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A Novel Approach to Algorithmic Encoding and Decoding by Tribonacci Matrices

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Abstract

In this paper, we discuss a new, effective technique for encoding and decoding text that uses Tribonacci numbers to boost security and make it harder to figure out the right keys needed for decryption. We focused on developing different algorithms and methods to protect different types of data, especially text messages, and making it very difficult for unauthorized people to access them and be able to break these codes. Strong data protection is needed for a variety of applications, and the suggested algorithm offers promising results in terms of encryption and decryption accuracy. This paper demonstrates the potential of Tribonacci matrix-based algorithms in advancing the landscape of secure communication protocols.

Keywords: Tribonacci numbers; Tribonacci matrices; Coding/decoding algorithm.

1. Introduction

Encoding and decoding are fundamental processes in various fields, communications, cryptography, and data storage. Innovations in mathematics that can effectively represent and manage data are frequently the foundation of progress in various fields. There is a lot of research on coding with Fibonacci and Lucas sequence, while the Tribonacci and Lucas sequence [1,2] has received relatively less attention. Our investigation focused on the encoding and decoding of data using the Tribonacci matrix. The Tribonacci number sequence is a number sequence with 3-term recurrence. The term Tribonacci was first used by Feinberg in 1963 [3]. Many fundamental traits were later examined [4,5].

The sequence $(t_n)_{n\geq 0}$ of Tribonacci numbers is defined by

$$t_n = t_{n-1} + t_{n-2} + t_{n-3}, \tag{1.1}$$

where $t_0 = t_1 = 0, t_2 = 1$ (see [6]).

The Tribonacci numbers are as follows:

$(t_n)_{n\geq 0} = 0, 0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, 927, 1705, 3136, 5768, , ...$ so on.

The Tribonacci negative number t_{-n} satisfies the recurrence relation

$$t_{-n} = \begin{vmatrix} t_{n+1} & t_{n+2} \\ t_n & t_{n+1} \end{vmatrix}, \tag{1.2}$$

here $t_0 = t_1 = 0, t_2 = 1$.

The Tribonacci matrix is defined by

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$$M=\begin{bmatrix}1&1&1\\1&0&0\\0&1&0\end{bmatrix}. \tag{1.3}$$

Using Tribonacci matrices, Basu recently created a new coding and decoding technique in [7]. This paper uses Tribonacci M-matrices to introduce a new coding/decoding algorithm. The fundamental tenet of our approach is to split the message matrix into block matrices of size 3×3. We have a safer coding/decoding method since we employ a mixed type algorithm and a separate numbered alphabet for each message. The number of block matrices in the message matrix determines the alphabet. Our approach will improve information security while also having a high degree of accuracy in data transmission through communication channels.

In [8], a new method of coding/decoding algorithms using Fibonacci Q −matrices were given. In [9], a new application to Coding Theory was introduced using Fibonacci Q-matrices and R-matrices. Sumerya Ucar, NihalTas, and Nihal Yilmaz Ozgur [10] discussed Pell coding and decoding methods with some Applications. The study by Manjusri Basu and Monojit Das has been a great inspiration for introducing this paper. Sumerya Ucar, NihalTas, and Nihal Yilmaz Ozgur presented different methods for encoding and decoding algorithms. Their methods depend on dividing the message square matrix into block matrices of size 3×3. Our method relies on the same approach but our method introduces a new encoding/decoding technique using Tribonacci numbers and introduces a different coding method. In [7], a new coding theory called Tribonacci coding theory is used based on the Tribonacci matrix M^k of order 3 as a coding matrix and its inverse matrix M^{-k} as a decoding matrix for any value of k, whereas the main idea of our method is the encryption of each message matrix with secure and more complex different keys.

Theorem 1.1: [7] The Tribonacci matrix M^n is given by:

$$M^n=\begin{bmatrix}t_{n+2}&t_{n+1}+t_n&t_{n+1}\\t_{n+1}&t_n+t_{n-1}&t_n\\t_n&t_{n-1}+t_{n-2}&t_{n-1}\end{bmatrix},\qquad \text{where } M=\begin{bmatrix}1&1&1\\1&0&0\\0&1&0\end{bmatrix}. \tag{1.4}$$

Proof: By **[7]**.

Some properties of M^n matrix 1.2 [7]

1. $M^n=M^{n-1}+M^{n-2}+M^{n-3}$.
2. $M^nM^l=M^lM^n=M^{n+1}\ (k,l=0,\pm1,\pm2,\pm3,\dots)$. (1.5)
3. $\det(M^n)=1$.

2. Main results:

In this paper, we present novel algorithms for encoding and decoding. To encoding the message, we put the massage in a matrix U of size $3m\times 3m$ adding $\beta=\left[\left|\frac{n}{2}\right|\right]+26$ (see [11,12]) for space between two words. We divide the message square matrix U into block matrices named $S_k(1\le k\le m^2)$ of size 3×3. This method is the encryption of each message matrix with different keys. For readability and simplicity, for $1\le k\le m^2$, let

$$S_k=\begin{bmatrix}s_1^k&s_2^k&s_3^k\\s_4^k&s_5^k&s_6^k\\s_7^k&s_8^k&s_9^k\end{bmatrix},\qquad L_k=\begin{bmatrix}L_1^k&L_2^k&L_3^k\\L_4^k&L_5^k&L_6^k\\L_7^k&L_8^k&L_9^k\end{bmatrix}.$$

We rewrite the elements of the matrix M^n as follows:

$$M^n = \begin{bmatrix} m_1 & m_2 & m_3 \\ m_4 & m_5 & m_6 \\ m_7 & m_8 & m_9 \end{bmatrix}.$$

Now, we define the following table mod 36 (This table can be expanded to the used characters in the message text).

