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Using Gravitational Search Algorithm for Solving Nonlinear Regression Analysis

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Abstract:

Evolutionary algorithms (EAs) provide a framework for dealing with a variety of large-scale multi-objective problems (MOPs) in the field of evolutionary algorithms, when applied to different problem types. Although computational strategies for dealing with nonlinear regression problems are difficult to apply, we used the gravity search algorithm and combined it with these regressions to estimate and interpret the parameters. Estimation parameters for nonlinear regression and gravitational search algorithm (EPNGSA), enabling us to access them consistently. Estimating the nonlinear estimation parameter using general estimating equations. It is necessary to use Chebyshev's strategy in the leader recruitment procedure, which leads to tackling (MOP) based on the Gravity Search Algorithm (GSA) and at the same time may lead to quick results. When building a leader library, the concept of dominance is crucial because it allows leaders, they choose to include less dense regions, thus producing an estimated Pareto front with a large diversity, which reduces global optimization challenges. The used and new method showed its effectiveness and was closer to the solution compared to other algorithms. This result was obtained using six standard nonlinear functions. GSA appears to be more productive than both Practical Swarm Optimization PSO and Bat Algorithm BAT.

Keyword: Non-Linear Degradation, Swarm Algorithm Practice, Estimation Factor

استخدام خوارزمية بحث الجاذبية لحل تحليل الانحدار غير الخطي

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الخلاصة

توفر الخوارزميات التطورية (EAS) إطارًا للتعامل مع مجموعة متنوعة من المسائل والمشكلات متعددة الأهداف (MOPs) على نطاق واسع في حقل الخوارزميات التطورية عند تطبيقها على أنواع من المسائل المختلفة على الرغم من أن الاستراتيجيات الحسابية للتعامل مع مسائل الانحدار غير الخطي صعبة التطبيق, لذلك قمنا باستخدام خوارزمية بحث الجاذبية (EPNGSA) ودمجها مع تحليل الانحدارات الخطية لتقدير وتفسير المعلمات مما يمكننا من الوصول إليها باستمرار. وتقدير معلمة التخمين غير الخطي باستخدام معادلات التقدير العامة. ومن الضروري استخدام استراتيجية تشبيبيشيف في إجراءات تعيين القادة، مما يؤدي إلى معالجة المشاكل متعددة الأهداف (MOP) وبالاستناد إلى خوارزمية بحث الجاذبية (GSA) مما قد يؤدي إلى نتائج سريعة عند بناء مكتبة القائد، يعد مفهوم الهيمنة أمرًا بالغ الأهمية لأنه يسمح للقادة المختارين

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بتضمين مناطق أقل كثافة، وبالتالي تنتج جبهة باريتو المقطرة ذات التنوع الكبير مما يقلل من تحديات التحسين العالمية. إن الطريقة المستخدمة والجديدة أظهرت فاعليتها وكانت أقرب للحل مقارنة بالخوارزميات الأخرى. تم الحصول على هذه النتيجة باستخدام ست وظائف قياسية غير خطية. يبدو أن GSA أكثر إنتاجية من كل من PSO و BAT.

1- Introduction

The investigation's current status is covered in the following section. The context is taken into account, and the study's objectives, questions, meaning, scope, and limitations are spelled out.

Nonlinear residuals are those in which the modeling formulation involves a nonlinear relationship between two or more of the model parameters. There are several scientific and commercial sectors that make use of nonlinear scenarios, which are utilized to model complex interrelationships between variables. Numerical approaches include an extensive range of representations for physical, biological, commercial, and econometric phenomena [1], [2] and [3] including progression, yield intensity, and dose-response equations.

The principle behind non-linear training is identical to that of frameworks based on linear degradation which is to associate an outcome y with a set of predictors $x = (x_1, x_2, \dots, x_k)$. Non-linear Inference is characterized by the fact that the foretelling equation is non-linearly dependent on any number of unknowable factors. Whenever there are empirical indications that the response-predictor association takes a non-linear shape, analysts turn to non-linear degradation. Non-Linear Degradation is employed whenever unlike the degradation approach, and this is frequently used to develop a purely econometric method, there are physical grounds to believe that the association involving the response with the variables that impact it follows a specified mathematical design.

$y = (x \cdot \beta) + \varepsilon$. $i = 1$. The ε_i are commonly believed to be uncorrelated with variances that are constant and median zero. In the nonlinear regression estimating scenario, the form is well-known, but the principles of β_1, \dots, β_p are unknown. Non-linear degradation functions often have unidentified variables that can be calculated using a normal linear model [4], [5]. By reducing the parameter $\beta = \sum_{i=1}^n e_i^2 = \sum_{j=1}^n (k_j - L(X_j \cdot N))^2$ (the total multiplied of mistakes foretelling), the following method can be used to derive the estimates of β_1, \dots, β_p . Nonlinear estimating problems that include optimizing the target function $S(\beta)$ are examples of efficiency issues.

Several articles have been written about the estimate of parameters in Non-linear degradation frameworks. Component estimation and mathematical modeling become more complex and difficult due to the nonlinearity hypothesis. As well as the restrictions imposed upon these (classic) approaches to estimating nonlinear parameters. It is hard to regulate by professionals and requires a lot of supplementary data to work properly. These difficulties originate from the objective function's inherent multimodality and the large number of parameters. Minimizing the mean squared deviation's function $S(\beta)$ as totaling conventional optimization techniques is notoriously difficult [6], [7].

Therefore, the acceptance of advanced meta-heuristic procedures is reliable, robust, and useful in conquering these obstacles because of the many benefits they offer, one of them is their ease of deployment. In this relook, we use a set of three meta-heuristic techniques, collectively known as the Gravitational Look for Algorithm (GSA), to address the problem of Non-linear multiple degradation.

