The Dynamics of Holling Type IV Prey Predator Model with Intra- Specific Competition

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Abstract

In this paper a prey-predator model involving Holling type IV functional response and intra-specific competition is proposed and analyzed. The local stability analysis of the system is carried out. The occurrence of a simple Hopf bifurcation is investigated. The global dynamics of the system is investigated with the help of the Lyapunov function and poincare-bendixson theorem. Finally, the numerical simulation is used to study the global dynamical behavior of the system. It is observed that, the system has either stable point or periodic dynamics.

Keywords: Holling type IV functional response, equilibrium points, stability and Hopf bifurcation.

ديناميكية نظام هولنك من النوع الرابع للفريسة–المفترس والمتضمن التناقس الضمني

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الخلاصة

في هذا البحث تم اقتراح وتحليل نظام الفريسة –المفترس والمتضمن دالة استجابة لهولنك ذات النوع الرابع التنافس بين افراد المفترس.السلوك الديناميكي المحلي للنظام درس .تفرع هوبف نوقش, كما تم مناقشة السلوك الديناميكي الشامل بمساعدة دالة اليابانوف ونظرية بينكاريه–بندكس, واخيرا تم استخدام المحاكاة العددية لدراسة سلوك الديناميكية الشاملة وقد لوحظ بان النظام يمتلك اما نقطة استقرار او ديناميكية دورية.

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1.Introduction

Variety of the mathematical models for interacting species incorporating different factors to suit the varied requirements are available in literature, a successful model is one that meets the objectives, explains what is currently happening and predicts what will happen in future. The first major attempt to predict the evolution and existence of species mathematically is due to the

American physical chemist Lotka (1925) and independently by the italian mathematician Volterra (1926), see ref. [1], which constitute the main them of the deterministic theory of population-dynamics in theoretical biology even today. Over the last few decades, many models for two or more interacting species have been proposed on the basis of Lotka-Volterra models by taking into account the effects of crowding, age structure, time delay, functional response, switching, etc. [2,3,4]. Prey-predator relationships is an interaction type which cover all kinds of natural enemies, including typical predators and their preys, hosts parasites, parasitoids, plants and herbivores. Despite important modeling work by Lotka-Volterra, there has been much less quantitative work on predation than competition. This mainly results from the fact that the dynamics of the two species prey-predator models is based on the trophic function (functional response) type and hence it is harder to incorporate into predation models than competition models. The functional response of predator to prey density refers to the change in the density of prey attacked per unit time per predator as the prey density changes [5]. These functions can be classified into three classes known as prey-dependent, ratio dependent and predator dependent [6]. The prey-dependent (including Holling types I-III given in [7]), prey-predator models have been extensively studied in literature see for example refs. [8-11]. However, the ratio dependent, which proposed originally by Arditie and Ginzburg [12], prey-predator model have been investigated by number of researchers [13-17] and the references their in. Finally predator-dependent, which originally proposed by Beddington [18] and De Angelis [19], prey-predator models have been also studied in literature [20] and the References their in.

Recently, Mainal Haque 2009 [21] has been proposed and study ratio-dependent prey-predator model involving intra-specific competition in predator species.

In this paper however, the Haque model is modified so that Holling type IV that suggested by Andrews [22] is used instead of ratio-dependent functional response. The new model has been studied analytically as well as numerically.

2. Mathematical models

Let $x(t)$ be the density of prey species at time t, $y(t)$ be the density of predator species at time t that consumes the prey species according to Holling type IV functional response then the dynamics of a prey–predator model can be represented by the following system of ordinary differential equations.

$$\begin{aligned}\frac{dx}{dt} &= x\left[a - bx - \frac{\alpha\gamma y}{x^2 + \gamma x + \gamma\beta}\right] = xf_1(x,y)\\ \frac{dy}{dt} &= y\left[-d + \frac{e\alpha\gamma x}{x^2 + \gamma x + \gamma\beta} - \delta y\right] = yf_2(x,y)\end{aligned} \quad (1)$$

with $x(0) \geq 0$ and $y(0) \geq 0$

Note that all the parameters of system (1) are assumed to be positive constants and can be described as following: a is the intrinsic growth rate of the prey population; d is the death rate of the predator population; the parameter b is the strength of intra-specific competition among the prey species; the parameter β can be interpreted as the half-saturation constant in the absence of any inhibitory effect; the parameter γ is a direct measure of the predator immunity from the prey; α is the maximum attack rate of the prey by a predator; e represents the conversion rate; and finally δ is the strength of intra-specific competition among the predator species .

