# Mathematical Model For Determination the Increase in Operational Cost of Transmission Line From HAD3 to QIM3 in Electric Power System

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## Abstract

The transformation of a physical system to mathematical base is very important due to analysis of the systems behavior. In this paper an electric power system is considered, we design mathematical model for the determination of the increase in operational cost of transmission line from Haditha Dam substation to Qa'im substation . We derived relations which the approximate distance for VARS transmission must satisfy with considering minimum losses in the system. MATLAB computer programming is used to obtain the numerical results. The developed mathematical model and the numerical results could be useful to electric power systems engineers.

**Keywords**: Mathematical model, Lossy transmission, reactive power transmission costs.

# نموذج رياضي لتحديد الزيادة في الكلفة التشغيلية لخط النقل من محطة HAD3 إلى محطة QIM3 في منظومة القدرة الكهربائية

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## الخلاصة

التحويل من النظام الفيزيائي إلى الأساس الرياضي هو مهم لأجل تحليل سلوك الأنظمة. في هذا البحث درسنا منظومة القدرة الكهربائية، و صممنا نموذج رياضي لتحديد الزيادة في الكلفة التشغيلية لخط النقل من محطة سدة حديثة في مدينة حديثة إلى محطة القائم في غرب العراق. اشتققنا علاقات لتقدير المسافة لنقل VARS والذي يجب إن يكون مقنع مع اعتبار أدنى حد للخسائر في المنظومة. البرمجة بلغة MATLAB استخدمت للحصول على النتائج العددية. النموذج الرياضي مع نتائجه العددية يمكن أن يكون مفيد لمهندسي منظومات القدرة الكهربائية.

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## 1. Introduction

Exact solution and simulation of various engineering problems, especially control engineering problems, depend on convenient mathematical models for elements and subsystems of the system considered. Process of the transformation of the system's behavior to mathematical basis is called "mathematical modeling" [1-3].

The social structures and the industrial development of any country depend primarily upon low cost and uninterrupted supply of electrical energy, Mehta et al. [4]. The process of modernization, increase in productivity, agriculture and industry basically depend upon the adequate supply of electrical energy, Gupta [5]. Generation of electrical energy is the conversion of energy available in different forms in nature to electrical energy. The ever increasing use of electrical energy for industrial, domestic and commercial purposes necessitated the bulk production of electrical energy. This bulk production is achieved with the help of suitable power production stations which are generally referred to as electric power generating stations or electric power plants. A generating station usually employs a prime mover coupled with an alternator to produce electric power. Electrical energy is generated at power stations which are usually situated far away from load centers.

Hence an extensive network of conductors between the power stations and the consumers is required. This network of conductors may be divided into two main components, called the transmission system and the distribution system. The transmission system is to deliver bulk power from power stations to load centers and large industrial consumers while the distribution system is to deliver power from substations to various consumers. Electrical energy must be transmitted and distributed to the point of use as soon as it is needed. Transmission lines and other materials are needed to achieve this purpose. Transmission lines are materials or media that are used to transmit electric energy and signals from one point to another, specifically from a source to a load. They can be regarded as a set of conductors being run from one place to another and supported on transmission towers. This involves connections between an electric generating plant and a substation which is several hundred kilometers away. The transmission and distribution stages are very important to electric power system because without these stages the generated power cannot get to the load centers not to talk of getting to the final consumers, Mehta et al. [4], Atandare [6] and Wadhwa [7].

## 2. Motivation for the Study

A lot of research work had been carried out by scientists and engineers on the generation of power, reliability of transmission systems and reduction of losses on transmission lines: Bamigbola et al [8] ,considered the characterization of optimal control model of electric power generating systems using two control variables, Aderinto[9] developed a mathematical model for electric power generating system using the optimal control approach with one control variable, Okafor et al [10] assessed the reliability of transmission systems in Nigeria by using the general reliability function and calculating the reliability indices for six 330KV transmission lines in Nigeria. Bagriyanik et al [11] used a fuzzy multi-objective optimization and genetic algorithm-based method to find optimum power system operating conditions. In addition to active power losses, series reactive power losses of transmission system are also considered as one of the multiple objectives. Onohaebi et al [12] considered the relationship of the effect of distance and loadings on power losses using the existing 28 bus, 330KV Nigerian transmission network as a case study in his empirical modeling of power losses as a function of line loadings and lengths in the Nigeria 330KV transmission lines, to mention a few. The mathematical models for the determination of voltages and currents on lossy electric power transmission lines has not been work upon by any of these researchers hence the need for this work. In this paper, we present the mathematical models for the determination of voltage and energy on lossy electric power transmission lines.