A	$\left[\left\|\frac{n}{2}\right\|\right]$	G	$\left[\left\|\frac{n}{2}\right\|\right]+6$	M	$\left[\left\|\frac{n}{2}\right\|\right]+12$	S	$\left[\left\|\frac{n}{2}\right\|\right]+18$	Y	$\left[\left\|\frac{n}{2}\right\|\right]+24$	$=$	$\left[\left\|\frac{n}{2}\right\|\right]+30$
B	$\left[\left\|\frac{n}{2}\right\|\right]+1$	H	$\left[\left\|\frac{n}{2}\right\|\right]+7$	N	$\left[\left\|\frac{n}{2}\right\|\right]+13$	T	$\left[\left\|\frac{n}{2}\right\|\right]+19$	Z	$\left[\left\|\frac{n}{2}\right\|\right]+25$	$\cdot$	$\left[\left\|\frac{n}{2}\right\|\right]+31$
C	$\left[\left\|\frac{n}{2}\right\|\right]+2$	I	$\left[\left\|\frac{n}{2}\right\|\right]+8$	O	$\left[\left\|\frac{n}{2}\right\|\right]+14$	U	$\left[\left\|\frac{n}{2}\right\|\right]+20$	β	$\left[\left\|\frac{n}{2}\right\|\right]+26$	0	$\left[\left\|\frac{n}{2}\right\|\right]+32$
D	$\left[\left\|\frac{n}{2}\right\|\right]+3$	J	$\left[\left\|\frac{n}{2}\right\|\right]+9$	P	$\left[\left\|\frac{n}{2}\right\|\right]+15$	V	$\left[\left\|\frac{n}{2}\right\|\right]+21$	(	$\left[\left\|\frac{n}{2}\right\|\right]+27$	5	$\left[\left\|\frac{n}{2}\right\|\right]+33$
E	$\left[\left\|\frac{n}{2}\right\|\right]+4$	K	$\left[\left\|\frac{n}{2}\right\|\right]+10$	Q	$\left[\left\|\frac{n}{2}\right\|\right]+16$	W	$\left[\left\|\frac{n}{2}\right\|\right]+22$	1	$\left[\left\|\frac{n}{2}\right\|\right]+28$	$	$\left[\left\|\frac{n}{2}\right\|\right]+34$
F	$\left[\left\|\frac{n}{2}\right\|\right]+5$	L	$\left[\left\|\frac{n}{2}\right\|\right]+11$	R	$\left[\left\|\frac{n}{2}\right\|\right]+17$	X	$\left[\left\|\frac{n}{2}\right\|\right]+23$	€	$\left[\left\|\frac{n}{2}\right\|\right]+29$	)	$\left[\left\|\frac{n}{2}\right\|\right]+35$

Now, we explain a new coding and decoding algorithms.

Coding Algorithm

1. Dived the matrix U into blocks S_k $(1 \leq k \leq m^2)$.
2. Find c " the number of the block matrices S_k $(1 \leq k \leq m^2)$".
3. Find n as follows:

$$n = \begin{cases} c, & c \leq 3 \\ c-2, & c > 3 \end{cases}.$$

4. Determine s_i^k $(1 \leq i \leq 9)$.
5. Compute $\det(S_k) \to d_k$.
6. Construct $W = \left[d_k, s_j^k\right]_{j \in \{1,2,3,4,5,6,8,9\}}$.
7. End of algorithm.

Decoding Algorithm

1. Compute M^n for chosen n.
2. Determine $m_i (1 \leq i \leq 9)$.
3. Compute the elements $L_1^k, L_2^k, L_3^k, L_4^k. L_5^k. L_6^k$ $(1 \leq k \leq m^2)$,

$$s_1^k m_1 + s_2^k m_4 + s_3^k m_7 \to L_1^k,$$
$$s_1^k m_2 + s_2^k m_5 + s_3^k m_8 \to L_2^k,$$
$$s_1^k m_3 + s_2^k m_6 + s_3^k m_9 \to L_3^k,$$
$$s_4^k m_1 + s_5^k m_4 + s_6^k m_7 \to L_4^k,$$
$$s_4^k m_2 + s_5^k m_5 + s_6^k m_8 \to L_5^k,$$
$$s_4^k m_3 + s_5^k m_6 + s_6^k m_9 \to L_6^k.$$

4. Determine $\det(C_k) \to b_k, \det(O_k) \to r_k, and$ $\det(T_k) \to h_k$, such that,

$$C_k = \begin{bmatrix} L_1^k & L_2^k \\ L_4^k & L_5^k \end{bmatrix}, O_k = \begin{bmatrix} L_3^k & L_1^k \\ L_6^k & L_4^k \end{bmatrix}, T_k = \begin{bmatrix} L_2^k & L_3^k \\ L_5^k & L_6^k \end{bmatrix}.$$

5. Compute the equation,

$$d_k.\det(M^n) = b_k(y_k m_3 + s_8^k m_6 + s_9^k m_9) + r_k(y_k m_2 + s_8^k m_5 + s_9^k m_8)$$
$$+h_k(y_k m_1 + s_8^k m_4 + s_9^k m_7).$$

6. Substitute for $y_k = s_7^k$.
7. Construct U.
8. End of algorithm.

Example 2.1 Encoded the message text:
" CONVERT FROM EURO TO USD (1€ = 1.05$)",
the message matrix $U$is:

$$U = \begin{bmatrix} C & O & N & V & E & R \\ T & \beta & F & R & O & M \\ \beta & E & U & R & O & \beta \\ T & O & \beta & U & S & D \\ \beta & (& 1 & € & = & 1 \\ \cdot & 0 & 5 & \$ &) & \beta \end{bmatrix}_{6\times 6}$$

