Parameters Estimation for Modified Weibull Distribution Based on Type One Censored Samplest

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Abstract
The three parameters distribution called modified weibull distribution (MWD) was introduced first by Sarhan and Zaindin (2009)[1]. In theis paper, we deal with interval estimation to estimate the parameters of modified weibull distribution based on singly type one censored data, using Maximum likelihood method and fisher information to obtain the estimates of the parameters for modified weibull distribution, after that applying this technique to asset of real data which taken for Leukemia disease in the hospital of central child teaching .

Keywords: Modified weibull distribution, Maximum likelihood estimation, interval estimation, Fisher information, survival function, hazard function, single type censored samples.

تقدير المعلمات التوزيع ويبل المعدل بالاعتماد على العينات المراقبة من النوع الأول

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الخلاصة
لقد اوجد الباحثان (sarhan and zaindin) في عام (٢٠٠٩) توزيع معدل عن توزيع ويبل سمي توزيع ويبل المعدل . في هذا البحث سوف تقدر معلمات الثلاثة لهذا التوزيع بالتقدير الفترة بالاعتماد على بيانات المراقبة من النوع الأول باستخدام طريقة الإمكان الأعظم ومن ثم استخدام مصفوفة معلومات (fisher) للحصول على التقدير الفترة للمعلمات الثلاثة في هذا التوزيع وبعد ذلك تم تقدير دالة الوفاة، ودالة البقاء، ودالة المخاطرة لهذا التوزيع وقد تم الحصول على النتائج العددية لجميع هذه المقدرات والدوال من خلال استخدام مجموعة من البيانات الحقيقية .

1-Introduction
The modified weiball distribution was first introduced by Sharan and Zaindin (2009) [1, 2]. This is a very important distribution that it can be used to describe several reliability models. This distribution contains one scale parameter α and two shape parameters λ and γ respectively

The authers in [1, 2] introduced modified weibull distribution and prove some basic properties. Zaindin [3] estimate parameters by MLEM based on type two censored data. He considered rank sampling to estimate parameters based on MLEM, AlHadhrami ;

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(2010), [4] presented estimators of parameter based on type one Censored data by using MLEM, Gasmi and Berzig ;(2011), [5] presented the hazard rate function of MWD is constant if γ=1, increasing if γ >1and decreasing if γ <1.The aim of this study is derive and estimate the unknown parameters of the MWD(α,λ,γ **)** for singly type one censored samples by using maximum likelihood method then derive and estimated asymptotic interval estimate for the unknown three parameters of MWD(α,λ,γ) using the fisher information matrix . As well as deriving and estimating the death density function, survival function and hazard function applied and fit this distribution to a set of real data to Calculate the values of estimators for parameters and compute the values estimate of survival and hazard function. The rest of paper is organized as follows. In section two definition and some properties of modified wediull distribution. In section three deriving point estimation for unknown parameters of MWD(α,λ,γ) .In section four deriving the interval point estimation for the unknown parameters of MWD(α,λ,γ) . In section five application of real data set compute the estimation survival function and hazard function. Finally we conclude the paper in section six.

2-Definition and Properties of MWD:

The cdf of the MWD(α,λ,γ) take follows from:

$$F(t;\alpha,\lambda,\gamma)=1-e^{(-\alpha t-\lambda t^{\gamma})} \quad t>0$$

The pdf of the MWD(α,λ,γ) is:

$$f(t;\alpha,\lambda,\gamma)=\begin{cases}(-\alpha t-\lambda t^{\gamma})e^{(-\alpha t-\lambda t^{\gamma})} & t>0\\ 0 & o.w\end{cases} \quad \ldots(2.1)$$

Where the parameter space is

$$\Omega=\{(\alpha,\lambda,\gamma);\alpha,\lambda\geq 0;\gamma>0\}$$

The mean of MWD(α,λ,γ) is:

$$\mu_1=E(t)=\sum_{i=0}^{n}\frac{(-\lambda)^i}{i!}\left[\frac{\Gamma(i\gamma+2)}{\alpha^{1+\gamma i}}+\frac{\lambda\gamma\Gamma(\gamma(1+i)+1)}{\alpha^{1+\gamma(1+i)}}\right]$$

