 Iraqi Journal of Science,Vol 53,No 1,2012, Pp. 125-129

TRIANGULAR MEMBERSHIP FUNCTIONS FOR SOLVING SINGLE AND MULTIOBJECTIVE FUZZY LINEAR PROGRAMMING PROBLEM.

Iraq Tariq Abbas
Departement of Mathematic, College of Science ,University of Baghdad.Baghdad-Iraq

Abstract

In this paper, fuzzy single and multiobjective linear programming models are presented. Both the objective function and the constraints are considered fuzzyly.The coefficient of the decision variable in the objective functions and the constraints, as well as the right-hand side of the constraints are assumed to be fuzzy numbers with triangular membership functions. The possibility programming approach is utilized to transform the fuzzy model into its crisp equivalent form, and then a suitable method will be used to solve the crisp problem.
Keyword: Fuzzy multiobjective linear programming, possibility programming

دالة الانتماء المثلثية لحل مسائل البرمجة الخطية المفردة والمتعددة الاهداف الضبابية

عراق طارق عباس
قسم الرياضيات، كلية العلوم ، جامعة بغداد.بغداد –العراق

الخلاصة

في هذا البحث،قدمنا نماذج برمجة خطية مفردة و متعددة الاهداف الضبابية.حيث ان كل من دالة الهدف والقيود عبارة عن دوال ضبابية.ان معاملات القرار في دالة الهدف والشروط،بالاضافة الى الطرف الايمن من القيود تم افتراضها اعداد ضبابية وبدالة انتماء مثلثية .ان خوارزمية برمجة الامكان قد اعتمدت في تحويل النموذج الضبابي الى الغير ضبابي المرادف له ،ومن ثم استخدمت طريقة عددية مناسبة في البرمجة الخطية لحل المسائل المناظرة.

1-Introduction

Linear programming has its applications in many fields of operation research which is concerned with the optimizations (minimization and/or maximization) of a linear function, while satisfying a set of linear equality and /or inequality constraints or restrictions.

In real situation, the available information in the system under consideration are not exact, therefore fuzzy linear programming was introduced and studied by many authors and were regularly treated by others .Zimmermann [1]proposed the first formulation of fuzzy liner programming. Fang and Hu[2]considered linear programming with fuzzy constraint coefficients.Vasant et.al[3] applied linear programming with fuzzy parameters for decision making in industrial production planning.Maleki and et al [4, 5]introduced a line ar programming problem with fuzzy variables and proposed a new method for solving these problems using anauxil iaryproblem.Mahdavi-Amiri and Nasseri [6] described duality theory for the fuzzy variable [LP] problem. In this paper, the possibility programming approach is utilized to transform the fuzzy single- objective and multiobjectve linear programming models to its crisp equivalent problem, according to its Iskander's modification [7,8].Then the crisp single and multiobjective linear programming model are solved using any linear programming method.

2- Definitions and Notations in fuzzy set theory

In this section, some of the fundamental definitions and basic concepts of fuzzy sets theory are given for completeness purpose:

Definition (1)[9].

A fuzzy set $\tilde{A}$ in X is a set of order pairs:

$\tilde{A} = \{(x,\ \mu_{\tilde{A}}(x)/x\in X\}$

$\mu_{\tilde{A}}(x)$ is called the membership function of x in $\tilde{A}$ which maps X into[0,1]. If $\sup_x\ \mu_{\tilde{A}}(x)=1$ the fuzzy set $\tilde{A}$ is called normal.

Definition (2)[9].

The support of a fuzzy set $\tilde{A}$ on X is the crisp set of all $x\in X$, such that $\mu_{\tilde{A}}(x)>0$.

Definition (3)[9].

The set of elements that belong to the fuzzy set $\tilde{A}$ on X at least to the degree α is called the α-cut set:

$\tilde{A}_\alpha = \{(x,\ \mu_{\tilde{A}}(x)\geq\alpha,\alpha\in[0,1]\}$.

Definition (4)[9].