Coding Algorithm

1. We divide the massage matrix U of size 6×6 into blocks S_k $(1 \le k \le m^2)$ of size 3×3.

$$S_1 = \begin{bmatrix} C & O & N \\ T & \beta & F \\ \beta & E & U \end{bmatrix}, \quad S_2 = \begin{bmatrix} V & E & R \\ R & O & M \\ R & O & \beta \end{bmatrix}, \quad S_3 = \begin{bmatrix} T & O & \beta \\ \beta & (& 1 \\ \cdot & 0 & 5 \end{bmatrix} \text{ and } S_4 = \begin{bmatrix} U & S & D \\ € & = & 1 \\ \$ &) & \beta \end{bmatrix}.$$

2. From step (1), the number of the block matrices S_k $(1 \le k \le m^2)$ is $c = 4$.
3. To find n, we have from step (2), $c = 4 > 3$, so $n = c - 2$, then $n = 2$. For $n = 2$ we can use the following character table for the message matrix U:

C	O	N	V	E	R	T	β	F
3	15	14	22	5	18	20	27	6
R	O	M	β	E	U	R	O	β
18	15	13	27	5	21	18	15	27
T	O	β	U	S	D	β	(	1
20	15	27	21	19	4	27	28	29
€	=	1	·	0	5	$	)	β
30	31	29	32	33	34	35	0	27

4. The elements of the block matrices S_k $(1 \le k \le 9)$:

$k=1$	s_1^k	3	$k=2$	s_1^k	22	$k=3$	s_1^k	20	$k=4$	s_1^k	21
	s_2^k	15		s_2^k	5		s_2^k	15		s_2^k	19
	s_3^k	14		s_3^k	18		s_3^k	27		s_3^k	4
	s_4^k	20		s_4^k	18		s_4^k	27		s_4^k	30
	s_5^k	27		s_5^k	15		s_5^k	28		s_5^k	31
	s_6^k	6		s_6^k	13		s_6^k	29		s_6^k	29
	s_7^k	27		s_7^k	18		s_7^k	32		s_7^k	35
	s_8^k	5		s_8^k	15		s_8^k	33		s_8^k	0
	s_9^k	21		s_9^k	27		s_9^k	34		s_9^k	27

5. Determinants of the block matrices S_k

$d_1 = \det(S_1) = -11065$

$d_2 = \det(S_2) = 3360$
$d_3 = \det(S_3) = -85$
$d_4 = \det(S_4) = 17132$

6. We construct $W = \left[d_k, s_j^k\right]_{j\in\{1,2,3,4,5,6,8,9\}}$,

$$W = \begin{bmatrix} -11065 & 3 & 15 & 14 & 20 & 27 & 6 & 5 & 21 \\ 3360 & 22 & 5 & 18 & 18 & 15 & 13 & 15 & 27 \\ -85 & 20 & 15 & 27 & 27 & 28 & 29 & 33 & 34 \\ 17132 & 21 & 19 & 4 & 30 & 31 & 29 & 0 & 27 \end{bmatrix}.$$

7. End of algorithm.

Decoding Algorithm

1. From Equation (1.4)

$$M^n = \begin{bmatrix} t_{n+2} & t_{n+1}+t_n & t_{n+1} \\ t_{n+1} & t_n + t_{n-1} & t_n \\ t_n & t_{n-1}+t_{n-2} & t_{n-1} \end{bmatrix}$$

for $n = 2$,

$$M^2 = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

2. The elements of M^2 are denoted by

$m_1 = 2$	$m_2 = 2$	$m_3 = 1$
$m_4 = 1$	$m_5 = 1$	$m_6 = 1$
$m_7 = 1$	$m_8 = 0$	$m_9 = 0$

3. Now we compute the elements $L_1^k, L_2^k, L_3^k, L_4^k, L_5^k, L_6^k$, for $(1 \leq k \leq 4)$.

$L_1^1 = 35$	$L_2^1 = 21$	$L_3^1 = 18$	$L_4^1 = 73$	$L_5^1 = 67$	$L_6^1 = 47$
$L_1^2 = 67$	$L_2^2 = 49$	$L_3^2 = 27$	$L_4^2 = 64$	$L_5^2 = 51$	$L_6^2 = 33$
$L_1^3 = 82$	$L_2^3 = 55$	$L_3^3 = 35$	$L_4^3 = 111$	$L_5^3 = 82$	$L_6^3 = 55$
$L_1^4 = 65$	$L_2^4 = 61$	$L_3^4 = 40$	$L_4^4 = 120$	$L_5^4 = 91$	$L_6^4 = 61$

4. We compute the matrices, $C_k = \begin{bmatrix} L_1^k & L_2^k \\ L_4^k & L_5^k \end{bmatrix}, O_k = \begin{bmatrix} L_3^k & L_1^k \\ L_6^k & L_4^k \end{bmatrix}, T_k = \begin{bmatrix} L_2^k & L_3^k \\ L_5^k & L_6^k \end{bmatrix}$, for $(1 \leq k \leq 4)$ then

$$C_1 = \begin{bmatrix} 35 & 21 \\ 73 & 67 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 67 & 49 \\ 64 & 51 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 82 & 55 \\ 111 & 82 \end{bmatrix}, \quad C_4 = \begin{bmatrix} 65 & 61 \\ 120 & 91 \end{bmatrix},$$

$$O_1 = \begin{bmatrix} 18 & 35 \\ 47 & 73 \end{bmatrix}, \quad O_2 = \begin{bmatrix} 27 & 67 \\ 33 & 64 \end{bmatrix}, \quad O_3 = \begin{bmatrix} 35 & 82 \\ 55 & 111 \end{bmatrix}, \quad O_4 = \begin{bmatrix} 40 & 65 \\ 61 & 120 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 21 & 18 \\ 67 & 47 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 49 & 27 \\ 51 & 33 \end{bmatrix}, \quad T_3 = \begin{bmatrix} 55 & 35 \\ 82 & 55 \end{bmatrix}, \quad T_4 = \begin{bmatrix} 61 & 40 \\ 91 & 61 \end{bmatrix}.$$

