delta_i \le I_B^{(i)}.F_i.P_s$ (similar component)

$(0.9853600123).(0.0137928833).(0.5)$

$\le \Delta_4 \le (0.9991410482).(0.0137928833).(0.5)$

$0.9921554901 \le \overline{F}^{(4)} \le 0.9922505302 \longrightarrow$

$\overline{F}^{(4)} \in [0.9921554901, 0.9922505302]$

$I_B^{(i)}.\overline{F}_i^*.F_i.P_s \le \Delta_i \le I_B^{(i)}.F_i.P_s$ (different component)

$(0.9991410482).(0.9999852825).(0.5).$

$(0.0137928833) \le \Delta_4 \le$

$(0.9991410482).(0.0137928833).(0.5)$

$0.9922504288 \le \overline{F}^{(4)} \le 0.9922505302 \longrightarrow$

$\overline{F}^{(4)} \in [0.9922504288, 0.9922505302]$

Example (3)

Consider a parallel system of three components. The component lifetimes are distributed according to weibull distribution. We try to add a similar standby component or a different one, which is distributed according to weibull distribution with scale parameter $\theta = 2000$, and shape parameter $\beta = 2.1$ to each component in the system (let t= 10 yr time).

Components	scale parameter	shape parameter	reli. Per 10 yrs
Comp.1	100	1.2	0.9388535888
Comp.2	150	0.87	0.9095563625
Comp.3	510	1.8	0.9991562954

$\overline{F} = 0.9999953341$

$F\overline{F}_i \le \Delta_i \le I_B^{(i)} F_i$

$(4.6659 \times 10^{-6}).(0.9388535888) \le \Delta_1 \le$

$(0.0611464112).(7.6307713 \times 10^{-5})$

$0.9999997147 \le \overline{F}^{(1)} \le 1 \longrightarrow$

$\overline{F}^{(1)} \in [0.9999997147, 1]$

$(4.6659 \times 10^{-6}).(0.9095563625) \le \Delta_2 \le$

$(0.0904436375).(5.15895084 \times 10^{-5})$

$0.999999578 \le \overline{F}^{(2)} \le 1 \longrightarrow$

$\overline{F}^{(2)} \in [0.999999578, 1]$

$(4.6659 \times 10^{-6}).(0.9991562954) \le \Delta_3 \le$

$(8.437046 \times 10^{-4}).(5.530303849 \times 10^{-3})$

$0.9999999961 \le \overline{F}^{(3)} \le 1 \longrightarrow$

$\overline{F}^{(3)} \in [0.9999999961, 1]$

The highest increase in reliability will be at component 3.

Now, we add a standby component which is distributed according to weibull distribution with scale parameter $\theta = 2000$, and shape parameter $\beta = 2.1$ to each component in the system .

$F\overline{F}_i^* \le \Delta_i \le I_B^{(i)} F_i$

$\overline{F}_i^* = 0.9999852825 \quad , \forall i$

$(4.6659 \times 10^{-6}).(0.9999852825) \le \Delta_1 \le$

$(0.0611464112).(7.6307713 \times 10^{-5})$

$0.9999999999 \le \overline{F}^{(1)} \le 1 \longrightarrow$

$\overline{F}^{(1)} \in [0.9999999999, 1]$

$(4.6659 \times 10^{-6}).(0.9999852825) \le \Delta_2 \le$

$(0.0904436375).(5.15895084 \times 10^{-5})$

$0.9999999999 \le \overline{F}^{(2)} \le 1$

$\overline{F}^{(2)} \in [0.9999999999, 1]$

$(4.6659 \times 10^{-6}).(0.99998528254) \le \Delta_3 \le$

$(8.437046 \times 10^{-4}).(5.530303849 \times 10^{-3})$

$0.9999999999 \le \overline{F}^{(3)} \le 1 \longrightarrow$

$\overline{F}^{(3)} \in [0.9999999999, 1]$

If we consider that the switching device is imperfect with $P_s = 0.5$. Then:

$F.\overline{F}_i.P_s \le \Delta_i \le I_B^{(i)}.F_i.P_s$ (similar component)

