Longitudinal and Transverse Electron Scattering Form Factors for ^{13}C Nucleus with Core-Polarization Effects

N. M.Adeeb, A. A. Alzubadi and O. A. Jalal*

Department of Physics, College of Science, University of Baghdad, Baghdad, Iraq

Abstract

Inelastic electron scattering have been studied for $\frac{3}{2}^{-}\frac{1}{2}(3.68MeV)$, $\frac{5}{2}^{-}\frac{1}{2}(7.55MeV)$ *and* $\frac{3}{2}^{-}\frac{3}{2}(15.11MeV)$ states in the ^{13}C nucleus. ^{4}He is considered as an inert core with nine nucleons out of it (the model space of nucleus). Form factors are calculated by using Cohen-Kurath interaction for 1p-shell model space with Modified Surface Delta Interaction (MSDI) as a residual interaction for higher configuration. The study of core-polarization effects on the form factors is based on microscopic theory, which combines shell model wave functions and configurations with higher energy as the first order perturbation. The radial wave functions for the single-particle matrix elements have been calculated with the harmonic oscillator potential and the oscillator length parameter b is chosen to reproduce the measured root mean square charge radius for nucleus under considered in this work. The inclusion of the core-polarization effects (the effects from out of the core) gives a good agreement with the experimental data.

Keywords: Longitudinal form factors, Transvers form factors, Calculated first-order core-polarization effects

عوامل التشكيل للإستطارة الألكترونية الطولية والمستعرضة لنواة الكربون-١٣مع تأثيرإستقطاب القلب

نادية محمد أديب، علي عبد اللطيف الزبيدي و عمر أياد جلال*

قسم الفيزياء، كلية العلوم، جامعة بغداد، بغداد، العراق.

الخلاصة:

في هذا البحث تمَّ دراسة الإستطارة الألكترونية غير المرنة للحالات $\frac{3}{2}^{-}\frac{1}{2}(3.68MeV)$، $\frac{3}{2}^{-}\frac{3}{2}(15.11MeV)$ $\frac{5}{2}^{-}\frac{1}{2}(7.55MeV)$......*and* في نواة الكربون-١٣،حيث فُرض القلب هليوم-٤ وتسع جسيمات موزعة في المدارات $1p_{3/2}$ و$1p_{1/2}$ والتي تشكل الفضاء الأنموذجي. تمَّ حساب عوامل التشكل في إطار نظرية القشرة بإستخدام تفاعل كوهين-كوراث مع تفاعل تشوه السطح المعدل (MSDI) كتفاعل بقية للتوزيعات العالية. تمَّ دراسة تأثير إستقطاب القلب على عوامل التشكل إستناداً الى النظرية المجهرية، والتي تربط الدوال الموجية لأنموذج الأغلفة مع التوزيعات ذات الطاقة العالية. الحسابات في العمل الحالي أُنجزت مع الدالة القطرية للمتذبذب التوافقي، حيث تمَّ إختيار معامل الحجم b بحيث يعيد إنتاج جذر معدل المربع لنصف قطر الشحنة المقاس عملياً. إدخال تأثير إستقطاب القلب أعطى نتائج ذات تطابق جيد مع المعطيات العملية.

*Email: omar.a.jalal@gmail.com

1. Introduction

Shell model takes a particularly important position, basically because the shell model is more fundamental framework based on a minimum number of assumptions. We are also aware of the fact that the shell model has been extremely successful in the description of the light nuclei at low excitation energies [1].The nuclear shell model continues to provide the main theoretical tool for understanding all properties of nuclei. It can be used in its simplest single particle form to provide qualitative understanding, but it also be used as a basis for much more complex and complete calculations. There appears to be limited within the near future to the expansion of its application [2].The interacting shell model is generally considered to be the most fundamental theory of the nucleus short of an explicit solution of the A-body problem, and the residual interaction among the valance nucleons plays an important role [3].The scattering of electron from nuclei gives the most precise information about nuclear size and charge distribution, and it has provided important information about the electromagnetic currents inside the nuclei. Electron scattering can provide a good test for such calculation since it is sensitive to the spatial dependence of the charge and current densities [4-6].The Coulomb form factors have been discussed for the stable sd-shell nuclei using sd-shell wave functions with phenomenological effective charges [7]. Shell model within a restricted model space succeeded in describing static properties of nuclei. For p-shell nuclei, Cohen-Kurath [8] interaction explains well the low-energy properties of p-shell nuclei. However, at higher-momentum transfer, it fails to describe the form factors. Radhi [9-12] has successfully proved that the inclusion of core polarization effects in the p-shell and sd-shell are very essential to improve the calculations of the form factors. Comparisons between calculated and measured longitudinal and transverse electron scattering form factors have long been used as stringent tests of models of nuclear structure [13-19]. Single-particle wave functions are used as a zero-th contribution and the effect of core polarization is included as a first-order perturbation theory with the modified surface delta interaction (MSDI) [20] as a residual interaction. The single-particle wave functions are those of the harmonic-oscillator (HO) potential and the oscillator length parameter b is chosen to reproduce the measured root mean square charge radius for nucleus under considered in this work [21].

