Reverse *-Centralizers on *-Lie Ideals

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Abstract

The purpose of this paper is to prove the following result : Let R be a 2-torsion free prime *-ring , U a square closed *-Lie ideal, and let T: R→R be an additive mapping. Suppose that 3T(xyx) = T(x) y*x* + x*T(y)x* + x*y*T(x) and $x^*T(xy+yx)x^* = x^*T(y)x^{*2} + x^{*2}T(y)x^*$ holds for all pairs x, y ∈ U , and T(u) ∈U, for all u∈U, then T is a reverse *-centralizer.

Keywords: Prime *-Ring, Semiprime *-Ring, *-Lie Ideal, Reverse *-Centralizer.

تـمركـزات *–العكسية في مـثـاليـات *– لــي

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الخلاصة:

الهدف من البحث هو برهان النتيجة الآتية : لتكن R *–حلقة أولية طليقة الالتواء من الدرجة الثانية و U *–مثالي لي مغلق تربيعيا في R و T: R→R دالة تجميعيه و T تحقق المعادلتين التاليتين لكل x,y في U

3T(xyx) = T(x) y*x* + x*T(y)x* + x*y*T(x) و

$x^*T(xy+yx)x^* = x^*T(y)x^{*2} + x^{*2}T(y)x^*$

اذا كان T(u) ∋ U لكل u ∋ U فان T تكون دالة تمركزات–* العكسية.

1.Introduction

Throughout, R will represent an associative ring with center Z(R). A ring R is n-torsion free, if $nx = 0$, $x \in$ R implies $x = 0$, where n is a positive integer. Recall that R is prime if aRb = (0) implies $a = 0$ or $b = 0$, and semiprime if aRa = (0) implies a =0. An additive mapping $x \to x^*$ on a ring R is called an involution if $(xy)^* = y^* x^*$ and $(x)^{**} = x$ for all $x, y \in$ R. A ring equipped with an involution is called *-ring (see [1]). As usual the commutator $xy - yx$ will be denoted by $[x,y]$. We shall use basic commutator identities $[xy, z] = [x, z]y + x[y, z]$ and $[x, yz] = [x, y]z + y[x, z]$ for all $x,y,z \in$R. Also we write xo $y=xy+yx$ for all $x, y \in$R (see [2]). An additive subgroup U of R is said to be a Lie-ideal if [u, r] ∈ U for all u ∈ U and r ∈ R. A Lie-ideal U on a *-ring R, which satisfies U*= U is called a *-Lie ideal [3]. If U is a Lie (resp.*-Lie) ideal of a *-ring R, then U is called a square closed Lie (resp. *-Lie) ideal if $u^2 \in$ U for all u ∈ U [3]. A left (right) centralizer of R is an additive mapping T: R→R which satisfies $T(xy) = T(x)y$ ($T(xy) = xT(y)$) for all $x, y \in$ R. A centralizer of R is an additive mapping which is both left and right centralizer. A left (right) Jordan centralizer of R is an additive mapping T: R→R which satisfies $T(x^2) = T(x)x$ ($T(x^2)= xT(x)$) for all $x \in$ R. A Jordan centralizer of R is an additive mapping which is both left and right Jordan centralizer (see [4-7]). Every centralizer is a Jordan centralizer. B. Zalar [4] proved the converse when R is 2- torsion free semiprime ring . Inspired by the above definition Majeed

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and Altay [8] define, a left (right) reverse *-centralizer of a *-ring R is an additive mapping T: $R\rightarrow R$ which satisfies $T(yx)=T(x)y^*$ ($T(yx)=x^*T(y)$) for all $x,y\in R$. A reverse *-centralizer of R is an additive mapping which is both left and right reverse *-centralizer. A left (right) Jordan *-centralizer of R is an additive mapping T: $R\rightarrow R$ which satisfies $T(x^2)=T(x)\, x^*$ ($T(x^2)=x^*T(x)$) for all $x\in R$. A Jordan *-centralizer of R is an additive mapping which is both left and right Jordan *-centralizer. Every reverse *-centralizer is a Jordan *-centralizer. Majeed and Altay [8] proved the converse when R is 2-torsion free semiprime *-ring. In this work we will study an Identity on a reverse *-centralizers of a 2-torsion free prime *-ring , U a square closed *-Lie ideal. We will prove in case T: $R\rightarrow R$ be an additive mapping, satisfies $3T(xyx) = T(x)\, y^*x^* + x^*T(y)x^* + x^*y^*T(x)$ and $x^*T(xy+yx)x^* = x^*T(y)x^{*2} + x^{*2}T(y)x^*$ holds for all pairs $x, y \in U$, and $T(u) \in U$, for all $u\in U$, then $T(yx)= T(x)y^* = x^*T(y)$ for all $x,y\in U$.