Now, we calculate the determinants $\det(C_k) \to b_k, \det(O_k) \to r_k$, and $\det(T_k) \to h_k$,

$b_1 = \det(C_1) = 812,$	$r_1 = \det(O_1) = -331,$	$h_1 = \det(T_1) = -219,$
$b_2 = \det(C_2) = 281,$	$r_2 = \det(O_2) = -483,$	$h_2 = \det(T_2) = 240,$
$b_3 = \det(C_3) = 619,$	$r_3 = \det(O_3) = -625,$	$h_3 = \det(T_3) = 155,$
$b_4 = \det(C_4) = -1405,$	$r_4 = \det(O_4) = 835,$	$h_4 = \det(T_4) = 81.$

5. We compute the elements y_k:

$$d_1.\det(M^2) = b_1(y_1 m_3 + s_8^1 m_6 + s_9^1 m_9) + r_1(y_1 m_2 + s_8^1 m_5 + s_9^1 m_8)$$

$$+h_1(y_1m_1+s_8^1m_4+s_9^1m_7),$$
$$d_2.\det(M^2)=b_2(y_2m_3+s_8^2m_6+s_9^2m_9)+r_2(y_2m_2+s_8^2m_5+s_9^2m_8)$$
$$+h_2(y_2m_1+s_8^2m_4+s_9^2m_7),$$
$$d_3.\det(M^2)=b_3(y_3m_3+s_8^3m_6+s_9^3m_9)+r_3(y_3m_2+s_8^3m_5+s_9^3m_8)$$
$$+h_3(y_3m_1+s_8^3m_4+s_9^3m_7),$$
and
$$d_4.\det(M^2)=b_4(y_4m_3+s_8^4m_6+s_9^4m_9)+r_4(y_4m_2+s_8^4m_5+s_9^4m_8)$$
$$+h_4(y_4m_1+s_8^4m_4+s_9^4m_7).$$
Thus,
$$-11065=812(y_1+5)-331(2y_1+5)\ -219(2y_1+5+21),$$
so,
$$y_1=27.$$
Thus,
$$3360=281(y_2+15)-483(2y_2+15)\ +240(2y_2+15+27),$$
so,
$$y_2=18.$$
Thus,
$$-85=619(y_3+33)-625(2y_3+33)\ +155(2y_3+33+34),$$
so,
$$y_3=32.$$
and,
$$17132=-1405y_4+835(2y_4)\ +81(2y_4+27),$$
so,
$$y_4=35.$$

6. Substitute for $y_k=s_7^k$, when $(1\le k\le 4)$,

$y_1=s_7^1=27$
$y_2=s_7^2=18$
$y_3=s_7^3=32$
$y_4=s_7^4=35$

7. We construct the block matrices S_k $(1\le k\le 9)$,

$$S_1=\begin{bmatrix}3&15&14\\20&27&6\\27&5&21\end{bmatrix},S_2=\begin{bmatrix}22&5&18\\18&15&13\\18&15&27\end{bmatrix},S_3=\begin{bmatrix}20&15&27\\27&28&29\\32&33&34\end{bmatrix},S_4=\begin{bmatrix}21&19&4\\30&31&29\\35&0&27\end{bmatrix}.$$

8. The message matrix U is:
$$U=\begin{bmatrix}3&15&14&22&5&18\\20&27&6&18&15&13\\27&5&21&18&15&27\\20&15&27&21&19&4\\27&28&29&30&31&29\\32&33&34&35&0&27\end{bmatrix}_{6\times 6}.$$
The text massage is:
$$U=\begin{bmatrix}C&O&N&V&E&R\\T&\beta&F&R&O&M\\\beta&E&U&R&O&\beta\\T&O&\beta&U&S&D\\\beta&(&1&€&=&1\\\cdot&0&5&\$&)&\beta\end{bmatrix}_{6\times 6}.$$

9. End of algorithm.

Example 2.2 To encode the message:
"LIFE BECAMES EASIER AND MORE BEAUTIFUL WHEN WE CAN SEE THE GOOD IN OTHER PEOPLE"
the message matrix U is:

$$U=\begin{bmatrix} L & I & F & E & \beta & B & E & C & O \\ M & E & S & \beta & E & A & S & I & E \\ R & \beta & A & N & D & \beta & M & O & R \\ E & \beta & B & E & A & U & T & I & F \\ U & L & \beta & W & H & E & N & \beta & W \\ E & \beta & C & A & N & \beta & S & E & E \\ \beta & T & H & E & \beta & G & O & O & D \\ \beta & I & N & \beta & O & T & H & E & R \\ \beta & P & E & O & P & L & E & \beta & \beta \end{bmatrix}_{9\times 9}.$$