## 3. Reactive Power

Is one of a class of power system reliability services collectively known as ancillary services, measured in volt-amperes reactive or VARs, ancillary services are essential for the reliable operation of the bulk power system. Reactive power flows when current leads or lags behind the voltage; typically, the current lags because of inductive loads like motors. Reactive power flow wastes energy and transmission capacity, and causes voltage droop. To correct this lagging power flow, leading reactive power (current leading voltage) is supplied to bring the current in phase with voltage [13, 14].

Reactive power can be supplied from either static or dynamic VAR sources. Static sources are typically transmission and distribution equipment, such as static VAR compensators or capacitors at substations, and their cost has historically been included in the revenue requirement of the transmission owner (TO), and recovered through cost-of-service rates. By contrast, dynamic sources are typically energy producers, including generators capable of producing both real and reactive power, and synchronous condensers, which produce only reactive power [15].

## 4. Economic Analysis

Transmission of both active and reactive power lead to losses in the system as mentioned in the introduction. Since active power is usually generated specifically to compensate for load demand, it is the reactive power that is controlled to achieve a reduction of losses in the system.

When a power system is being designed and the parameters are yet to be determined, it is a generally accepted must to compensate for the predicted reactive power demand at the consuming end so as to reduce losses in the system. This reduction of the total transmitted power allows for the use of smaller conductors for transmission, leading to the reduction of system construction costs. Because of the expensive nature of the compensation equipment, the cost is also taken into account in determining the most economically justified distance for reactive power transmission.

## 5. Added Increase in Cost of Systems Equipment

The total current in any system element of a three phase network is given as:

$$I = \frac{\sqrt{p^2+q^2}}{\sqrt{3}\,V} = \frac{p\sqrt{1+tan^2\theta}}{\sqrt{3}\,V} \ldots\ldots(1)$$

Where:

p : active power

q : reactive power

The cross section area of a power transmission conductor is given as,

$$F = \frac{I}{J} = \frac{p\sqrt{1+tan^2\theta}}{\sqrt{3}\,VJ} \ldots\ldots \qquad (2)$$

Where J is the current density of the conductor in A/mm2.

The total cost of transmission line per km due to the added losses is [17]:

$$B_L = (b_{0L}L + b_LFL) = (b_{0L} + b_LF)L \ldots (3)$$

Where,

$b_L$: a variable constant reflecting increase in cost of conductor, \$/km.mm$^2$.

$b_{0L}$ : a fixed cost component of the conductor, \$/km.

L ,km : total length of the conductor.

The substitution of equation (2) into equation (3) gives the following equation (4),

$$B_L = \left(b_{0L} + \frac{b_L\,p\sqrt{1+tan^2\theta}}{\sqrt{3}\,VJ}\right)L \ldots\ldots.(4)$$

The equation of total cost without transmission reactive power (i.e., transmission purely active power, that is θ = 0) is:

$$B_{L2} = \left(b_{0L} + \frac{b_L\,p\sqrt{1+tan^2 0}}{\sqrt{3}\,V\,J}\right)L \ldots\ldots.(5)$$

The additional cost due to the transmission of apparent power as compared with the transmission of purely active power is expressed as:

$$\Delta B_L = \frac{b_L\,pL(\sqrt{1+tan^2\theta}-1)}{\sqrt{3}\,V\,J} \ldots\ldots..(6)$$

The final equation, Increased cost per unit of reactive power transmission is given by:

$$B_{Lu} = \frac{\Delta B_L}{q} = \frac{\Delta B_L}{p\,tan\theta} = \frac{b_L\,L(\sqrt{1+tan^2\theta}-1)}{\sqrt{3}\,V\,J\,tan\theta} \ldots(7)$$

That is, about VARs transmitted throw transmission line .