Obviously, the interaction function in the right hand side of system (1) are continuously differentiable function on $R_+^2 = \{(x,y) \in R^2 : x \geq 0, y \geq 0\}$ and hence they are Lipschitzian therefore for any initial value $((x(0), y(0)) \in R_+^2$ the solution of system (1) exists and is unique. Further all the solutions of system (1) with non-negative initial conditions is

uniformly bounded as shown in the following theorem

Theorem 1: All the solutions of system (1) which initiate in R_+^2 are uniformly bounded.

Proof:

Let $((x(t), y(t))$ be any solution of the system (1) with non negative initial condition (x_0, y_0). According to the first equation of system (1) we have

$$\frac{dx}{dt} \le x(a - bx)$$

Then by solving this differential inequality we obtain that

$$x(t) \le \frac{ax_0}{ae^{-at} + bx_0 - x_0 be^{-at}}$$

Thus $\lim_{t\to\infty}$ Sup $x(t) \le M$ where $M = \max\left\{\frac{a}{b}, x_0\right\}$

Define the function: $W(t) = x(t) + \frac{1}{e}y(t)$

So the time derivative of $w(t)$ along the solution of the system (1)

$$\frac{dW}{dt} = \frac{dx}{dt} + \frac{1}{e}\frac{dy}{dt}$$

$$\frac{dW}{dt} \le (a + d)x - d(x + \frac{1}{e}y)$$

$$\frac{dW}{dt} + dW \le (a + d).M$$

Again by solving the above linear differential inequality we get

$$W(t) \le \frac{(a+d)}{d}M + W(0)e^{-dt} - \frac{(a+d)M}{d}e^{-dt}$$

Consequently, for $t \to \infty$ we have

$$0 \le W(t) \le \frac{(a+d)M}{d}$$

Hence all solution of system (1) enter the region

$$\Omega = \left\{(x(t), y(t)) \in R_+^2 : x(t) + \frac{1}{e}y(t) \le \frac{(a+d)M}{d} + \varepsilon \text{ for any } \varepsilon > 0\right\}$$

■

3. Existence and Local Stability analysis of system (1) with persistence

In this section, the existence and local stability analysis of all possible non –negative equilibrium points of system (1) are investigated. There are three non-negative equilibrium points of system (1) the existence and the stability analysis for each of them are as follows:

(1) The trivial equilibrium point $E_0 = (0,0)$ always exists.

(2) The equilibrium point $E_1 = \left(\frac{a}{b}, 0\right)$ always exists, as the prey population grows to the carrying capacity in the absence of predation.

(3) There is no equilibrium point on $y - axis$ as the predator population dies in the absence of its prey.

(4) The positive equilibrium point $E_2 = (x^*, y^*)$ exists in the interior of the first quadrant if and only if there is a positive solution to the following set of algebraic nonlinear equations:

$$a - bx - \frac{\alpha\gamma y}{(x^2 + \gamma x + \gamma\beta)} = 0 \quad (2a)$$

$$-d + \frac{e\alpha\gamma x}{(x^2 + \gamma x + \gamma\beta)} - \delta y = 0 \quad (2b)$$

From $(2b)$ we have

$$y = \frac{-d(x^2 + \gamma x + \gamma\beta) + e\alpha\gamma x}{\delta(x^2 + \gamma x + \gamma\beta)} \quad (3)$$

Clearly, $y > 0$ if the following condition holds

$$e\alpha\gamma > \frac{d(x^2 + \gamma x + \gamma\beta)}{x} \quad (4)$$

Now by substituting 3 in 2a and then simplifying the resulting term we obtain that

$$f(x) = A_5x^5 + A_4x^4 + A_3x^3 + A_2x^2 + A_1x + A_0 \quad (5)$$

Where

$$A_5 = -b\delta < 0$$

$$A_4 = \delta(a - 2b\gamma)$$

$$A_3 = \gamma\delta(2a - b\gamma - 2b\beta)$$

$$A_2 = \gamma(2\beta\delta(a - b\gamma) + a\gamma^2\delta + \alpha d)$$

$$A_1 = \gamma^2(\beta\delta(2a - b\beta) + \alpha(d - e\alpha))$$

$$A_0 = a\beta^2\gamma^2\delta + \alpha\beta\gamma^2 d > 0$$

Therefore the positive equilibrium point $E_2 = (x^*, y^*)$, where x^* is a positive root of equation (5) that satisfies condition 4 while $y^* = y(x^*)$ that given in Equation 3 exists uniquely in $Int\ R_+^2$ provided that condition 4 holds a long with one set of the following sets of conditions