2. The Main Result

We now give the main result of this paper.

Theorem 2.1: Let R be a 2-torsion free prime *-ring , U a square closed Lie ideal, such that $U=U^*$ and let T: $R\rightarrow R$ are additive mappings. Suppose that $3T(xyx) = T(x)\, y^*x^* + x^*T(y)x^* + x^*y^*T(x)$ and $x^*T(xy+yx)x^* = x^*T(y)x^{*2} + x^{*2}T(y)x^*$ holds for all pairs $x, y \in U$, and $T(u) \in U$, for all $u\in U$, then T is a reverse *-centralizers.

To proof the above theorem we need the following lemmas.

Lemma 2.2.[9]: If $U \not\subset Z$ is a Lie ideal of a 2-tortion free prime ring R and a , b $\in$ R such that $aUb = \{0\}$, then either a=0 or b=0 .

Lemma 2.3: Let R be a 2-tortion free prime ring , U be a square closed *-Lie ideal of R. Suppose that the relation $axb + bxc = 0$ holds for all $x \in U$ and some a , b , c $\in$ U. In this case $(a + c)xb = 0$ is satisfied for all $x \in U$.

Proof: If $U \subset Z$ the prove is clear, so assume that $U \not\subset Z$, putting (4xby) for x in the relation

$axb + bxc = 0$ for all $x \in U$,

we obtain,

$axbyb + bxbyc = 0$ for all $x,y \in U$,

On the other hand right multiplication the first relation by yb gives

$axbyb + bxcyb = 0$ for all $x, y \in U$,

Subtracting the above relation, we obtain

$bx(byc - cyb) = 0$ for all $x, y \in U$,

from above relation we can get two relation ,first come by substitution (4ycx) for x and second by left multiplication by cy, so we obtain

$bycx(byc - cyb) = 0$ for all $x\ y \in U$,

And,

$cybx(byc - cyb) = 0$ for all $x\ y \in U$,

Subtracting the two above relation, we get

$(byc - cyb)x(byc - cyb) = 0$ for all $x,y \in U$,

which gives by Lemma2.2, $byc = cyb$, $y \in U$. Therefore bxc can be replaced by cxb in first relation which gives $(a + c)xb = 0$, $x \in U$.

Lemma 2.4.[8.Corollary (1.2.4)]: Let R be a 2-torsion free prime *-ring and let T: $R \rightarrow R$ be an additive mapping such that $2T(x^2) = T(x)x^* + x^*T(x)$ holds for all $x \in R$. In this case, T is a reverse *-centralizer.

Now will give the prove of theorem 2.1.

Proof. of Theorem (2.1):

$3T(xyx) = T(x)\, y^*x^* + x^*T(y)x^* + x^*y^*T(x)$
for all $x, y \in U$ (1)

And,

$x^*T(xy + yx)x^* = x^*\, T(y)\, x^{*2} + x^{*2}T(y)x^*$
for all $x, y \in U$ (2)

(i) If $U \not\subset Z(R)$

After replacing x by x + z in (1), we obtain

$3T(xyz+zyx) = T(x)\, y^*z^* + T(z)\, y^*x^* + x^*T(y)z^* + z^*T(y)x^* + z^*y^*T(x) + x^*y^*T(z)$,
for all $x, y, z \in U$ (3)