The variance of the MWD(α,λ,γ) is:

$$\sigma^2=\operatorname{var}(t)=$$

$$\sum_{i=0}^{n}\frac{(-\lambda)^i}{i!}\left[\frac{\Gamma(i\gamma+3)}{\alpha^{3+i\gamma}}+\frac{\lambda\gamma\Gamma((i+1)\gamma+2)}{\alpha^{2+(1+i)\gamma}}\right]$$

$$-\sum_{i=0}^{n}\frac{(-\lambda)^{2i}}{(i!)^2}\left[\frac{\Gamma(i\gamma+2)}{\alpha^{2+i\gamma}}+\frac{\lambda\gamma\Gamma((i+1)\gamma+1)}{\alpha^{1+(1+i)\gamma}}\right]^2$$

The survival function of MWD(α,λ,γ) takes following form:

$$s(t;\alpha,\lambda,\gamma)=e^{(-\alpha t-\lambda t^{\gamma})} \quad , t\geq 0 \ldots (2.2)$$

The hazard rate function of MWD(α,λ,γ) is:

$$h(t;\alpha,\lambda,\gamma)=(\alpha+\lambda\gamma t^{\gamma-1}) \quad , \quad t\geq 0$$

3-Parameters Estimation:

In this section, we use the maximum likelihood method to derive interval estimation for the unknown parameters of the MWD(α,λ,γ) based on Fisher information matrix which consist of the life times that failed before say $0\leq t_1\leq t_2\leq \ldots\leq t_r\leq t_0$.where r is the number of units failed before t_0 and the number of units r that failed before t_0 and n is total number of observation.

3-1-Point Estimation:

The likelihood function of $0\leq t_1\leq t_2\leq \ldots\leq t_r$ is:

$$L=\frac{n!}{(n-r)!}\prod_{i=1}^{r} f(t;\alpha,\lambda,\gamma)[s(t_0,\alpha,\lambda,\gamma)]^{n-r} \quad (3.1)$$

$$,0\leq t_1\leq t_2\leq \ldots\leq t_r\leq t_0$$

Where, $s(t_0;\alpha,\lambda,\gamma)$ is the corresponding survival distribution; substituting (2.1), (2.2) in to (3.1) we get:

$$L=\frac{n!}{(n-r)!}\left[\prod_{i=1}^{r}(\alpha+\lambda\gamma t^{\gamma})e^{(-\alpha t-\lambda t_0^{r})^{n-r}}\right]$$

$$\left[e^{(-\alpha t_0-\lambda t_0^{r})}\right]^{n-r}$$

Where $a=\dfrac{n!}{(n-r)!}$

$$L=a^r\left[\prod_{i=1}^{r}(\alpha+\lambda\gamma t_i^{\gamma-1})e^{(-\alpha t-\lambda t_0^{r})}\left[e^{(-\alpha t_0-\lambda t_0^{\gamma})^{n-r}}\right]\right] \ldots(3.2)$$

Taking logarithm for equation (3.2) we get :

$$\ln L = r\ln a + \sum_{i=1}^{r}\ln(\alpha+\lambda\gamma t_i^{\gamma-1}) - \alpha\sum_{i=1}^{r} t_i \qquad (3.3)$$
$$-\lambda\sum_{i=1}^{r} t_i^{\gamma} - (n-r)(\alpha t_0 + \lambda t_0^{\gamma})$$

We take the first derivative (3.3) with respect to α,λ,γ and equating each equation to zero, then we get three nonlinear respectively as follows:

$$\frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^{r}\frac{1}{(\alpha+\lambda\gamma t_i^{\gamma-1})} - \sum_{i=1}^{r} t_i - (n-r)t_0$$

$$\frac{\partial \ln L}{\partial \lambda} = \sum_{i=1}^{r}\frac{\gamma t_i^{\gamma-1}}{(\alpha+\lambda\gamma t_i^{\gamma-1})} - \sum_{i=1}^{r} t_i^{\gamma} - (n-r)t_0^{\gamma}$$

