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## Determination and evaluation of the orbital transition methods between two elliptical earth orbits

Farid M. Mahdi<sup>1</sup>, Abdul-Rahman H. Salih<sup>2</sup>, Majeed M. Jarad<sup>1</sup>

<sup>1</sup>Department of Physics, College of science, University of Anbar, Anbar, Iraq

<sup>2</sup>Department of Astronomy and Space, College of science, University of Baghdad, Baghdad, Iraq

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### Abstract

To transfer a satellite or a spacecraft from a low parking orbit to a geosynchronous orbit, one of the many transition methods is used. All these methods need to identify some orbital elements of the initial and final orbits as perigee and apogee distances. These methods compete to achieve the transition with minimal consumption of energy, transfer time and mass ratio consumed ( $\Delta m/m$ ), as well as highest accuracy of transition. The ten methods of transition used in this project required designing programs to perform the calculations and comparisons among them.

The results showed that the evaluation must depend on the initial conditions of the initial orbit and the satellite mechanical exception as well as the target orbit. The most efficient methods of transition in terms of energy required were, sequentially, methods 10, 1, 8, 9, and 2, whereas the least efficient in terms of energy consumption, fuel and transition time were, sequentially, methods 5,6, and 7. Method 3 was the most efficient when the orbit needed to change the inclination with the transition. The first phase of multi-stage transition is the most energy consuming.

**Keywords:** Transition orbit, Hohmann orbit, Barking orbit

### تحديد وتقييم طرائق الانتقال المدارية بين مدارين ارضية بيضوية

فريد محمد مهدي<sup>1</sup> ، عبد الرحمن حسين صالح<sup>2</sup> ، مجيد محمد جراد<sup>1</sup>

<sup>1</sup>قسم الفيزياء ، كلية العلوم ، جامعة الأنبار ، الأنبار ، العراق

<sup>2</sup>قسم الفلك والفضاء ، كلية العلوم ، جامعة بغداد ، بغداد ، العراق

### الخلاصة

لنقل قمر صناعي أو مركبة فضائية من مدار انتظار منخفض إلى مدار متزامن مع الأرض يتم استخدام واحدة من العديد من طرائق الانتقال. تتنافس هذه الطرق في تحقيق الانتقال بأقل طاقة مستهلكة ووقت نقل ممكن وادنى نسبة كتلة مستهلكة ( $\Delta m/m$ ) بالإضافة إلى دقة انتقال عالية. تتطلب الطرق العشر للانتقال المستخدمة في هذا المشروع تصميم برامج لإجراء العمليات الحسابية والمفاضلة بين هذه الطرائق جميعها. أظهرت النتائج أن التقييم يجب أن يعتمد على الظروف الأولية للمدار الأولي والدفع الميكانيكي للأقمار الصناعية وكذلك المدار المستهدف. أفضل الطرائق للانتقال من حيث الطاقة المطلوبة للانتقال هي حسب

التسلسل: (10 ، 1 ، 8 ، 9 ، 2). أسوأ طرائق الانتقال من حيث استهلاك الطاقة والوقود ووقت الانتقال هي حسب التسلسل (6، 5، 7). الطريقة الثالثة هي الأفضل عندما يحتاج المدار إلى تغيير الميل مع الانتقال.

## 1.Introduction

When a spacecraft is launched to the outer space or to orbiting around the Earth in a high earth orbit (HEO) or a super high earth orbit (SHEO), it is placed in the beginning in an initial orbit called parking orbit (about 180-200 km in altitude). It is a temporary orbit that is used during the launching of a satellite then lifting it to the final orbit by the use of transfer orbits. The transfer and placement of the spacecraft to a geostationary orbit, which conforms with the equatorial plane, requires a large amount of fuel[1].

The satellite or spacecraft is transferred to a final orbit in which it is planned to spend its life or accomplish its mission by using different types of transfer orbits depending on the target orbit type, coordinates, orbital elements and mission. By trying to consume a lowest possible level of energy or time to transition, this energy is consumed to transfer a satellite in two phases, when leaving the initial orbit and at reaching the final orbit. The transfer orbit can be defined in orbital mechanics as an intermediate elliptical orbit that is used to move a satellite or other objects from an orbit to another, as in Figure-1. There are several types of transfer orbits that vary in their required energy efficiency and speed of transfer [2, 3].

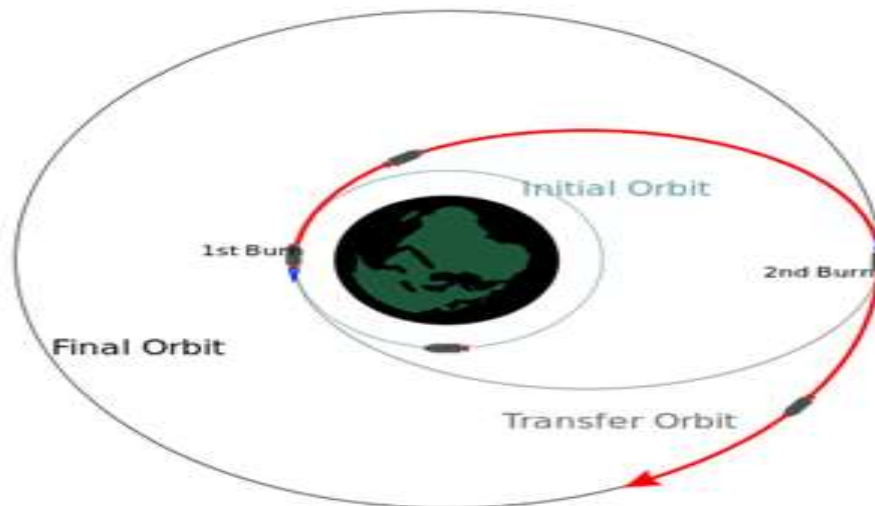


Figure 1-The  $\Delta v$  in Hohmann Transfer between two circular orbits [4].