In order to achieve the aforementioned objectives, the paper is divided into eight sections; each section represents a different stage in the research process.

2- Non-Linear Degradation

Nonlinear degradation is a special case of degradation analysis. Econometric methodology is one of the most often used measuring methods for examining the association between two or more parameters [8], [9]. It is an indispensable tool for precise analysis and thorough use of investigational data. $x = (x_1, x_2, \dots, x_k)$ and the dependent or response measurement, Y , to show the relationship between autonomous or predictor observations (marked by the letter I) as well as dependent or predicting measurements (identified by the letter P).

To achieve this, we will develop a regressive model, $y = f(x, \beta) + \varepsilon$, where y is the totally reliant variable, x is a vector of separate from variables, β is a vector of characteristics, ε is a constant, and s is a set of parameters that has zero deviations from average and corresponds to [1,2]. The two most common kinds of degradation evaluations are linear and nonlinear. The factor of determinant [10], [11] constitutes a very popular statistical deductive technique when the degradation product f is linear degradation. In 1894, Sir Francis Galton was the first one who presents the theory of linear degradation [10]. However, a few linear structures are appropriate; Consequently, a nonlinear quantitative technique is usually utilized, where f is nonlinear in β [j]. Non-linear parameters and characteristics, non-linear measures, and non-linear characteristics may all be included in non-linear degradation analysis. If the properties of the framework remain non-linear, despite the forecast's linear sections, the model is called a non-linear deterioration scenario, [12], [13]. Whenever data needs to be converted to fit a linear interpolation as the the Non-linear Resistance approach is very helpful for evaluating scientific information. Consequently, linear degradation is regarded as an aspect of non-linear degradation, which is a more general concept [14], [15]. Inductive, biological, physical, and economic data, building, mathematics, and management, are just a few of the disciplines that use nonlinear frameworks. The creation of nonlinear frameworks is a recent and intriguing area of study in mathematical applications. The challenge of creating a multiple linear degradation model will almost certainly be encountered by a relook for in arithmetic or any other scientific field. In the literature, numerous nonlinear approaches have been created defined and effectively applied to a wide range of real-world situations pertaining to a wide range of relook for questions in a wide range of quantitative modelling domains. However, many scenarios have not yet been nonlinearly described due to the complexity of the situations or their implacability in terms of statistics and mathematical concepts [16], [2]. We shall examine two examples of nonlinear regressors next. Numerical approaches have a broad spectrum of applications in reality.

Table 1: Frameworks of Non-Linear Dependent variables $\beta_1, \beta_2, \beta_3$ and β_4 . [16]

N	Problems Name	Function
1	Meryer1	$\frac{\beta_1 \beta_2 x_1}{1 + \beta_1 x_1 + \beta_2 x_2}$
2	Meryer4	$\beta_3 (e^{-\beta_1 x_1} + e^{\beta_2 x_2})$
3	Meryer7	$\beta_1 + \beta_2 e^{\beta_3 x}$
4	Militky4	$\beta_1 e^{\beta_3 x} + \beta_2 e^{\beta_2 x}$
5	Militky5	$\beta_1 x^{\beta_2} + \beta_3^{\beta_2/x}$
6	Gompertz	$\beta_1 e^{-e^{(\beta_2 - \beta_3 x)}}$

3- Classical Estimation Methods

The two approaches that are most common for estimating parameters are the Mean Square Error (MSE) and the Least Square Error (LSE), according to [17], [18]. MSE is favored by

relook for ers and is thought to have superior statistical features. Additionally, the LSE approach only subjects data to a totalming of squared residuals as the target goal. LSE is incredibly practical, which is why practitioners adore it. In this study, kernel density estimation and least squares were used to estimate the parameters of four non-linear degradation frameworks.

4- Gravitational Search Algorithm Technique (GSA)

The Gravitational Optimization Technique (GSA) was suggested by [19] and [20] to address problems with effectiveness. The wider public's heuristic technique depends on gravity law and mass relationships. Agents, or solutions in the GSA communities, connect to other agents through gravity. Every agent's effectiveness in the community was evaluated using its mass. The most suitable response has historically been the one with the higher mass. The force of gravity in each neutrino attracts other neutrinos (i.e. a neutral subatomic particle with a mass close to zero and half-integral spin, rarely reacting with normal matter. Three kinds of neutrinos are known, associated with the electron, muon, and tau particle.), and the gravitational force between both of them is inversely proportional to the distance, R , between them and directly relates to the standard deviation of their masses. Granules are merely a subset of their overall community and are inversely proportional to the distance between it, according to the law of gravitation. The law of motion states that every mass's current velocity is exactly the product of its proportionate prior velocity and its fluctuating acceleration. The rate of acceleration or alteration in the velocity of any substance is equal to the force applied to it multiplied by its mass.

The masses of the things obey gravity in a few things ways:

$$F_{ij} = G \frac{M_{aj} \times M_{pj}}{R^2} \quad \dots(1)$$

$$a_i = \frac{F_{ij}}{M_{ii}} \quad \dots(2)$$

Where M_{aj} and M_{pj} represent the active gravitational mass of particle i and passive gravitational mass of particle j , respectively, and M_{ii} represents the inertia mass of particle i . Furthermore, the next velocity of an agent is considered as a fraction of its current velocity added to its acceleration. Therefore, its position and its velocity could be calculated as follows:

$$V_i^d(t+1) = rand_i \times V_i^d(t) + a_i(t) \quad \dots(3)$$

$$X_i^d(t+1) = X_i^d(t) + V_i^d(t+1) \quad \dots(4)$$

where $rand_i$ is a uniform random variable in the interval $[0,1]$. We use this random number to give arandomized characteristic to the search.