$$a < \min\{b\gamma, \frac{b\gamma}{2} + b\beta\}$$
$$a\gamma\delta + \alpha d < 2\beta\delta(b\gamma - a) \quad (6a)$$

$$2a > b\gamma + 2b\beta$$
$$a > b\gamma \ \ or \ \ a\gamma\delta + \alpha d > 2\beta\delta(b\gamma - a) > 0 \quad (6b)$$
$$d > e\alpha \ \ or \ \ \beta\delta(2a - b\beta) > \alpha(e\alpha - d) > 0$$

$$a < 2b\gamma$$
$$2a < b\gamma + 2b\beta$$
$$a\gamma\delta + \alpha d > 2\beta\delta(b\gamma - a) \quad (6c)$$
$$\beta\delta(2a - b\beta) + \alpha(d - e\alpha) > 0$$

Note that, conditions 6a - 6c guarantee that Equation .5 has unique positive root.

Now, we will discuss the local dynamical behavior of the solution of system 1 near these equilibrium points. First we need to compute the varitional matrix of eigenvalues. Assume that $J(x, y)$

Where

$$J(x,y) = \begin{pmatrix} x\frac{\partial f_1}{\partial x} + f_1 & x\frac{\partial f_1}{\partial y} \\ y\frac{\partial f_2}{\partial x} & y\frac{\partial f_2}{\partial y} + f_2 \end{pmatrix} \quad (7)$$

$$\frac{\partial f_1}{\partial x} = -b + \frac{\alpha\gamma y(2x + \gamma)}{(x^2 + \gamma x + \gamma\beta)^2}$$

$$\frac{\partial f_1}{\partial y} = \frac{-\alpha\gamma}{(x^2 + \gamma x + \gamma\beta)}$$

$$\frac{\partial f_2}{\partial x} = \frac{e\alpha\gamma(\beta\gamma - x^2)}{(x^2 + \gamma x + \gamma\beta)^2} \text{ and } \frac{\partial f_2}{\partial y} = -\delta$$

Accordingly, by substituting the equilibrium points $E_i, i = 0,1,2$ in 7 and then computing the eigenvalues, for $J(E_i), i = 0,1,2$ respectively the following results are obtained:

The varitional matrix of system (1) at E_0 is $J(E_0) = \begin{pmatrix} a & 0 \\ 0 & -d \end{pmatrix}$ clearly the eigenvalues of $J(E_0)$ are $\lambda_{0x} = a > 0$ and $\lambda_{0y} = -d < 0$, where $\lambda_{0x}, \lambda_{0y}$ represent the eigenvalues of $J(E_0)$ that describe the dynamics in the $x-$ direction and $y-$ direction respectively. Hence E_0 is a saddle point.

The varitional matrix of system (1) at $E_1 = \left(\frac{a}{b}, 0\right)$ is given by

$$J(E_1) = \begin{pmatrix} -a & \frac{-ab\alpha\gamma}{a^2 + ab\gamma + b^2\gamma\beta} \\ 0 & -d + \frac{abe\alpha\gamma}{a^2 + ab\gamma + b^2\gamma\beta} \end{pmatrix}$$

So, the eigenvalues are $\lambda_{1x} = -a < 0$

and $\lambda_{1y} = -d + \frac{abe\alpha\gamma}{a^2 + ab\gamma + b^2\gamma\beta}$

Therefore, E_1 is locally asymptotically stable if and only if

$$\frac{abe\alpha\gamma}{a^2 + ab\gamma + b^2\beta\gamma} < d \quad (8a)$$

While $E_1 = (\frac{a}{b}, 0)$ is saddle point provided that

$$\frac{abe\alpha\gamma}{a^2 + ab\gamma + b^2\beta\gamma} > d \quad (8b)$$

The varitional matrix of the system (1) at the positive equilibrium point $E_2 = (x^*, y^*)$ is given as follows

$$J(E_2) = \begin{pmatrix} -bx^* + \frac{\alpha\gamma x^* y^*(2x^* + \gamma)}{R^2} & \frac{-\alpha\gamma x^*}{R} \\ \frac{e\alpha\gamma y^*(\beta\gamma - x^{*2})}{R^2} & -\delta y^* \end{pmatrix} = (a_{ij})_{2\times 2} \quad (9)$$