In this work, the amount of energy ( $\Delta v$ ) required, the ratio of mass ( $\Delta m/m$ ) consumed and the time required to transition ( $T_{tran.}$ ) from a low earth orbit (200 km) in altitude to a synchronous orbit (35786 km) in altitude will be calculated in the same eccentricity (0.01), and the results will be compared.

## 2. Calculation methods of transition orbits

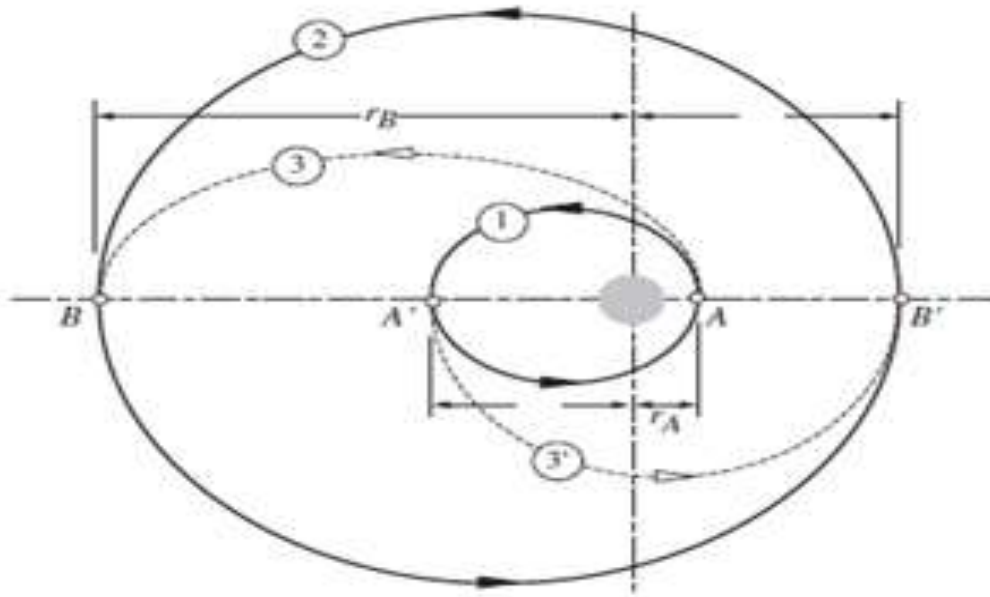
### 2.1 Hohmann Transfer orbit on same plan:

The Hohmann transfer orbit is an elliptical orbit used to transfer between two circular or elliptical orbits of different radii in the same plane with lowest possible amount of energy and highest fuel efficiency. It is a fairly slow process that is mostly used for transferring spacecrafts in shorter distances [4, 5].

Figure- 2 shows two hohmann transfer orbits between two coaxial elliptical orbits, from perigee to apogee and from apogee to perigee, represented in the orbits 3,3. In the first transfer orbit 3, the following equations are used [4]:

$$h_1 = \sqrt{2\mu} \sqrt{\frac{r_A r_{A'}}{r_A + r_{A'}}} \quad (1)$$

$$h_2 = \sqrt{2\mu} \sqrt{\frac{r_B r_{B'}}{r_B + r_{B'}}} \quad (2)$$



**Figure 2**-Hohmann transfer orbit between two elliptical orbits (from perigee and from apogee) [4].

$$h_3 = \sqrt{2\mu} \sqrt{\frac{r_A r_B}{r_A + r_B}} \tag{3}$$

Where  $h_1, h_2, h_3$  are the angular momentum in initial, final and transfer orbits (3), respectively, and  $\mu = GM$  is the standard Earth gravitational parameter. To calculate the velocity in point A, B in the three orbits, the following equations are applied [4]:

$$v_{A1} = \frac{h_1}{r_A} \quad , \quad v_{A3} = \frac{h_3}{r_A} \tag{4}$$

$$v_{B2} = \frac{h_2}{r_B} \quad , \quad v_{B3} = \frac{h_3}{r_B} \tag{5}$$

$$\Delta v_A = |v_{A3} - v_{A1}| \quad , \quad \Delta v_B = |v_{B2} - v_{B3}| \tag{6}$$

$$\Delta v_{total} = \Delta v_A + \Delta v_B \tag{7}$$

Where  $v_{A1}, v_{B2}$  are the velocities in perigee of the initial orbit and apogee of the final orbit,  $v_{A3}, v_{B3}$  are the velocities in perigee and apogee of the transfer orbit, respectively. Steps 1 to 7 are repeated with a different altitude  $r_A$  to calculate the required velocity of transfer at any altitude.