2. General Theory

In the plane-wave Born approximation (PWBA), the differential cross-section for the scattering of an electron from a nucleus of charge (Ze) and mass (M) into a solid angle (dΩ) is given by [22]:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} f_{rec} \sum_J \left|F_J(q,\theta)\right|^2 \qquad \text{….. (1)}$$

where

$$F_J^2(q,\theta) = \left(\frac{q_\mu}{q}\right)^4 \left|F_J^L(q)\right|^2 + \left[\frac{q_\mu^2}{2q^2} + \tan^2(\theta/2)\right]\left|F_J^T(q)\right|^2 \qquad (2)$$

and

$$\left|F_J^T(q)\right|^2 = \left|F_J^{El}(q)\right|^2 + \left|F_J^{mag}(q)\right|^2 \qquad \text{……….. (3)}$$

The form factor of multipolarity(J) as a function of momentum transfer is written in terms of the reduced matrix elements of the transition operator as [22]:

$$\left|F_J^\eta(q)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)}\left|\left\langle J_f \left\| T_J^\eta(q) \right\| J_i\right\rangle\right|^2 \qquad \text{…….(4)}$$

The nuclear states have a well-defined isospin. So using the Wigner-Eckart theorem in isospin space, the form factor can be written in terms of the matrix element reduced both in total angular momentum (spin) (J) and isospin (T) (triple-bar matrix elements). Also, in the realistic calculation of the form factor, it is necessary to take into account the effects of finite size (f.s), center of mass (c.m) motion and Coulomb distortion of electron waves. Thus, the form factor for a given multipolarity(J) can be written in terms of the matrix elements reduced both in spin and isospin spaces as:

$$\left|F_J^\eta(q)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)}\left|\sum_{T=0,1}(-1)^{T_f-T_{zf}}\begin{pmatrix} T_f & T & T_i \\ -T_{Zf} & M_T & T_{Zi}\end{pmatrix}\left\langle \Gamma_f \left\| T_{J,T}^\eta(q) \right\| \Gamma_i\right\rangle\right|^2$$
$$\times\left|F_{c.m}(q)\right|^2 \times \left|F_{f.s}(q)\right|^2 \qquad \text{……….. (5)}$$

The multipolarity(J) in the last equation is resterictted by angular momentum selection rule: or

$$\left|J_i - J_f\right| \le J \le J_i + J_f \qquad \Delta(J_i, J_f, J) \quad (6)$$

and

$$\left|T_i - T_f\right| \le T \le T_i + T_f \qquad \text{……….. (7)}$$

The parity selection rules (for same parity)

$$\Delta\pi^{El} = (-1)^{J} \qquad \text{……….. (8 - a)}$$

$$\Delta\pi^{mag} = (-1)^{J+1} \qquad \text{……….. (8 - b)}$$

3. Core-Polarization Effects

The core polarization effect on the form factor is based on a microscopic theory that combines shell-model wave functions and configurations with higher energy as first order perturbations; these are called "core-polarization effects"

The reduced matrix elements of the electron scattering operator consist of two parts, one is the "Model space" matrix elements and the other is the "Core-polarization" matrix elements

$$\left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle = \left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{MS} + \left\langle \Gamma_f \left\| \delta\hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{CP} \qquad (9)$$

$\left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{MS}$ is the model-space matrix elements

$\left\langle \Gamma_f \left\| \delta\hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{CP}$ is the core-polarization matrix elements

$\left|\Gamma_i\right\rangle$ and $\left|\Gamma_f\right\rangle$ are described by the model-space wave functions.