Where $R = (x^{*2} + \gamma x^* + \gamma\beta)$

Note that according to the stability theorem for the two dimensional dynamical system, $E_2 = (x^*, y^*)$ is locally asymptotically stable provided that

Trace $(J(E_2)) = T = a_{11} + a_{22} < 0$

$\text{Detriment}(J(E_2)) = D = a_{11}a_{22} + a_{12}a_{21} > 0$

Now since

$$T = \frac{1}{R^2}\left[-(bx^* + \delta y^*)R^2 + \alpha\gamma x^* y^*(2x^* + \gamma)\right] \quad (10a)$$

$$D = \frac{x^* y^*}{R^3}\left[e\alpha^2\gamma^2(\gamma\beta - x^{*2}) + \delta R\left(bR^2 - \alpha\gamma y^*(2x^* + \gamma)\right)\right] \quad (10b)$$

Therefore the positive equilibrium point $E_2 = (x^*, y^*)$ of system (1) is locally asymptotically stable in $Int\ R_+^2$ under the following necessary and sufficient conditions

$$\alpha\gamma x^* y^*(2x^* + \gamma) < (bx^* + \delta y^*)R^2 \quad (11a)$$

$$b\delta R^3 + e\alpha^2\gamma^2(\gamma\beta - x^{*2}) > \alpha\gamma\delta y^*(2x^* + \gamma)R \quad (11b)$$

In the following the persistence of the system (1) is studied. It well known that the system is said to be persists if and only if each species is persist.

Mathematically, this is means that, system (1) is persists if the solution of the system with positive initial condition does not have omega limit sets on the boundary planes of its domain.

However, biologically means that, all the species are survivor. In the following theorem the persistence condition of the system (1) is established using the Gard and Hallam technique [23].

Theorem 2: The system (1) is uniformly persist provided that condition $(8b)$ holds.

Proof:

consider the following function, $\sigma(x, y) = x^{s_1} y^{s_2}$ where $s_i, i = 1,2$ undetermined positive constants. Obviously, $\sigma(x, y)$ is C^1 positive function defined on R^2, and $\sigma(x, y) \to 0$, if $x \to 0$ or $y \to 0$. Now since

$$\psi(x, y) = \frac{\sigma'(x, y)}{\sigma(x, y)} = s_1\frac{x'}{x} + s_2\frac{y'}{y}$$

Therefore

$$\psi(x, y) = s_1\left[a - bx - \frac{\alpha\gamma x}{(x^2 + \gamma x + \gamma\beta)}\right] + s_2\left[-d + \frac{e\alpha\gamma x}{(x^2 + \gamma x + \gamma\beta)} - \delta y\right]$$

Note that, since $E_0 = (0,0)$ and $E_1 = \left(\frac{a}{b}, 0\right)$ are the only possible omega limit sets of the solution of system (1) on the boundary of $Int\ R_+^2$, in addition

$\psi(E_0) = s_1 a - s_2 d$

and $\psi(E_1) = s_2(-d + \frac{e\alpha\gamma ab}{a^2 + \gamma ab + \gamma\beta b^2}) > 0$

Clearly $\psi(E_0) > 0$ for all sufficiently large positive value of s_1 with respect to s_2, while $\psi(E_1) > 0$, for all values of s_2 under condition $(8b)$. Hence σ represents persistence function and system (1) is uniformly persistent. ■

Theorem (3): Assume that condition $(11b)$ holds then the System (1) possesses a Hopf bifurcation, near the positive equilibrium point E_2 at the parameter value $\alpha^* = \frac{(bx^* + \delta y^*)R^2}{\gamma x^* y^*(2x^* + \gamma)}$

Proof:

According to the varitional matrix (9), the eigenvalues of $J(E_2)$ can be written as

$$\lambda x, y = \frac{T}{2} \pm \frac{1}{2}\sqrt{T^2 - 4D}$$

Here T and D are given by equation (10a) and $(10b)$ respectively.

Cleary $D > 0$ under condition (10b) and $T = 0$ at $\alpha = \alpha^*$ then we obtain

$$\lambda_{x,y} = \pm\frac{1}{2}\sqrt{-4D} = \pm i\sqrt{D}$$

further

$$\left.\frac{\partial\,\mathrm{Re}(\lambda_{x,y})}{\partial\alpha}\right|_{\alpha=\alpha^*} = \left.\frac{\partial}{\partial\alpha}(\frac{T}{2})\right|_{\alpha=\alpha^*} = \gamma x^* y^*(2x^* + \gamma) \neq 0$$

So according to hopf bifurcation theorem the proof is completed. ■

4. Global dynamical behavior of the system (1)

In this section the global stability for the equilibrium points of system (1) is investigated in the $Int.R_+^2$ by using the Lyapunov method as shown in the following theorems.