Time of transition is calculated by using the third Kepler law for half period:

$$T = 0.5 \left( \frac{2\pi}{\sqrt{\mu}} a^{\frac{3}{2}} \right) \tag{8}$$

Where  $T$  is the time of transfer,  $a$  is the semi major axis of the transfer orbit. To calculate the mass ratio ( $\Delta m/m$ ) the following equation is used [6,7] :

$$M_{prop} = \frac{\Delta m}{m_o} = 1 - \left( e^{-\frac{\Delta v}{g_o I_{sp}}} \right) \tag{9}$$

where  $M_{prop}$  is the propellant mass fraction that is the part of the initial total mass that is spent as working mass,  $g_o$  is the standard earth gravity, and  $9.80665 \text{ m/s}^2 \cdot I_{sp}$  is the specific impulse in seconds. These processes are repeated for orbit 3 to compare between these two orbits.

### 2.2 Hohmann Transfer with changing the inclination

The orbital inclination ( $i$ ) is the angle between the orbit plane and the Earth's equatorial plane. It is determined by the latitude of the launch site and the launch azimuth. The inclination must be reduced to zero to obtain a geosynchronous orbit. The required velocity for a plane change is proportional to the instantaneous velocity. The inclination and eccentricity are usually changed together in a single maneuver at apogee, where velocity is lowest. The required  $\Delta v$  for an inclination change at either the ascending or descending nodes of the orbit is calculated as follows [6,8]:

$$\Delta v = 2v \sin \frac{\Delta i}{2} \tag{10}$$

The inclination change is combined with the orbital circularization by apogee burn to reduce the total  $\Delta v$  for the two maneuvers. The combined  $\Delta v$  is the vector sum of the inclination change  $\Delta v$  and the circularization  $\Delta v$ . The total  $\Delta v$  in a combined maneuver will always be less than that in two maneuvers. The combined  $\Delta v$  can be calculated as follows [9]:

For transition from perigee of the initial to apogee of the target orbit:

$$\Delta v_{A(A')} = \sqrt{v_{A1(A'1)}^2 + v_{A3(A'3)}^2 - 2 v_{A1(A'1)} v_{A3(A'3)} \cos \Delta i} \tag{11}$$

For transition from apogee of the initial to perigee of the target orbit:

$$\Delta v_{B(B')} = \sqrt{v_{B2(B'2)}^2 + v_{B3(B'3)}^2 - 2 v_{B2(B'2)} v_{B3(B'3)} \cos \Delta i} \tag{12}$$

Where  $v_{A1orA'1}$  is the velocity magnitude at the perigee or apogee of the initial orbit,  $v_{A3orA'3}$  is the velocity of the transfer orbit 3 or 3' in point  $A$  or  $A'$ ,  $v_{B2(B'2)}$  is the velocity magnitude at the perigee (apogee) of the final orbit, and  $v_{B3(B'3)}$  is the velocity of the transfer orbit 3 (3')  $B(B')$ .

### 2.3 The Bi-elliptic transfer orbit

The **Bi-elliptic transfer orbit** is the orbital maneuver that transfers a spacecraft from one orbit to another. From Figure- 3, the Bi-elliptic transfer consists of two half-elliptical orbits (2 and 3). In the initial orbit (1), a first burn expands  $\Delta v$  to boost the spacecraft into the first transfer orbit (2) with an apoapsis at some point ( $B$ ) away from the central body [9].

At this point ( $B$ ), a second burn sends the spacecraft into the second elliptical transfer orbit number 3 with periapsis at the periapsis of the final orbit number 4, where a third burn is performed at point ( $C$ ), injecting the spacecraft into the desired orbit. This requires one more engine burn than a Hohmann transfer orbit [9].

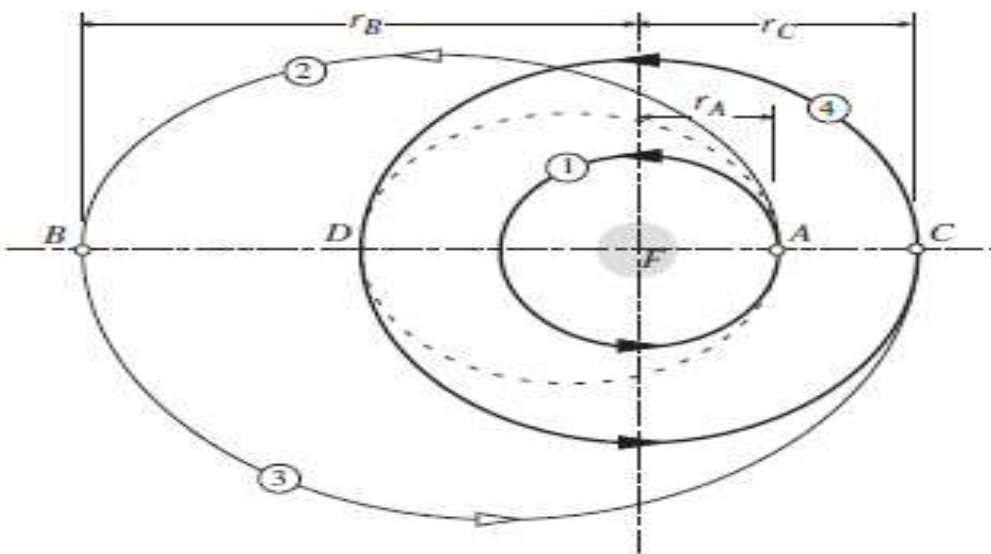


Figure 3-The Bi-Elliptic transfer orbit [4].

