Conclusion Empirical Equations of Asymmetry Parameter for Magic Nuclei in N for (n,α) Reaction of Incident Neutron Energy (14.5 MeV)

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Abstract

In this study, the magic nuclei is divided into two groups, one of them is light group and the other is middle group, it was calculated shell corrections for all nuclei, and also it was concluded the relationship between cross sections for nuclear reactions (n,α) and the mass number (A) for all nuclei to incident neutrons (14.5 MeV). We found empirical equations to asymmetry parameter (N-Z)/A as function of mass number and for that two groups:

for A=38 to A=40 light nuclei. $(N-Z)/A = -0.0263A + 1.0534$

for A=50 to A=89 middle nuclei. $(N-Z)/A = 0.0001A^2 + 0.0151A - 0.408$

for A=90 to A=144 middle nuclei. $(N-Z)/A = -9e^{-5A^2} + 0.0221A - 1.0711$

Keywords: asymmetry, magic nuclei, neutron, nuclear reactions.

استنتاج معادلات تجريبية لبرامتر النوى السحرية ذات اللاتناظر في التفاعلات النووية (n,α) عند طاقة النيوترونات الساقطة (14.5 MeV)

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الخلاصة

في هذه الدراسة ,النوى السحرية قسمت إلى مجموعتين واحدة منها مجوعة خفيفة والأخرى مجموعة متوسطة, كما ان تصحيحات القشرة لجميع النوى التي قيد الدراسة قد تم حسابها في هذا البحث,وكذلك تضمنت الدراسة العلاقة بين المقاطع العرضية للتفاعلات النووية (n,α) لكل النوى عند طاقة النيوترونات الساقطة (14.5 MeV). تم إيجاد المعادلات التجريبية لبارمتر اللاتناظر $(N-Z)/A$ كذلك للعدد الكتلي وللمجموعتين :

for A=38 to A=40 light nuclei. $(N-Z)/A = -0.0263A + 1.0534$

for A=50 to A=89 middle nuclei. $(N-Z)/A = 0.0001A^2 + 0.0151A - 0.408$

$(N-Z)/A = -9e^{-5A^2} + 0.0221A - 1.0711$ for A=90 to A=144 middle nuclei

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Introduction

More than 50 years after the publication of the first empirical formulas for the predication of neutron reaction cross sections (Levkovskij, 1957,1958) systematic are successfully used for the evaluation of reaction cross sections [1]. The nuclear reaction systematic is widely for the reaction cross section evaluation supplementing result of the measurement and calculation by theoretical models [2].

According to Previously obtained cross sections for many nuclei significantly vary with the mass number A, neutron number N and proton number Z of the target nucleus [3]. The evaluated nuclear cross-section data and required for understanding the binding energy systematic, nuclear structure.

Theory

The nuclear reaction systematic is widely used for the reaction cross section evaluation supplementing result of measurement and calculations by theoretical models. A large number of empirical and semi-empirical cross sections formulas with different parameters have been proposed by several authors in the literature [4,5,6]

The attributable effects to the asymmetry parameter, S=(N-Z)/A, as well as to the isotopic, isotonic and odd-even properties of nuclei have been observed in the cross section data. Then lately, semi-empirical formulas for the $(P,\alpha)(P,n\alpha)$ and (P,nP) reaction cross sections have been derived by using analytical expressions describing the equilibrium and non-equilibrium particle emission in nuclear reactions. These systematic are based on analytical formulas derived from the pre-equilibrium exaction model, evaporation model (that means the Weisskopf treatment is an application that relates the probabilities to go from a state I to another d and vice versa through the density of states in the two systems as equation $P_{i\to d}\rho(i)=P_{d\to i}\rho(d)$), and semi-empirical mass formula.