Theorem 4: Suppose that $E_1 = \left(\frac{a}{b}, 0\right)$ is locally asymptotically stable, then it's globally asymptotically stable if the following condition holds:

$$d \geq \frac{ae\alpha}{b\beta} \quad (12)$$

Proof:

The proof is based on a Lyapunov direct method. Consider the following positive definite function

$$V_1(x, y) = c_1(x - \hat{x} - \hat{x}\ln\frac{x}{\hat{x}}) + c_2 y$$

Clearly $V_1; R^2 \to R$ is a continuously differential function and $V_1(\hat{x},0) = 0$ with $V_1(x, y) > 0$ $\forall (x, y) \neq (\hat{x},0)$

Now

$$\frac{dV_1}{dt} = c_1(\frac{x-\hat{x}}{x})\frac{dx}{dt} + c_2\frac{dy}{dt}$$

$$\frac{dV_1}{dt} = c_1(x-\hat{x})[-b(x-\hat{x}) - \frac{\alpha\gamma y}{(x^2+\gamma x+\gamma\beta)}] - c_2 dy + \frac{c_2 e\alpha\gamma xy}{(x^2+\gamma x+\gamma\beta)} - c_2\delta y^2$$

$$\frac{dV_1}{dt} = -c_1 b(x-\hat{x})^2 - \frac{\alpha\gamma xy}{(x^2+\gamma x+\gamma\phi)}(c_1 - c_2 e) - c_2\delta y^2 - y(c_2 d - \frac{c_1\alpha\gamma\hat{x}}{(x^2+\gamma x+\gamma\beta)})$$

choose $c_1 = 1$ and $c_2 = \frac{1}{e}$

$$\frac{dV_1}{dt} \leq -b(x-\hat{x})^2 - y\left(\frac{d}{e} - \frac{\gamma\alpha\hat{x}}{(x^2+\gamma x+\gamma\beta)}\right)$$

Now, if $d \geq \frac{ae\alpha}{b\beta}$ then we get $\frac{dV_1}{dt} < 0$ Hence V_1 is a Lyapunov function therefore the equilibrium point $E_1 = (\hat{x},0)$ is globally asymptotically point. ■

Now, the global stability of the positive equilibrium point is investigated in the following theorem.

Theorem 5: Assume that the positive equilibrium point $E_2 = (x^*, y^*)$ of the system (1) is locally asymptotically stable in $Int\ R_+^2$. Then it is a globally asymptotically stable for any initial point in the region

$$B = \left\{(x, y) \in R^2 : x > 0, y > 0, bx + \delta y > \frac{\alpha\gamma xy(2x+\gamma)}{(x^2+\gamma x+\gamma\beta)^2}\right\} \quad (13)$$

Proof:

consider the following function

$$H(x, y) = \frac{1}{xy}, h_1(x, y) = xf_1(x, y)$$

$$h_2(x, y) = yf_2(x, y)$$

Clearly, $H(x, y) > 0$ for all $(x, y) \in Int.R_+^2$ and its C^1 function in $Int.R_+^2$ of xy-plane. Further, it is observed that

$$\Delta(x, y) = \frac{\partial(Hh_1)}{\partial x} + \frac{\partial(Hh_2)}{\partial y}$$

$$\Delta(x, y) = \frac{-1}{xy}(\delta y + bx) + \frac{\alpha\gamma(2x+\gamma)}{(x^2+\gamma x+\gamma\beta)^2}$$

Obviously $\Delta(x, y)$ does not change the sign and is not identically zero in the regain B. and hence according to Bendixson-Dulic [24] criteria, there is no periodic dynamic

Also, since E_2 is a unique positive equilibrium point for system (1) in $Int.R_+^2$ and all solution of system (1) initiated in R_+^2 are uniformly bounded, then according to Poincare-Bendixson theorem E_2 is globally asymptotically stable. ■

5. Numerical analysis

In this section the global dynamics of system (1) is studied numerically. The system (1) is solved numerically for different sets of parameters and for different sets of initial condition, by using six order Runge-Kutta method with predictor-corrector method, and then the attracting sets and their time series are drown as shown below.

Now, for the following set of hypothetical parameters

$$a = 1, b = 0.2,\ \alpha = 0.50, \gamma = 0.75, \beta = 1,\ d = 0.01, e = 0.75, \delta = 0.01 \quad (14)$$

The attracting sets along with their time series of system (1) are drown in Figure (1a) – (1b).