$$\frac{\partial \ln L}{\partial \gamma} = \sum_{i=1}^{r}\frac{\lambda[\gamma t_i^{\gamma-1}\ln t_i + t_i^{\gamma-1}]}{(\alpha+\lambda\gamma t_i^{\gamma-1})} - \sum_{i=1}^{r} t_i^{\gamma}$$
$$-(n-r)\lambda t_0^{\gamma}\ln t_0$$

$$\sum_{i=1}^{r}\frac{1}{(\hat\alpha+\hat\lambda\hat\gamma t_0^{\hat\gamma-1})} - \sum_{i=1}^{r} t_i - (n-r)t_0 = 0 \quad \dots (3.4)$$

$$\sum_{i=1}^{r}\frac{\hat\gamma t_i^{\hat\gamma-1}}{\hat{(}\hat\alpha+\hat\lambda\hat\gamma t_i^{\hat\gamma-1})} - \sum_{i=1}^{r} t_i^{\gamma} - (n-r)t_0^{\hat\gamma-1} = 0 \quad \dots (3.5)$$

$$\sum_{i=1}^{r}(\frac{\hat\lambda[\hat\gamma t_I^{\hat\gamma-1}\ln t + t_i^{\hat\gamma-1}{}_i]}{(\hat\alpha+\hat\lambda\hat\gamma t_i^{\hat\gamma-1})} - \sum_{i=1}^{r} t_i^{\hat\gamma} - \quad \dots \qquad (3.6)$$
$$(n-r)\lambda t_0^{\hat\gamma}\ln t_0 = 0$$

To find the maximum likelihood estimations for α,λ,γ we must solve the system of three nonlinear equation (3.4),(3.5),(3.6) by using the iterative method. Such as Newtone –Raphson method to obtain the solution which is as follows:

$$\begin{bmatrix}\alpha_{i+1}\\ \lambda_{i+1}\\ \gamma_{i+1}\end{bmatrix} = \begin{bmatrix}\alpha_i\\ \lambda_i\\ \gamma_i\end{bmatrix} - J_i^{-1}\begin{bmatrix} f(\alpha)\\ g(\lambda)\\ z(\gamma)\end{bmatrix} \quad \dots \qquad (3.7)$$

$$f(\alpha) = \sum_{i=1}^{r}\frac{1}{(\alpha+\lambda\gamma t_i^{\gamma-1})} - \sum_{i=1}^{r} t_i - (n-r)t_0$$

$$g(\lambda) = \sum_{i=1}^{r}\frac{\gamma t_i^{\gamma-1}}{(\alpha+\lambda\gamma t_i^{\gamma-1})} - \sum_{i=1}^{r} t_i^{\gamma} - (n-r)t_0^{\gamma}$$

$$z(\gamma) = \sum_{i=1}^{r}\frac{\lambda[\gamma t_i^{\gamma-1}\ln t_i + t_i^{\gamma-1}]}{(\alpha+\lambda\gamma t_i^{\gamma-1})} - \sum_{i=1}^{r} t_i^{\gamma} -$$
$$(n-r)\lambda t_0^{\gamma}\ln t_0$$

Thus, J_i^{-1} is Jacobean matrix which is defined as follows:

$$J_i^{-1} = \begin{bmatrix} \frac{\partial f(\alpha)}{\partial\alpha} & \frac{\partial f(\alpha)}{\partial\lambda} & \frac{\partial f(\alpha)}{\partial\gamma}\\ \frac{\partial g(\lambda)}{\partial\alpha} & \frac{\partial g(\lambda)}{\partial\lambda} & \frac{\partial g(\lambda)}{\partial\gamma}\\ \frac{\partial z(\gamma)}{\partial\alpha} & \frac{\partial z(\gamma)}{\partial\lambda} & \frac{\partial z(\gamma)}{\partial\gamma}\end{bmatrix}$$

$$\frac{\partial f(\alpha)}{\partial\alpha} = \sum_{i=1}^{r}\frac{-1}{(\alpha+\lambda\gamma t_i^{\gamma-1})^2}$$

$$\frac{\partial f(\alpha)}{\partial\lambda} = \sum_{i=1}^{r}\frac{-\gamma t_i^{\gamma-1}}{(\alpha+\lambda\gamma t_i^{\gamma-1})^2}$$