The three required changes in velocity ( $\Delta v$ ) can be obtained directly from equation 4. To Calculate  $\Delta v$  in a Bi-elliptic transfer orbit, we must track the variations of orbits of the satellite as well as its burn position and radius, starting from the initial orbit with perigee distance  $r_{p1}$ . A Prograde burn puts the spacecraft on the first elliptical transfer orbit. The magnitude of the required  $\Delta v$  for this burn is calculated as follows [9,10]:

$$\Delta v_1 = \sqrt{\frac{2\mu}{r_{p1}} - \frac{\mu}{a_{t1}}} - \sqrt{\frac{\mu}{r_{p1}}} \tag{13}$$

The magnitude of the required  $\Delta v$  for the second elliptical transfer orbit is:

$$\Delta v_2 = \sqrt{\frac{2\mu}{r_B} - \frac{\mu}{a_{t2}}} - \sqrt{\frac{2\mu}{r_B} - \frac{\mu}{a_{t1}}} \quad (14)$$

$$a_{t1} = \frac{r_{p1} + r_B}{2} \quad (15)$$

$$a_{t2} = \frac{r_{p2} + r_B}{2}$$

Where  $a_{t1}, a_{t2}$  are the semi major axes of the two elliptical transfer orbits,  $r_{p1}, r_{p2}$  are the perigee distances of the initial and the final orbits.  $r_B$  is the common apoapsis radius of the two transfer ellipses and is a free parameter of the maneuver.

Lastly, when the final elliptical orbit with perigee distance  $r_{p2}$  is reached, a retrograde burn changes the trajectory to the final target orbit. The final retrograde burn requires  $\Delta v$  of the following magnitude:

$$\Delta v_3 = \sqrt{\frac{2\mu}{r_{p2}} - \frac{\mu}{a_{t2}}} - \sqrt{\frac{\mu}{r_{p2}}} \quad (16)$$

The total  $\Delta v$  is calculate by

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 + \Delta v_3 \quad (17)$$

The time required to perform each phase of the transfer is half the orbital period of each transfer ellipse calculate from the third kipler law, and the total time of bi-elliptical transfer ( $t_{total}$ ) equals the sum of the times required for each half-orbit, as follows:

$$t_{total} = \frac{\pi}{\sqrt{\mu}} \sqrt{a_{t1}^3 + a_{t2}^3} \quad (18)$$

## 2.4 Geosynchronous transfer orbit (GTO)

This orbit is a Hohmann transfer orbit used to reach geosynchronous or geostationary orbits. GTO is a highly elliptical Earth orbit with an apogee of 42,164 km which corresponds to the geostationary altitude. Perigee can be at a few hundred kilometers above the Earth's surface [10, 11].

In some cases of Hohmann transfer, single, long-duration burns can be less efficient, and there may also be limitations on a spacecraft's engines that prevent them from firing for too long. A series of engine burns are used at apogee instead of one long burn on the same plan. The total  $\Delta v$  is equal to the sum of  $\Delta v$  values of all transfer orbits.

We suggest other methods that include two types of sequential transition. The first one is the direct transfer from the perigee of the initial orbit to the apogee of the final orbit with a high eccentricity, using a single burn. The perigee is then raised by several burns until we reach the perigee of the final orbit. The second transition is the transition of each perigee and apogee of the initial orbit to stages placed in the third or fifth of the distance from the final orbit. This process is repeated several times until reaching the final synchronous orbit.

The relationships 1 to 9 are used to calculate the  $\Delta v$  at each phase, and the  $\Delta v_{total}$  is the sum of  $\Delta v$  values in each phase.

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 + \Delta v_3 + \dots \quad (19)$$

The process in the previous paragraph is repeated using an initial point of transfer of apogee vice perigee.

## 2.5 The Super synchronous transfer orbit

The orbit with a period greater than the synchronous orbit and with apogee higher than the geosynchronous orbit is a super synchronous orbit. It is a band of near-circular Geocentric orbits beyond the Geosynchronous belt with a perigee altitude at the geosynchronous orbit.

The super synchronous orbits are used for the transfer of communication satellites intended for geosynchronous orbits. In this way, the launch vehicle places the satellite into a super synchronous elliptical Geostationary transfer orbit, an orbit with a somewhat larger apogee than the more typical Geostationary transfer orbit (GTO) typically utilized for communication satellites. This technique was used to launch and transfer orbit injections for very high apogees of about 90,000 kilometers, then the propulsion built into the satellite is used to reduce the apogee and circularize the orbit to a geostationary orbit [12].

The relationships 1 to 9 are used to calculate the  $\Delta v$  at each phase, and the  $\Delta v_{total}$  is the sum of  $\Delta v$  in each phase, as in equation 19 [12].

