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# Construct the Character Table of $\mathbf{D}_{7 \mathrm{~h}}$ Point Group and Apply it for Classify the (3N-6) Modes of Vibration for the [7] Cyclacene (Linear) Monoring Molecule Rehab Majed Kubba <br> Department of Chemistry, College of Science, University of Baghdad, Jadiriya, Baghdad, Iraq 


#### Abstract

The character table or the table of eigen values for the $\mathrm{D}_{7 \mathrm{~h}}$ point group, which is not found in the literature was constructed. The constructed table is used to classify all modes of the vibrational frequencies of the IR and Raman spectra for any compound or molecule that has the $\mathrm{D}_{7 \mathrm{~h}}$ symmetry in the Point Group. For that, it was used for classifying all the modes of vibrational frequencies of the [7] cyclacene (linear) monitoring molecule, possessing $\mathrm{D}_{7 \mathrm{~h}}$ symmetry. The constructed table was used to classify the symmetry species for the total degrees of freedom, total degrees of transition, total degrees of rotation and for the total degrees of modes of vibration ( $3 \mathrm{~N}-6$ ) for this tube. The character table was also used to determine the active and inactive modes of vibration in the IR and Raman spectra.Also,determining the polarized and non-polarized vibrational frequencies in the Raman spectrum, and used for describing the complete physical structure of the studied molecule after extracting its geometric shape. The methods of quantum mechanics calculations PM3 and DFT were used for this purpose applying the Gaussian 09 program.


Keywords: Character table, monoring molecule, modes of vibration, PM3, DFT.


الخلاصة

$$
\begin{aligned}
& \text { تم بناء جدول القيم الذاتية (character table) لمجموعة النقطة D }{ }_{\text {( }}^{\text {( }} \text { غير موجود في الادبيات، المعتمد عليه } \\
& \text { عند استخراج تماثلات الانماط الاهتزازية، والذي يمكن الاستفاده منه لغرض إستخراج وتصنيف جميع انماط } \\
& \text { الحركة الاهتزازية في طيف الاشعة تحت الحمراء و طيف رامان لأي مركب أو جزيئة تمتلك التماثلم } \\
& \text { مجموعة النقطة (Point Group). وقد تم تطبيقه على انبوب نانو احادي الحلقة نوع } \mathrm{C} \text { (جزيئة }
\end{aligned}
$$

$$
\begin{aligned}
& \text { الحرية للحركات الدورانية ولدرجات الحرية للحركات الانتقالية و لارجات الحرية لانماط الحركة الاهتزازية بعدد }
\end{aligned}
$$

$$
\begin{aligned}
& \text { رامان و انماط الترددات الاهتزازية المستقطبة و غير المستقطبة في طيف رامان. أمكن أيضا وصف الصورة } \\
& \text { الغيزياوية التامة لهذا الانبوب بعد استخراج الثثكل الهندسي التوازني لهذا الانبوب وفق طريقة حسابات ميكانيك } \\
& \text { الكم DFT وبإستخدام برنامج } 09 \text { Gaussian وفقا لتماثله من أطوال وزوايا التآصر والقيم الطاقية وتوزيع }
\end{aligned}
$$

## Introduction

Each molecule has a Symmetry Group that specifies ownership or non-possession of the five symmetry elements: E (Identity), the proper rotation axis ( n ), the level of symmetry (the plane of symmetry), the axis of rotation (rotation reflection axis) (n) and inversion center (i). The group is similar to symbols called SchÖenflies symbols, such as $\mathrm{C}_{1}, \mathrm{C}_{2 \mathrm{v}}, \mathrm{D}_{2 \mathrm{~h}} \ldots$..etc [1-2]. Each point group returns all the molecules that have the same analog geometrical form, the same symmetry elements. Using the Character Table in the literature [3] and the Molecular Group, the undifferentiated analog type can be assigned to a particular vibration pattern [4-5], where 3N-6 is produced from the basic vibrational patterns of each nonlinear molecule. The effectiveness of these patterns can also be determined in the Raman IR spectra. These patterns are polarized or nonpolarized in the Raman spectra and are located relative to the surface of symmetry, as they are at the surface level of symmetry. In case the molecule has a homogeneous surface $\left(\sigma_{h}\right)$ [6].
The idea of symmetry is familiar, and the relationship of the symmetry of an object to its aesthetic appeal has been understood from the earliest ages. The aim of the scientist was to make the idea of symmetry quantitative so that they could use the symmetry properties of a molecule to simplify many of the problems concerning the structure (internal coordinates of bonds and angles) of molecules. Symmetry dealing mainly with applications that concern isolated, finite molecules, but the extension to objects that can be regarded as infinite (e.g., crystal lattices) is not difficult. It is possible to classify the symmetry of any molecule in terms of the symmetry operations that can be carried out on that molecule. It could be seen that symmetry operations involve;
1-the rotation (a simple rotation or a proper rotation) of the molecule about a certain axis, a rotation axis is given the symbol $C_{n}$, where n is an integer, and (360/n) is the minimum rotation necessary to give an equivalent configuration (such an axis is said to be an $n$-fold axis or an axis of order $n$ ).
2-The mirror planes of the molecule (including horizontal planes of symmetry $\sigma_{\mathrm{h}}$, so called because it is perpendicular to the main axis or axis of the highest order, a vertical planes of symmetry $\sigma_{v}$, contains the highest order axis of the molecule or any object. Such planes are known as dihedral planes, $\sigma_{\mathrm{d}}$, if they bisect the angles between the $\sigma_{\mathrm{v}}$ planes or bisect the angles between two secondary axes of $\mathrm{C}_{2}$.
3-The center of inversion of the object (including the identity as a special case of rotation by $0^{\circ}$ or $360^{\circ}$ ) or reflections. The operation of inversion can be regarded as a composite movement comprising rotation by $180^{\circ}$ and reflection in a plane perpendicular to this axis. The symmetry operation of improper rotation consists of a clockwise rotation about an axis, followed by a reflection in a plane perpendicular to the axis.
4- The associated symmetry element is a rotation-reflection axis (sometimes known as an alternating axis of symmetry). The symbol for such an axis is $S_{m}$ where the angle of rotation is $(360 / n)^{0}$, and the operation of rotation by $\left(\mathrm{mx}(360 / \mathrm{n})^{0}\right.$ with m reflections in the perpendicular plane is given the symbol $\mathrm{S}_{\mathrm{n}}{ }^{\mathrm{m}}$.
5- Finally, the identity of the object $E$, which could be regarded as rotation by $360^{\circ}$. The essential property of such operations is that they bring the object into an equivalent configuration. Such a configuration cannot be distinguished from the original one, but it need not be identical with it, as some equivalent parts of the object may have been interchanged.
The symbols for point groups (the collections of symmetry operations possessed by molecules) used usually are the SchÖnflies symbols; these are used in most applications of group theory, except crystallography, where an alternative nomenclature, the international system, is used [7-9].