The empirical reaction cross sections of reaction induced by fast neutrons can be approximately expressed as follows [7].

$$\sigma(n,x)=C\sigma_{ne}\exp[as]\ldots\ldots\ldots\ldots(1)$$

Where x represents particle of the reaction produced, σ_{ne} is the neutron non-elastic cross section,

$$\sigma_{ne}=\pi r_0^2\left(A^{1/3}+1\right)^2$$

and the coefficients C and a are the fitting parameters determined from least-square method for different reaction the neutron non-elastic cross section is given by πR^2 where R is the nuclear radius.

The exponential term represents the escape of the reaction products from a compound nucleus [8].

Method

The (n,α) reaction cross section, obtained from the analysis of experimental data for 120 nuclei with $Z\geq 18$ used to obtain the cross sections and any parameters others as in table-1.

For target nuclei with atomic number $(18\leq Z\leq 50)$ [9].

$$\sigma(n,\alpha)=\pi r_0^2\left(A^{1/3}+1\right)^2 \exp\begin{pmatrix}-160.6S^2-9.54131\times10^{-5}A^2+0.10398f_{ch,p}\\ -1.4934\end{pmatrix}\ldots(2)$$

Thus:

$$f_{sh,p}=1/0.10398\left[\ln\sigma(n,\alpha)/\pi r_0^2\left(A^{1/3}+1\right)^2+160.6S^2+9.54131\times10^{-5}A^2+1.4934\right](3)$$

Table 1-The (n,α) reaction cross sections at the incident energy 14.5 MeV for stable nuclei with mass numbers from 18 to 83 and ^{99}T obtained from the analysis of experimental data, and cross section calculated using the systematic $(\sigma \pm \Delta\sigma)$ [7].

Z	A	Cross-section(mb)	Z	A	Cross-section(mb)	Z	A	Cross-section(mb)
18	36	142.0	42	100	2.86 ± 0.29	62	149	4.15
18	38	61.3	43	99	6.06 ± 0.61	62	150	3.21 ± 0.34
18	40	10.8±1.7	44	96	32.4	62	152	1.81 ± 0.23
19	39	13.0±26.0	44	98	16.9	62	154	0.833 ± 0.083
19	40	103.0	44	99	15.4	63	151	2.27 ± 1.27
19	41	33.3 ±3.3	44	100	8.24	63	153	1.56 ± 0.16
20	40	128.0 ± 13.0	44	101	7.35	64	152	3.78
20	42	72.7	44	102	6.46 ± 0.82	64	154	2.33
20	43	58.7	44	104	3.12 ± 0.31	64	155	2.87
20	44	27.7±2.8	45	103	6.00	64	156	3.84 ± 1.91
20	46	3.54	46	102	17.6	64	157	1.88
20	48	1.25±0.33	46	104	8.93	64	158	1.84 ± 0.47
21	45	56.4±5.6	46	105	8.40	64	160	0.64
22	46	76.5±16.9	46	106	5.28 ± 0.53	65	159	2.22 ± 0.29
22	47	66.9	46	108	2.73 ± 0.27	66	156	3.79
22	48	34.0±3.9	46	110	0.84	66	158	2.71
22	49	19.6	47	107	6.79	66	160	1.94
22	50	8.63 ±0.86	47	109	3.29	66	161	2.30
23	50	38.0±1.5	48	106	18.6	66	162	1.93 ± 0.34
23	51	16.5±1.7	48	108	9.83	66	163	1.47
24	50	99.7±10.0	48	110	4.88	66	164	1.11 ± 0.30
24	52	36.2±3.6	48	111	4.61	67	165	0.763 ± 0.079
24	53	44.2±3.7	48	112	2.54 ± 0.36	68	162	3.30
24	54	11.8±1.2	48	113	2.14	68	164	2.35
25	55	24.0±2.4	48	114	0.889 ± 0.089	68	166	1.63
26	54	91.1±9.1	48	116	0.46	68	167	1.87
26	56	43.9±4.4	49	113	4.45 ± 0.47	68	168	1.66 ± 0.26
26	57	31.0±4.0	49	115	2.39 ± 0.24	68	170	0.613 ± 0.097
26	58	19.8±2.0	50	112	44.9 ± 10.0	69	169	1.50 ± 0.13
27	59	31.4±3.1	50	114	5.50	70	168	2.73
28	58	109.0±11.0	50	115	5.08	70	170	2.02
28	60	62.8±6.3	50	116	1.88 ± 0.24	70	171	2.28
28	61	44.4±4.2	50	117	2.45 ± 0.19	70	172	1.68 ± 0.33
28	62	23.4±2.3	50	118	1.18 ± 0.12	70	173	1.46
28	64	3.77±0.38	50	119	0.999 ± 0.087	70	174	1.16 ± 0.14
29	63	44.4±4.4	50	120	0.450 ± 0.045	70	176	0.547 ± 0.120
29	65	8.69±1.48	50	122	0.221 ± 0.024	71	175	1.33
30	64	36.4±10.1	50	124	0.079 ± 0.013	71	176	2.23 ± 0.55
30	66	30.2	51	121	3.08	72	174	2.42
30	67	27.1	51	123	1.81	72	176	1.77
30	68	9.02±0.90	52	120	7.09	72	177	1.98
30	70	3.69	52	122	4.72	72	178	1.86 ± 0.37
31	69	20.2±2.0	52	123	5.66	72	179	1.28
31	71	5.06±0.78	52	124	3.08	72	180	0.848 ± 0.151