Coding Algorithm

1. We divide the massage matrix U of size 9×9 into blocks S_k $(1\le k\le m^2)$ of size 3×3.

$$S_1=\begin{bmatrix} L & I & F \\ M & E & S \\ R & \beta & A \end{bmatrix},\quad S_2=\begin{bmatrix} E & \beta & B \\ \beta & E & A \\ N & D & \beta \end{bmatrix},\quad S_3=\begin{bmatrix} E & C & O \\ S & I & E \\ M & O & R \end{bmatrix}$$
$$S_4=\begin{bmatrix} E & \beta & B \\ U & L & \beta \\ E & \beta & C \end{bmatrix},\quad S_5=\begin{bmatrix} E & A & U \\ W & H & E \\ A & N & \beta \end{bmatrix},\quad S_6=\begin{bmatrix} T & I & F \\ N & \beta & W \\ S & E & E \end{bmatrix},$$
$$S_7=\begin{bmatrix} \beta & T & H \\ \beta & I & N \\ \beta & P & E \end{bmatrix},\quad S_8=\begin{bmatrix} E & \beta & G \\ \beta & O & T \\ O & P & L \end{bmatrix} \ and \ S_9=\begin{bmatrix} O & O & D \\ H & E & R \\ E & \beta & \beta \end{bmatrix}.$$

2. From step (1), the number of the block matrices S_k $(1\le k\le m^2)$ is $c=9$
3. To find n, we have from step (2), $c=9>3$, so $n=c-2$, then $n=7$. For $n=7$ we can use the following character table for the message matrix U:

L	I	F	E	β	B	E	C	O
14	11	8	7	29	4	7	5	17
M	E	S	β	E	A	S	I	E
15	7	21	29	7	3	21	11	7
R	β	A	N	D	β	M	O	R
20	29	3	16	6	29	15	17	20
E	β	B	E	A	U	T	I	F
7	29	4	7	3	23	22	11	8
U	L	β	W	H	E	N	β	W
23	14	29	25	10	7	16	29	25
E	β	C	A	N	β	S	E	E
7	29	5	3	16	29	21	7	7
β	T	H	E	β	G	O	O	D
29	22	10	7	29	9	17	17	6
β	I	N	β	O	T	H	E	R
29	11	16	29	17	22	10	7	20
β	P	E	O	P	L	E	β	β

29	18	7	17	18	14	7	29	29

4. The elements of the block matrices S_k $(1 \leq k \leq 9)$:

	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
s_1^k	14	7	7	7	7
s_2^k	11	29	5	29	3
s_3^k	8	4	17	4	23
s_4^k	15	29	21	23	25
s_5^k	7	7	11	14	10
s_6^k	21	3	7	29	7
s_7^k	20	16	15	7	3
s_8^k	29	6	17	29	16
s_9^k	3	29	20	5	29

	$k = 6$	$k = 7$	$k = 8$	$k = 9$
s_1^k	22	29	7	17
s_2^k	11	22	29	17
s_3^k	8	10	9	6
s_4^k	16	29	29	10
s_5^k	29	11	17	7
s_6^k	25	16	22	20
s_7^k	21	29	17	7
s_8^k	7	18	18	29
s_9^k	7	7	14	29

5. We calculate determinants of the block matrices S_k $(1 \leq k \leq 9)$.

$d_1 = det(S_1) = -1747$
$d_2 = det(S_2) = -21454$
$d_3 = det(S_3) = 2396$
$d_4 = det(S_4) = -569$
$d_5 = det(S_5) = 7644$
$d_6 = det(S_6) = 1183$
$d_7 = det(S_7) = 1653$
$d_8 = det(S_8) = 63$
$d_9 = det(S_9) = -7513$

6. We construct $W = \left[d_k, s_j^k\right]_{j \in \{1,2,3,4,5,6,8,9\}}$,

$$W = \begin{bmatrix} -1747 & 14 & 11 & 8 & 15 & 7 & 21 & 29 & 3 \\ -21454 & 7 & 29 & 4 & 29 & 7 & 3 & 6 & 29 \\ 2396 & 7 & 5 & 17 & 21 & 11 & 7 & 17 & 20 \\ -569 & 7 & 29 & 4 & 23 & 14 & 29 & 29 & 5 \\ 7644 & 7 & 3 & 23 & 25 & 10 & 7 & 16 & 29 \\ 1183 & 22 & 11 & 8 & 16 & 29 & 25 & 7 & 7 \\ 1653 & 29 & 22 & 10 & 29 & 11 & 16 & 18 & 7 \\ 63 & 7 & 29 & 9 & 29 & 17 & 22 & 18 & 14 \\ -7513 & 17 & 17 & 6 & 10 & 7 & 20 & 29 & 29 \end{bmatrix}.$$

7. End of algorithm.

Decoding Algorithm

1. from Equation (1.4),

$$M^n = \begin{bmatrix} t_{n+2} & t_{n+1}+t_n & t_{n+1} \\ t_{n+1} & t_n+t_{n-1} & t_n \\ t_n & t_{n-1}+t_{n-2} & t_{n-1} \end{bmatrix}$$

for $n = 2$,

$$M^7 = \begin{bmatrix} 44 & 37 & 24 \\ 24 & 20 & 13 \\ 13 & 11 & 7 \end{bmatrix}.$$

2. The elements of M^7 are denoted by:

$m_1 = 44$	$m_2 = 37$	$m_3 = 24$
$m_4 = 24$	$m_5 = 20$	$m_6 = 13$
$m_7 = 13$	$m_8 = 11$	$m_9 = 7$