## The Utilized Programs

Both Gaussian 09 program of Pople et al. [10] and the Molek9000 program of P. Bischof, Heidelberg, were applied throughout the present work.

## Results and discussion

The intrinsic value table of the $\mathrm{D}_{7 \mathrm{~h}}$ group was built on the knowledge of the exact concept of the elements and processes of symmetry that belong to this group, from the knowledge of the supposed number of symmetries, the nature of the returned analogue items and the analogue classes of rotational and transitional movements, Spectral infrared and spectrum Raman. The construction of this table also requires a deep knowledge of the concept of symmetries, which include vibrational patterns located at the surface of the molecule and outside, and the determination of the rotation angle of the axes. It is also very important to predict the intrinsic values produced by the transformation matrix solution for
the symmetry operations of the point set concerned. The constructed table below represents all of the expected symmetry elements and operations represented for the $D_{7 h}$ point group.

Table 1- The constructed character table of $\mathrm{D}_{7 \mathrm{~h}}$ point group

| $\mathrm{D}_{7 \mathrm{~h}}$ | E | $2 \mathrm{C}_{7}$ | $2 \mathrm{C}_{7}{ }^{2}$ | $2 \mathrm{C}_{7}{ }^{3}$ | 7C ${ }_{2}$ | $\sigma_{\text {h }}$ | $2 \mathrm{~S}_{7}$ | $2 S_{7}{ }^{3}$ | $2 S_{7}{ }^{5}$ | 76. | IR | Raman |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}^{\prime} 1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{x}^{2}-\mathrm{y}^{2}, \mathrm{z}^{2}$ |
| $\mathbf{A}^{\prime}{ }^{\prime}$ | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | $\mathbf{R}_{\mathbf{z}}$ |  |
| $\mathbf{E}_{1}$ | 2 | $\begin{gathered} \hline 2 \cos \\ 360 / 7 \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | $\begin{gathered} \hline 2 \cos \\ 360 / 7 \mathrm{x} 3 \end{gathered}$ | 0 | 2 | $\begin{gathered} \hline 2 \cos \\ 360 / 7 \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | $\begin{gathered} \hline 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | 0 | Tx, Ty |  |
| $\dot{E}_{2}$ | 2 | $\begin{gathered} 2 \cos \\ 360 / 7 \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | 0 | 2 | $\begin{gathered} 2 \cos \\ 360 / 7 \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | 0 |  |  |
| $\dot{E}_{3}$ | 2 | $\begin{gathered} \hline 2 \cos \\ \mathbf{3 6 0 / 7} \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | $\begin{gathered} \hline 2 \cos \\ 360 / 7 \times 3 \\ \hline \end{gathered}$ | 0 | 2 | $\begin{gathered} \hline 2 \cos \\ 360 / 7 \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | 0 |  | $x^{2}-y^{2}, x y$ |
| $\mathrm{A}^{\prime \prime}$ | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 |  |  |
| $\mathrm{A}^{\prime \prime}$ | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1 | Tz |  |
| $\mathrm{E}^{\prime \prime}{ }_{1}$ | 2 | $\begin{gathered} \text { 2cos } \\ \text { 360/7 } \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | $\begin{gathered} \hline 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | 0 | -2 | $\begin{gathered} 2 \cos \\ 360 / 7 \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | 0 | $\mathbf{R}_{x}, \mathbf{R}_{\mathbf{y}}$ | xz, yz |
| $\mathrm{E}^{\prime \prime}$ | 2 | $\begin{gathered} 2 \cos \\ \mathbf{3 6 0} / 7 \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | $\begin{gathered} \hline 2 \cos \\ 360 / 7 \times 3 \\ \hline \end{gathered}$ | 0 | -2 | $\begin{gathered} \hline 2 \cos \\ 360 / 7 \\ \hline \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \\ \hline \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | 0 |  |  |
| $E^{\prime \prime}{ }_{3}$ | 2 | $\begin{gathered} 2 \cos \\ 360 / 7 \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | $\begin{gathered} \hline 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | 0 | -2 | $\begin{gathered} 2 \cos \\ 360 / 7 \end{gathered}$ | $\begin{gathered} 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | $\begin{gathered} \hline 2 \cos \\ 360 / 7 \times 3 \end{gathered}$ | 0 |  |  |