32	**70**	**35.4**	**52**	**125**	**3.84**	**73**	**181**	**0.644 ± 0.064**
32	**72**	**17.4±3.7**	**52**	**126**	**1.88 ± 0.38**	**74**	**180**	**1.88**
32	**73**	**12.1**	**52**	**128**	**1.17**	**74**	**182**	**1.20**
32	**74**	**7.89±3.14**	**52**	**130**	**0.66**	**74**	**183**	**1.41**
32	**76**	**2.52±0.29**	**53**	**127**	**2.69**	**74**	**184**	**0.789 ± 0.079**
33	**75**	**10.7±1.1**	**54**	**124**	**7.10**	**74**	**186**	**0.542 ± 0.054**
34	**74**	**38.7**	**54**	**126**	**5.01**	**75**	**185**	**1.41 ± 0.50**
34	**76**	**16.5**	**54**	**128**	**3.43**	**75**	**187**	**0.529 ± 0.053**
34	**77**	**15.3**	**54**	**129**	**4.24**	**76**	**184**	**2.17**
34	**78**	**7.41±0.78**	**54**	**130**	**2.28**	**76**	**186**	**1.54**
34	**80**	**3.29±0.33**	**54**	**131**	**2.91**	**76**	**187**	**1.76**
34	**82**	**0.70**	**54**	**132**	**1.47**	**76**	**188**	**0.94**
35	**79**	**12.5±1.2**	**54**	**134**	**0.90**	**76**	**189**	**1.15**
35	**81**	**4.48±0.59**	**54**	**136**	**0.53**	**76**	**190**	**0.678 ± 0.085**
36	**78**	**41.1**	**55**	**133**	**1.43 ± 0.27**	**76**	**192**	**0.22**
36	**80**	**19.4**	**56**	**130**	**5.81**	**77**	**191**	**0.828 ± 0.437**
36	**82**	**8.06**	**56**	**132**	**4.11**	**77**	**193**	**0.44**
36	**83**	**7.09**	**56**	**134**	**2.81**	**78**	**190**	**1.72**
36	**84**	**3.03**	**56**	**135**	**5.58 ± 1.53**	**78**	**192**	**1.06**
36	**86**	**1.00**	**56**	**136**	**1.87**	**78**	**194**	**1.19 ± 0.20**
37	**85**	**5.97±0.60**	**56**	**137**	**2.96 ± 0.83**	**78**	**195**	**0.81**
37	**87**	**2.02**	**56**	**138**	**2.32 ± 0.23**	**78**	**196**	**0.560 ± 0.096**
38	**84**	**21.3**	**57**	**138**	**2.94**	**78**	**198**	**0.16**
38	**86**	**9.26**	**57**	**139**	**1.98 ± 0.20**	**79**	**197**	**0.373 ± 0.037**
38	**87**	**8.12**	**58**	**136**	**4.71**	**80**	**196**	**1.05**
38	**88**	**3.54**	**58**	**138**	**3.30**	**80**	**198**	**0.66**
39	**89**	**6.27+0.63**	**58**	**140**	**2.28**	**80**	**199**	**0.83**
40	**90**	**13.7+1.4**	**58**	**142**	**5.97 ± 0.60**	**80**	**200**	**0.461 ± 0.461**
40	**91**	**11.8**	**59**	**141**	**3.03**	**80**	**201**	**0.50**
40	**92**	**9.32+0.93**	**60**	**142**	**5.58 ± 0.56**	**80**	**202**	**0.19**
40	**94**	**4.63+0.46**	**60**	**143**	**5.23**	**80**	**204**	**0.07**
40	**96**	**2.32+0.33**	**60**	**144**	**4.05 ± 0.57**	**81**	**203**	**0.39**
41	**93**	**8.85+0.89**	**60**	**145**	**3.67**	**81**	**205**	**0.519 ± 0.082**
42	**92**	**25.7+2.6**	**60**	**146**	**3.42 ± 0.34**	**82**	**204**	**0.722 ± 0.082**
42	**94**	**13.0+2.6**	**60**	**148**	**1.56 ± 0.33**	**82**	**206**	**0.556 ± 0.056**
42	**95**	**12.9+1.7**	**60**	**150**	**0.73**	**82**	**207**	**0.366 ± 0.033**
42	**96**	**7.98+1.77**	**62**	**144**	**5.63**	**82**	**208**	**0.395 ± 0.075**
42	**97**	**8.41+0.99**	**62**	**147**	**5.61**	**83**	**209**	**0.590 ± 0.059**
42	**98**	**6.02+0.60**	**62**	**148**	**3.54**			