3. Now, we compute the elements $L_1^k, L_2^k, L_3^k, L_4^k, L_5^k, L_6^k$,

$L_1^1 = 984$	$L_2^1 = 826$	$L_3^1 = 535$	$L_4^1 = 1101$	$L_5^1 = 926$	$L_6^1 = 598$
$L_1^2 = 1056$	$L_2^2 = 883$	$L_3^2 = 573$	$L_4^2 = 1483$	$L_5^2 = 1246$	$L_6^2 = 808$
$L_1^3 = 649$	$L_2^3 = 546$	$L_3^3 = 352$	$L_4^3 = 1279$	$L_5^3 = 1074$	$L_6^3 = 696$
$L_1^4 = 1056$	$L_2^4 = 883$	$L_3^4 = 573$	$L_4^4 = 1725$	$L_5^4 = 1450$	$L_6^4 = 937$
$L_1^5 = 679$	$L_2^5 = 572$	$L_3^5 = 368$	$L_4^5 = 1431$	$L_5^5 = 1202$	$L_6^5 = 779$
$L_1^6 = 1336$	$L_2^6 = 1122$	$L_3^6 = 727$	$L_4^6 = 1725$	$L_5^6 = 1447$	$L_6^6 = 936$
$L_1^7 = 1934$	$L_2^7 = 1623$	$L_3^7 = 1052$	$L_4^7 = 1748$	$L_5^7 = 1469$	$L_6^7 = 951$
$L_1^8 = 1121$	$L_2^8 = 938$	$L_3^8 = 608$	$L_4^8 = 1970$	$L_5^8 = 1655$	$L_6^8 = 1071$
$L_1^9 = 1234$	$L_2^9 = 1035$	$L_3^9 = 671$	$L_4^9 = 868$	$L_5^9 = 730$	$L_6^8 = 471$

4. We compute the matrices, $C_k = \begin{bmatrix} L_1^k & L_2^k \\ L_4^k & L_5^k \end{bmatrix}, O_k = \begin{bmatrix} L_3^k & L_1^k \\ L_6^k & L_4^k \end{bmatrix}, T_k = \begin{bmatrix} L_2^k & L_3^k \\ L_5^k & L_6^k \end{bmatrix}$, then

$$C_1 = \begin{bmatrix} 984 & 826 \\ 1101 & 926 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1056 & 883 \\ 1483 & 1246 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 649 & 546 \\ 1279 & 1074 \end{bmatrix},$$

$$C_4 = \begin{bmatrix} 1056 & 883 \\ 1725 & 1450 \end{bmatrix}, \quad C_5 = \begin{bmatrix} 679 & 572 \\ 1431 & 1202 \end{bmatrix}, \quad C_6 = \begin{bmatrix} 1336 & 1122 \\ 1725 & 1447 \end{bmatrix},$$

$$C_7 = \begin{bmatrix} 1934 & 1623 \\ 1748 & 1469 \end{bmatrix}, \quad C_8 = \begin{bmatrix} 1121 & 938 \\ 1970 & 1655 \end{bmatrix}, \quad C_9 = \begin{bmatrix} 1234 & 1035 \\ 868 & 730 \end{bmatrix},$$

$$O_1 = \begin{bmatrix} 535 & 984 \\ 598 & 1101 \end{bmatrix}, \quad O_2 = \begin{bmatrix} 573 & 1056 \\ 808 & 1483 \end{bmatrix}, \quad O_3 = \begin{bmatrix} 352 & 649 \\ 696 & 1279 \end{bmatrix},$$

$$O_4 = \begin{bmatrix} 573 & 1056 \\ 937 & 1725 \end{bmatrix}, \quad O_5 = \begin{bmatrix} 368 & 679 \\ 779 & 1431 \end{bmatrix}, \quad O_6 = \begin{bmatrix} 727 & 1336 \\ 936 & 1725 \end{bmatrix},$$

$$O_7 = \begin{bmatrix} 1052 & 1934 \\ 951 & 1748 \end{bmatrix}, \quad O_8 = \begin{bmatrix} 608 & 1121 \\ 1071 & 1970 \end{bmatrix}, \quad O_9 = \begin{bmatrix} 671 & 1234 \\ 471 & 868 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 826 & 535 \\ 926 & 598 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 883 & 573 \\ 1246 & 808 \end{bmatrix}, \quad T_3 = \begin{bmatrix} 546 & 352 \\ 1074 & 696 \end{bmatrix},$$

$$T_4 = \begin{bmatrix} 883 & 573 \\ 1450 & 937 \end{bmatrix}, \quad T_5 = \begin{bmatrix} 572 & 368 \\ 1202 & 779 \end{bmatrix}, \quad T_6 = \begin{bmatrix} 1122 & 727 \\ 1447 & 936 \end{bmatrix},$$

$$T_7 = \begin{bmatrix} 1623 & 1052 \\ 1469 & 951 \end{bmatrix}, \quad T_8 = \begin{bmatrix} 938 & 608 \\ 1655 & 1071 \end{bmatrix}, \quad T_9 = \begin{bmatrix} 1035 & 671 \\ 730 & 471 \end{bmatrix}.$$

Now, we calculate the determinants $\det(C_k) \to b_k, \det(O_k) \to r_k,$ and $\det(T_k) \to h_k$,

$b_1 = \det(C_1) = 1758$	$r_1 = \det(O_1) = 603$	$h_1 = \det(T_1) = -1462$
$b_2 = \det(C_2) = 6287$	$r_2 = \det(O_2) = -3489$	$h_2 = \det(T_2) = -494$
$b_3 = \det(C_3) = -1308$	$r_3 = \det(O_3) = -1496$	$h_3 = \det(T_3) = 1968$

$b_4 = \det(C_4) = 8025$	$r_4 = \det(O_4) = -1047$	$h_4 = \det(T_4) = -3479$
$b_5 = \det(C_5) = -2374$	$r_5 = \det(O_5) = -2333$	$h_5 = \det(T_5) = 3252$
$b_6 = \det(C_6) = -2258$	$r_6 = \det(O_6) = 3579$	$h_6 = \det(T_6) = -1777$
$b_7 = \det(C_7) = 4042$	$r_7 = \det(O_7) = -338$	$h_7 = \det(T_7) = -1915$
$b_8 = \det(C_8) = 7395$	$r_8 = \det(O_8) = -2831$	$h_8 = \det(T_8) = -1642$
$b_9 = \det(C_9) = 2440$	$r_9 = \det(O_9) = 1214$	$h_9 = \det(T_9) = -2345$