A fuzzy set $\tilde{A}$ on X is said to be convex if

$\mu_{\tilde{A}}\ (\lambda x+\ (1-\ \lambda)\ y)\geq\min\ \{\mu_{\tilde{A}}(x),\mu_{\tilde{A}}(y)\}$, x, $y\in R$, and $\lambda\in[0,\ 1]$. Note that, a fuzzy set is convex if all its α-cuts are convex.

Definition (5) [9].

A fuzzy number $\tilde{a}$ is a convex normalized fuzzy set on the real line R, such that:

1) There exist at least one $x_0\in R$ with $\mu_{\tilde{a}}(x_0)=1$.

2) $\mu_{\tilde{a}}(x)$ is piecewise continuous.

A fuzzy number $\tilde{a}$ is a triangular fuzzy number **see** (Figure 1**)** if the membership function may be show as:

$$\mu_{\tilde{A}}(x)= \begin{cases} \frac{X-a}{b-a} & a\leq x\leq b \\ \frac{c-x}{c-b} & b\leq x\leq c \\ 0 & O.W \end{cases}$$

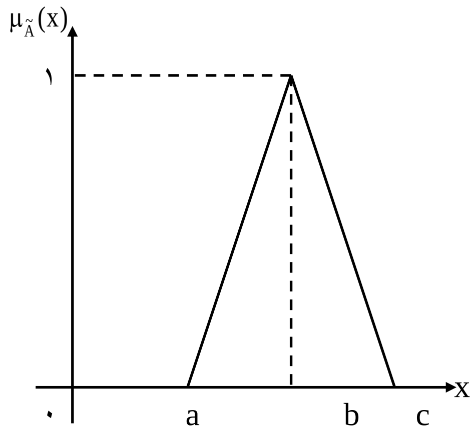

Figure 1: The Triangular membership function

Definition (6) [9].

A fuzzy number $\tilde{M}$ is of LR-type if there exist tow functions, L (called the Lift function), R(called the right function) ,such that $L(x)\leq \mu_{\tilde{M}}(x)\leq R(x)$, $\forall x\in X$ (Universal set) and a scalars a>0, b>0, with:

$$\mu_{\tilde{M}}(x) = \begin{cases} L(\frac{m-x}{a}), & \text{for } x\leq m \\ R(\frac{x-m}{b}), & \text{for } x\geq m \end{cases}$$

When $m\in R$ which is called the mean value of $\tilde{M}$,a and b are called the left and right spreads of m, respectively. Symbolically $\tilde{M}$ is denoted by $(m, a, b)_{LR}$.

Now, in applications, the representation of a fuzzy number in terms of its membership function is so difficult to use, therefore a suitable approach for representing the fuzzy number in terms of itsα-level sets is given, as in the following remark:

Remark (1): A fuzzy number $\tilde{M}$ may be uniquely represented in terms of its α-level sets, as the following closed intervals of the real line: $M_\alpha =\ [m-\sqrt{1-\alpha}\ ,\ m+\sqrt{1-\alpha}\ ]$ or $M_\alpha =\ [\alpha m,\ \frac{1}{\alpha}m]$ Where m is the mean value of $\tilde{M}$ and $\alpha\in(0,\ 1]$. This fuzzy number may be written as $M_\alpha =[\underline{\tilde{M}},\ \overline{\tilde{M}}\ ]$, ,where $\underline{\tilde{M}}$, refers to the greatest lower bound of M_α and $\overline{\tilde{M}}$ the least upper bound of M_α.