Where $f_{sh,p}$ represent shell correction (that's mean the shell and pairing correction energies are calculated for heavy and super heavy nuclei, also the shell and pairing correction energies, the signatures of the magic numbers appear at the spherical shell closures Z=82, 114,146 and N=126, 184, 228 and 308. There are also signatures for some other shell closures at, e.g., Z=108 and N=162 which appear only when the deformation degrees of freedom is taken into account) [10].

For target nuclei with $Z > 50$ [9],

$$\sigma(n,\alpha) = \pi r_0^2 \left(A^{1/3} + 1 \right)^2 A^{\frac{-1}{3}} \times \left(-2.194P + 1.0307 \times 10^{-2} f_{sh,p} + 0.57038 \right)^3 \quad (4)$$

Thus:

$$f_{sh,p} = \frac{1}{1.0307} \times 10^{-2} \left[\frac{\left\{ \frac{\sigma(n,\alpha)}{\pi r_0^2 \left(A^{\frac{1}{3}} + 1 \right)^2 A^{\frac{-1}{3}}} \right\}^{\frac{1}{3}} +}{2.1946P - 0.57038} \right] \quad ....(5)$$

where: σ(n,α) = Cross section of the (n,α) reaction

S = (N – Z +1) / A.

P = (N – Z + 0.5) / A.

$f_{sh,p} = (dw_n - \delta_n) - (dw_\delta - \delta_\alpha)$,where

dw_n and dw_α are shell corrections (Myers and Swiatecki, 1966, 1967) for nuclei (Z,A) and (Z-2, A-3) respectively , taken from Lgnatyuk (2003).Also δ_n and δ_α are pairing correction for nuclei (Z,A) and (Z-2,A-3) calculated as follows

$\delta = 12A^{\frac{-1}{2}}$ for even -even nuclei.