Sometimes, the GSA's major step can be described up as described below:

Step 1. Starting out the gravitational acceleration equations G_0 , a is the first step in the look for process. G_0 and a are started at the start of the process, and their principles will decrease as the look for progresses. T represents the overall total of iterations.

Step 2. A beginning community of N people was formed at random, and each attorney's orientation was selected as follows:

$$X_i(t) = (X_i^1(t), X_i^2(t), \dots, X_i^n(t)), \quad i=1, \dots, N, \quad \dots(5)$$

Step 3: Repeat this procedure up until the cessation of operations goals are achieved.

A. Recognized the best and worst agents, at this point all agents were evaluated according to the community .

B. As previously stated, the gravitational factor was modified.

C. Determine the energy using the following method:

$$F_{ij}^d = G(t) \frac{M_{aj}(t) \times M_{pj}(t)}{R_{ij}(t) + \epsilon} (X_j^d(t) - X_i^d(t)) \quad \dots(6)$$

$$R_{ij}(t) = \frac{X_i(t) \cdot X_j(t)}{2} \quad \dots(7)$$

The distance measured by the Euclidian separating agents i and j is represented by $R_{ij}(t)$.

D. To give a stochastic characteristic to our algorithm , we suppose that the total force that acts on a gent i in a dimension d be a randomly weighted sum of d^{th} components of the forces exerted from other agents:

$$F_i^d(t) = \sum_{j=1, j \neq i}^N rand_j F_{ij}^d(t) \quad \dots(8)$$

whereas $rand_j$ is a arbitrary symbol in the range $[0,1]$.

One way to perform a good compromise between exploration and exploitation is to reduce the number of agents with lapse of time in Eq. (8). Hence, we propose only a set of agents with bigger mass apply their force to the other. However, we should be careful of using this policy because it may reduce the exploration power and increase the exploitation capability.

E. To determine the inertial mass, you can use the formula:

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \quad \dots(9)$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^N m_j(t)} \quad \dots(10)$$

Where $fit_i(t)$ represent the fitness value of the agent i at time t , and , $worst(t)$ and $best(t)$ are defined as follows (for a minimization problem):

$$best(t) = \min fit_j(t) \quad \dots(11)$$

$$worst(t) = \max fit_j(t) \quad \dots(12)$$

F. An additional technique was used to compute the acceleration of the agent:

$$a_i^d(t) = \frac{L_i^d(t)}{M_{ii}(t)} \quad \dots(13)$$

G. The formula used shows how the location and speed of agent i are established.

H. The each iteration count is increased until the termination conditions are satisfied.

Step 4. The fastest and best answer is found in step four.

5- Practical Swarm Optimization (PSO)

The coordinated movement of fish schools and avian flocks served as the inspiration for the PSO metaheuristics [21]. The PSO is a swarm of matter that represents a potential solution to the issue at hand. Particles "flow" around the problem's hyperdimensional look for space, and adjustments to their positions The social cognitive tendency of people to imitate the accomplishments of others is the basis for the search for space. Every individual within a community, specifically a community of particles, has distinct life experiences and the ability to assess their worth. Because they are sociable creatures, they are aware of how well their neighbors have behaved. Those two types of information stand for the social (cultural transmission) and cognitive (individual learning) components, respectively. As a result, each person makes a decision while considering the social and cognitive factors, which leads to the

community (the swarm) to engage in a spontaneous action and the flowchart below in Figure (1) we can see all the details [22].

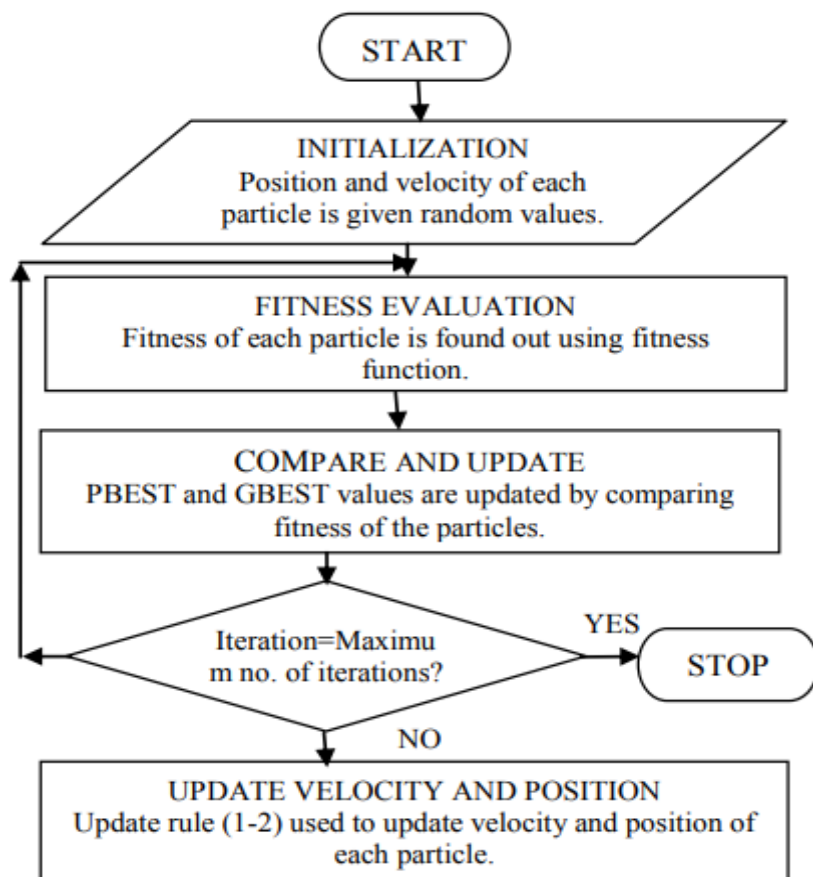


Figure 1: Flow Diagram of PSO

6- Bat Algorithm

The SI family includes the biologically-inspired Bat Algorithm (BAT). In 2010, Xin-She - Yang developed the Bat Algorithm [23] and [24], which is now widely used. Bats employ a type of echolocation using a sonic signal called sonar echolocation to navigate their environments and identify potential dangers. Yang focused on the following three guidelines for proper bat execution:

Although the volume and wavelength can vary, bats always fly at the same arbitrary speed and regularity toward the same fixed location. So bats naturally adjust their frequencies to sound like their prey. In addition, all bats rely on acoustic signals to gauge how far away an object is.