3-Possibility Programming Method in Fuzzy Single Objective Functions

Consider the formulation of the fuzzy single objective linear programming model as:

$$\text{Maximize} \sum_{j=1}^{n} \tilde{c}_j x_j \quad \text{........(1)}$$

Subject to:

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i, \quad i=1, 2\ldots m, \ldots\ldots (2)$$

$$x_{j\geq 0}, \quad j=1, 2 \ldots n, \quad \ldots\ldots\ldots.. (3)$$

Where x_j, j=1,2,….n, are non-negative decision variable $\tilde{c}_j$ is the fuzzy coefficient of the jth decision variable ,$\tilde{a}_{ij}$ represents the fuzzy coefficient of the jth decision variable in the ith constraint, while $\tilde{b}_i$ is the fuzzy right-hand side in the ith constraint .Hence for simplicity $\tilde{c}_j$, $\tilde{a}_{ij}$ and $\tilde{b}_i$ are considered to be of triangular fuzzy numbers i.e., using remark(1)above $\tilde{c}_j$= $[c_j-\sqrt{1-\alpha}, c_j, c_j+\sqrt{1-\alpha}]$, $\tilde{a}_{ij}=[a_{ij}-\sqrt{1-\alpha}, a_{ij}, a_{ij}+\sqrt{1-\alpha}]$, $\tilde{b}_i=[b_i-\sqrt{1-\alpha}, b_i, b_i+\sqrt{1-\alpha}]$

Thus, according to the triangular fuzzy numbers, the
equivalent crisp model for the fuzzy model (1)-(3), is given below:

$$\text{Maximize} \quad Z=\sum_{j=1}^{n}[(1-\alpha)\bar{c}_j+\alpha c_0]x_j$$

Subject to:

$$\sum_{j=1}^{n}[(1-\alpha)\underline{a}_{ij}+\alpha a_{ijo}]x_j \leq (1-\alpha)\bar{b}_i+\alpha b_{i0}$$

Where α is a predetermined value of the minimum required possibility, $\alpha \in (0, 1]$

As an illustrated, consider the following example:

<u>Example (1)</u>:-

Maximize $Z=\tilde{3}\, x_1+\tilde{2}\, x_2$

Subject to:

$$-\tilde{1}\, x_1+\tilde{2}\, x_2 \leq \tilde{4}$$
$$\tilde{3}\, x_1+\tilde{2}\, x_2 \leq \tilde{14}$$
$$\tilde{1}\, x_1-\tilde{1}\, x_2 \leq \tilde{3}$$
$$x_1 \geq 0,\ x_2 \geq 0$$

The fuzzy single objective linear programming in triangular form takes the form:

Maximize $(3-\sqrt{1-\alpha}, 3, 3+\sqrt{1-\alpha})\, x_1+(2-\sqrt{1-\alpha}, 2, 2+\sqrt{1-\alpha})x_2$

Subject to:

$(1-\sqrt{1-\alpha}, 1, 1+\sqrt{1-\alpha})x_1+(2-\sqrt{1-\alpha}, 2, 2+\sqrt{1-\alpha})\, x_2 \leq (4-\sqrt{1-\alpha}, 4, 4+\sqrt{1-\alpha})$

$(3-\sqrt{1-\alpha}, 3, 3+\sqrt{1-\alpha})\, x_1+(2-\sqrt{1-\alpha}, 2, 2+\sqrt{1-\alpha})\, x_2 \leq (14-\sqrt{1-\alpha}, 14, 14+\sqrt{1-\alpha})$

$(1-\sqrt{1-\alpha}, 1, 1+\sqrt{1-\alpha})\, x_1-(1-\sqrt{1-\alpha}, 1, 1+\sqrt{1-\alpha})\, x_2 \leq (3-\sqrt{1-\alpha}, 3, 3+\sqrt{1-\alpha})$

Thus, the equivalent crisp single objective linear programming in the case of possibility programming is stated as:

Maximize $[(1-\alpha)(3+\sqrt{1-\alpha})+3\alpha]\, x_1+[(1-\alpha)(2-\sqrt{1-\alpha})+2\alpha)]\, x_2$

Subject to:

$-[(1-\alpha)(1-\sqrt{1-\alpha}+\alpha)]\, x_1+[(1-\alpha)(2-\sqrt{1-\alpha}+2\alpha)]\, x_2 \leq (1-\alpha)(4+\sqrt{1-\alpha}+4\alpha)$

$[(1-\alpha)(3-\sqrt{1-\alpha})+3\alpha]x_1+[(1-\alpha)(2-\sqrt{1-\alpha}+2\alpha]x_2 \leq (1-\alpha)(14+\sqrt{1-\alpha}+14\alpha)$

$[(1-\alpha)(1-\sqrt{1-\alpha}+\alpha)]\, x_1-[(1-\alpha)(1-\sqrt{1-\alpha}+\alpha]\, x_2 \leq (1-\alpha)(3+\sqrt{1-\alpha}+3\alpha)$

$x_1, x_2 \geq 0.$

Following, Table (1) which represents the solution of the above example.