$\delta = 0$ for nuclei with odd and $\delta = -12A^{\frac{-1}{2}}$ for odd-odd nuclei, $r_0 = 1.3$ fm, which corresponds to the πr_0^2 value equal to 53.093mb; N,Z and A are number of neutrons , protons and nucleon in the target nuclei, respectively

Results and discussion

In this work, I used table-1- to calculate asymmetry parameter for magic nucleus in N for (n,α) reaction of incident neutron energy 14.5 MeV, perhaps table-1,contains stable nuclei with atomic numbers from 18 to 83 and by using this table, we can make another table as below:

Table 2-The (n,α) reaction cross sections of the incident energy 14.5MeV for stable magic nuclei by N with mass number from 38 to 144

Z Proton number	A Mass number	N magic number	Cross section (mb)	$(N-Z)/A$ Asymmetry parameter	Type of nuclei
18	38	20	61.3	0.0526315	Light nuclei
19	39	20	130.0	0.025641	Light nuclei
20	40	20	128	0	Light nuclei
22	50	28	8.63	0.12	Middle nuclei
23	51	28	16.5	0.1	Middle nuclei
24	52	28	36.2	0.076923	Middle nuclei
36	86	50	1.0	0.1627907	Middle nuclei
37	87	50	2.02	0.1494252	Middle nuclei
38	88	50	3.54	0.1363636	Middle nuclei
39	89	50	6.27	0.1235955	Middle nuclei
40	90	50	13.7	0.222222	Middle nuclei
42	92	50	25.7	0.0869565	Middle nuclei
54	136	82	0.53	0.2058823	Middle nuclei
56	138	82	2.32	0.1884058	Middle nuclei
57	139	82	1.98	0.1798561	Middle nuclei
58	140	82	2.28	0.1714285	Middle nuclei
62	144	82	5.63	0.1388888	Middle nuclei

In this work, we classify it into two groups (light and middle group) for N magic number. We calculate $(f_{sh,p})$ shell corrections for nuclei in table-2-, by using eq.3 for target nuclei with atomic number $18 \le Z \le 50$, and by using eq.5 for target nuclei with $Z > 50$, in this calculation we neglected $(\Delta\sigma)$ as in table-3

Table 3- Shell corrections for stable magic nuclei by N with mass number from 38 to 144

Z	A	N magic) (number	Cross section (mb)	$f_{sh,p}$ (shell corrections)	Type of nuclei
18	38	20	61.3	1.6345663-|	Light nuclei
19	39	20	130.0	0.027350355-	Light nuclei
20	40	20	128	2.8451304-	Light nuclei
22	50	28	8.63	0.019259822+	Middle nuclei
23	51	28	16.5	2.913895807-	Middle nuclei
24	52	28	36.2	2.45829597-	Middle nuclei
36	86	50	1.0	2.550481554-	Middle nuclei
37	87	50	2.02	2.682027149-	Middle nuclei
38	88	50	3.54	3.474632727-	Middle nuclei
39	89	50	6.27	3.501685853-	Middle nuclei
40	90	50	13.7	0.885166081+	Middle nuclei
42	92	50	25.7	2.907819129-	Middle nuclei
54	136	82	0.53	14.32759523-	Middle nuclei
56	138	82	2.32	20.32889876-	Middle nuclei
57	139	82	1.98	21.84052023-	Middle nuclei
58	140	82	2.28	23.89585427-	Middle nuclei
62	144	82	5.63	32.82566652-	Middle nuclei

By using table-2 when A=38 to A=40 empirical equation by least-square fitting method:

$$\sigma(n,\alpha) = -35.35A^2 + 2790.6A - 54938.$$

The relationship between cross section (mb) and mass number A, is shown in figure.1 for light nuclei

And by using table-2, when A=50 to A=144 the relationship between $\sigma(n,\alpha)$ and mass number (A) is shown in figure .2 for middle nuclei.