The model-space matrix elements are expressed as the sum of the product of the one-body density matrix elements (OBDM) times the single-particle matrix elements which are given by:

$$\left\langle \Gamma_f \left\| \hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{MS} = \sum_{\alpha,\beta} OBDM(\Gamma_i,\Gamma_f,\alpha,\beta) \left\langle \alpha \left\| \hat{T}^{\eta}_{\Lambda} \right\| \beta \right\rangle_{MS} \qquad \text{…..(10)}$$

The core-polarization matrix element can be written as follows

$$\left\langle \Gamma_f \left\| \delta\hat{T}^{\eta}_{\Lambda} \right\| \Gamma_i \right\rangle_{cp} = \sum_{\alpha,\beta} OBDM(\Gamma_i,\Gamma_f,\alpha, \qquad \text{……….. (11)}$$

According to first – order perturbation theory, the reduced single – particle matrix element of the one body operator is expressed as the sum of three terms, a model space matrix element and two core – polarization matrix elements [20]

$$(\alpha \| \hat{T}^{\eta}_{\Lambda} \| \beta) = \left\langle \alpha \left\| \hat{T}^{\eta}_{\Lambda} \right\| \beta \right\rangle_{MS} + \left\langle \alpha \left\| \hat{T}^{\eta}_{\Lambda} \frac{Q}{E-H^{(0)}} V_{res} \right\| \beta \right\rangle_{CP} + \left\langle \alpha \left\| V_{res} \frac{Q}{E-H^{(0)}} \hat{T}^{\eta}_{\Lambda} \right\| \beta \right\rangle_{CP} \qquad \text{……….. (12)}$$

The first term is the zero- order contribution. The second and third terms are the first – order contribution which give the higher – energy configuration.

4. Results and Discussion

Carbon nucleus is one of the 1p-shell nuclei. For the conventional many particles shell-model, ^{13}C is considered as the ^{4}He core and nine nucleons outside the core distributed over $1p_{3/2} 1p_{1/2}$ shell space.

The wave functions of the single-particle for transitions under consideration are those of the harmonic oscillator potential, the size parameter (b) is chosen to reproduce the measured root mean square charge radius for nucleus under considered in this work (b = 1.628fm). The core polarization effects are calculated with the (MSDI) as a residual interaction. The parameters of the (MSDI) are denoted by A_T, B and C [20], where T indicates the isospin (0 or 1). These parameters are taken to be A_0 = A1 = B = (25/A) MeV and C = 0, where A is the mass number, as a first set. Or taken to be A_0 = A1 = (35/A) MeV and B = C = 0, as a second set. The first set used in the transitions to the $\frac{3}{2}^{-}\frac{1}{2}$ (3.68 MeV) state, and for the longitudinal transition to $\frac{5}{2}^{-}\frac{1}{2}$ (7.55 MeV) State. The second set used in the transverse transition to $\frac{5}{2}^{-}\frac{1}{2}$ (7.55 MeV) state, and in the transitions to $\frac{3}{2}^{-}\frac{3}{2}$ (15.11 MeV) State.

1. The $\frac{3}{2}^{-}\frac{1}{2}$ (3.68MeV) State:

The nucleus is excited from the
denoted by $J^{\pi}{}_i T_i = \frac{1}{2}^{-}\frac{1}{2}$ by the incide
the $J_f T_f = \frac{3}{2}^{-}\frac{1}{2}$ with excitation energy 3.68 MeV.

The theoretical and experimental electron scattering form factors are shown
the C2 transition. The theoretica
that the core polarization contribution gives a very good agreement with the experimental data [21] in all range of momentum transfer. By compare this result with that of the model space, it can be seen that the result of the core polarization plus to that of the model space is quite close to the experimental data in all momentum transfer range. This may be interpreted as the values of momentum transfers q in the all range of momentum transfers have their effects in the core polarization quite well which enhance the result of the 1p-shell relative to the data.

Figure 2 shows the transverse transition of multi-polarity M1 and E2 in the 1p-shell model space. We note that the theoretical result don't close the experimental ones. Also the range of momentum transfer less than 1.5 fm^{-1} the M1 transition predominate, while in the range of momentum transfer up to 1.5 fm^{-1} the E2 transition predominate.