In conclusion, the contributor suggested that the volume be adjusted from loudest to quietest rather than the other way around. In order to replicate the variation in the radiance and pulse emissions from combustion that bats experience throughout hunting, BATA makes use of automatic zooming throughout its look foring phase [25].

Pseudo code for Bat algorithm components are explained in this manner:

Pseudo code for Bat algorithm

Objective function $f(x)$, $x = (x_1, \dots, x_d)^T$

Initialize the bat population x_i ($i = 1, 2, \dots, n$) and v_i

Define pulse frequency f_i at x_i

Initialize pulse rates r_i and the loudness A_i

```

while (t < Max number of iterations)
Generate new solutions by adjusting frequency,
and updating velocities and locations/solutions [see equations in Yang (2010)]
if (rand > ri)
Select a solution among the best solutions
Generate a local solution around the selected best solution
end if
Generate a new solution by flying randomly
if (rand < Ai & f(xi) < f(x*))
Accept the new solutions
Increase ri and reduce Ai
end if
Rank the bats and find the current best x*
end while

```

End

7- The Proposed Method (EPNGSA)

The article will begin with a glossary of MOP terms. Then, the structure of the proposed estimation parameter of nonlinear regression with gravitational look for algorithm method is displayed. The process for allocating grades will be discussed afterward. Lastly, we will cover the mechanisms of ambient adjustment and the strategies for mating, we can see all the details in Figure (2) below .

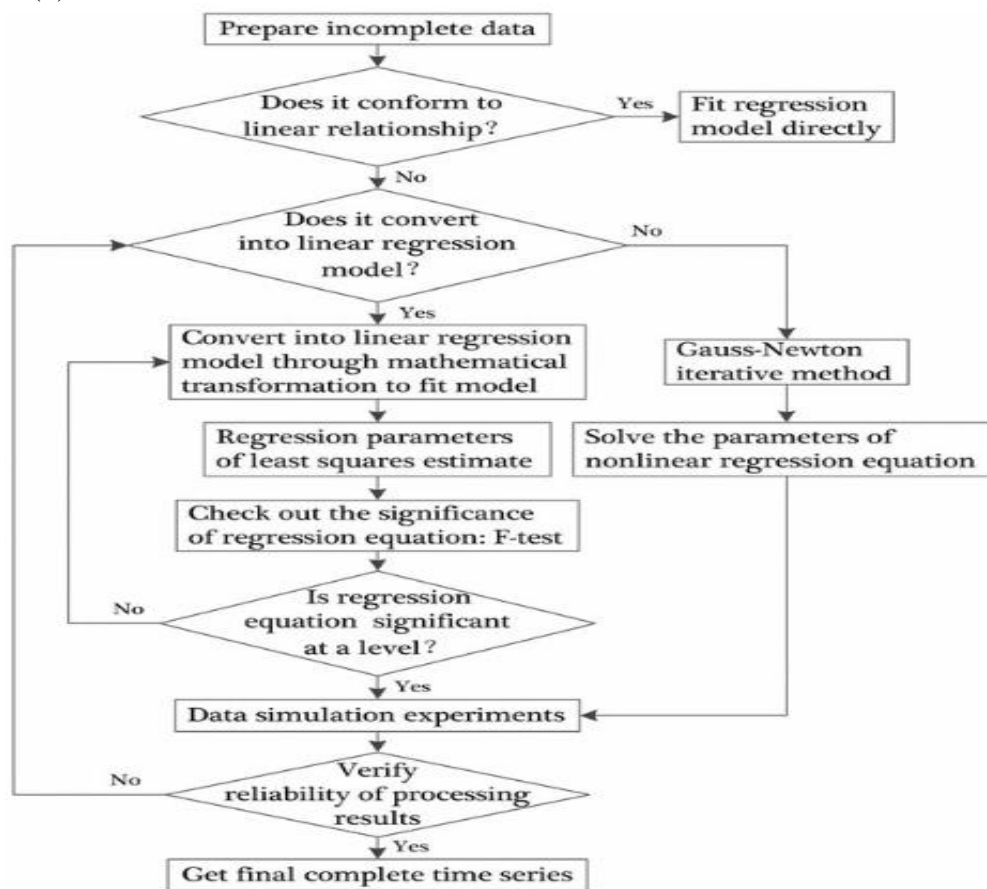


Figure 2: Flowchart of EPNGSA

Moreover, there it was no systematic investigation on the relationships between the attributes and convergence rates. To generate a better approximation strategy, and predict Non-linear

degradation Frameworks, the relook presents a hybrid method called EPNGSA by adding two extra components, repository and leader, uncovered in the Multi-Objective Practical Swarm Optimization MOPSO algorithm proposed by [26], [27]. The primary function of the repository is neither to store and retrieve the most important found neither dominated nor under influenced Pareto solutions that are best. The central processing unit (CPU) of the storage is also presented.