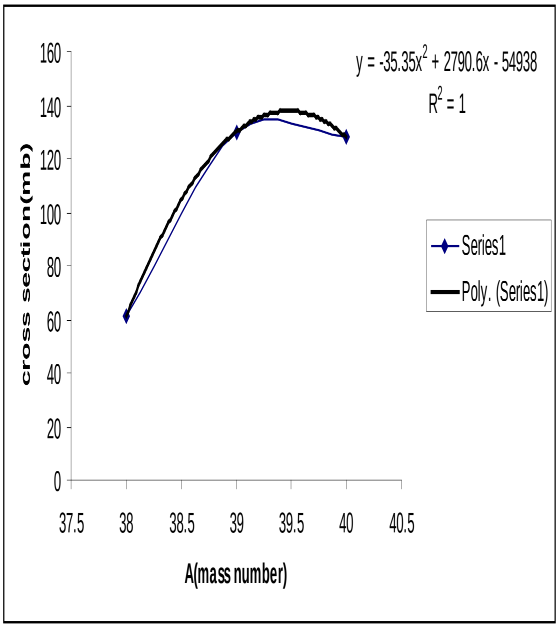

Figure 1-the relationship between cross-section and mass number for light nuclei.

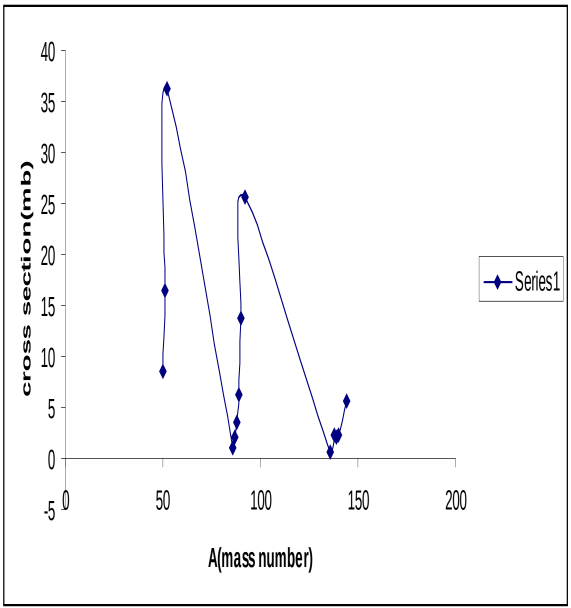

Figure 2- The relationship between $\sigma(n,\alpha)$ and mass number (A) for middle nuclei.

I want to study figure.2 correctly, so I divide it into tow regions, to get suitable empirical equations.

$\sigma(n,\alpha) = -0.4082A^2 + 55.364A - 1741$ for A= 50 to A=86, and

$\sigma(n,\alpha) = -0.0179A^2 + 3.949A - 200.3$ for A =86 to A = 144, as in figure 2.1 and figure 2.2 .

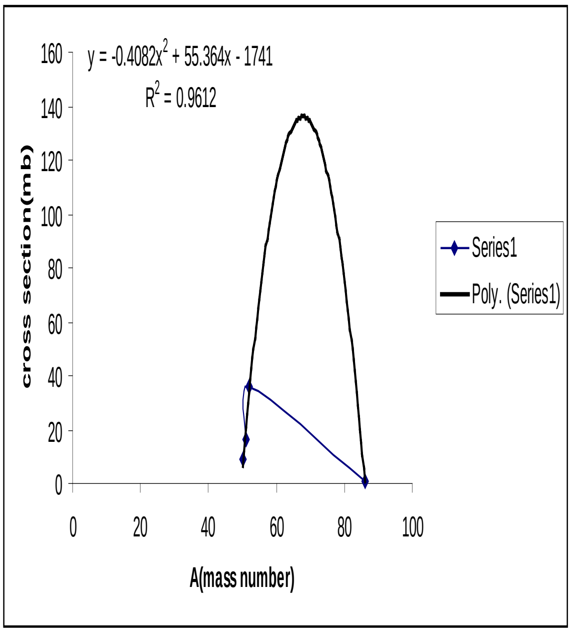

Figure 2.1-The relationship between $\sigma(n,\alpha)$ and A=50 to A=86 middle nuclei.