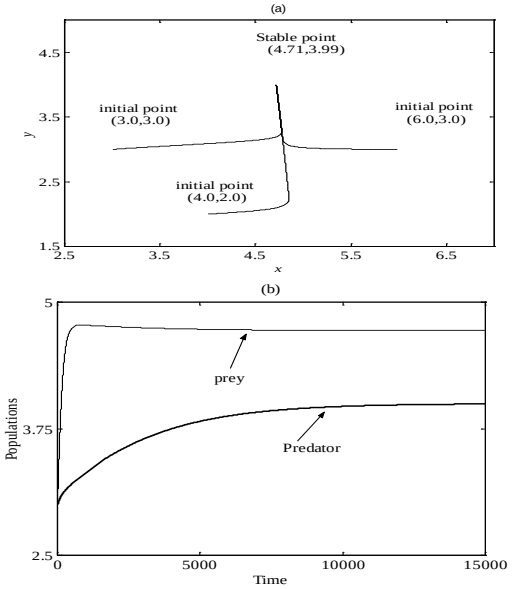
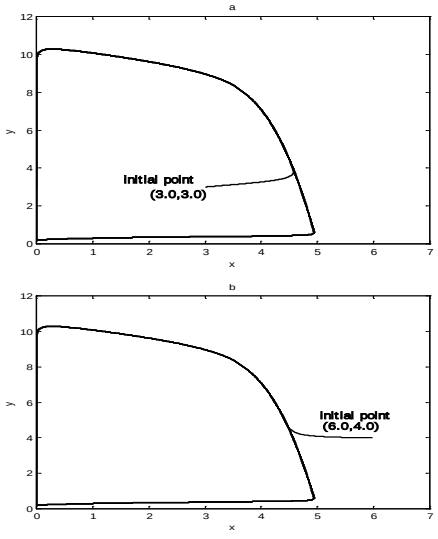


Figure 1-(a) Globally asymptotically stable positive point of system (1) for the data given in Equation.14 starting from different initial values. (b) Time series of the attractor given by Figure 1(a).

Clearly, as shown in figure 1a-1b, the system (1) approaches asymptotically to the positive equilibrium point $E_2 = (4.71,3.99)$ from different initial points. However, for the parameters values given by Equation. 1 with the maximum attack rate parameter $\alpha = 0.75$, system (1) approaches asymptotically to the globally stable limit cycle, as shown in the following figureure.



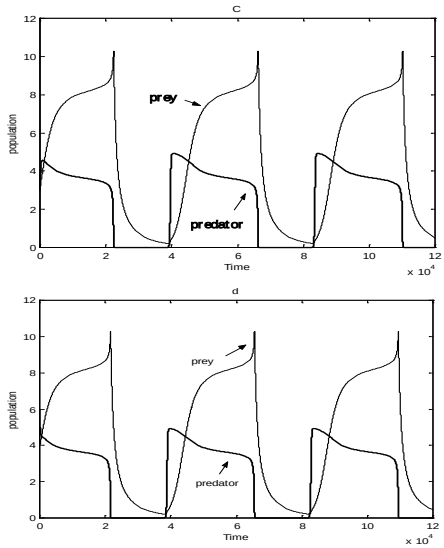


Figure 2-Globally asymptotically stable limit cycle of system(1) for the data given by Equation 14 with $\alpha = 0.75$. (a) The solution approaches tolimit cycle from inside (b) The solution approaches to limit cycle from outside (c) Time series for figure 1a (d) Time series for figure 1b.

Further analysis shows that, for the parameter $\alpha < 0.75$ with the rest of parameters as given in Equation. 14, system (1) has a globally asymptotically stable positive point, while for $\alpha \geq 0.75$ the system (1) approaches to periodic dynamic. Now, in order to investigate the effect of other parameters on the dynamics of system (1), the system (1) is solved numerically with varying the other parameters in two different cases.

Case 1: In which the system (1) has asymptotically stable point. Thus Equation. 14 will be used here.

Case 2: In which the system (1) has periodic dynamic, and hence Equation 14 with $\alpha = 0.75$ will be used in this case.

Now, the effects of varying the intrinsic growth rate of the prey, the parameter a, on the dynamical behavior of system (1) is studied. It is observed that, for case 1, the system (1) approaches asymptotically to periodic dynamic for $a \leq 0.6$, see for example figure 3, otherwise it approaches asymptotically to stable positive point. However, in case 2, the system (1) has asymptotically stable point in $IntR_+^2$ for $a > 0.6$, as shown in figure 4.