Letting y = x and z = y in (3) gives

$3T(x^2y+yx^2) = T(x)x^*y^* + T(y)x^{*2} + x^*T(x)y^* + y^*T(x)x^* + y^*x^*T(x) + x^{*2}T(y)$
for all $x , y \in U$ (4)

After replacing x by 3x and z by $2x^3$ in (3) and using (1), we obtain

$9T(xyx^3+x^3yx) = 3T(x)y^*x^{*3} + 3T(x^3)y^*x^* + 3x^*T(y)x^{*3} + 3x^{*3}T(y)x^* + 3x^{*3}y^*T(x) + 3x^*y^*T(x^{*3}) = 3T(x)y^*x^{*3} + T(x)x^{*2}y^*x^* + x^*T(x)x^*y^*x^* + x^{*2}T(x)y^*x^* + 3x^*T(y)x^{*3} + 3x^{*3}T(y)x^* + x^*y^*T(x)x^{*2} + x^*y^*x^*T(x)x^* + x^*y^*x^{*2}T(x) + 3x^{*3}yT(x)$
for all $x, y \in U$ (5)

Replacing y by $3(x^2y + yx^2)$ in (1) and using (4), we obtain

$9T(xyx^3+ x^3yx) = 3T(x)\, x^{*2}y^*x^* + 3T(x)y^*x^{*3} + x^*T(x)x^*y^*x^* + x^*\, T(y)\, x^{*3} + x^{*2})T(x)y^*x^* + x^*y^*T(x)\, x^{*2} + x^{*3}T(y)x^* + x^*y^*x^*\, T(x)\, x^* + 3\, x^{*3}y^*)T(x) + 3x^*y^*x^{*2}T(x)$
for all $x, y \in U$ (6)