Where
$\mathrm{D}_{7 \mathrm{~h}}$ : SchÖnflies symbol (Point group).
$E$ : The identity
$\mathrm{C}_{7}, \mathrm{C}_{2}$ : The symmetry axes of proper rotation
$\mathrm{S}_{7}$ : The symmetry axis of improper rotation (rotation-reflection axis).
$\sigma_{h}$ : The horizontal plane of symmetry.
$\sigma_{\mathrm{v}}$ : The vertical plane of symmetry.
IR: Infra-red (the active modes of vibration in Infra-red spectrum).
Raman: (the active modes of vibration in Raman spectrum).
A: One dimension matrix.
E: Two dimension matrix
$\mathrm{A}^{\prime}{ }_{1}, \mathrm{~A}^{\prime}{ }_{2}, \dot{E}_{1}, \dot{E}_{2}, \dot{E}_{3}$ : Symmetry species with respect to $\sigma_{\mathrm{h}}$ (irreducible representations of Mullican symbols).
$A_{1}^{\prime \prime}, A_{2}^{\prime \prime}, E_{1}{ }_{1}, 9 E_{2}{ }_{2}, E_{3}{ }_{3}$ : Antisymmetry species with respect to $\sigma_{h}$ (irreducible representations of Mullican symbols).
$\mathrm{R}_{\mathrm{x}}, \mathrm{R}_{\mathrm{y}}, \mathrm{R}_{\mathrm{z}}$ : Rotation toward $\mathrm{x}, \mathrm{y}$ and z axis respectively.
$\mathrm{T}_{\mathrm{x}}, \mathrm{T}_{\mathrm{y}}, \mathrm{T}_{\mathrm{z}}$ : Translation toward $\mathrm{x}, \mathrm{y}$ and z axis respectively.
$x^{2}-y^{2}, z^{2}$ : For polarizad symmetry species,
$x^{2}-y^{2}, x y, x z, y z$ : For despolarized symmetry species.
There aren't found the symmetry species labels from the results calculations of $\mathrm{D}_{7 \mathrm{~h}}$ point group. This is where the projection operator comes in. The formula for a projection operator is:
$\mathrm{a}_{\mathrm{i}}=(\mathbf{1} / \mathbf{g}) \sum n \mathrm{X}_{\mathrm{R}} \mathrm{X}_{\mathrm{i}}$
Where $\mathbf{a}_{\mathbf{i}}$ is the projection operator for the irreducible representation, or the number of the symmetry species.
$\mathbf{g}$ : is the number of symmetry operations for the concerned point group.
$\mathbf{n}$ : the number of repeatable of the symmetry operation.
$\boldsymbol{X}_{\boldsymbol{R}}:$ is the character of operation $R$ in the irreducible representations/ un-shifted atoms.
$\boldsymbol{X}_{i}$ : is the transformation number.
In practice, we need to take one of the basis vectors (or functions) and carry out all of the symmetry operations of the point group on that vector (or function). Then we multiply the result for each symmetry operation by the character of that operation in the irreducible representation for which we are forming the symmetry species. Finally, these products are all summed, and the result gives the required total degree of freedom ( $\boldsymbol{\Gamma}_{\text {tot }}$.).
For the [7] cyclacene (linear) monoring molecule Figure-1, the number of the symmetry species can be calculated depending on the constructed character table of $\mathrm{D}_{7 \mathrm{~h}}$ point group and by applying equation 1 $[1,2]$ on the geometrical structures of the studied molecule:


Figure 1- Equilibrium geometry structure of [7] cyclacene (linear) monoring molecule: a) Two dimensions, b) Three dimensions
$\mathbf{a}_{(\overline{\mathbf{A}} 1)}=(1 / 28)[(3 \times 42 \times 1 \times 1)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(7 \times 6 \times 1 \times 1)]=(126+42) / 28=168 /$ $28=6 \mathbf{Q}$ (normal coordinates for the vibration frequencies)
$\mathbf{a}_{(\mathbf{A} 2)}=(1 / 28)[(3 \times 42 \times 1 \times 1)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(7 \times 6 \times 1 \times(-1))]=(126-42) / 28=84 /$ $28=3=\mathbf{1 A}^{\prime}{ }_{\mathbf{2}}\left(\mathbf{R}_{\mathbf{z}}\right)+\mathbf{2 Q}$ (vibration frequencies)
$\left.\mathbf{a}_{(\text {É1 } 1)}=(1 / 28)[(3 \times 2 \times 42 \times 1 \times 1)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0))\right]=(126 \times 2) / 28=9=\mathbf{1 E}_{\mathbf{1}}\left(\mathbf{T}_{\mathbf{x}}\right.$, $\left.\mathrm{T}_{\mathrm{y}}\right)+8 \mathrm{Q}$ (vibration frequencies)
$\left.\mathbf{a}_{(\text {É } 2)}=(1 / 28)[(3 \times 2 \times 42 \times 1 \times 1)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0))\right]=(126 \times 2) / 28=9 \mathbf{Q}$ (vibration frequencies)
$\left.\mathbf{a}_{(\dot{\mathbf{E}} 3)}=(1 / 28)[(3 \times 2 \times 42 \times 1 \times 1)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0))\right]=(126 \times 2) / 28=9 \mathbf{Q}$ (vibration frequencies)
$\mathbf{a}_{\left(\mathbf{A}^{\prime \prime}\right)}=(1 / 28)[(3 \times 42 \times 1 \times 1)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(-42)]=(126-42) / 28=84 / 28=$ $3 Q$ (vibration frequencies)
$\mathbf{a}_{\left(\mathbf{A}^{\prime \prime}\right)}=(1 / 28)[(3 \times 42 \times 1 \times 1)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(-42)]=(126-42) / 28=84 / 28=$ $3 Q$ (vibration frequencies)
$\mathbf{a}_{\left(\mathbf{A}^{\prime \prime}\right)}=(1 / 28)[(3 \times 42 \times 1 \times 1)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(7 \times 6 \times 1 \times(+1))]=(126+42) / 28=$ $3=1 \mathrm{~A}_{2} \mathbf{2}^{\left(T_{\mathrm{z}}\right)+5 Q \text { (vibration frequencies) }}$
$\left.\mathbf{a}_{(\mathbf{E}}{ }^{\prime \prime}{ }_{10}=(1 / 28)[(3 \times 2 \times 42 \times 1 \times 1)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0))\right]=(126 \times 2) / 28=9=\mathbf{1 E}_{\mathbf{1}}{ }_{\mathbf{1}}$ $\left(R_{x}, R_{y}\right)+8 Q$ (vibration frequencies)
$\left.\mathbf{a}_{(\dot{\mathbf{E}} 2)}=(1 / 28)[(3 \times 2 \times 42 \times 1 \times 1)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0))\right]=(126 \times 2) / 28=9 \mathbf{Q}$ (vibration frequencies)
$\left.\mathbf{a}_{(\text {É } 3)}=(1 / 28)[(3 \times 2 \times 42 \times 1 \times 1)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0)+(0))\right]=(126 \times 2) / 28=9 \mathbf{Q}$ (vibration frequencies)
The net results are:
$\Gamma_{\text {tot }}=3 N=3 \times 42=126$
$\Gamma_{\text {tot }}=6 \mathrm{~A}^{\prime}{ }_{1}+3 \mathrm{~A}^{\prime}{ }_{2}+9 \mathrm{E}_{1}+9 \mathrm{E}_{2}+9 \mathrm{E}_{3}+3 \mathrm{~A}^{\prime \prime}{ }_{1}+6 \mathrm{~A}^{\prime \prime}{ }_{2}+9 \mathrm{E}^{\prime \prime}{ }_{1}+9 \mathrm{E}^{\prime \prime}{ }_{2}+9 \mathrm{E}^{\prime \prime}{ }_{3}$
$\Gamma_{\text {tot }}$ : Total degrees of freedom.
N : Number of atoms of the calculated molecule.
$\Gamma_{\text {Rot }}$ : Total degrees of rotation.
$\Gamma_{\text {Rot }}=A^{\prime}{ }_{2}\left(\mathbf{R}_{z}\right)+E^{\prime \prime}{ }_{1}\left(R_{x}, R_{y}\right)=3$
$\Gamma_{\text {Trans. }}=1 \mathrm{~A}_{2}^{\prime \prime}\left(\mathrm{T}_{\mathrm{z}}\right)+\mathbf{1 E}_{\mathbf{1}}\left(\mathrm{T}_{\mathrm{x}}, \mathrm{T}_{\mathrm{y}}\right)=3$
$\Gamma_{\text {Trans }}$ : Total degrees of translation
$\Gamma_{\text {vib }}$ : Total degrees of vibration
$\Gamma_{\text {vib. }}=\Gamma_{\text {tot }}\left(\Gamma_{\text {Rot }}+\Gamma_{\text {Trans. }}\right)=3 \mathrm{~N}-6=126-6=120$
$\Gamma_{\text {vib. }}=6 A^{\prime}{ }_{1}+2 A^{\prime}{ }_{2}+8 \dot{E}_{1}+9 E_{2}+9 \dot{E}_{3}+3 A^{\prime \prime}{ }_{1}+5 A^{\prime \prime}{ }_{2}+8 E^{\prime \prime}{ }_{1}+9 E^{\prime \prime}{ }_{2}+9 E^{\prime \prime}{ }_{3}=120$
$\Gamma_{+\sigma h}$ : Symmetric modes of vibrations (with respect to $\sigma \mathrm{h}$ ).
$\Gamma_{+\sigma h}=6 A^{\prime}{ }_{1}+2 A^{\prime}{ }_{2}+8 E_{1}+9 \dot{E}_{2}+9 \dot{E}_{3}=60$
': In-plane of themolecule.
$\Gamma_{-\sigma h}$ :Asymmetric modes of vibrations (with respect to $+\sigma h$ ).
$\Gamma_{- \text {бh }}:=3 A_{1}{ }_{1}+5 \mathrm{~A}_{2}+8 \mathrm{E}_{1}{ }_{1}+9 \mathrm{E}_{2}{ }_{2}+9 \mathrm{E}_{3}{ }_{3}=60$
": Out of-plane of the molecule.
To study the vibration motions of [7] cyclacene monoring molecule, one has to define its geometric parameters and has to distinguish between the axial $\mathrm{CCa}(\mathrm{C}-\mathrm{Caxial})$ and circumferential $\mathrm{CC}_{\mathrm{c}}(\mathrm{C}-$ Ccircumferential) bonds. Figure-2, Table-2.This was done after calculating the geometrical optimization of
the molecule using PM3 and DFT method with the level of (B3LYP/ 6-31G and 6-311G bases set) [11-17]. Table 1 shown the following relationship:

- Length of circumferences bonds $<$ length of axial bonds.
- K(force constant) of the circumferences bonds $>\mathrm{K}$ (force constant) of the axial bonds.
- Zero value of cis dihedral angle bonds for CCCC indicates that the surface included carbon atoms is
a planar surface, unlike the surface included the CCCH atoms, it is not planar surface (of different values than zero for its cis dihedral angle bond).
- Lower value for $\Delta \mathrm{E}_{\text {номо-цимо }}$ of the [7] cyclacene nanotube molecule indicates to be a very good conductor for electronic charge [18].


Figure 2- Repetitive sections of bonds and angles of [7]cyclacene (zigzag or linear); monoring molecule of $\mathrm{D}_{7 \mathrm{~h}}$ point group, with the numbering of atoms

Table 2- PM3 and DFT calculations of the bond distances, angles, dihedral angles and some of the physical properties of the linear [7]cyclacene monoring molecule.

| Bond length ( $\AA$ ) and bond angles (deg.) and dihedral angle bond (deg.) description | PM3 | $\begin{gathered} \text { DFT [19-21] } \\ \text { B3LYP/ 6-31G } \end{gathered}$ | $\begin{gathered} \hline \text { DFT [19-21] } \\ \text { B3LYP/ 6- } \\ 311 \mathrm{G} \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| C2-H1 | 1.09539 | 1.08670 | 1.08341 |
| C2-C3, C2-C4 (circumferences bonds) | 1.40838 | 1.41815 | 1.41564 |
| C3-C5, C4-C6 (axial bonds) | 1.44490 | 1.47358 | 1.47347 |
| <C3C2H1, <C4C2H1 | 118.19543 | 118.19189 | 118.19539 |
| <C3C2C4 | 117.59046 | 117.44495 | 117.59047 |
| < H 1 C 2 C 3 C 5 (trans) | -178.06437 | -178.06437 | -178.06513 |
| $<\mathrm{H1C2C3C10}$ (cis) | 31.72869 | 23.23397 | 23.23212 |
| <C2C4C6C11, < C2C3C5C12 (trans) | -157.80963 | -154.71581 | -154.64502 |
| <C2C3C5C7, < C2C4C6C7 (cis) | 0.0000 | 0.0000 | 0.0000 |
|  |  |  |  |
| Molecular formula | $\mathrm{C}_{28} \mathrm{H}_{14}$ |  |  |
| Point group | $\mathrm{D}_{7 \mathrm{~h}}$ |  |  |
| M.wt. (gm/mol) | 350.4 |  |  |
| $\Delta \mathrm{H}_{\mathrm{f}}^{0}(\mathrm{kcal} / \mathrm{mol})$ | 340.074 | --------- | --------- |
| $\mathrm{E}_{\text {total }}(\mathrm{a} . \mathrm{u})$ | 1075.07816 | -1075.78040 | -1075.07816 |
| Electronic energy (eV) | -32416.59007 | --------- |  |
| Ionization potential (eV) | 4.4323 | 4.1443 | 4.43229 |
| $\mathrm{E}_{\text {номо ( }}(\mathrm{eV}$ ) | -6.9296 | -4.1443 | -4.43229 |
| $\mathrm{E}_{\text {LUMO }}(\mathrm{eV})$ | -2.5028 | -2.5021 | -2.82134 |
| $\Delta \mathrm{E}_{\text {номо-иммо }}=\mathrm{E}_{\text {LUмо }}-\mathrm{E}_{\text {номо }}(\mathrm{eV})$ | 4.4268 | 1.6422 | 1.6109 |
| Dipole moment (Debye) | 0.0000 | 0.0000 | 0.0000 |
| Diameter ( A ) | 5.57186 | 5.63296 | 5.62630 |
| Length (A) | 4.97913 | 5.00636 | 4.99476 |

## Classification of vibration frequencies

The infrared spectrum of the (3N-6) normal modes of vibration for the [7] cyclacene monoring molecule was calculated and classified (symmetrically and equivalently) according to the character table of $\mathrm{D}_{7 \mathrm{~h}}$ point group. They are $60\left(\mathbf{2 A}_{\mathbf{2}}, \mathbf{6} \mathbf{A}^{\prime}, \mathbf{9} \mathbf{E}_{\mathbf{3}}, \mathbf{9} \mathbf{E}_{\mathbf{2}}, \mathbf{8} \mathbf{E}_{\mathbf{1}}\right)$ symmetric modes of vibrations (with respect to $\left.\sigma_{h}\right)$ and $60\left(\mathbf{6}^{\prime \prime} \mathbf{A}_{\mathbf{2}}, \mathbf{2}^{\prime \prime} \mathbf{A}_{1}, \mathbf{9}^{\prime \prime} \mathbf{E}_{\mathbf{3}}, \mathbf{9}^{\prime \prime} \mathbf{E}_{\mathbf{2}}, \mathbf{8}^{\prime} \mathbf{E}_{\mathbf{1}}\right)$ asymmetric modes of vibrations (with respect to $\sigma_{h}$ ), Table 3. Relative to the $\sigma_{h}$ reflection the modes of vibration are classified to:

## $a$ - Modes of vibrations with $\left(+\sigma_{h}\right)$ symmetry operation

They are 60 modes of vibration in number, of which 18 are Raman active ( $\mathbf{6 A ^ { \prime }} \mathbf{1}$ (polarized) $+\mathbf{9 E}_{3}{ }^{\prime}$ (depolarized) ), and 16 IR active ( $8 \mathbf{E}_{\mathbf{1}}^{\prime}$ ). The rest are both IR and Raman inactive.
$\Gamma_{+\sigma h}=6 A^{\prime}{ }_{1}+2 A^{\prime}{ }_{2}+8 E_{1}+9 \dot{E}_{2}+9 \dot{E}_{3}=60$
$\Gamma_{+\sigma h}$ : Symmetric modes of vibrations (with respect to $+\sigma_{h}$ ).
They are classified as the following:
The ( $\mathbf{C - H}$ ) stretching vibrations
They are 14 modes of vibration. The frequency values range from ( $3066-3067 \mathrm{~cm}^{-1}$ ) on using PM3 method, and range from ( $3056-3066 \mathrm{~cm}^{-1}$ ) on using DFT method Table 3, showing the following correlations:
$v_{\text {sym }}\left(\mathrm{CH}_{\text {str. }}\right)\left(3066 \mathrm{~cm}^{-1}\right)<v_{\text {asym }}\left(\mathrm{CH}_{\text {str. }}\right)\left(3067 \mathrm{~cm}^{-1}\right)(\mathrm{PM} 3)$
$\mathbf{v}_{\text {sym }}\left(\mathrm{CH}_{\text {str. }}\right)\left(3066 \mathrm{~cm}^{-1}\right)>v_{\text {asym }}\left(\mathrm{CH}_{\text {str. }}\right)\left(\mathbf{3 0 5 6 c m}^{-1}\right)(\mathrm{DFT})$
The highest intensity is ( $70.6065 \mathrm{~km} / \mathrm{mol}$ ) for $v_{64}$.

## The CC stretching vibrations

They are 13 modes of vibration of (Nc-1) in number, where Nc : is the number of carbon atoms in the molecule. The frequency values range from ( $1638-1681 \mathrm{~cm}^{-1}$ ) on using PM3 method, and range from (1584-1474 $\mathrm{cm}^{-1}$ ) on using DFT method Table 3, showing the following correlations:
$v_{\text {sym }}\left(\mathrm{CC}_{\text {str. }}\right)\left(1659 \mathrm{~cm}^{-1}\right)<v_{\text {asym }}\left(\mathrm{CC}_{\text {str }}\right)\left(1661 \mathrm{~cm}^{-1}\right)(\mathrm{PM} 3)$
$\boldsymbol{v}_{\text {sym }}\left(\mathrm{CC}_{\text {str }}\right.$.) $\left(1515 \mathrm{~cm}^{-1}\right)>\nu_{\text {asym }}\left(\mathrm{CC}_{\text {str. }}\right)\left(1507 \mathrm{~cm}^{-1}\right)$ (DFT)
The highest intensity is $(1.4881 \mathrm{~km} / \mathrm{mol})$ for $v_{11-12}$, Table-3.

## Ring (CCC) stretching vibrations

The CCC modes of vibration (unlike the C-C vibration modes), are not located at definite C atoms as can be seen from the atomic displacement vectors Figure-2. Their frequencies are usually lower than those of $\mathrm{C}-\mathrm{C}$ vibration modes.
According to the calculated results, the range of these frequencies is $\left(1402-1600 \mathrm{~cm}^{-1}\right)$ on using PM3 method, and range from ( $1387-1411 \mathrm{~cm}^{-1}$ ) on using DFT method.Thehighest intensity is ( 45.1853 $\mathrm{km} / \mathrm{mol}$ ) for $v_{65}$, Table-3.
The bending ( $\delta \mathrm{CH}$ ) vibrations
They are 14 modes of vibration of $\left(\mathrm{N}_{\mathrm{H}}\right)$ in number, where $\mathrm{N}_{\mathrm{H}}$ is the number of hydrogen atoms in the molecule. The frequency values range from (1121-1337 $\mathrm{cm}^{-1}$ ) on using PM3 method, and range from $\left(1035-1311 \mathrm{~cm}^{-1}\right)$ on using DFT method.The highest intensity is $(2.8088 \mathrm{~km} / \mathrm{mol})$ for $v_{15-16}$, Table-3.

## The bending ( $\delta C C C$ ) vibrations

The frequency values range from ( $492-1419 \mathrm{~cm}^{-1}$ ) on using PM3 method, and range from (488$1321 \mathrm{~cm}^{-1}$ ) on using DFT method. The highest intensity is $(0.0974 \mathrm{~km} / \mathrm{mol})$ for $v_{19-20}$, Table 3 .