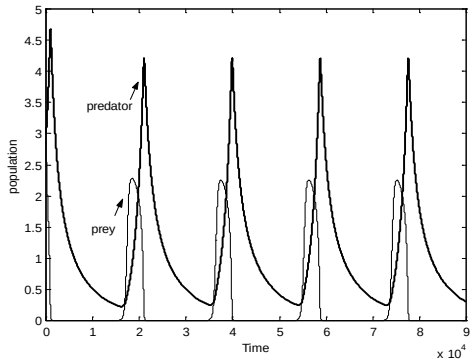


Figure 3- Time series for periodic dynamic of system (1) in case 1 with. $a = 0.5$.

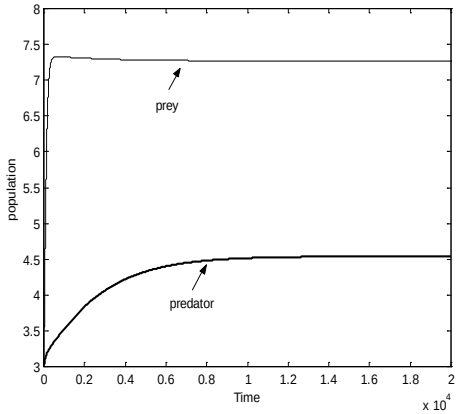


Figure. 4-Time series for stable point attractor of system (1) in case 2 with $a = 1.5$

The effect of varying the intra specific competition of the predator species on the dynamical behavior of system (1) shows that: in case 1 for $\delta \le 0.005$ the system becomes periodic as shown in the typical figureure given by Figure 5, while it is still approaches to positive equilibrium point otherwise. In case 2, it is observed that the system (1) approaches asymptotically to a positive equilibrium point when $\delta > 0.005$ as shown in figure 6, while it has periodic dynamic otherwise.

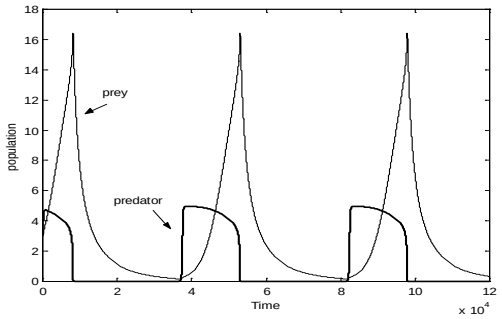


Figure 5- Time series for periodic dynamics of system (1) in case 1 with $\delta = 0.003$

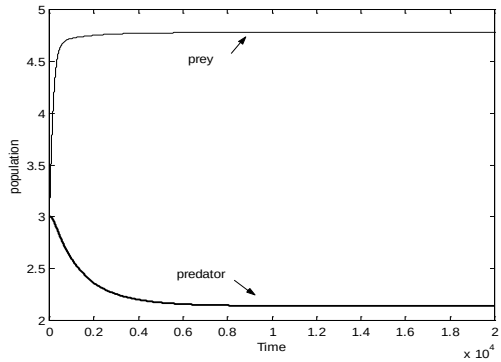


Figure 6- Time series for stable point of system (1) in case 2 with $\delta = 0.03$

Further, the effect of death rate of predator species on the dynamical behavior of system (1) is also studied. It is observed that, in case 1, system (1) still has an asymptotically stable point in the $IntR_+^2$, for $d \le 0.03$ while for $d > 0.03$ there is an extinction in a predator species and the system (1) approaches asymptotically to $E_1 = \left(\frac{a}{b}, 0\right)$ as shown in figure 7. Moreover, in case 2, for $d \le 0.01$ the system (1) have periodic attractor while it has a stable point in the $IntR_+^2$ when $d > 0.01$.

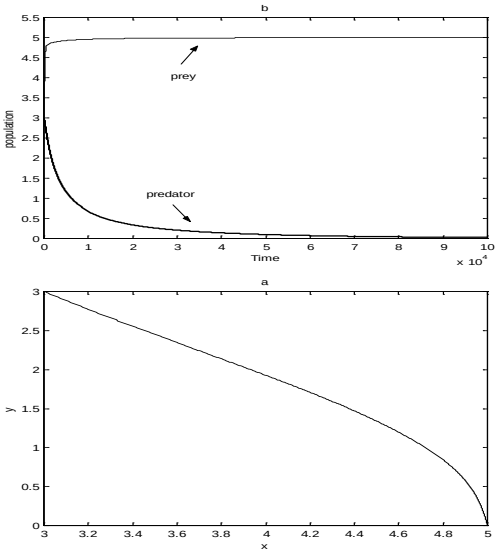


Figure 7- (a) The system (1) approaches as- ymptotically to stable point $E_1 = (5,0)$ for case 1 with $d = 0.04$ starting from (3, 3), (b) Time series of the attractor given by figure 1 (a).