Subtracting (6) from (5), we obtain

$T(x)x^{*2}y^*x^* + x^*y^*x^{*2}T(x) - x^{*3}T(y)x^* - x^*T(y)x^{*3} = 0$
for all $x, y \in U$ (7)
Replacing y by 6xyx in (4), we obtain
$9T(x^3yx+xyx^3) = 3T(x)x^{*2}y^*x^* + T(x)y^*x^{*3} + x^*T(y)x^{*3} + x^*y^*T(x)x^{*2} + 3x^*T(x)x^*y^*x^* + 3x^*y^*x^*T(x)x^* + x^{*2}T(x)y^*x^* + x^{*3}T(y)x^* + x^{*3}y^*T(x) + 3x^*y^*x^{*2})T(x)$
for all $x,y \in U$ (8)
On the other hand by replacing z by $6x^3$ in (3), we obtain
$9T(x^3yx+xyx^3) = 3T(x)y^*x^{*3} + T(x)x^{*2}y^*x^* + x^*T(x)x^*y^*x^* + x^{*2}T(x)y^*x^* + 3x^*T(y)x^{*3} + 3x^{*3}T(y)x^* + x^*y^*T(x)x^{*2} + x^*y^*x^*T(x)x^* + x^*y^*x^{*2}T(x) + x^{*3}y^*T(x)$
for all $x,y \in U$ (9)
Comparing (8) and (9), we arrive at
$T(x)y^*x^{*3} - T(x)x^{*2}y^*x^* + x^*T(y)x^{*3} - x^*T(x)x^*y^*x^* - x^*y^*x^{*2}T(x) + x^{*3}y^*T(x) - x^*y^*x^*T(x)x^* + x^{*3}T(y)x^* = 0$
for all $x, y \in U$ (10)
From (7) and (10), we obtain
$T(x)y^*x^{*3}-x^*T(x)x^*y^*x^*+x^{*3}y^*T(x)-x^*y^*x^*T(x)x^*=0$, for all $x, y \in U$ (11)
Replacing y by 2xy in the above relation gives
$T(x)y^*x^{*4}-x^*T(x)x^*y^*x^{*2}+x^{*3}y^*x^*T(x)-x^*y^*x^{*2}T(x)x^*=0$, for all $x,y\in U$ (12)
On the other hand right multiplication of (11) by x* gives
$T(x)y^*x^{*4}-x^*T(x)x^*y^*x^{*2}+x^{*3}y^*T(x)x^*-x^*y^*x^*T(x)x^{*2}=0$, for all $x, y \in U$ (13)
Subtracting (13) from (12) gives
$x^{*3}y^*[T(x), x^*] - x^*y^*x^*[T(x), x^*]x^* = 0$,
for all $x, y \in U$ (14)
Left multiplication of (14) by T(x) gives
$T(x)x^{*3}y^*[T(x),x^*]-T(x)x^*y^*x^*[T(x), x^*]x^*=0$
for all $x, y \in U$. (15)
Replacing y by 2yT(x)* in (14) gives,
$x^{*3}T(x) y^*[T(x), x^*]-x^*T(x)y^*x^*[T(x),x^*]x^*=0$
for all $x, y \in U$ (16)
After subtracting (15) from (16), we arrive at
$[T(x),x^{*3}]y^*[T(x),x^*]-[T(x),x^*]y^*x^*[T(x),x^*]x^*=0$ for all $x,y \in U$ (17)
In the above relation let
$a = [T(x), x^{*3})]$, $b = [T(x), x^*]$,
$c = -x^*[T(x), x^*]x^*$ and $z = y^*$
From the above substitutions, we have
$azb + bzc = 0$.
We apply Lemma 2.3 to the above relation to obtain
$\{[T(x),x^{*3}] - x^*[T(x), x^*]x^*\}y^*[T(x), x^*] = 0$,
for all $x, y \in U$,
this reduces to
$\{[T(x), x^*]x^{*2} + x^{*2}[T(x), x^*]\}y^*[T(x), x^*] = 0$,
for all $x, y \in U$ (18)
Right multiplication of the above relation by x^{*2} gives
$\{[T(x),x^*]x^{*2}+x^{*2}[T(x), x^*]\}y^*[T(x), x^*]x^{*2}=0$
for all $x , y \in U$ (19)
After replacing y by $2x^2y$ in (18), we get
$\{[T(x),x^*]x^{*2}+x^{*2}[T(x),x^*]\}y^*x^{*2}[T(x), x^*]=0$
for all $x, y\in U$ (20)
Adding (19) to (20), we obtain
$\{[T(x), x^*]x^{*2} + x^{*2}[T(x), x^*]\}y^*\{[T(x), x^*]x^{*2} + x^{*2}[T(x), x^*]\}= 0$ for all $x,y\in U$
Using Lemma 2.2, we get
$[T(x),x^*]x^{*2}+x^{*2}[T(x), x^*]=0$, for all $x \in U$ (21)
Replacing y by 2xy in (14) gives
$x^{*3}y^* x^*[T(x), x^*] - x^*y^*x^{2*}[T(x), x^*]x^*= 0$
for all $x,y\in U$ (22)
Replacing y* by $2[T(x), x^*]y^*$ in the above relation gives
$x^{*3}[T(x), x^*] y^* x^*[T(x), x^*] - x^*[T(x), x^*] y^*x^{2*}[T(x), x^*]x^*= 0$ for all $x,y \in U$ (23)
In the above relation let
$a = x^{*3}[T(x),x^*]$, $b = x^*[T(x),x^*]$,
$c = - x^{*2}[T(x),x^*]x^*$ and $z = y^*$
From the above substitutions, we have
$azb + bzc = 0$.
We apply Lemma 2.3 to the above relation to obtain
$\{x^{*3}[T(x),x^*]-x^{*2}[T(x),x^*]x^*\}y^*x^*[T(x),x^*]=0$
for all $x, y\in U$ (24)
Replacing y by $2x^2y$ in the above relation gives
$\{x^{*3}[T(x),x^*]-x^{*2}[T(x),x^*]x^*\}y^*x^{*3}[T(x),x^*]=0$
for all $x,y\in U$ (25)
On the other hand replacing y by 2xy in relation (24) and right multiplying of this relation by x* gives
$\{x^{*3}[T(x),x^*]-x^{*2}[T(x),x^*]x^*\}y^*x^{*2}[T(x),x^*]x^*=0$
for all $x, y \in U$. (26)
Subtracting (26) from (25) gives
$\{x^{*3}[T(x),x^*]-x^{*2}[T(x),x^*]x^*\}y^*\{x^{*3}[T(x),x^*] -x^{*2}[T(x),x^*]x^*\}=0$ for all $x,y\in U$
Also by using Lemma 2.2, we get
$x^{*3}[T(x),x^*]-x^{*2}[T(x),x^*]x^* = 0$
for all $x \in U$ (27)
Right multiplication of (21) by x* gives
$[T(x), x^*]x^{*3} + x^{*2}[T(x), x^*]x^*= 0$
for all $x \in U$ (28)
According to (27) and (28), we have
$[T(x), x^*]x^{*3} + x^{*3}[T(x),x^*] = 0$
for all $x \in U$ (29)
Left multiplication of (22) by [T(x), x*] gives
$[T(x), x^*] x^{*3}y^* x^*[T(x), x^*] - [T(x), x^*] x^*y^*x^{2*}[T(x), x^*]x^*= 0$
for all $x, y \in U$ (30)