## b- Modes of vibrations with ( $-\boldsymbol{\sigma}_{h}$ ) symmetry operations

They are 60 modes of vibration in number, of which 18 are Raman active ( $\mathbf{P E}^{\mathbf{\prime}} \mathbf{1}_{\mathbf{1}}$ (depolarized) $)$, and 5IR active $\left(\mathbf{5} \mathbf{A}_{\mathbf{2}}\right)$. The rest are both IR and Raman inactive.
$\Gamma_{-\sigma h}$ :Asymmetric modes of vibrations (with respect to $-\sigma_{h}$ ).
$\Gamma_{- \text {бh }}:=3 A^{\prime \prime}{ }_{1}+5 A^{\prime \prime}{ }_{2}+8 E_{1}{ }_{1}+9 E^{\prime \prime}{ }_{2}+9 E^{\prime \prime}{ }_{3}=60$
They are classified as the following:

## The $(\gamma \mathrm{CH})$ out of planeof the molecule bending vibrations

They are 14 modes of vibration. The frequency values range from ( $859-996 \mathrm{~cm}^{-1}$ ) on using PM3 method, and range from ( $851-957 \mathrm{~cm}^{-1}$ ) on using DFT method. The highest intensity is ( 275.5610 $\mathrm{km} / \mathrm{mol}$ ) for $\mathrm{v}_{17-18}$, Table-3.
The ( $\gamma \mathrm{CC}$ ) and ( $\gamma \mathrm{CCC}$ ) out of plane of the molecule bending vibrations
They have the lowest values for modes of vibration. The frequency values range from $\left(146-821 \mathrm{~cm}^{-1}\right)$ on using PM3 method, and range from ( $151-801 \mathrm{~cm}^{-1}$ ) on using DFT method. The highest intensity is $(40.6089 \mathrm{~km} / \mathrm{mol})$ for $v_{21-22}$, Table-3.

The intensity for the active modes of vibration is obtained only for the IR spectrum when using the Gaussian calculations program.Figure-3 shown the graphical pictures of some vibration modes for [7]cyclacene (zigzag) nanotube molecule as calculated applying the DFT (B3LYP/ 6-31G) method.Scheme 1 shows theIR spectrum for [7]cyclacene (zigzag) molecule as calculated applying the (DFT/ 6-31G) method.
Note: There isn'tany description for symmetry species or modes of vibration could be obtained from the result calculations of the IR vibration spectrum on using PM3 neither DFT method.

Table 3-The calculated vibration frequencies and IR absorption intensities for [7]cyclacene (linear) molecule using PM3 and DFT methods


| $\mathrm{A}_{1}{ }^{\prime \prime}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{v}_{61}$ | CC ${ }_{\text {Strr }}$ axial | 1562 | 1486 | ---- |
| $v_{62}$ | $\delta \mathrm{CH}$ (Rock) + סring (clockanticlock) | 1121 | 1035 | ------- |
| $v_{63}$ | $\gamma$ ring ( $\gamma \mathrm{CCC}$ ) clockanticlock | 453 | 428 | ------- |
| $\mathbf{A}_{2}{ }^{\prime \prime}$ |  |  | IR active |  |
| $v_{64}$ | CH ${ }_{\text {Str }}$. | 3064 | 3063 | 70.6065 |
| $v_{65}$ | סring elongation | 1419 | 1321 | 45.1853 |
| $v_{66}$ | $\gamma \mathrm{CH}$ (Twist.) | 951 | 897 | 664.795 |
| $\mathrm{v}_{67}$ | $\gamma$ ring ( $\gamma$ CCC) ( Puck.) | 790 | 801 | 152.1273 |
| $\mathrm{v}_{68}$ | $\gamma \mathrm{ring}(\gamma \mathbf{C C C})($ Puck. $)$ | 379 | 358 | 16.5256 |
| $\mathbf{E}_{1}{ }^{\prime \prime}$ |  |  | Raman active |  |
| $\mathrm{v}_{69}, 70$ | $\mathrm{CH}_{\text {Str }}$. | 3064 | 3056 | ---- |
| $\mathrm{v}_{71,72}$, | CC ${ }_{\text {Str}}$. | 1638 | 1574 | ------ |
| $\mathrm{v}_{73,74}$ | $\delta \mathrm{CH}$ (Sciss)+ $\mathrm{CCC}_{\text {str}}$. | 1402 | 1483 | ------- |
| $\mathrm{v}_{75,75}$ | 8CH | 1145 | 1209 | ------- |
| $\mathrm{v}_{77}, 78$ | $\gamma \mathrm{CH}$ (Wag.) | 942 | 911 | ------- |
| $\mathrm{v}_{79,80}$ | $\gamma$ ring $+\gamma \mathbf{C H}$ (Twist.) | 802 | 814 | ------- |
| $\mathrm{v}_{81,82}$ | סring (8CCC) | 625 | 642 | ---- |
| $\mathrm{V}_{83},{ }_{84}$ | $\gamma$ ring ( $\gamma \mathrm{CCC}$ ) | 306 | 326 | ------- |
| $\mathbf{E}_{2}{ }^{\prime \prime}$ |  |  |  |  |
| $V_{85}{ }^{\text {\% }} 86$ | $\mathrm{CH}_{\text {Str}}$. | 3065 | 3057 | ------- |
| $\mathrm{V}_{87}$, 88 | CC ${ }_{\text {Strr }}$. | 1649 | 1584 | -- |
| $\mathrm{V}_{89,90}$ | $\mathrm{CCC}_{\text {str}}$. | 1415 | 1408 | ---- |
| $\mathrm{v}_{91}, 92$ | $\delta \mathbf{C H}$ (Sciss.) + ¢ring | 1196 | 1262 | ---- |
| $v_{93,94}$ | $\gamma \mathrm{CH}$ (Twist.) | 902 | 907 | ------- |
| $\nu_{95,96}$ | $\gamma \mathrm{CH}$ (Twist.) | 859 | 851 | ---- |
| $\mathrm{V}_{97}, 98$ | $\gamma$ ring ( $\gamma \mathrm{CC}$ ) (Puck.) | 688 | 715 | -- |
| $v_{99,100}$ | $\gamma \mathrm{ring}(\gamma \mathrm{CCC})($ Puck.) | 394 | 431 | ----- |
| $\mathrm{v}_{101}, 102$ | $\gamma \mathrm{ring}(\gamma \mathrm{CCC})($ Puck.) | 146 | 151 | ----- |
| $\mathbf{E}_{3}{ }^{\prime \prime}$ |  |  |  |  |
| $\mathrm{V}_{103,104}$ | CH ${ }_{\text {str}}$. | 3065 | 3063 | ------- |
| $v_{105} 106$ | CC ${ }_{\text {Str}}$. | 1681 | 1574 | ------- |
| $v_{107,108}$ | CC ${ }_{\text {str }}$ | 1541 | 1515 | ------- |
| $v_{109}{ }^{1110}$ | ¢ring $+\delta \mathbf{C H}$ (Sciss.) | 1220 | 1299 | ------- |
| $v_{111}, 112$ | $\gamma \mathrm{CH}$ (Wag.) | 872 | 897 | ------- |
| $v_{113}, 114$ | סring (8CCC) | 813 | 815 | ------- |
| $\mathrm{v}_{115}, 116$ | $\gamma \mathrm{ring}(\gamma \mathrm{CCC})($ Puck.) | 678 | 679 | ------- |
| $v_{117}, 118$ | סring ( $8 \mathbf{C C C}$ ) | 492 | 488 | ---- |
| $v_{119,120}$ | $\gamma \mathrm{ring}(\gamma \mathrm{CCC})($ Puck.) | 308 | 326 | ------- |