$$\frac{\partial f(\alpha)}{\partial\gamma} = \sum_{i=1}^{r}\frac{-\lambda[\gamma t_i^{\gamma-1}\ln t_i + t_i^{\gamma-1}]}{(\alpha+\lambda\gamma t_i^{\gamma-1})^2}$$

$$\frac{\partial g(\lambda)}{\partial\alpha} = \sum_{i=1}^{r}\frac{-\gamma t_i^{\gamma-1}}{(\alpha+\lambda\gamma t_i^{\gamma-1})^2}$$

$$\frac{\partial g(\lambda)}{\partial\lambda} = \sum_{i=1}^{r}\frac{-\gamma^2 t_i^{2(\gamma-1)}}{(\alpha+\lambda\gamma t_i^{\gamma-1})^2}$$

$$\frac{\partial g(\lambda)}{\partial\gamma} = \sum_{i=1}^{r}\frac{-\lambda\gamma[\gamma t_i^{2(\gamma-1)}\ln t_i + t_i^{2(\gamma-1)}]}{(\alpha+\lambda\gamma t_i^{\gamma-1})^2}$$
$$-\sum_{i=1}^{r}\frac{-\lambda[\gamma t_i^{\gamma-1}\ln t_i + t_i^{\gamma-1}]}{(\alpha+\lambda\gamma t_i^{\gamma-1})} - \sum_{i=1}^{r} t_i^{\gamma}\ln t_i - (n-r)t_0^{r}\ln t_0$$

$$\frac{\partial z(\gamma)}{\partial\alpha} = \sum_{i=1}^{r}\frac{-\lambda[\gamma t_i^{\gamma-1}\ln t_i + t_i^{\gamma-1}]}{(\alpha+\lambda\gamma t_i^{\gamma-1})^2}$$

$$\frac{\partial z(\gamma)}{\partial\lambda} = \sum_{i=1}^{r}\frac{-\lambda\gamma[\gamma t_i^{2(\gamma-1)}\ln t_i + t_i^{2(\gamma-1)}]}{(\alpha+\lambda\gamma t_i^{\gamma-1})^2}$$
$$-\sum_{i=1}^{r}\frac{-\lambda[\gamma t_i^{\gamma-1}\ln t_i + t_i^{\gamma-1}]}{(\alpha+\lambda\gamma t_i^{\gamma-1})} - \sum_{i=1}^{r} t_i^{\gamma}\ln t_i - (n-r)t_0^{r}\ln t_0$$

$$\frac{\partial z(\gamma)}{\partial\gamma}=\sum_{i=1}^{r}\frac{\lambda[\gamma t_i^{\gamma-1}(\ln t_i)^2+2t_i^{\gamma-1}\ln t_i]}{(\alpha+\lambda\gamma t_i^{\gamma-1})}$$
$$-\sum_{i=1}^{r}\frac{\lambda^2[\gamma t_i^{\gamma-1}\ln t_i+t_i^{\gamma-1}]^2}{(\alpha+\lambda\gamma t_i^{\gamma-1})^2}-\sum_{i=1}^{r}t_i^{\gamma}(\ln t_i)^2$$
$$-(n-r)t_0^{r}(\ln t_0)^2$$

The absolute values of the difference between the new founded vales of parameters initial values are error terms, where the error are vary small value and calculate as follows:

$$\begin{bmatrix}\varepsilon(\alpha)_{i+1}\\ \varepsilon(\lambda)_{i+1}\\ \varepsilon(\gamma)_{i+1}\end{bmatrix}=\left\|\begin{bmatrix}\alpha_{i+1}\\ \lambda_{i+1}\\ \gamma_{i+1}\end{bmatrix}-\begin{bmatrix}\alpha_i\\ \lambda_i\\ \gamma_i\end{bmatrix}\right\|$$

4- Interval Estimation:

In this section we derive the approximated Fisher information matrix which is needed it to find confidence interval of the parameters based on the asymptotic distribution of their MLEM. Let $I(\theta)$ be the Fisher information matrix for the vectors of unknown parameters $\theta=\theta_1,\theta_2,\theta_3)$

Let $\theta_1=\alpha,\theta_2=\lambda,\theta_3=\gamma$

The elements of 3*3 matrix $I(\theta),I_{rs}(\theta)$ $,r,s\in[1,2,3]$, can be approximated by $I(\theta)$

$$I_{rs}(\theta)=\frac{-\partial^2\ln(t;\theta)}{\partial\theta_r\partial\theta_s}$$