Now, about the VARs transmission increases the apparent power and hence the rating of transformers.

The transformer rating of one transformer substation $S_{T1}$ is:

$$S_{T1} = \sqrt{p^2 + q^2} = p\sqrt{1 + \tan^2\theta} \ldots\ldots (8)$$

$$S_{T2} = \frac{p\sqrt{1 + \tan^2\theta}}{1.4} \ldots\ldots\ldots\ldots (9)$$

For a two – transformer substation $S_{T2}$ , the rating of each transformer is approximately sixty percent of the total load ( i.e., S/1.4) [16,17].

The total cost of transformer for a one-transformer substation due to the added losses is:

$B_T$ = initial cost + add cost by increasing reactive power

= initial cost + add cost for VA * no. of VA

i.e.

$$B_T = b_{OT} + b_T S = b_{OT} + b_T p\sqrt{1 + \tan^2\theta} \ldots (10)$$

Where :

$b_{OT}$: initial cost of transformer in \$ .

$b_T$ : additional cost per additional VA .

S : value of increasing in VA .

The equation of total cost without transmission reactive power (i.e., transmission purely active power) is:

$$B_{T2} = b_{OT} + b_T\, p\sqrt{1 + \tan^2 0} \ldots\ldots (11)$$

The additional cost due to the transmission of apparent power as compared with the transmission of purely active power is expressed as:

$$\Delta B_L = b_T\, p\left(\sqrt{1 + \tan^2\theta} - 1\right) \ldots\ldots (12)$$

Therefore, the rise in cost per unit of reactive power transmission is, for a one-transformer substation:

$$B_{Tu1} = \frac{\Delta B_T}{q} = \frac{\Delta B_T}{p\tan\theta} = \frac{b_T\, P\left(\sqrt{1 + \tan^2\theta} - 1\right)}{P\tan\theta}$$
$$= \frac{b_T\left(\sqrt{1 + \tan^2\theta} - 1\right)}{\tan\theta} \ldots\ldots\ldots\ldots\ldots (13)$$

and for a two transformer substation:

$$B_{Tu2} = \frac{\Delta B_T}{p\tan\theta} = \frac{b_T\left(\sqrt{1 + \tan^2\theta} - 1\right)}{1.4\tan\theta} \ldots (14)$$

## 6. Active Power Loss

Active power loss in the line is:

$$\Delta P_L = I^2 R_L = \frac{p^2 + q^2}{V^2} R_L$$
$$= \frac{p^2}{V^2}\left(1 + \tan^2\theta\right) R_L \ldots\ldots\ldots (15)$$

For a balanced active power in the system, the generated output at the power station should be increased to meet the extra active power loss due to the transmission of VARS. Such an increase in the generated output is considered economically permissible if the cost due to the additional power loss does not exceed the cost of installing and maintaining the compensating VARS equipment at the consuming end [18], [21], i.e.,

$$K_a \Delta P_L \leq K_r Q_r \ldots\ldots\ldots\ldots \quad (16)$$

Where:

$K_a$ : Cost / kW of generated output, \$ / kW,

$K_r$ : Cost / kVAr of VARS compensation equipment , \$ / kVAr,

$Q_r$ : kVAr rating of reactive power equipment , kVAr.

## 7. Reactive power loss

Reactive power transmission leads to voltage drop in transmission lines, The reactive power loss is:

$$\Delta Q_L = I^2 X_0 L = \frac{p^2}{V^2}\left(1 + \tan^2\theta\right) X_0 L \ldots (17)$$

Where, $X_0$ is unit reactance of the line, Ω / km.

This loss is taken into account in the reactive power balance in the system. As such, the installed VARS source in the system should be increased to compensate for the loss.

## Note that

VARS transmission from the generator has technical constraints.

VARS transmission is considered economical if the cost of generation at the power station, (including losses in the system) is less than or equal to the cost ( excluding losses in the system) of installing VARS compensating equipment at the consuming end [18, 20], i.e.,

$$K_Q\left(Q_{beg} + \Delta Q_L\right) \leq K_r Q_r \ldots\ldots\ldots\ldots (18)$$

Where:

$K_Q$ : cost energy loss due to VARS transmission,\$/kWh,

$Q_{beg}$ : VARS output at the beginning of the line, kVAr.