Pseudo-code of the EPNGSA

Set $k := 0$ and velocity = 0 $\mu=0.1, r_0 = 0.5, A = 0.6$.
 Start the point at arbitrary P_i for n . population ;
 Determine the first neighborhood's fitness standards : $f(P)$;
 Locate the approaches that are not dominating and begin the collection in them as well.
 While (The requirements for removal are not fulfilled)
 Practical Swarm Optimization PSO Steps
 $M = M_{\min} + (M_{\min} - M_{\max}) * \text{rand}$
 $V_{ij}^{t+1} = V_{(ij)}^t + C_1 R_1 (B_{BEST_{ij}} - X_{ij}^t) + C_2 R_2 (G_{BEST_{ij}} - X_{ij}^t)$
 $X_i^{t+1} = X_i^t + V_i^{t+1}$
 If $\text{rand} > r$
 $P_{\text{leader}2} = \text{Choose the King (preserve)}$
 $X_{\text{new}} = X_{(t)} + \text{rand} * (X_{\text{leader}2} - X_{(t)})$
 End
 if X_{new} dominated on $X_{(t)}$ & ($\text{rand} < A$)
 $X_{(t)} = X_{\text{new}}$
 End
 If $\text{rand} < \left(\frac{1-(k-1)}{\text{Max iteration}-1}\right)^{1/\mu}$
 $S = \text{Mutation}(P_{(t)})$
 if X_{new} dominated on $X_{(t)}$ & ($\text{rand} < A$)
 $P_{(t)} = S$
 End
 End
 Identify the non-dominated answers.
 Rebuild the repository with the acquired information. Non-dominated solutions in the event that the repository is full.
 To remove one of the current archive participants, use the grid technique.
 Update the file system with the most recent solution.
 If some recently uploaded solutions to the archive are situated elsewhere other than the hypercube
 Adjust the grids to reflect the novel answer or possibilities.
 End if
 Increase r and reduce A
 Set $k := k + 1$;
 End While

8- Simulation for Results and Analysis

8.1 Execution Behaviors

The EPNGSA is calculated using data from quantitative associations and is then compared with different algorithms. Pareto charts showing the outcomes of meaningful comparisons are

shown. The mathematical concepts of Gravitational Distancing (GD), Inverted Gravitational Distancing (IGD), and Hyper Volume (HV) [28] are used to make quantitative comparisons. GD , or gravitational distance. It is acceptable to use the phrase metric when referring to the (GD) [29] since it analyzes the structure of the sets' typical disparities of Q (where Q represents criminality that is assigned to the P^* set, which is also known as the IGD metric) as it continues with P : in evaluating whether or not arrangements of Q can be included with the configuration of P^* .

$$GD = \frac{\sum_{n=1}^Q d_i^p}{Q} \quad \dots (14)$$

$$IGD = \frac{1}{Q} (\sum_{i=1}^Q \min (\sqrt[p]{\sum_{i=1}^Q (d_i^p)})) \quad \dots (15)$$

Inverted transmissible distance (IGD) is the mean distance within each possible solution in the dataset being tested and the closest the fluid within the nearest set of solutions; this indicates an expansion of options. The IGD estimation, also known as the Pareto front (PF) P^* evaluation, measures the cohesion of movement of acquired buildings via concepts of scattering and enhancing.

The identification of excessive volume (Hyper volume) (A) calculates how much of the objective space a PF approximation pitifully commands. Hyper volume utilizes a reference point v^* which means an upper bound overall objective. v^* Is characterized as the most noticeably terrible objective esteems found in A (for example v^* is ruled by all arrangements in A). Utilizing the Lévesque measure (Λ), the hyper volume is characterized as:

$$HV(A) = \Lambda(\cup \{x \mid a < x < v^*, a \in A\}) \quad \dots (16)$$

Table (2) lists the IGD 's residual consequences. Table (2) displays the outcomes of employing hyper volume HV . The final paragraph displays the p-estimation. Powerful text style reveals a factually significant difference in the two subsequent matched $t - test$ between $EPNGSA$ and other approaches. Pseudo-code for each of the five estimations being taken into consideration, calculate the difference between PF valid and PF estimated.

8.2 Multi-Objective Test Functions

Six benchmarking functions are compared and utilized to validate the proposed $EPNGSA$ algorithm in order to demonstrate its effectiveness. The examination's testing features capacity of nonlinear functions is shown in detail in Table (2) through the sex problems.

8.3 Decision Space

The variety of possibilities that are open to us is represented by the choice spectrum. The choices we provide will be the sole factor in determining the criterion's weights. A similar issue Deciding space allows for the definition of consequently. For instance, while developing items, we make decisions about the design requirements (with regard to cost), each of which affects the quality metrics (expectations) that we employ to evaluate it. The real Pareto approach: is a statistical technique for determining the most important choices among a large number of possibilities [30].

8.4- Results and Discussion

The accuracy of the provided method is the focus of this essay. The suggested method ($EPNGSA$) is run in Matlab (version 2020), and depending on the subject under discussion, registration takes a few seconds to less than a second. Several factors are used to test it, such as the total community ($n=20, 40, 80, 160, \text{ and } 200$), the quantity of objectives M , and the size D (i.e., the number of n).

The usefulness of the suggested technique in balancing of accessibility and diversity is confirmed by the results of the experiment. The numerous studies showed how well a theoretical policy balanced proximity and variety. On the other hand, scientists have

developed numerous mathematical estimation techniques for non-linear degradation employing Hybrid algorithms for computation.

Applying the new EPNGSA calculations along with further PSO and BAT computation, we first announce the MSE demonstration in this section.

The mean square error of the comparison prediction values obtained by different computations under contrasting settings for the algorithms (BAT and PSO) based on the first model (Meyer (7)) of Non-Linear Degradation is presented in tables (2 and 3): The model's parameters that were selected are as follows. $(\beta_1, \beta_2 \text{ and } \beta_3) = (600, 1.5, \text{ and } 1.5)$ and $(700, 2, 1)$. Specifically, the best-estimated qualities are in blue color.

According to these data tables, the EPNGSA algorithm outperforms all other approaches in terms of accuracy for every sample since it has a lower MSE.