Then the observed information matrix is given by:

$$I=\begin{bmatrix}\frac{-\partial^2\ln L(t;\alpha,\lambda,\gamma)}{\partial\alpha^2} & \frac{-\partial^2\ln L(t;\alpha,\lambda,\gamma)}{\partial\alpha\partial\lambda} & \frac{-\partial^2\ln L(t;\alpha,\lambda,\gamma)}{\partial\alpha\partial\gamma}\\ \frac{-\partial^2\ln L(t;\alpha,\lambda,\gamma)}{\partial\lambda\partial\alpha} & \frac{-\partial^2\ln L(t;\alpha,\lambda,\gamma)}{\partial\lambda^2} & \frac{-\partial^2\ln L(t;\alpha,\lambda,\gamma)}{\partial\lambda\partial\gamma}\\ \frac{-\partial^2\ln L(t;\alpha,\lambda,\gamma)}{\partial\gamma\partial\alpha} & \frac{-\partial^2\ln L(t;\alpha,\lambda,\gamma)}{\partial\gamma\partial\lambda} & \frac{-\partial^2\ln L(t;\alpha,\lambda,\gamma)}{\partial\gamma^2}\end{bmatrix}$$

We approximate then the variance-covariance matrix by inverting the information matrix:

$$V=\begin{bmatrix}\mathrm{var}(\hat\alpha) & \mathrm{cov}(\hat\alpha,\hat\lambda) & \mathrm{cov}(\hat\alpha,\hat\gamma)\\ \mathrm{cov}(\hat\lambda,\hat\alpha) & \mathrm{var}(\hat\lambda) & \mathrm{cov}(\hat\lambda,\hat\gamma))\\ \mathrm{cov}(\hat\gamma,\hat\alpha) & \mathrm{cov}(\hat\gamma,\hat\lambda) & \mathrm{var}(\hat\gamma)\end{bmatrix}$$

Then the MLEM $(\hat\alpha,\hat\lambda,\hat\gamma)$ is asymptotically normal distribution which given as

$$\begin{bmatrix}\hat\alpha\\ \hat\lambda\\ \hat\gamma\end{bmatrix}N\left(\begin{bmatrix}\alpha\\ \lambda\\ \gamma\end{bmatrix},\begin{bmatrix}\mathrm{var}(\alpha) & \mathrm{cov}(\alpha,\lambda) & \mathrm{cov}(\alpha,\gamma)\\ \mathrm{cov}(\lambda,\alpha) & \mathrm{var}(\lambda) & \mathrm{cov}(\lambda,\gamma)\\ \mathrm{cov}(\gamma,\alpha) & \mathrm{cov}(\gamma,\lambda) & \mathrm{var}(\gamma)\end{bmatrix}\right)\text{...(4.3)}$$

If we use the equation (4.3), approximate 100(1-α)% confidence intervals for the parameters α,λ,γ that given respectively as follows:

$$\hat\alpha_{lour}=\alpha-Z_\alpha\sqrt{\mathrm{var}(\hat\alpha)}\quad \hat\alpha_{upper}=\alpha+Z_\alpha\sqrt{\mathrm{var}(\hat\alpha)}$$
$$\hat\lambda_{lour}=\lambda-Z_\lambda\sqrt{\mathrm{var}(\hat\lambda)}\ ,\ \hat\lambda_{upper}=\lambda+Z_\lambda\sqrt{\mathrm{var}(\hat\lambda)}$$
$$\hat\gamma_{lour}=\gamma-Z\gamma\sqrt{\mathrm{var}(\hat\gamma)}\ ,\ \hat\gamma_{upper}=\gamma+Z\gamma\sqrt{\mathrm{var}(\hat\gamma)}$$

Where Z_θ is the upper percentile value of the stranded normal distribution.