Finally, the system (1) still has the same dynamical behavior as in case 1, for all values of γ, β and e , however the effect of these parameters on the dynamical behavior of system (1) in the case 2 is shown below.

It is observed that for $\gamma < 0.75$ the system (1) has an asymptotically stable positive point as shown in figure 8. While it still has periodic attractor otherwise.

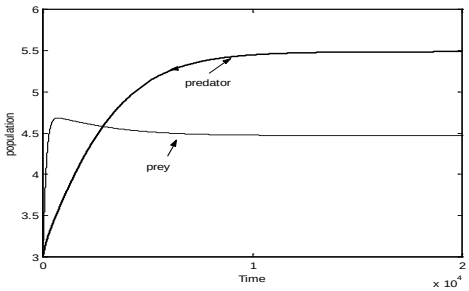


Figure 8- Time series for stable point of the system (1) in case 2 with $\gamma = 0.60$

For $\beta > 1.1$ the system (1) has an asymptotically stable positive point as shown in the typical figureure given by figure 9 while it still has periodic attractors for $\beta \le 1.1$

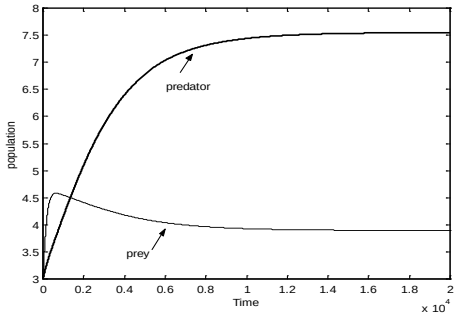


Figure 9- Time series for stable point of the system (1) in case 2 with $\beta = 1.5$

Finally, for $e < 0.75$ the system approaches asymptotically to a positive equilibrium point see for example figure 10, while it has periodic dynamics when $e \ge 0.75$

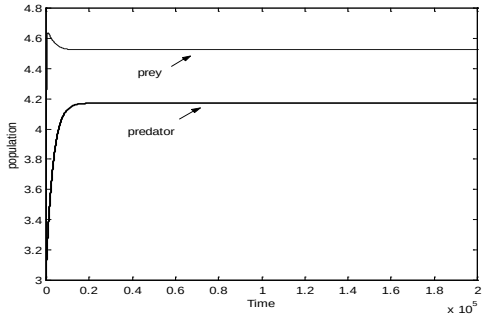


Figure 10- Time series for stable point of the system (1) in case 2 with $e = 0.50$

6. Conclusion

In this paper, a mathematical model consisting of a Holling type IV prey predator model with intra specific competition has been studied analytically as well as numerically.

The condition for the system (1) to be uniformly bounded and persistence have been derived. The local as well as global stability of the proposed system has been studied. The occurrence of a Hopf-bifurcation in system (1) is investigated.

The effect of each parameter on the dynamical behavior of system (1) is studied numerically and the trajectories of the system are drowned. According to these figureures the following conclusions are obtained.

1. As the intrinsic growth rate of the prey species increases then the system (1) approaches to an asymptotically stable positive equilibrium point, otherwise the system has periodic dynamics. So this parameter has a stabilizing effect on the system.
2. The intra specific competition parameter ($i.e.\delta$) has the same effects, as that of a, on the dynamics of the system (1).
3. The death rate of the predator species ($i.e.$ d) has the same effect, as those of a and δ on the dynamical behavior of system (1). But, it is observed that increasing this parameter further than a specific value wills causes extinction of predator species and then the trajectory of system (1) approaches to equilibrium point $E_1 = \left(\frac{a}{b},0\right)$. Hence in such case the system loss the persistence.
4. When the system (1) approaching to the positive equilibrium point, then it is observed that the parameters γ, β and e have no effect on the dynamical behavior of the system.
5. When the system (1) has a periodic dynamics then it is observed that, decreasing the parameters γ or e , where γ represents the direct measure of the predator immunity and e represents the conversion rate, causes stabilizing the system. While increasing the parameter β will has stabilizing effect on the system.

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