Table (1)

α	0.1	0.5	o.75	0.9	1
Z	30.4844	19.2137	15.6527	14.4008	14
x_1	4.3622	3.0368	2.6666	2.5401	3.1188
x_2	4.7911	3.8365	3.4444	3.2978	2.3216

It is remarkable then when α=1, then the solution of the fuzzy linear programming is the same solution of the crisp problem.

<u>Example (2):-</u>

Maximize $Z=\tilde{8}\, x_1+\tilde{6}\, x_2$

Subject to:

$$\tilde{4}\, x_1+\tilde{2}\, x_2 \leq \tilde{60}$$
$$\tilde{2}\, x_1+\tilde{4}\, x_2 \leq \tilde{48}$$
$$x_1 \text{and } x_2 \geq 0$$

The fuzzy single objective linear programming in Triangular form will be:

Maximize $(8-\sqrt{1-\alpha}, 8, 8+\sqrt{1-\alpha})\, x_1+(6-\sqrt{1-\alpha}, 6, 6+\sqrt{1-\alpha})\, x_2$

Subject to:

$(4\sqrt{1-\alpha}, 4, 4+\sqrt{1-\alpha})x_1+(2-\sqrt{1-\alpha}, 2, 2+\sqrt{1-\alpha})x_2$
$\leq=(60-\sqrt{1-\alpha}, 60, 60+\sqrt{1-\alpha})$

$(2-\sqrt{1-\alpha},2,2+\sqrt{1-\alpha})x_1+(4-\sqrt{1-\alpha},4,4+\sqrt{1-\alpha})x_2\leq$

$=(48-\sqrt{1-\alpha},48,48+\sqrt{1-\alpha})$

x_1 and $x_2 \geq 0$

Thus, the equivalent crisp single objective linear programming in the case of possibility programming is stated as:

Maximize $[(1-\beta)(8+\sqrt{1-\alpha})+8\beta]\,x_1+[(1-\beta)(6+\sqrt{1-\alpha})+6\beta]\,x_2$

Subject to:

$[(1-\alpha)(4-\sqrt{1-\alpha})+4\alpha]x_1+[(1-\alpha)(2-\sqrt{1-\alpha})+2\alpha]x_2 \leq(1-\alpha)(60+\sqrt{1-\alpha})+60\alpha$

$[(1-\alpha)(2-\sqrt{1-\alpha})+2\alpha]x_1[(1-\alpha)(4-\sqrt{1-\alpha})+4\alpha]x_2\leq(1-\alpha)(48+\sqrt{1-\alpha})+48\alpha$

x1and $x_2, \geq 0$.

Following, (Table 2) which represents the solution of the above example.

Table 2

α	0.1	0.5	0.75	0.9	1
Z	117.7883	125.1845	129	131.3223	132
x_1	10.1168	11.1038	5.6600	11.9113	12
x_2	4.1168	5.1038	5.6600	5.9113	6

It is remarkable then when α=1, then the solution of the fuzzy linear programming is the same solution of the crisp problem.