5-Result and Discussion:

In this paper, based on real data for the Leukememia disease, we chose this type of disease because it is widespread and deadly those days in Iraq and this type of diseases has failure time (death time) occurs which is interesting phenomenon in this paper. The data for the Leukemia disease, are collected from the educational hospital in Bagdad .The time of study point in this paper determined from (1-4-2012) until (31-12-2012),the means the duration time of study is constant and fixed for the (9) Months or (275) days, the number of patients in the experiment for the Above duration time is (50).The (15) patients were dead (35) patients remain alive during the time of the study. When applying the test statistic (chi-square) in order to fit modified weibull distribution it the data, is found that the calculated value is (9.331645); when comparing this value with tabulated value (14.07) we find out that the calculated value isless than the value tabulated at level of significant (0.05) with (7) degrees of freedom that means the data is distributed according to modified weibull distribution.

The null and alternative hypotheses are as follows:

H_0 : The survival time data has modified weibull distribution.
H_1 : The survival time data has not modified weibull distribution.

Using a MATLAB program, we've got the following estimated parameters values

$\hat{\alpha} = 0.1733$

$\hat{\lambda} = 0.0005$

$\hat{\gamma} = 1.0068$

Now, the calculated variance-covariance matrix using the observed Fisher information matrix as follows:

$$I(\alpha,\lambda,\gamma) = \begin{bmatrix} 0.00452 & -.000279 & -0.00043 \\ -0.00279 & 0.00000003 & -0.07214 \\ -0.00043 & -0.07214 & 0.03278 \end{bmatrix}$$

The above values are calculated using the modified weibull distribution to get numerical values, and the lower and upper Values for the estimated parameters are:

$\hat{\alpha}_{lour} = 0.04153$, $\hat{\alpha}_{upper} = 0.30507$

$\hat{\lambda}_{lour} = 0.00016$, $\hat{\lambda}_{upper} = 0.00084$

$\hat{\gamma}_{lour} = 0.065193$, $\hat{\gamma}_{upper} = 1.36166$

These values are used to find the for for probability death density function f(t), f(t), survival function s(t), hazard function h(t) which available in table 1 .

6-Conclusion:

Note we can make the following comments for the results in the last table:

1-Noting that the values of lower and upper death density function f(t) are decreasing gradually with increasing failure times for the Leukemia patients in the hospital .The means there is an opposite Relationship between failure time and death density function.

2- Noting that the values of lower and upper survival function s(t) are decreasing gradually with increasing the failure times for Leukemia patients in the hospital, that means there is an opposite relationship between failure times and death density function.

3-Noting that the values of lower and upper hazard function h(t) are decreasing gradually with increasing the failure times for lower bounded $\hat{\gamma} = 0.065193$

While the hazard function h(t) are increasing gradually with increasing the failure times for upper bounded where $\hat{\gamma} = 1.36166$

Table 1- Estimated values for the function f(t), s(t) , h(t)

Failure time	f(t)		h(t)		s(t)	
	Lower	upper	Lower	upper	Lower	Upper
12	0.042219908	0.270686178	0.042964934	0.305891162	0.884910098	0.982659682
21	0.041434616	0.247093186	0.042710966	0.30607537	0.807295231	0.970116564
43	0.039916864	0.197604221	0.042450314	0.306372734	0.644979789	0.940319635
53	0.039295339	0.178440426	0.042385589	0.306475281	0.582234322	0.927091965
54	0.039235332	0.176658463	0.042380096	0.306484717	0.576402194	0.925796209
60	0.038876199	0.166176174	0.042349485	0.306539664	0.542103336	0.917985182
86	0.037390931	0.127466464	0.042252943	0.306744098	0.415546591	0.884930822
121	0.035527038	0.089190404	0.042171977	0.306963997	0.290556566	0.842432345
143	0.034415002	0.071222041	0.042135681	0.307082064	0.231931623	0.816766249
169	0.033155437	0.054610258	0.042101493	0.307207274	0.17776356	0.787512154
170	0.033107071	0.054042176	0.042100297	0.307211931	0.175911711	0.786385691
178	0.032731399	0.049808479	0.042091264	0.307247761	0.162111774	0.777629281
207	0.031404902	0.037018372	0.04206258	0.30736999	0.120435872	0.746612423
238	0.026956539	0.030051731	0.042037287	0.307489013	0.087666675	0.714887276
252	0.023354299	0.029460772	0.042027286	0.307539577	0.075939167	0.705045637

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