Adding relations (23) and (30) and using (29), we obtain

$\{[T(x),x^*]x^*+x^*[T(x),x^*]\}y^*x^{*2}[T(x),x^*]x^*= 0$

for all $x,y\in U$ (31)

Using (27) we obtain from the above relation

$\{[T(x),x^*]x^*+x^*[T(x),x^*]\}y^*x^{*3}[T(x),x^*]= 0$

for all $x,y \in U$ (32)

Left multiplication of (32) by x^{*2} gives

$\{x^{*2}[T(x),x^*]x^*+x^{*3}[T(x),x^*]\}y^*x^{*3}[T(x),x^*]=0$

for all $x, y \in U$

According to (27) one can replace $x^{*2}[T(x),x^*]x^*$ by $x^{*3}[T(x),x^*]$ in the above relation. Thus, we have

$x^{*3}[T(x),x^*]\, y^*x^{*3}[T(x),x^*]= 0$, for all $x, y \in U$

Hence, we obtain

$x^{*3}[T(x),x^*]=0$ for all $x \in U$ (33)

Because of (29), we have

$[T(x), x^*]x^{*3} = 0$, for all $x \in U$ (34)

Replacing y^* by $2[T(x), x^*]y^*$ in (14) gives

$x^{*3}[T(x), x^*]y^*\ [T(x), x^*] - x^*[T(x), x^*]y^*\ x^*[T(x), x^*]x^* = 0$

for all $x,y\in U$ (35)

Using (33) the above relation reduces to

$x^*[T(x), x^*]y^*\ x^*[T(x), x^*]x^* = 0$

for all $x,y\in U$ (36)

Replacing y^* by $2x^*y^*$ in (36) gives

$x^*[T(x), x^*]\ x^*\ y^*\ x^*[T(x), x^*]x^* = 0$

for all $x, y \in U$

Therefore,

$x^*\ [T(x), x^*]x^* = 0$, for all $x \in U$ (37)

Putting $x + y$ for x in (37), we obtain

$x^*[T(x),x^*]y^* + x^*[T(x), y^*]x^* + x^*[T(y), x^*]x^* + y^*[T(x), x^*]x^* + x^*[T(x),y^*]y^* + x^*[T(y),x^*]y^*+ y^*[T(x),x^*]y^* + x^*[T(y),y^*]x^* + y^*[T(x), y^*]x^* + y^*[T(y),x^*]x^* + x^*[T(y), y^*]y^* + y^*[T(x), y^*]y^* + y^*[T(y), x^*]y^* + y^*[T(y), y^*]x^* =0$

for all $x, y \in U$ (38)

Putting $-x$ for x in the above relation and combining the relation so obtained with (38), we obtain

$x^*[T(x),y^*]y^*+ x^*[T(y),x^*]y^*+ y^*[T(x),x^*]y^*+ x^*[T(y),y^*]x^*+y^*[T(x),y^*]x^*+ y^*[T(y),x^*]x^*=0$

for all $x, y \in U$ (39)