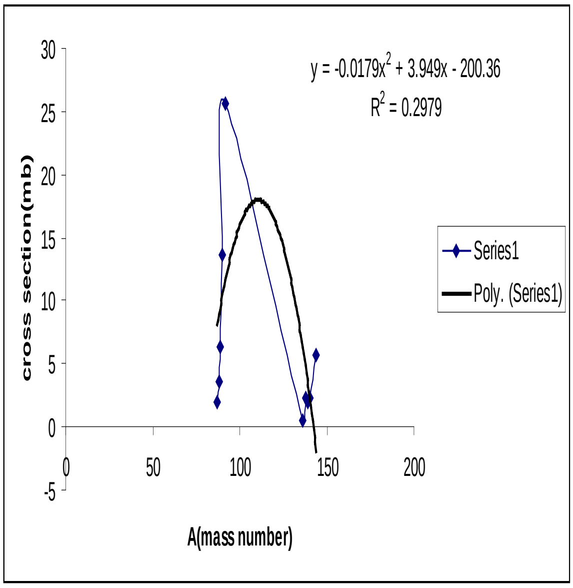

Figure 2.2-The relationship between $\sigma(n,\alpha)$ and A=86 to A=144 middle nuclei.

Conclusions

1. We conclude empirical equations by least-square fitting method to calculate asymmetry parameter for magic nucleus in N for (n,α) reaction of incident neutron energy 14.5 MeV by using table 2 as shown below:

$\frac{(N-Z)}{A} = -0.0263A + 1.0524$ for A=38 to A=40 as in figure 3 for light nuclei.

$\frac{(N-Z)}{A} = -0.0001A^2 + 0.0151A - 0.408$ for A=50 to A=89 as in Figure 4 for middle nuclei.

$\frac{(N-Z)}{A} = -9e^{-5A^2} + 0.0221A - 1.0711$ for A=90 to A=144 as in Figure 5 for middle nuclei.

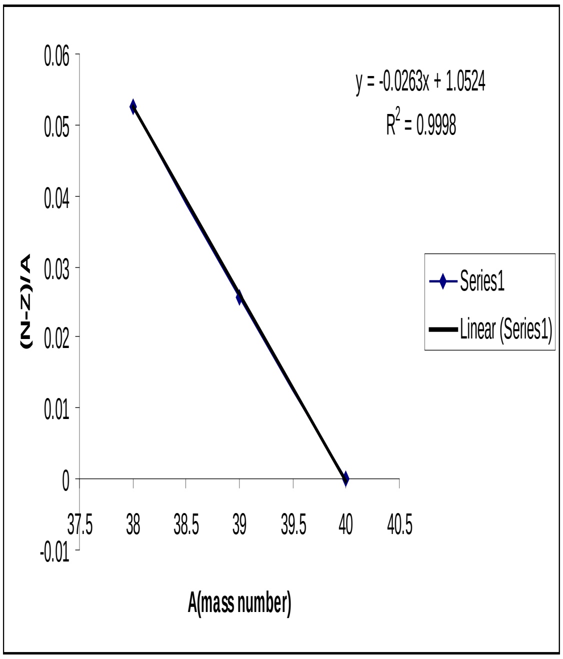

Figure 3- Asymmetry parameter for A=38 to A=40 light nuclei.