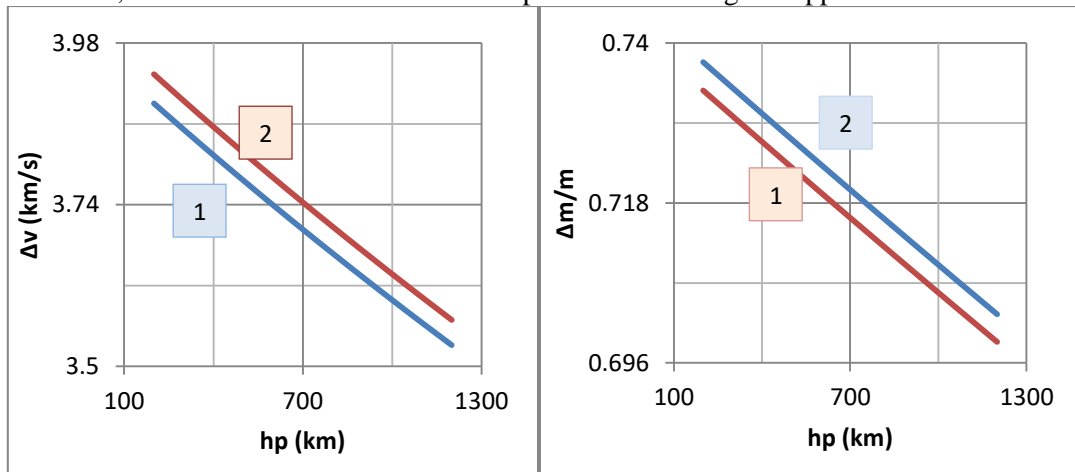
**3. The initial conditions and the transition methods**

Ten computer programs have been constructed to calculate the  $\Delta v$ ,  $\Delta m/m$  and period T required to transfer the satellite from low Earth orbit (200,300,---,1200 km in perigee altitude and 332.888, 434.908,---, 1410.909 km in apogee altitude) with eccentricity (0.01) to a geosynchronous orbit (42164 km in perigee altitude) and eccentricity (0.01), with variations in these parameters with various altitudes. These methods are described as follows:

1. Hohmann transfer from perigee of initial orbit to apogee of final orbit. (method 1)
2. Hohmann transfer from apogee of initial orbit to perigee of final orbit. (method 2)
3. Hohmann transfer from perigee of initial orbit to apogee with changing the inclination of final orbit. (method 3)
4. Hohmann transfer from perigee with change the inclination of initial orbit to apogee of final orbit. (method 4)
5. Bi-elliptical transfer from perigee of initial orbit to apogee of final orbit. (method 5)
6. Hohmann transfer from perigee of initial orbit to apogee of next orbit and by stages to arrive to the final orbit (method 6)
7. Hohmann transfer from apogee of initial orbit to perigee of next orbit by stages to arrive to the final orbit. (method 7)
8. Hohmann transfer from apogee of initial orbit to perigee of final orbit with increasing the perigee by stages to arrive to the final orbit. (method 8)
9. Hohmann transfer from perigee of initial orbit to apogee of final orbit with increasing the apogee by stages to arrive to the final orbit. (method 9)
10. Hohmann transfer from perigee of initial orbit to super synchronous orbit with reducing the apogee of final orbit by stages. (method 10)

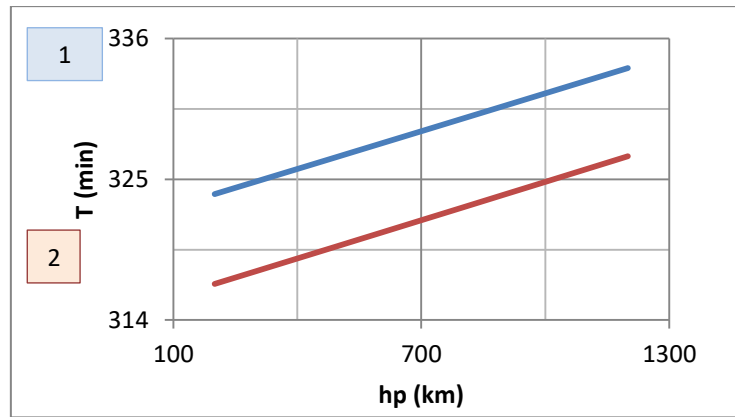
**4. Results and discussion**

**4.1 Methods 1 and 2:** Figures- 1 and 2 represent the transition from the perigee of the initial orbit to the apogee of the final orbit and from the apogee of the initial orbit to the perigee of the final orbit. The figures show that the value of the change in velocity decreases linearly with the increase in height of the initial orbit. The transition in the first method requires less energy and less fuel mass as compared to the transition in the second method in all altitudes (200 - 1200 km). Figure- 3 shows that the transition in the second method can be accomplished in less time than that in the first method . This is because the semi major axis of the second transition orbit is smaller than that of the first transition orbit, on which the transition time is dependent according to Kippler third law.