$\gamma$. Out of-plane of the molecule., $\delta:$ In-plane of the molecule.
(Breath.): ring breathing mode, (Puck.): ring puckering mode, (Sciss.): CH scissoring mode.(Rock.):
CH rocking mode. (Twist.): CH twisting mode, (Wag.): CH wagging Mode.
Scaling factors: 0.96 (CH str.) for all DFT (B3LYP/6-31G) frequencies, [22].


Figure 3- The graphical pictures of some IR modes of vibration for [7]cyclacene (linear) molecule as calculated applying Gauss view program


Scheme 1- IR spectrum for [7]cyclacene (zigzag) molecule as calculated applying the (DFT/ 6-31G) method.

The arrow in the center of the molecule indicates the direction of the dipole moment of the active mode of vibration in the infrared spectrum].
And similarly to the single wall carbon nanotubes (SWCNs) [18, 23-26], the calculations of electronic charge for [7]cyclacene (linear) molecule (regardless of the calculation method, PM3, DFT (B3LYP/631G) or DFT (B3LYP/6-311G)) also shown that the Mulliken electronic charge densities are mainly concentrated at the outer circumferential carbon atoms, and the axial carbon atoms have diminishing charges. The H atoms are of positively charged, whereas the C atoms are of the negative charge, Figure-4.


Figure 4- Mulliken distribution of charge density at the atoms of [7]cyclacene (linear) monoring molecule as calculated by using (PM3, DFT (B3LYP/6-31G and DFT (B3LYP/6-311G) methods.

## Conclusion

The important of symmetry is giving as the following:

- Ideas about symmetry are of great important in connection with both theoretical and experimental
studies of molecular structures.
- The basic principles of symmetry are applied in quantum mechanics, spectroscopy, and structural determination.
- When symmetry is present, certain calculations are simplified, if symmetry is taken into account.
- Aspects of symmetry also determine whether a molecule can be optically active or whether it may have a dipole moment.
- Modern treatments of rotation, vibration, and electronic spectroscopy of molecules, all make extensive use of group theory.


## The important of the character table (table of eigen values) is for:

-Determining the symmetry species of the total degrees of freedom(transition, rotation, and vibration).

- Identify the active and inactive modes of vibration in the IR and Raman spectra.
- Identify the polarized and non-polarized modes of vibration in the Raman spectra.
-Determiningthe in-plane and out of plane modes of vibration of the molecule.
-Determining of many physical properties related to compounds and molecules and simplified the determination of the lengths, angles and electronic charge at atoms of the molecules.
- The construction character table of the $\mathrm{D}_{7 \text { h }}$ point group in this study, which is not found in the literature, and their symmetry species are not included in the Gaussian program could be used forextracting and classifying the symmetry species forthe total degrees of freedom. (degrees of transition movements, degrees of the rotation movements, and degrees of modes of vibration with the number of ( $3 \mathrm{~N}-6$ )for the monoring molecule (zigzag nanotube molecule)).
- The complete physical image of $\mathrm{D}_{7 \mathrm{~h}}$ nanotube could be described from the calculated bond lengths and angles, energy values and the distribution of the charges after obtaining the optimization geometric shape according to the method of calculations of quantum mechanics DFT using the Gaussian 09 program.


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