Equations (17) and (18) are used to establish approximately the economic justifiable distance for VARS transmission by putting $Q_{beg} = Q_r$ (considering only power loss) as:

$$K_Q(Q_r + \Delta Q_L) = K_r Q_r$$

$$K_Q \Delta Q_L = K_r Q_r - K_Q Q_r$$

$$\Delta Q_L = \frac{(K_r - K_Q)Q_r}{K_Q}$$

Since $Q_r$ is expressed as $q = p\ tan\theta$ then

$$\Delta Q_L = \frac{(K_r - K_Q)p\ tan\theta}{K_Q}$$

By putting this equation in (17) , we get ;

$$\frac{(K_r - K_Q)p\ tan\theta}{K_Q} = \frac{p^2}{V^2}(1 + tan^2\theta)X_0 L$$

$$L = \frac{(K_r - K_Q)V^2 p\ tan\theta}{K_Q X_0 p^2 (1 + tan^2\theta)} \approx \frac{(K_r - K_Q)V^2 p\ tan\theta}{K_Q X_0 p^2 tan^2\theta}$$

then,

$$L_r = \frac{(K_r - K_Q)V^2}{K_Q X_0 p\ tan\theta} \dots\dots\dots. \quad (19)$$

**8. Energy loss :**

Energy loss is expressed as:

$$\Delta A_Q = \Delta P_Q \xi_Q \dots\dots. \quad (20)$$

Where, $\xi_Q$ is average time / year, corresponding to the total time for VARS transmission, hr.

The energy loss leads to an increase in the use of fuel at the generating station. This increase in operational cost is obtained as [16,18, 20]:

$$C_F = \beta\sigma\Delta A_Q$$
$$= \beta\sigma\frac{p^2}{V^2}tan^2\theta\ R_L\ \xi_Q \dots\dots(21)$$

Where,

$\beta$ : cost of fuel , \$/m$^3$ .

$\sigma$ : cubic metre of extra fuel used due to the compensation for the transmission of reactive power.

The cross-sectional area of the conductor is (from equation 2):

$$F = \frac{p\sqrt{1 + tan^2\theta}}{\sqrt{3}\,VJ} = \frac{p}{\sqrt{3}\,V\,J\,cos\theta} \dots\dots\dots(22)$$

also,

$$F = \rho\frac{L}{R_L} \dots\dots\dots\dots\dots \quad (23)$$

From equation (22) and equation (23) we get:

$$R_L = \frac{L\rho\sqrt{3}\,V\,J\,cos\theta}{p} \dots\dots\dots \quad (24)$$

Where, $\rho$ = resistivity of conductor, Ω mm$^2$/km. Hence,

$$C_F = \beta\sigma\frac{p^2}{V^2}tan^2\theta\ \frac{L\rho\sqrt{3}\,V\,J\,cos\theta}{p}\ \xi_Q$$
$$= \beta\sigma\frac{p}{V}tan^2\theta\ L\rho\sqrt{3}\,J\,cos\theta\ \xi_Q \dots\dots(25)$$

Now, we calculate $C_F$ ( increase in operational cost ) for transmission line from Haditha Dam substation to Qa'im substation in West of Iraq, for conductor of type Lark with 132 KV single circuit overhead line have cross section(F)=248mm$^2$ , L=1km , $\beta\sigma$ =0.001\$ , $\rho$=0.0365 Ωmm$^2$/km , p=35000MW and V=132kV with different value of θ and $\xi_Q$. The MATLAB computer results in Table (1) and Figure (1) are obtained on the basis of these data.