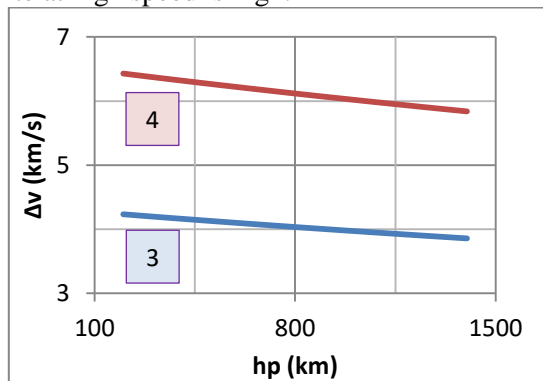
**Figure 1-**variation of  $\Delta v$  with altitude

**Figure 2-**variation of  $\Delta m/m$  with altitude

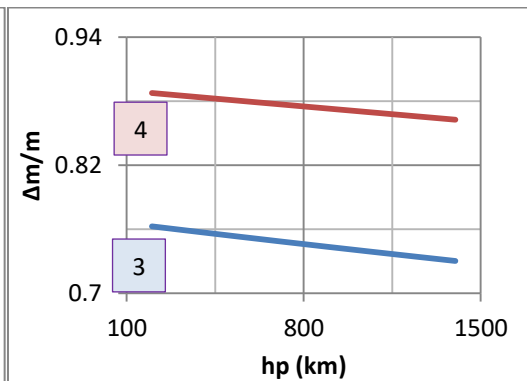


**Figure 3**-variation of transition time with altitude

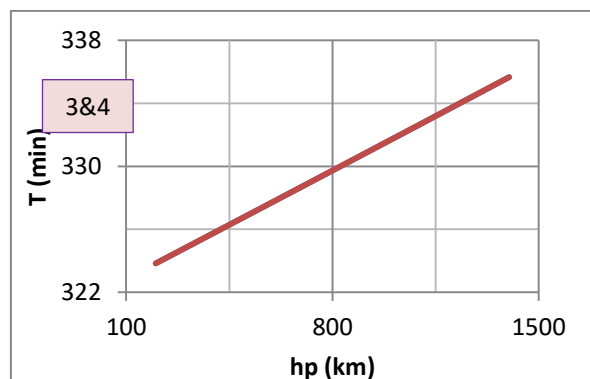
**4.2 Methods 3 and 4:** Figures-(4, 5) show that the change in speed is decreased linearly with the increase in the height of the initial orbit. The third method which represents the change of inclination in the final orbit showed much higher efficiency than method 4 which represents the change in the inclination in the initial orbit. Method 3 requires less energy (about 0.38) and less fuel consumption (about 0.125) than method 4 of all the heights of the initial orbit, while the transition time is the same in both (Figure-6). The reason behind this major difference in energy is that changing the direction upon high speed requires more energy than upon lower speed, because the angular momentum of the satellite at high speed is high.



**Figure 4**-variation of  $\Delta v$  with altitude



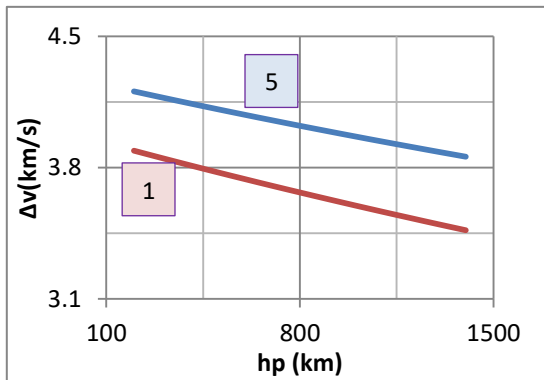
**Figure 5**-variation of  $\Delta m/m$  with altitude



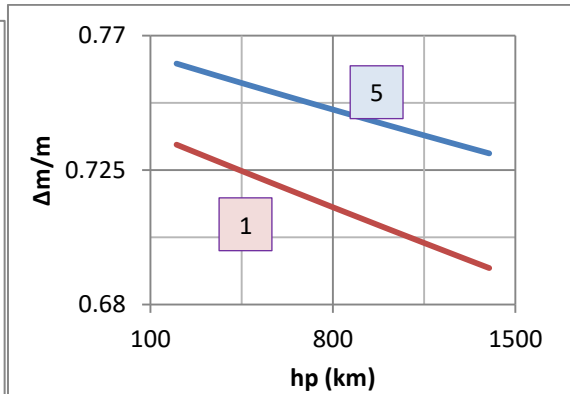
**Figure 6**- variation of Period with altitude

**4.3 Methods 5 and 1:** Figures-(7 and 8) show the linear decrease in the value of change in velocity with the increase of the height of the initial orbit. We also found that method 5 requires more energy and fuel consumption than method 1, because method 5 requires the transfer of the satellite through a transitional orbit above the final orbit. Also, the change of velocity in method 5 is performed in three

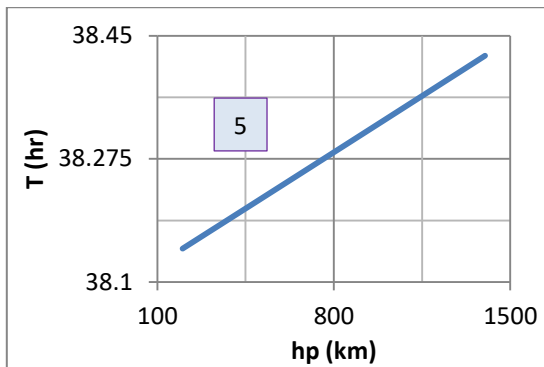
stages, which also requires much more time than that in the other methods Figures-(9 and 10). We believe that if the final orbit was higher, the energy required for transition in method 5 would be lower than that in the Hohmann method for the same height. This difference is due to the fact that the change in velocity in the second and third phases would be very little because of the convergence between the transition orbit and the final orbit .