Utilizing various design requirements, the companies EPNGSA method's usefulness is being examined. Whenever an algorithm is compared to other algorithms that solve the same problem, its full worth becomes apparent. In order to compare the novel EPNGSA algorithm with the PSO and BAT algorithms, we use the MSE expectations. The same standard functions are used to solve each algorithm utilizing variance assumptions.

The efficiency of the MSE measure with the structure of Meyer (7) is shown in Tables (2 and 3) in comparison to the new EPNGSA algorithm with PSO and BAT algorithms. The new EPNGSA algorithm performs average mean in the computation of its median and calculation mean, while each of the other algorithms succeeds better in terms of rank and requirements.

TABLE 2: EPNGSA, PSO, BAT, and MSE Comparative Results Using $\beta_1=600, \beta_2=1.5$ and $\beta_3=1.5$

n	Techniques	Important events	β_1	β_2	β_2	MSE
20	GSA	Estimated	5.246	1.227	1.588	1.417
		MSE	1.056	6.129	3.423	NaN
	PSO	Estimated	6.145	1.480	1.506	2.246
		MSE	9.176	1.766	1.424	NaN
	BAT	Estimated	5.773	1.404	1.515	1.640
		MSE	1.316	6.699	6.669	NaN
40	GSA	Estimated	5.455	1.273	1.543	9.897
		MSE	4.556	3.682	1.125	NaN
	PSO	Estimated	5.774	1.480	1.503	4.814
		MSE	5.107	8.934	4.435	NaN
	BAT	Estimated	6.025	1.741	1.489	1.031
		MSE	2.958	9.598	6.022	NaN
80	GSA	Estimated	5.070	1.467	1.509	4.128
		MSE	1.013	1.928	1.454	NaN
	PSO	Estimated	6.273	1.429	1.506	3.673
		MSE	1.213	4.982	4.006	NaN
	BAT	Estimated	4.484	1.569	1.497	2.153
		MSE	1.793	9.103	3.601	NaN
160	GSA	Estimated	5.590	1.154	1.570	9.830
		MSE	1.532	6.446	2.749	NaN
	PSO	Estimated	5.192	1.472	1.501	1.681
		MSE	3.426	6.284	1.098	NaN
	BAT	Estimated	1.730	1.406	1.507	7.912
		MSE	7.609	2.715	9.216	NaN
200	GSA	Estimated	6.512	1.681	1.495	1.350
		MSE	7.990	3.787	5.517	NaN
	PSO	Estimated	4.666	1.457	1.506	6.960
		MSE	2.069	1.705	6.726	NaN
	BAT	Estimated	2.890	1.540	1.500	9.933
		MSE	1.400	1.070	3.570	NaN

Table 3: EPNGSA, PSO, BAT, and MSE Comparative Results Using $\beta_1=700$, $\beta_2=1.7$ and $\beta_3=1.6$

n	Techniques	Important events	β_1	β_2	β_2	MSE
20	GSA	Estimated	5.61	1.35	1.72	5.35
		MSE	3.94	2.66	5.09	NaN
	PSO	Estimated	7.01	1.46	1.71	2.73
		MSE	4.06	1.18	4.08	NaN
	BAT	Estimated	5.24	1.72	1.69	1.22
		MSE	2.76	1.36	2.53	NaN
40	GSA	Estimated	5.28	1.35	1.78	9.43
		MSE	3.14	7.75	2.74	NaN
	PSO	Estimated	6.23	1.26	1.75	2.99
		MSE	2.70	3.82	8.91	NaN
	BAT	Estimated	4.53	1.82	1.70	1.43
		MSE	1.64	1.01	6.31	NaN
80	GSA	Estimated	5.13	1.62	1.72	5.11
		MSE	3.82	3.25	9.42	NaN
	PSO	Estimated	5.18	1.63	1.70	2.74
		MSE	1.15	1.98	7.75	NaN
	BAT	Estimated	4.88	1.67	1.69	1.67
		MSE	6.56	8.05	4.79	NaN
160	GSA	Estimated	4.93	9.29	1.81	2.07
		MSE	4.47	9.26	2.81	NaN
	PSO	Estimated	5.99	1.47	1.71	1.00
		MSE	4.80	1.37	6.97	NaN
	BAT	Estimated	3.03	1.49	1.71	4.37
		MSE	2.22	1.22	5.03	NaN
200	GSA	Estimated	4.39	1.54	1.71	1.13
		MSE	8.65	1.66	9.22	NaN
	PSO	Estimated	3.58	1.53	1.70	5.57
		MSE	2.03	5.84	1.73	NaN
	BAT	Estimated	4.43	1.73	1.70	6.97
		MSE	3.57	6.09	1.32	NaN

9- Convergence Graphs

Consolidation graphs have been created for things like the test results to demonstrate how soon the standard evaluation converges with the number of iterations. There have been 100,000 iterations for all datasets. The figures below show how our recommended new method strategy can help you find the ideal value more rapidly.

Figures (3) through (7) provide a clearer visual representation of all the comparison studies on several Non-linear degradation frameworks with various sample sets for $n = 20, 40, 80, 160,$ and 200 . These graphs demonstrate that, compared to other frameworks, meta-heuristic methods provide the lowest MSE principles. Nevertheless, efficiency was superior when evaluating BAT and PSO.