5. We compute the elements y_k for $(1 \leq k \leq 9$,

$$d_k.\det(M^7) = b_k\left(y_k m_3 + s_8^k m_6 + s_9^k m_9\right) + r_k\left(y_k m_2 + s_8^k m_5 + s_9^k m_8\right)$$
$$+h_k\left(y_k m_1 + s_8^k m_4 + s_9^k m_7\right),$$

$$(-1747)(1) = 1758(24y_1 + 377 + 21) + 603(37y_1 + 580 + 33)$$
$$- 1462(44y_1 + 696 + 39),$$

so,

$$y_1 = 20.$$

$$(-2145)(1) = 6287(24y_2 + 78 + 203)3489(37y_2 + 120 + 319)$$
$$-494(44y_2 + 144 + 377),$$

then

$$y_2 = 16,$$

$$(2396)(1) = -1308(24y_3 + 221 + 140) - 1496(37y_3 + 340 + 220)$$
$$+1968(44y_3 + 408 + 260),$$

thus

$$y_3 = 15.$$

$$(-569)(1) = 8025(24y_4 + 377 + 35) - 1047(37y_4 + 580 + 55)$$
$$-3479(44y_4 + 696 + 65),$$

then

$$y_4 = 7.$$

$$(7644)(1) = -2374(24y_5 + 208 + 203) - 2333(37y_5 + 320 + 319)$$
$$+3252(44y_5 + 384 + 377),$$

therefore,

$$y_5 = 3.$$

$$(1183)(1) = -2258(24y_6 + 91 + 49) + 3579(37y_6 + 140 + 77)$$
$$-1777(44y_6 + 168 + 91),$$

so,

$$y_6 = 21.$$

$$(1653)(1) = 4042(24y_7 + 234 + 49) - 338(37y_7 + 360 + 77)$$
$$-1915(44y_7 + 432 + 91),$$

hence

$$y_7 = 29.$$

$$(63)(1) = 7395(24y_8 + 234 + 98) - 2831(37y_8 + 360 + 154)$$
$$-1642(44y_8 + 432 + 182),$$

then

$$y_8 = 17$$

and

$$(-7513)(1) = 2440(24y_9 + 377 + 203) + 1214(37y_9 + 580 + 319)$$
$$-2345(44y_9 + 696 + 377),$$

hence

$$y_9 = 7.$$

6. Substitute for $y_k = s_7^k$, when $(1 \leq k \leq 9)$,

$y_1 = s_7^1 = 20,$	$y_4 = s_7^4 = 7,$	$y_7 = s_7^7 = 29,$
$y_2 = s_7^2 = 16,$	$y_5 = s_7^5 = 3,$	$y_8 = s_7^8 = 17,$
$y_3 = s_7^3 = 15,$	$y_6 = s_7^6 = 21,$	$y_9 = s_7^9 = 7.$

7. We construct the block matrices S_k $(1 \leq k \leq 9)$,

$$S_1 = \begin{bmatrix} 14 & 11 & 8 \\ 15 & 7 & 21 \\ 20 & 29 & 3 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 7 & 29 & 4 \\ 29 & 7 & 3 \\ 16 & 6 & 29 \end{bmatrix}, \quad S_3 = \begin{bmatrix} 7 & 5 & 17 \\ 21 & 11 & 7 \\ 15 & 17 & 20 \end{bmatrix},$$

$$S_4 = \begin{bmatrix} 7 & 29 & 4 \\ 23 & 14 & 29 \\ 7 & 29 & 5 \end{bmatrix}, \quad S_5 = \begin{bmatrix} 7 & 3 & 23 \\ 25 & 10 & 7 \\ 3 & 16 & 29 \end{bmatrix}, \quad S_6 = \begin{bmatrix} 22 & 11 & 8 \\ 16 & 29 & 25 \\ 21 & 7 & 7 \end{bmatrix},$$

$$S_7 = \begin{bmatrix} 29 & 22 & 10 \\ 29 & 11 & 16 \\ 29 & 18 & 7 \end{bmatrix}, \quad S_8 = \begin{bmatrix} 7 & 29 & 9 \\ 29 & 17 & 22 \\ 17 & 18 & 14 \end{bmatrix}, \quad S_9 = \begin{bmatrix} 17 & 17 & 6 \\ 10 & 7 & 20 \\ 7 & 29 & 29 \end{bmatrix}.$$

8. The Message matrix U is:

$$U = \begin{bmatrix} 14 & 11 & 8 & 7 & 29 & 4 & 7 & 5 & 17 \\ 15 & 7 & 21 & 29 & 7 & 3 & 21 & 11 & 7 \\ 20 & 29 & 3 & 16 & 6 & 29 & 15 & 17 & 20 \\ 7 & 29 & 4 & 7 & 3 & 23 & 22 & 11 & 8 \\ 23 & 14 & 29 & 25 & 10 & 7 & 16 & 29 & 25 \\ 7 & 29 & 5 & 3 & 16 & 29 & 21 & 7 & 7 \\ 29 & 22 & 10 & 7 & 29 & 9 & 17 & 17 & 6 \\ 29 & 11 & 16 & 29 & 17 & 22 & 10 & 7 & 20 \\ 29 & 18 & 7 & 17 & 18 & 14 & 7 & 29 & 29 \end{bmatrix}_{9\times 9}$$