4-Possibility programming in fuzzy multiobjective Functions

Consider the formulation of the fuzzy multiobjective linear programming:

$$\text{Maximize} \sum_{j=1}^{n} \tilde{c}_{rj}x_j,\ r=1,2\ldots p \qquad \ldots (4)$$

Subject to:

$$\sum_{j=1}^{n} \tilde{a}_{ij}x_j \leq \tilde{b}_i,\quad i=1,2\ldots m \qquad \ldots (5)$$

$$x_j \geq 0,\qquad j=1,2\ldots n \qquad \ldots (6)$$

Where x_j, j=1, 2….n, are non-negative decision variable $\tilde{c}_{rj}$ is the fuzzy coefficient of the jth decision variable in the rth objective function. $\tilde{a}_{ij}$ represents the fuzzy coefficient of the jth decision variable in the ith constraint, while $\tilde{b}_i$ is the fuzzy right-hand side in the ith constraint .Hence $\tilde{c}_{rj}$, $\tilde{a}_{ij}$ and $\tilde{b}_i$ are consider to be triangular fuzzy numbers [11],i.e., $\tilde{c}_{rj}=[\tilde{c}_{rj}-\sqrt{1-\alpha},\tilde{c}_{rj},\tilde{c}_{rj}+\sqrt{1-\alpha}]$, $\tilde{a}_{ij}=[\tilde{a}_{ij}-\sqrt{1-\alpha},\tilde{a}_{ij},\tilde{a}_{ij}+\sqrt{1-\alpha}]$, $\tilde{b}_i=[\tilde{b}_i-\sqrt{1-\alpha},\ ,\tilde{b}_i+\sqrt{1-\alpha}]$.Thus, according to triangular fuzzy numbers, the equivalent crisp model for the fuzzy model (4)-(6) is given below:-

$$\text{Maximize } Z=\sum_{j=1}^{n}[(1-\alpha)\bar{c}_{rj}+\alpha c_{r0}]x_j$$

Subject to:

$$\sum_{j=1}^{n}[(1-\alpha)+\underline{a}_{ij}+\alpha a_{ij0}]x_j \leq (1-\alpha)+\bar{b}_i+\alpha b_{i0}$$

Where α is a predetermined value of the minimum required possibility, $\alpha\in(0,1]$.

Now, an illustrated example will be considered.

Example (3):

Maximize $Z_1(x)=\tilde{2}\,x_1+\tilde{1}\,x_2$

$Z_2(x)=\tilde{3}\,x_1-\tilde{2}\,x_2$

Subject to:

$\tilde{2}\,x_1+\tilde{5}\,x_2\leq\tilde{60}$

$\tilde{1}\,x_1+\tilde{1}\,x_2\leq\tilde{18}$

$\tilde{3}\,x_1+\tilde{1}\,x_2\leq\tilde{44}$

$\tilde{1}\,x_2\leq\tilde{10}$

$x_1, x_2\geq 0$

The fuzzy multiobjective linear programming in triangular form will be:

$Z_1=(2-\sqrt{1-\alpha},\ 2,\ 2+\sqrt{1-\alpha})\ x_1+(1-\sqrt{1-\alpha},\ 1,$

$1+\sqrt{1-\alpha}$) x_2

$Z_2=$ $(3-\sqrt{1-\alpha}$, 3, $3+\sqrt{1-\alpha})x_1-(2-\sqrt{1-\alpha}$, 2, $2+\sqrt{1-\alpha})$ x_2

Subject to:

$(2-\sqrt{1-\alpha},\ 2,\ 2+\sqrt{1-\alpha})\ x_1+\ (5-\sqrt{1-\alpha},\ 5,\ 5+\sqrt{1-\alpha})\ x_2\leq(60-\sqrt{1-\alpha},\ 60,\ 60+\sqrt{1-\alpha})$

$(1-\sqrt{1-\alpha},1,1+\sqrt{1-\alpha})x_1+(1-\sqrt{1-\alpha},1,1+\sqrt{1-\alpha})x_2\leq(18-\sqrt{1-\alpha},18,18+\sqrt{1-\alpha})$

$(3-\sqrt{1-\alpha},\ 3,\ 3+\sqrt{1-\alpha})\ x_1(1-\sqrt{1-\alpha},\ 1,\ 1+\sqrt{1-\alpha})\ x_2\leq(44-\sqrt{1-\alpha},\ 44,\ 44+\sqrt{1-\alpha})$