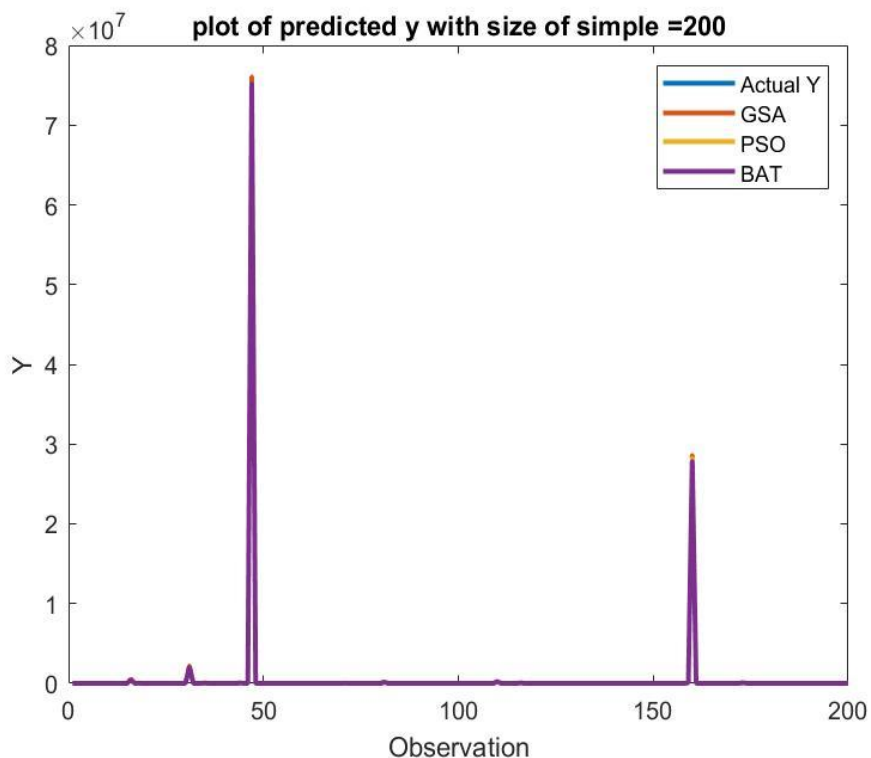


Figure 3: Comparison of the Module one's EPNGSA, BAT, PSO, and GSA methods where the data set size is 200

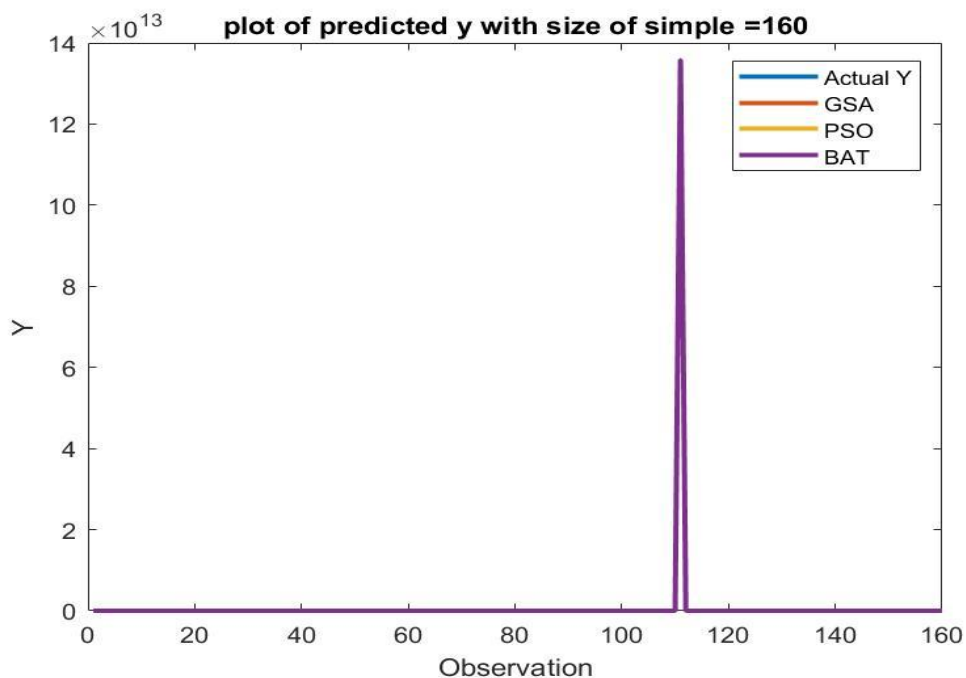


Figure 4: Comparison of the Module one's EPNGSA, BAT, PSO, and GSA methods where the data set size is 160