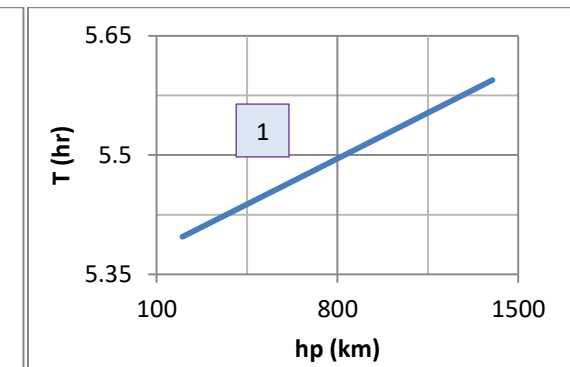
**Figure 7**-variation of  $\Delta v$  with altitude



**Figure 8**- variation of  $\Delta m/m$  with altitude



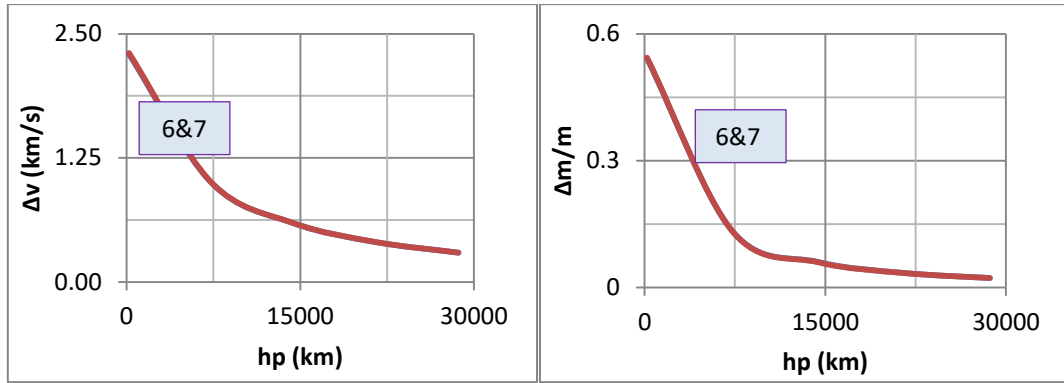
**Figure 9**-variation the transfer time with altitude



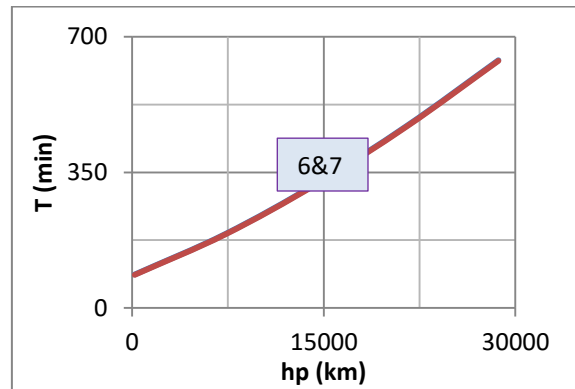
**Figure 10**-variation the transfer time with altitude

**4.4 Methods 6 and 7:** Figures- 11, 12, and 13 show that the values of change in velocity, mass, and transition time, respectively, are very comparable between the two methods, while the change follows a negative exponential curve. The maximum values of energy and mass required for the transition lay in the first phase of it, due to the nearness of the satellite to the ground and the effect of gravity which decreases with the increase of the height of the initial orbit. While, the transition time increased significantly with the increase in altitude due to the low velocity of the satellite in its orbit, which is inversely proportional to the size of the orbit and the semi major axis of the orbit upon a fixed eccentricity. We believe that the reason for this convergence of the change values is the small difference in the height between the successive phases of transition comparative with the height of the final orbit.





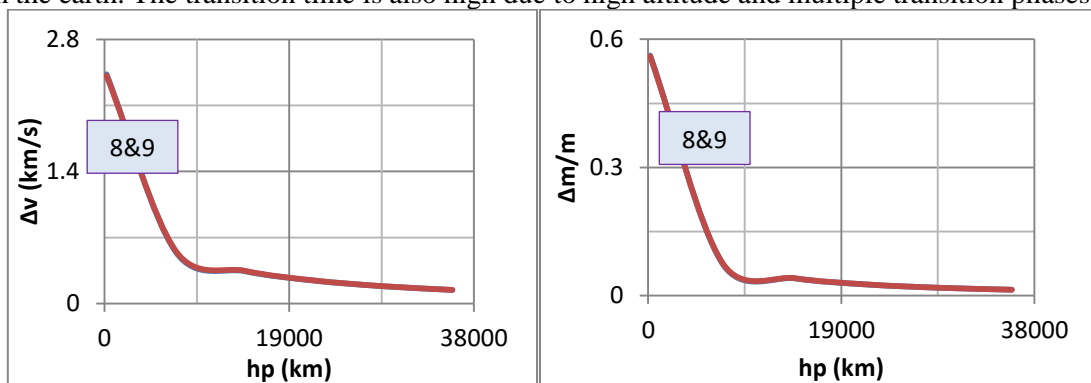
**Figure 11**-variation of  $\Delta v$  with altitude **Figure 12**-variation of  $\Delta m/m$  with altitude