After comparing (38) and (39), we have

$x^*[T(x),x^*]y^* + x^*[T(x), y^*]x^* + x^*[T(y), x^*]x^* + y^*[T(x), x^*]x^* + x^*[T(y), y^*]y^* + y^*[T(x), y^*]y^* + y^*[T(y), x^*]y^* + y^*[T(y), y^*]x^* =0$

for all $x, y \in U$ (40)

Replacing x by $2x$ in the above relation and subtracting the relation so obtained from the above relation multiplied by 8, we obtain

$x^*[T(y),y^*]y^*+ y^*[T(x),y^*]y^*+ y^*[T(y), x^*]y^*+ y^*[T(y), y^*]x^* =0$

for all $x, y \in U$for all $x, y \in U$ (41)

Comparing (40) and (41), we obtain

$x^*[T(x),x^*]y^*+ x^*[T(x), y^*]x^*+ x^*[T(y),x^*]x^* + y^*[T(x), x^*]x^* =0$

for all $x, y \in U$ (42)

Right multiplication of (42) by $x^{*2}[T(x), x^*]$ and using (33) gives

$x^*[T(x),x^*]y^*\ x^{*2}[T(x), x^*]=0$

for all $x, y \in U$ (43)

Left multiplication of (43) by x^* gives

$x^{*2}[T(x),x^*]y^*\ x^{*2}[T(x), x^*]=0$ for all $x, y \in U$

Hence,

$x^{*2}\ [T(x), x^*] = 0$, for all $x \in U$ (44)

Because of (21), we also have

$[T(x), x^*]x^{*2} = 0$, for all $x \in U$ (45)

Right multiplication of (42) by $x^*[T(x), x^*]$ gives because of (44)

$x^*[T(x), x^*]y^*x^*[T(x), x^*] = 0$, for all $x, y \in U$

Therefore,

$x^*[T(x), x^*] = 0$, for all $x \in U$ (46)

Left multiplication of (42) by $[T(x), x^*]x^*$ and use of (45) gives

$[T(x), x^*]x^*y^*[T(x), x^*]x^* = 0$ for all $x, y \in U$

Hence, we get

$[T(x), x^*]x^* = 0$ for all $x \in U$ (47)

From (47) one obtains (see the proof of (39))

$[T(x), y^*]x^*+[T(y),x^*]x^*+[T(x),x^*]y^*= 0$

for all $x,y\in U$ (48)

Right multiplication of the above relation by $[T(x), x^*]$ use of (46) gives

$[T(x), x^*]y^*[T(x), x^*] = 0$ for all $x, y \in U$

Therefore, we obtain

$[T(x), x^*] = 0$, for all $x \in U$ (49)

Now, we will prove that

$T(xy + yx) = T(y)x^* + x^*T(y)$

for all $x,y\in U$ (50)

In order to prove the above relation, we need to prove the following relation

$[A(x, y), x^*] = 0$ for all $x,y\in U$ (51)

where $A(x, y)$ stands for $T(xy + yx) - T(y)x^* - x^*T(y)$. With respect to this notation equation (2) can be rewritten as,

$x^*A(x, y)x^* = 0$ for all $x,y\in U$ (52)

Replacing x by $x + y$ in relation (49) gives

$[T(x), y^*] + [T(y), x^*] = 0$ for all $x,y\in U$ (53)

After replacing y by $2(xy + yx)$ in (53) and using (49), we obtain

$x^*[T(x), y^*] + [T(x),y^*]x^* + [T(xy+yx),x^*] = 0$

for all $x, y \in U$

According to (53) we can replace in the above relation $[T(x) , y^*]$ by $- [T(y) , x^*]$. We then have