$(1-\sqrt{1-\alpha},\ 1,\ 1+\sqrt{1-\alpha})\ x_2\leq\ (10-\sqrt{1-\alpha},\ 10,\ 10+\sqrt{1-\alpha})$

Thus, the equivalent crisp multi objective linear programming in the case of possibility programming is stated as:

$Z_1=[(1-\alpha)(2+\sqrt{1-\alpha}+2\alpha)]x_1+[(1-)(1+\sqrt{1-\alpha}+\alpha)]x_2$

$Z_2=\ [(1-\ \alpha)(3+\sqrt{1-\alpha}+3\alpha)]x_1-[(1-\ \alpha)\ 2+\sqrt{1-\alpha}+2\alpha]\ x_2$

Subject to:

$[(1-\alpha)\ (2-\sqrt{1-\alpha}+2\alpha)]\ x_1[(1-\alpha)\ (5-\sqrt{1-\alpha}+5\alpha)]\ x_2\leq(1-\alpha)\ (60+\sqrt{1-\alpha}+60\alpha)$

$[(1-\alpha)\ (1-\sqrt{1-\alpha}+\alpha)]\ x_1+\ [(1-\alpha)\ (1-\ \sqrt{1-\alpha})]\ x_2\leq\ (1-\alpha)\ (18+\sqrt{1-\alpha}+18\alpha)$

$[(1-\alpha)(3-\sqrt{1-\alpha}+3\alpha)]x_1+[(1-\alpha)(1-\sqrt{1-\alpha})+\alpha]x_2\leq(1-\alpha)(44+\sqrt{1-\alpha}+44\alpha)$

$[(1-\ \alpha)\ \ (1-\sqrt{1-\alpha}+\alpha)]\ \ x_2\leq\ \ (1-\ \alpha)\ (10+\sqrt{1-\alpha}+10\alpha)$

$x_1,x_2\geq 0.$

Following (Table 3) which represents the solution of the above example with different valueα:

Table 3

α	0.1	0.5	0.75	0.9	1
Z_1	59.4628	39.6666	32.6141	30.1362	29.3333
Z_2	20.8993	16.7596	15.34.78	14.8335	14.6666
X_1	20.8993	16.7596	15.3478	14.8335	14.6666
X_2	1.2519	3.7003	2.6529	0	1.7756

It is remarkable they when α=1, then the solution of the fuzzy linear programming is the same solution of the crisp problem.

References:

1. Zimmermann,H. J.**1978**.Fuzzy program ming and linear programming with seve ral objective function. *Fuzzy Sets and Systems* **1**: 45-55.
2. Fang, S.C.and C.F.Hu.**1999.**Linear prog ramming with fuzzy coefficients in cons traint.Computer *Mathematics Applied*. **37**: 63-76.
3. Vasant,P., R. Nagarajan, and S.Yaacab. **2004.**Decision making in industrial prod uction planning using fuzzy linear progr amming. *IMA, Journal of Management Mathematics* **15**: 53-65.
4. Maleki,H.R.**2002.** Ranking functions an d their Applications to fuzzy linear prog ramming. *Far East Journal. Mathematic s. Science*. **4**: 283-301.
5. Maleki,H.R. M.Tata and M. Mashinchi. **2000.** Linear programming with fuzzy v ariables.*Fuzzysets and Systems* **109**: 21-33.
6. MahdaviAmiri ,N.and S.H. Nasseri**.200 5**.Duality in fuzzy variable linear progra mming. 4th World Enformatika Conference, WEC'05, Istanbul, Turkey 24-26, June.
7. Iskander, M.G.**2004.** A possibility programming approaches for stochastic fuzzy multiobjective linear fraction programs. *Computer and Mathematics with applications*. **48**: 1603-1609.
8. Iskander, M.G.**2002**. Comparison of fuzzy numbers using possibility programming. *Computer and mathematics with application*. **43**:833-840.
9. Bellman, R.E. and Zadeh, L.A. **1970.**Decision making in a fuzzy environment. *Management . Science.* **17**:141-164.