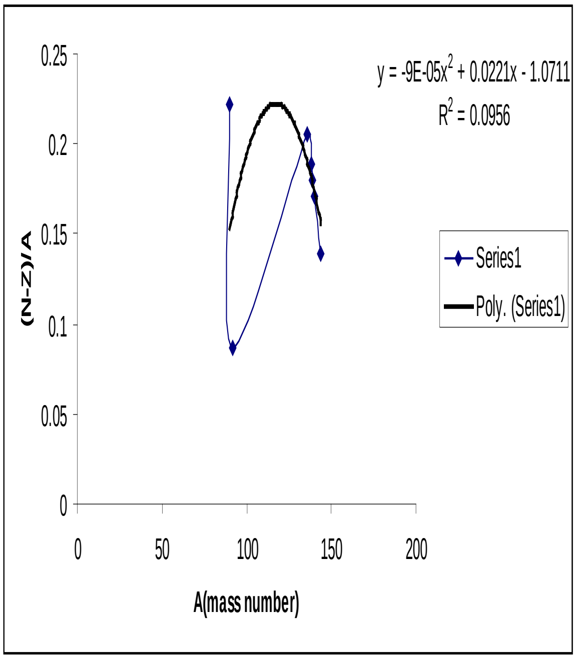

Figure 5- Asymmetry parameter for A=90 to A=144 middle nuclei.

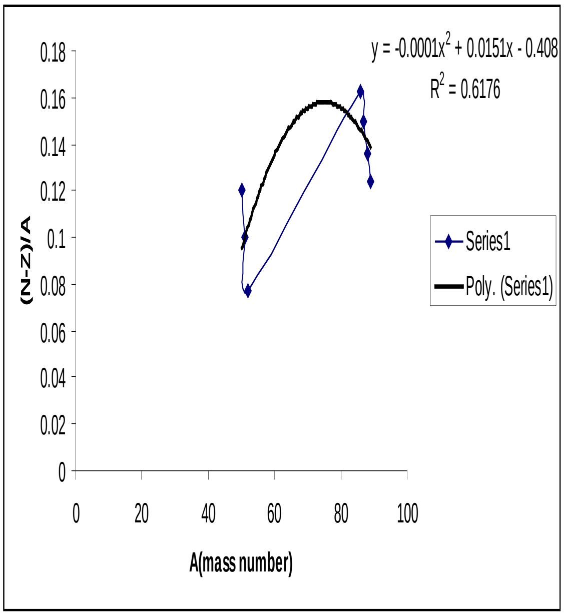

Figure 4- Asymmetry parameter for A=50 to A=89 middle nuclei.

2. The magic nuclei which have Z or N equal to (2,8,28,50,82,126), in this study we choose the nuclei which have (N) magician number because the required information was found table 2, and the reason for choosing this nuclei to add knowlage about probability of making nuclear reaction for that nuclei from type $\sigma(n,\alpha)$ and cause of known behavior to that nuclei from asymmetry parameter and also shell corrections, $(f_{sh,P})$, and as aresult for that we divide this nuclei into two groups: one of them expresses the group light nuclei $(A \le 40)$ and another group expresses middle nuclei $(40 < A < 140)$.

3. From table -2- we release that light group one, get high values for cross section reaction $\sigma(n,\alpha)$ of incident neutron energy 14.5 MeV compared with low values for the same cross sections for middle nuclei, in another words the areas interactions in energy between light nuclei and incident neutron are a very high which simplify and occur the nuclear reaction (n,α).

4. Also the factor of shell corrections light nuclei bigger than the middle nuclei as shown in table 3.

5. The asymmetry parameter for light nuclei proportional linear with mass number as in

figure.3, also the values asymmetry parameter small while cross sections is high, we found the asymmetry parameter values for middle nuclei are big to compared with light nuclei (figure 4,figure 5), that means the asymmetry parameter effecting with cross sections values for that nuclei.

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