**Figure 13**- variation of Period with altitude

**4.5 Methods 8 and 9:** Figure-(14 and 15) show the decreased change in the speed required and mass consumed to transfer rapidly with increasing height, as well as the stability of these values at high altitudes (more than 10,000 km). Figure- 16 demonstrates the convergence of the values of the change in velocity, the consumed mass, and the transition time in these two methods by increasing the perigee or apogee of the orbits until reaching the final orbit. The reason for this convergence of values is similar to that mentioned in the preceding paragraph.

**4.6 Method 10:** Method 10 represents the transition from an altitude of 200 km to the final orbit with a height of 35786 km by multiple transition stages. The transition from 200 to 83622 km from the surface of the earth occurs in one stage, then transition from 83622 to 36637 km occurs by several stages in the height. The total change in the velocity required to complete all phases of the transition is 3.865444 km / s, as in Table- 1. The maximum value of the consumed energy is in the first-order phase transition and the other stages require low transition energy due to the distance of the satellite from the earth. The transition time is also high due to high altitude and multiple transition phases.



**Figure 14**- variation of  $\Delta v$  with altitude **Figure 15**-variation of  $\Delta m/m$  with altitude

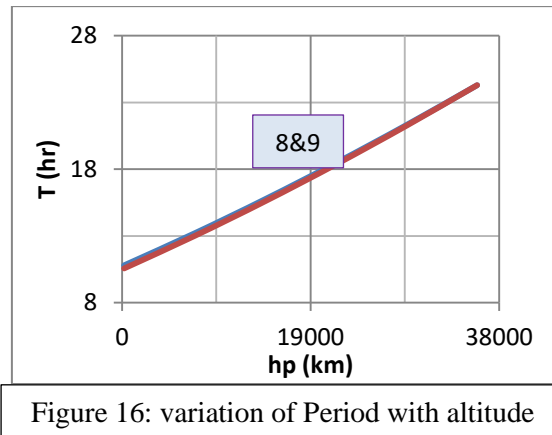


Figure 16: variation of Period with altitude

**Table 1**-variation of  $\Delta v$ ,  $\Delta m/m$  and transfer time for ten of transition methods.

No. of stage	$h_p$ (km)	$\Delta v$ (km/s)	$\Delta m/m$	Transition time(day)
1	200-83289	3.367232	0.681617	1.956685
2	73892.14	0.064923	0.021825	1.751759
3	64497.28	0.077211	0.025902	1.554574
4	55098.42	0.093496	0.031279	1.36532
5	45701.56	0.115609	0.038533	1.184476
6	36304.7	0.146973	0.048728	1.012407
sum		3.865444	0.847884	8.090811

**Table 2**-variation of  $\Delta v$ ,  $\Delta m/m$  and transfer time for ten types of transition methods.

s	No. of transition method	$\Delta v$ (km/s)	$\Delta m/m$	Transfer time
1	<b>1</b>	3.890425	0.733486	5.397315 hr
2	<b>2</b>	3.93375	0.737382	5.280384 hr
3	<b>3</b>	4.23122	0.762637	5.3972613 hr
4	<b>4</b>	6.42885	0.887536	5.3972613 hr
5	<b>5</b>	4.206561	0.760639	1.589479 day
6	<b>6</b>	4.584733	0.789510533	1.187182 day
7	<b>7</b>	4.602376	0.790769008	1.182425 day
8	<b>8</b>	3.890486	0.7334922	4.326620 day
9	<b>9</b>	3.895136	0.73392629	4.294771 day
10	<b>10</b>	3.865444	0.847884	8.090811 day

**5. Conclusions**

1. The most efficient transition methods in terms of energy required are in the sequence of 10, 1, 8, 9 and 2.
2. The most efficient transition methods in terms of fuel consumption are in the sequence of 1, 8, 9, 2 and 3.
3. The most efficient transition methods in terms of the required transition time are, sequentially 2, 3, 4 and 1.
4. Method 3 was the most efficient when changes in the orbit inclination and transition are required.
5. If technical or orbital barriers to simultaneous transition exist, the most efficient methods of sequential transition in terms of energy consumption are in the sequence of 10, 8 and 9. However, since the transition time is very long compared to other methods, these methods are not suitable for manned flights.

6. The first phase of multi-stage transition is the most energy consuming. This can be bypassed if the launch vehicle can deliver the satellite above the first stage of the transition.
7. The least efficient methods of transition in terms of energy consumption, fuel and time transition are methods 5, 6 and 7, because the implementation of these methods is performed in stages, which needs more energy, fuel and time.

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