Figure 3 illustrates the core polarization effect on the 1p-shell model space results which appears in the figure 2. Inspection of this figure revels that the core polarization plus to the model space results consent the model space results in the range of momentum transfer less than 0.5 fm^{-1}, while in the range between 0.5 fm^{-1} to 1.5 fm^{-1} the core polarization effect clearly enhance the model space results. In the range up to 1.5 fm^{-1} the core polarization effect doesn't enhance the model space result.

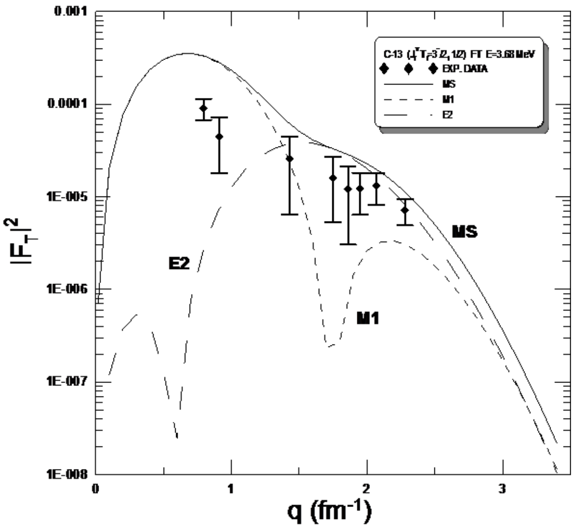

Figure 2- Transverse electron scattering form factors of multipolarity (M1+E2) without core-polarization effects for the excited state $(\frac{3^-}{2}\frac{1}{2})$ of the 3.68 MeV in ^{13}C. The experimental data are taken from reference [21].

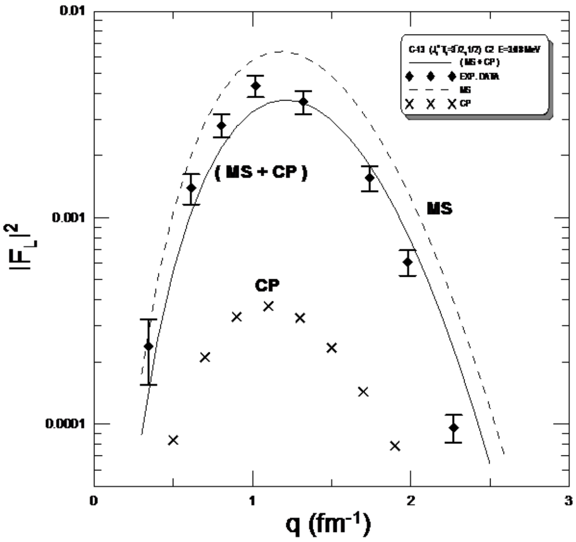

Figure 1- Longitudinal electron scattering (C2) form factors with core-polarization effects for the excited state $(\frac{3^-}{2}\frac{1}{2})$ of the 3.68 MeV in ^{13}C. The experimental data are taken from reference [21].

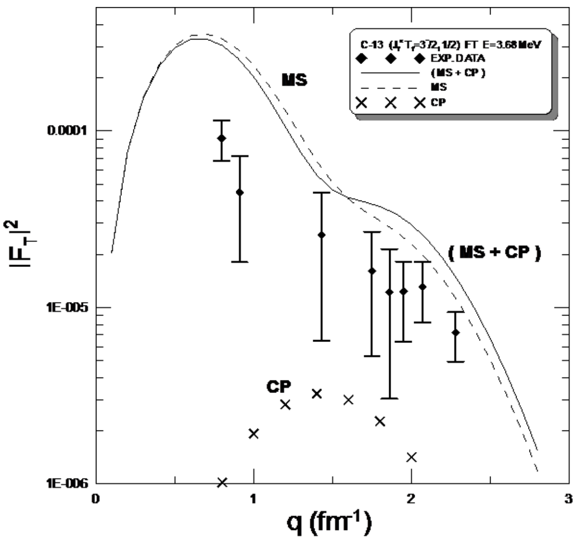

Figure 3- Transverse electron scattering form factors with core –polarization effects for the excited state $(\frac{3^-}{2}\frac{1}{2})$ of the 3.680 MeV in ^{13}C. The experimental data are taken from reference [21].

2. The $\frac{5^-}{2}\frac{1}{2}$ (7.55 MeV) State:

The C2 electron scattering form factors for the transition $\frac{1}{2}\ \frac{1}{2} \rightarrow \frac{5}{2}\ \frac{1}{2}$ are illustrated in figure 4. The results show that the core-polarization effect gives a good agreement with the experimental data for the all values of momentum transfer (q).