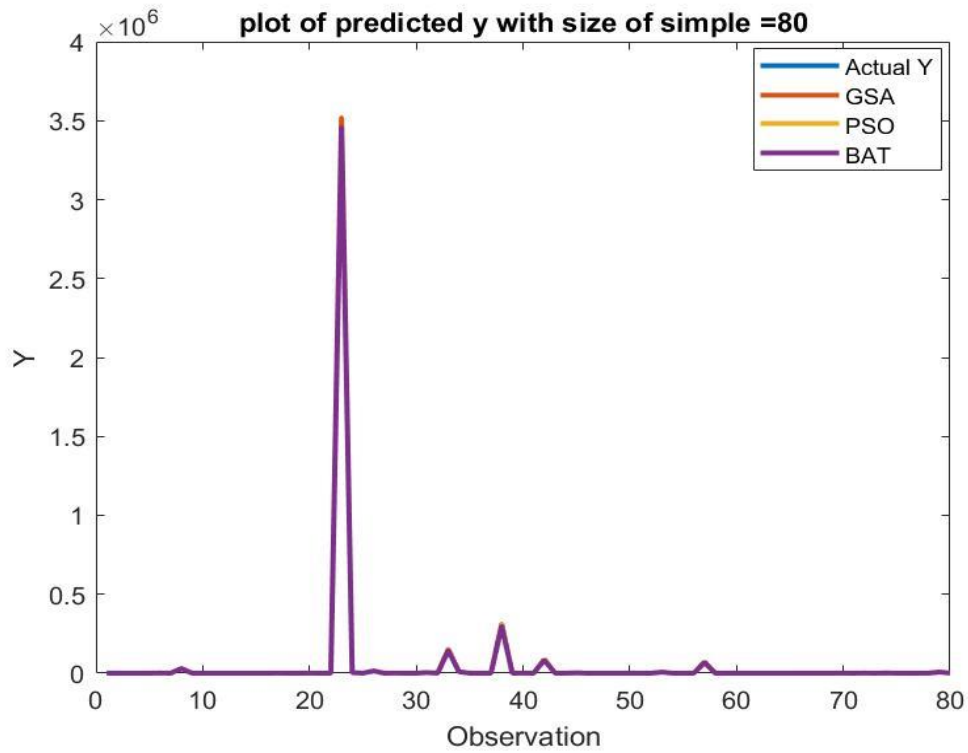


Figure 5 : Comparison of the Module one's EPNGSA, BAT, PSO, and GSA methods where the data set size is 80

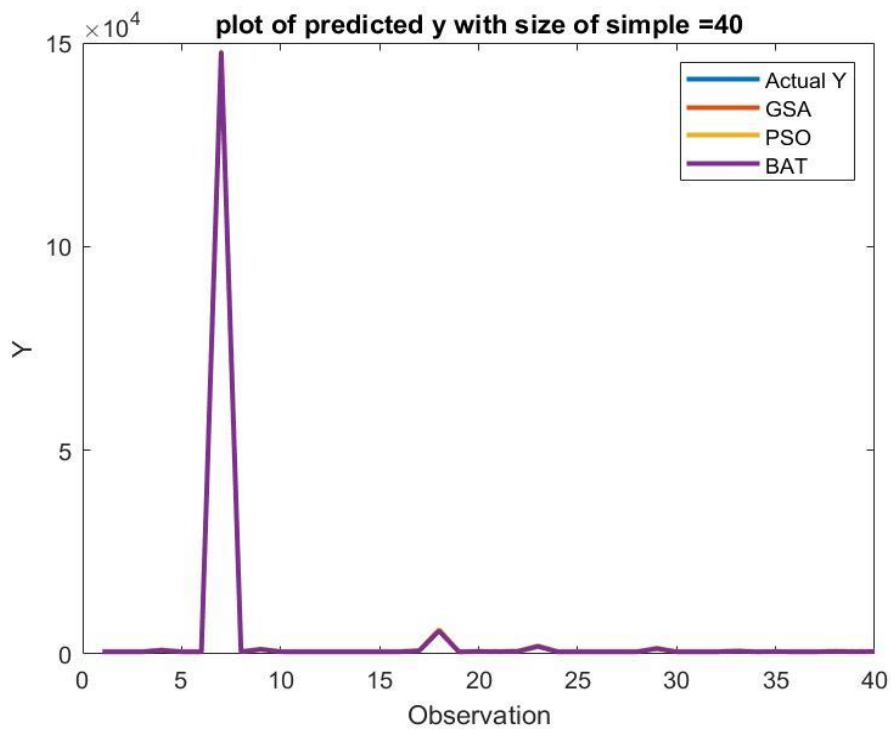


Figure 6: Comparison of the Module one's EPNGSA, BAT, PSO, and GSA methods where the data set size is 40

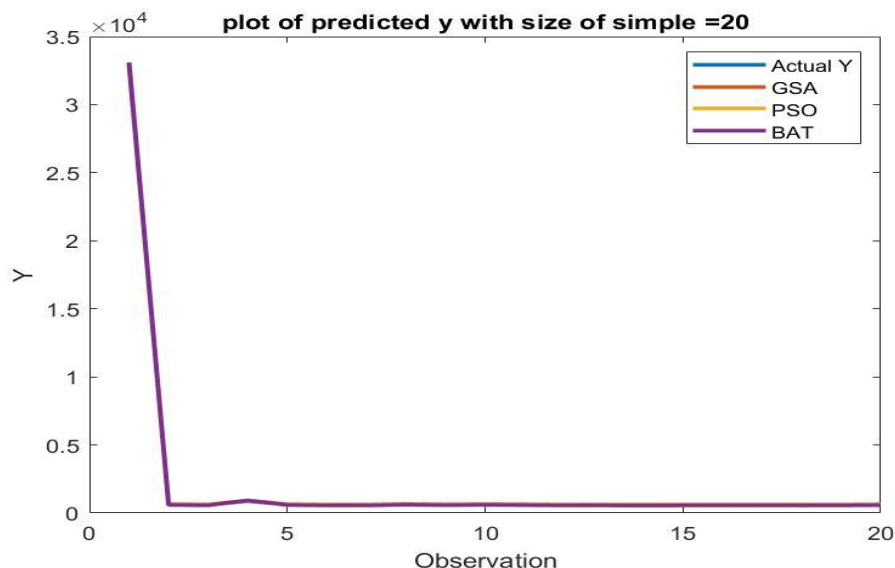


Figure 7: Comparison of the Module one's EPNGSA, BAT, PSO, and GSA methods where the data set size is 20

8-Conclusion and Recommendation

This paper was devoted to solving Non-linear degradation in the field of the combinatorial optimization problem. Firstly, we formalize the sectional reexaminations carried out in every section. For doing so, we provide an explanation of the way we address the Non-linear degradation issues that were initially discussed in this paper. Next, we provide some guidelines that go into detail about our suggested algorithm, EPNGSA. Furthermore, as a substitute method for estimating Non-linear degradation frameworks, three Meta-heuristic algorithms were employed. Six different types of non-linear degradation frameworks are utilized each with a variable number of components. Furthermore, various samples ($n= 20, 40, 80, 160, \text{ and } 200$) were employed.

Finally, it is crucial to conduct additional tests and comparisons of the suggestions. The results of this reexamination lead us to the conclusion that the suggested algorithms may be constructed in several ways.

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