The text massage is:

$$U = \begin{bmatrix} L & I & F & E & \beta & B & E & C & O \\ M & E & S & \beta & E & A & S & I & E \\ R & \beta & A & N & D & \beta & M & O & R \\ E & \beta & B & E & A & U & T & I & F \\ U & L & \beta & W & H & E & N & \beta & W \\ E & \beta & C & A & N & \beta & S & E & E \\ \beta & T & H & E & \beta & G & O & O & D \\ \beta & I & N & \beta & O & T & H & E & R \\ \beta & P & E & O & P & L & E & \beta & \beta \end{bmatrix}_{9\times 9}$$

9. End of algorithm.

Example 2.3 To encode the message:
" ENCODING"
the message matrix $U$is:

$$U = \begin{bmatrix} E & N & C \\ O & D & I \\ N & G & \beta \end{bmatrix}_{3\times 3}.$$

Coding Algorithm

1. Since the massage matrix U of size 3×3, so we have only one block,

$$S_1 = \begin{bmatrix} E & N & C \\ O & D & I \\ N & G & \beta \end{bmatrix}.$$

2. From step (1), the number of the block matrices S_k $(1 \le k \le m^2)$ is $c = 1$.
3. To find n, we have from step (2), $c = 1 < 3$, so $n = c$, then $n = 1$. For $n = 1$, we can use the following character table for the message matrix U:

E	N	C	O	D	I	N	G	β
4	13	2	14	3	8	13	6	26

4. The elements of the block matrices S_1:

$k = 1$								
s_1^k	s_2^k	s_3^k	s_4^k	s_5^k	s_6^k	s_7^k	s_8^k	s_9^k
4	13	2	14	3	8	13	6	26

5. Determinant of the block matrix S_1.

$$\boxed{d_1 = \det(S_1) = -3170}$$

6. We construct $W = \left[d_k, s_j^k\right]_{j \in \{1,2,3,4,5,6,8,9\}}$,

$$W = [-3170 \quad 4 \quad 13 \quad 2 \quad 14 \quad 3 \quad 8 \quad 6 \quad 26].$$

7. End of algorithm.

Decoding Algorithm

1. From Equation (1.4),

$$M^n = \begin{bmatrix} t_{n+2} & t_{n+1} + t_n & t_{n+1} \\ t_{n+1} & t_n + t_{n-1} & t_n \\ t_n & t_{n-1} + t_{n-2} & t_{n-1} \end{bmatrix}$$

for $n = 1$,

$$M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

2. The elements of M^2 are denoted by

$m_1 = 1$	$m_2 = 1$	$m_3 = 1$
$m_4 = 1$	$m_5 = 0$	$m_6 = 0$
$m_7 = 0$	$m_8 = 1$	$m_9 = 0$

3. Now, we compute the elements $L_1^k, L_2^k, L_3^k, L_4^k, L_5^k, L_6^k$,

$L_1^1 = 17$	$L_2^1 = 6$	$L_3^1 = 4$	$L_4^1 = 17$	$L_5^1 = 22$	$L_6^k = 14$

4. We compute the matrices, $C_k = \begin{bmatrix} L_1^k & L_2^k \\ L_4^k & L_5^k \end{bmatrix}, O_k = \begin{bmatrix} L_3^k & L_1^k \\ L_6^k & L_4^k \end{bmatrix}, T_k = \begin{bmatrix} L_2^k & L_3^k \\ L_5^k & L_6^k \end{bmatrix}$, for $k = 1$, then

$$C_1 = \begin{bmatrix} 17 & 6 \\ 17 & 22 \end{bmatrix}, \qquad O_1 = \begin{bmatrix} 4 & 17 \\ 14 & 17 \end{bmatrix}, \qquad T_1 = \begin{bmatrix} 6 & 4 \\ 22 & 14 \end{bmatrix}.$$

Now, we calculate the determinants $\det(C_k) \to b_k, \det(O_k) \to r_k$, and $\det(T_k) \to h_k$, for $k = 1$

$b_1 = \det(C_1) = 272$	$r_1 = \det(O_1) = -170$	$h_1 = \det(T_1) = -4$

5. We compute the elements y_k for $k = 1$:

$$d_1.\det(M) = b_1(y_1 m_3 + s_8^1 m_6 + s_9^1 m_9) + r_1(y_1 m_2 + s_8^1 m_5 + s_9^1 m_8) + h_1(y_1 m_1 + s_8^1 m_4 + s_9^1 m_7).$$

Thus,

$$(-3170)(1) = 272y_1 - 170(y_1 + 26) - 4(y_1 + 6),$$

Therefore,

$$y_1 = 13.$$

6. Substitute for $y_k = s_7^k$, when $(k = 1)$,

$$y_1 = s_7^1 = 13.$$

7. We construct the block matrix S_1,

$$S_1 = \begin{bmatrix} 4 & 13 & 2 \\ 14 & 3 & 8 \\ 13 & 6 & 26 \end{bmatrix}.$$

8. The Message matrix U is:

$$U = \begin{bmatrix} 4 & 13 & 2 \\ 14 & 3 & 8 \\ 13 & 6 & 26 \end{bmatrix}_{3\times 3}.$$

The text massage is:

$$U = \begin{bmatrix} E & N & C \\ O & D & I \\ N & G & \beta \end{bmatrix}_{3\times 3}.$$

9. End.

3. Conclusions:

After reviewing and comparing many studies on coding theory, encryption techniques, and encoding/ decoding algorithms, we presented a new method for encoding/ decoding a message using an algorithm that uses Tribonacci sequence. This algorithm divides the message matrix into blocks of size $\mathbf{3 \times 3}$. Our results demonstrate that using the Tribonacci matrix provides a unique and effective approach for practical applications in data transmission and storage. However, further research is needed to fully understand its capabilities and limitations. One of the avenues we are exploring in the future is to develop the Lucas Tribonacci coding. By investigating the properties of the Lucas sequence and its corresponding matrix, we may discover other new and innovative methods for encoding and decoding data.

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