The transverse electron scattering for this transition involves E2 and M3 multipoles according to the parity selection rules. But the transition M3 has neglected because weakness of this transition. Therefore, in figure 5 only E2 transition was depicted. In this figure the core polarization effect enhance the p-shell results in the momentum transfer region lies between 0.9 and 2 fm^{-1}.

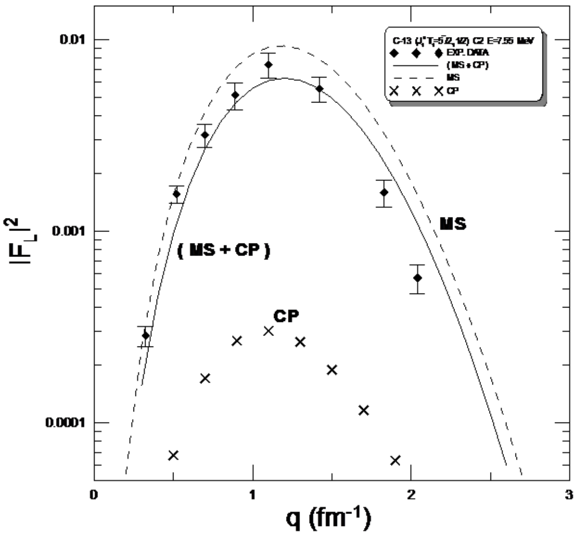

Figure 4- Longitudinal electron scattering (C2) form factors with core-polarization effects for the excited state ($\frac{5^-}{2}\frac{1}{2}$) of the 7.55 MeV in ^{13}C. The experimental data are taken from reference [21].

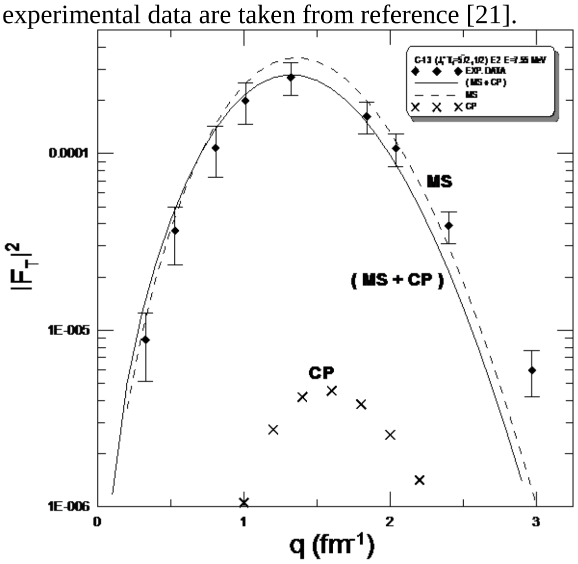

Figure 5- Transverse electron scattering (E2) form factors with core-polarization effects for the excited state ($\frac{5^-}{2}\frac{1}{2}$) of the 7.55 MeV in ^{13}C. The experimental data are taken from reference [21].

3. The $\frac{3^-}{2}\frac{3}{2}$ (15.11 MeV) State:

The theoretical and experimental electron scattering form factors are shown in figure 6 for the C2 transition of $\frac{3^-}{2}\frac{3}{2}$ state. The theoretical results show that the core polarization contribution gives a good agreement with the experimental data in all range of momentum transfer.

The transverse forms factors for the last excitation energy was studied in the figures 7 and 8. Figure 7 shows the p-shell results, which illustrate that these results, in range of momentum transfer less than 1 fm^{-1} the E2 transition predominate. While, in the range up to 1 fm^{-1} the M1 transition is predominate. Figure 8 shows that the core polarization plus to model space results concedes the p-shell results at momentum transfers region less than 1 fm^{-1}. While, the core polarization effects reduce the p-shell results at the momentum transfer region lies between (1 and 2) fm^{-1}. In the momentum transfers up to 2 fm^{-1} the core polarization effect doesn't enhance the model space result.