$(4.6659\times10^{-6}).(0.9991562954).(0.5)\le\Delta_3\le$

$(5.530303849\times10^{-3}).(8.437046\times10^{-4}).(0.5)$

$0.9999976651\le\bar{F}^{(3)}\le 0.9999976671$

$\bar{F}^{(3)}\in[0.9999976651,0.9999976671]$

$F.\bar{F}_i^{*}.P_s\le\Delta_i\le I_B^{(i)}.F_i.P_s$ (different component)

$(4.6659\times10^{-6}).(0.9999852825).(0.5)\le\Delta_1\le$

$(0.0611464112).(7.6307713\times10^{-5}).(0.5)$

$0.999997667\le\bar{F}^{(1)}\le 0.9999976671$

$\bar{F}^{(1)}\in[0.999997667,0.9999976671]$ ⟶

$(4.6659\times10^{-6}).(0.9999852825).(0.5)\le\Delta_2\le$

$(0.0904436375).(5.15895084\times10^{-5}).(0.5)$

$0.999997667\le\bar{F}^{(2)}\le 0.9999976671$

$\bar{F}^{(2)}\in[0.9999997667,0.9999976671]$ ⟶

$(4.6659\times10^{-6}).(0.9999852825).(0.5)\le\Delta_3\le$

$(5.530303849\times10^{-3}).(8.437046\times10^{-4}).(0.5)$ ⟶

$0.999997667\le\bar{F}^{(3)}\le 0.999997667$

$\bar{F}^{(3)}\in[0.999997667,0.9999976671]$

Example (4)

Consider a series system of three components. The component lifetimes are distributed according to exponential distribution with parameters 0.001, 0.005, 0.002, respectively. We try to add similar s standby components to each component in the system (let t= 10 yr time).

$\bar{F}=0.923116346$

$$\Delta_i=\sum_{m=1}^{s}\frac{(\alpha_i t)^m}{m!}.e^{-\alpha_i t}.I_B^{(i)}$$

$$\Delta_1=\sum_{m=1}^{s}\frac{(\alpha_1 t)^m}{m!}.e^{-\alpha_1 t}.I_B^{(1)}$$

$$\Delta_1=\sum_{m=1}^{3}\frac{(0.01)^m}{m!}.e^{-0.01}.e^{-0.07}$$ (three standby components available)

$\bar{F}^{(1)}=0.9323938191$

$$\Delta_2=\sum_{m=1}^{s}\frac{(\alpha_2 t)^m}{m!}.e^{-\alpha_2 t}.I_B^{(2)}$$

$$\Delta_2=\sum_{m=1}^{3}\frac{(0.05)^m}{m!}.e^{-0.05}.e^{-0.03}$$

$\Delta_2=0.04732894434$

$\bar{F}^{(2)}=0.9704452903$

$$\Delta_3=\sum_{m=1}^{s}\frac{(\alpha_3 t)^m}{m!}.e^{-\alpha_3 t}.I_B^{(3)}$$

$$\Delta_3=\sum_{m=1}^{3}\frac{(0.05)^m}{m!}.e^{-0.02}.e^{-0.06}$$

$\Delta_3=0.01864818102$

$\bar{F}^{(3)}=0.9417645267$

The highest increase will be if we add three iid components to the second component of the system.

Example (5)

Consider a parallel system of three components. The component lifetimes are distributed according to exponential distribution with parameters 1, 0.05, 2, respectively. We try to add similar s standby components to each component in the system (let t= 10 yr time).

$\bar{F}=0.60654852$

$$\Delta_i=\sum_{m=1}^{s}\frac{(\alpha_i t)^m}{m!}.e^{-\alpha_i t}.I_B^{(i)}$$

$$\Delta_1=\sum_{m=1}^{3}\frac{(10)^m}{m!}.e^{-10}.I_B^{(1)}$$

$\Delta_1=4.049055547\times10^{-3}$

$\bar{F}^{(1)}=0.610597581$

$$\Delta_2=\sum_{m=1}^{3}\frac{(0.5)^m}{m!}.e^{-0.5}.I_B^{(2)}$$

$\Delta_2=0.391699932$

$\bar{F}^{(2)}=0.998248452$

$$\Delta_3=\sum_{m=1}^{3}\frac{(20)^m}{m!}.e^{-20}.I_B^{(3)}$$

$\Delta_3=1.259697315\times10^{-6}$

$\bar{F}^{(3)}=0.606549779$

So, the highest increase will be when we add three standby components to the second component in the system.

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