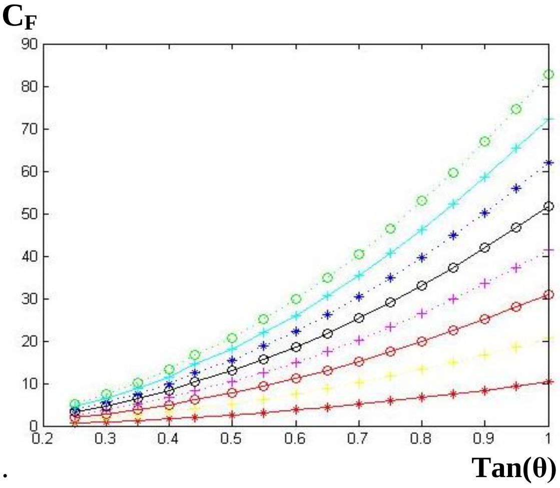

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**Figure 1-**The value of of $C_F$ with different value of tan(θ) & $\xi_Q$

**Table 1-**The value of $C_F$ with different value of tan(θ) & $\xi_Q$

| Tan($\theta$) | $C_F$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\xi = 1000$ | $\xi = 2000$ | $\xi = 3000$ | $\xi = 4000$ | $\xi = 5000$ | $\xi = 6000$ | $\xi = 7000$ | $\xi = 8000$ |
| 1 | 10.3349 | 20.6698 | 31.0046 | 41.3395 | 51.6744 | 62.0093 | 72.3442 | 82.679 |
| 0.95 | 9.3272 | 18.6545 | 27.9817 | 37.3089 | 46.6362 | 55.9634 | 65.2906 | 74.617 |
| 0.90 | 8.3713 | 16.7425 | 25.1138 | 33.4850 | 41.8563 | 50.2275 | 58.5988 | 66.970 |
| 0.85 | 7.4670 | 14.9339 | 22.4009 | 29.8678 | 37.334 | 44.8017 | 52.2687 | 59.735 |
| 0.80 | 6.6143 | 13.2287 | 19.8430 | 26.4573 | 33.0716 | 39.6860 | 46.3003 | 52.914 |
| 0.75 | 5.8134 | 11.6267 | 17.4401 | 23.2535 | 29.0669 | 34.8802 | 40.6936 | 46.507 |
| 0.70 | 5.0641 | 10.1282 | 15.1923 | 20.2564 | 25.3205 | 30.3846 | 35.4486 | 40.512 |
| 0.65 | 4.3665 | 8.7330 | 13.0995 | 17.4660 | 21.8324 | 26.1989 | 30.5654 | 34.931 |
| 0.60 | 3.7206 | 7.4411 | 11.1617 | 14.8822 | 18.6028 | 22.3233 | 26.0439 | 29.764 |
| 0.55 | 3.1263 | 6.2526 | 9.3789 | 12.5052 | 15.6315 | 18.7578 | 21.8841 | 25.010 |
| 0.50 | 2.5837 | 5.1674 | 7.7512 | 10.3349 | 12.9186 | 15.5023 | 18.0860 | 20.669 |
| 0.44 | 2.0928 | 4.1856 | 6.2784 | 8.3713 | 10.4641 | 12.5569 | 14.6497 | 16.742 |
| 0.40 | 1.6536 | 3.3072 | 4.9607 | 6.6143 | 8.2679 | 9.9215 | 11.5751 | 13.228 |
| 0.35 | 1.2660 | 2.5320 | 3.798 | 5.0641 | 6.3301 | 7.5961 | 8.8622 | 10.128 |
| 0.30 | 0.9301 | 1.8603 | 2.7904 | 3.7206 | 4.6507 | 5.5808 | 6.5110 | 7.4411 |
| 0.25 | 0.6459 | 1.2919 | 1.9378 | 2.5837 | 3.2297 | 3.8756 | 4.5215 | 5.1674 |

It is clear that from Table (1) the changeability of average times which corresponding to the total time for VARS transmission ($\xi_Q$) for the same coefficient power ( tanθ) are effectiveness in operational cost ($C_F$ ) of transmission line because the resistance and fuel used.

## 9. Conclusion

This paper has established a mathematical model that could be used to help determine the economically justified distance for VARS transmission and to the determine the increase in operational cost of transmission line from Haditha Dam substation to Qa'im substation .The computer results are satisfactory and could be of guidance to consulting and power systems design engineers who have to justify their projects technically and economically, especially in choosing conductor and equipment sizes / ratings.

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