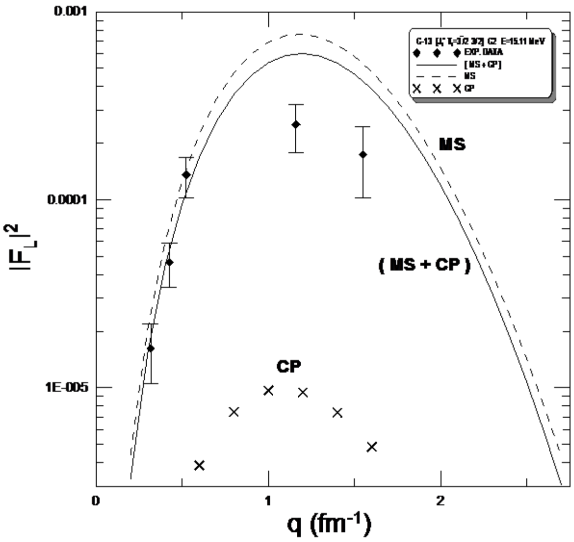

Figure 6- Longitudinal electron scattering (C2) form factors with core-polarization effects for the excited state ($\frac{3^-}{2}\frac{3}{2}$) of the 15.11 MeV in ^{13}C. The experimental data are taken from reference [21].

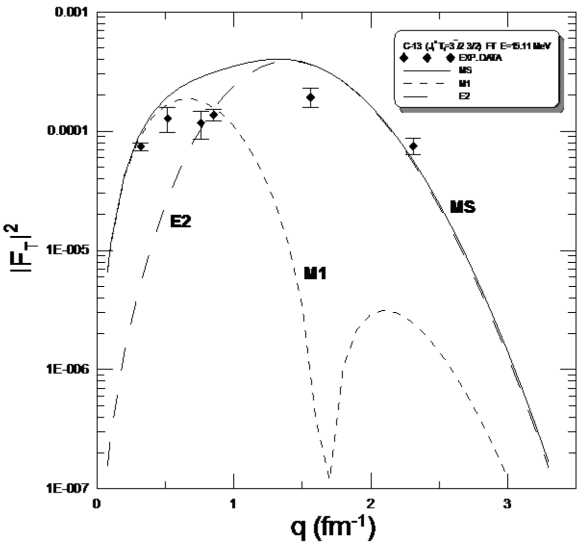

Figure 7- Transverse electron scattering of (M1+E2) form factors without core-polarization effects for the excited state ($\frac{3}{2}^{-}\frac{3}{2}$) of the 15.11 MeV in ^{13}C. The experimental data are taken from reference [21].

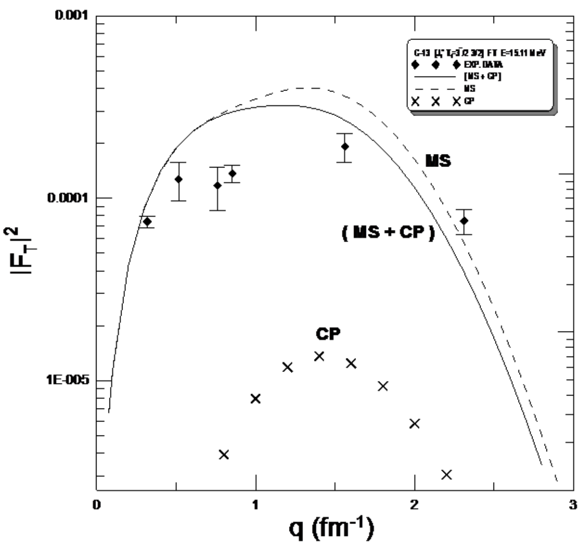

Figure 8- Transvers electron scattering form factors with core-polarization effects for the excited state ($\frac{3}{2}^{-}\frac{3}{2}$) of the 15.11 MeV in ^{13}C. The experimental data are taken from reference [21].

5. Conclusion

1. From figures 1, 4, and 6 the effect of core polarization enhances the results of p-shell in the all momentum transfers region for all longitudinal transitions under consideration in this study.
2. From figures 3, 5, and 8 the effect of core polarization enhances the results of p-shell in the low momentum transfers region only, for all transverse transitions under consideration in this study.
3. From figures 2 and 7 the M1 transition is predominate in the low momentum transfers. While, E2 transition is predominate in the high momentum transfers region.
4. In the low energy transition and in the longitudinal transition of the medium energy transition the suitable MSDI parameters are to be A_0 = A1 = B = (25/A) MeV and C = 0. While, in the transverse transition of the medium energy and in the high energy transitions the suitable MSDI parameters are taken to be A_0 = A1 = (35/A) MeV and B=C=0.

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