$[T(xy+yx),x^*] - x^*[T(y),x^*] - [T(y),x^*]x^* = 0$
for all $x, y \in U$
This can be written in the form
$[T(xy+yx) - T(y)x^* - x^*T(y) , x^*] = 0$,
for all $x, y \in U$
The proof of relation (51) is therefore complete.
Replacing x by x + z in (52) and using (52) gives
$x^*A(x,y)z^* + x^*A(z, y)x^* + z^*A(x, y)x^* + z^*A(z, y)x^* + z^*A(x, y)z^* + x^*A(z, y)z^* = 0$
for all $x, y, z \in U$
After replacing x for −x in the above relation and adding the relation so obtained to the above relation, we arrive at:
$x^*A(x,y)z^*+x^*A(z,y)x^* + z^*A(x,y)x^* = 0$
for all $x,y,z \in U$
Right multiplication of the above relation by $A(x, y)x^*$ and using (52) gives
$x^*A(x, y) z^*A(x, y)x^* = 0$, for all $x,y,z\in U$ (54)
Using (54), the above relation can be written in the form
$x^*A(x, y)z^*x^*A(x, y) =0$, for all $x,y,z \in U$ (55)
Therefore, we obtain
$x^*A(x, y) = 0$ for all $x,y\in U$ (56)
From (51) and (56), we also get
$A(x, y) x^* = 0$ for all $x,y\in U$ (57)
Replacing x by x + z in (57) gives
$A(x, y) z^* + A(z, y)x^* = 0$ for all $x, y, z \in U$
Right multiplication of the above relation by $A(x, y)$ and using (56) gives
$A(x, y) z^*A(x, y) = 0$ for all $x,y, z \in U$
Therefore,
$A(x, y) = 0$ for all $x,y \in U$
The proof of (50) is therefore complete.
(ii) If $U \subset Z(R)$
Right multiplication of relation(2) by r $A(x, y)$, $r\in R$ and by primness of R, we get
$x^*A(x, y) = 0$ for all $x,y \in U$
Replacing x by x + z in above relation gives
$z^*A(x, y) + x^*A(z, y) = 0$ for all $x, y, z \in U$
Left multiplication of the above relation by $A(x, y)$, we get
$A(x, y)z^*A(x, y) = 0$, for all $x, y, z \in U$
Right multiplication of the above relation by r z^*, $r\in R$ and by primness of R, we get
$A(x, y)z^* = 0$, for all $x, y, z \in U$
By the primness of R, we get (50)
In particular when y = x (50) reduces to
$2T(x^2) = T(x)x^* + x^*T(x)$ for all $x \in U$
By Lemma.2.4. We obtain T is a reveres *-centralizer , which completes the proof.

The following corollary is clear from theorem(2.1).

Corollary 2.5: Let R be a 2-torsion free prime *-ring , and let $T: R\rightarrow R$ are additive mappings. Suppose that $3T(xyx) = T(x) y^*x^* + x^*T(y)x^* + x^*y^*T(x)$ and $x^*T(xy+yx)x^* = x^*T(y)x^{*2} + x^{*2}T(y)x^*$ holds for all pairs $x, y \in R$, then T is a reveres *-centralizer.

References

1. Herstein, I. N**. 1969.** *Topics in ring theory*, University of Chicago Press,3-20.
2. Herstein, I. N**. 1976.** *Rings with involution***,** University of Chicago press, Chicago, 2-11.
3. Awtar,R**. 1973.** Lie ideals and Jordan structures in prime rings with derivations, *Proc. Amer. Math.* Soc. 41, pp:67-74.
4. Zalar,B**., 1991.** On centralizers of semiprime rings, *Comment. Math. Univ. Carolinae* 32, pp:609- 614.
5. Vukman,J**. 1999.** An identity related to centralizers in semiprime rings, *Comment. Math. Univ. Carolinae* 40, pp:447-456.
6. Vukman, J**. 2001.** Centralizers on semiprime rings, *Comment. Math. Univ. Carolinae* 42, pp:237-245.
7. Vukman, J. and Kosi-Ulbl, I**. 2003.** On centralizers of semiprime rings, *Aequationes Math*. 66, pp:277 - 283.
8. Majeed,A. and Altay,A**. 2009.** Additive mappings on prime and semiprime ring with involution.Baghdad University (Iraq). Msc. thesis. Advisor: A.H. Majeed,1, pp:19-25.
9. Bergen,J. , Herstein, I.N. and Ker, J.W**. 1981.**Lie ideals and derivation of prime rings , *J. Algebra* 71, pp:259-267.