Unsteady Heat Transfer Analysis on The MHD Flow of A Second Grade Fluid in A Channel with Porous Medium

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Abstract
The aim of this paper is to analyzed unsteady heat transfer for magnetohydrodynamic (MHD) flow of a second grade fluid in a channel with porous medium. The equations which was used to describe the flow are the momentum and energy, these equations were written to get thier non dimentional form. Homotopy analysis method (HAM) is employed to obtain a semi-analytical solutions for velocity and heat transfer fields. The effect of each dimensionless parameter upon the velocity and temperature distributions is analyzed and shown graphically by using MATHEMATICA package.

Keywords: Unsteady Flow, Magnetic Field, Homotopy Analysis Method

الجريان اللا مستقر و تحليل انتقال الحرارة في حقل مغناطيسي لمائع من الرتبة الثانية في قناة ذات وسط مسامي

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الخلاصة
الهدف من دراسة هذا البحث هو تحليل انتقال الحرارة والجريان اللا مستقر لمائع من الدرجة الثانية في قناة ذات وسط مسامي تحت تأثير حقل مغناطيسي. المعادلات التي استخدمت لوصف الجريان اللامستقر هي معادلات الحركة والطاقة, كتبناهذه المعادلات لنحصل على الشكل اللابعدي لها. استخدمت طريقة هوموتوبي التحليلية لنحصل على الحل التحليلي لحقلي السرعة و انتقال الحرارة. تأثير كل معلمة لابعدية على حقلي توزيع السرعة و الحرارة حُلل ووُضح بيانياً باستخدام برنامج ماثماتيكا.

1. Introduction

Within the past fifty years, many problems dealing with the flow of Newtonian and non-Newtonian fluids through porous channels have been studied by engineers and mathematicians. The analysis of such flows finds important applications in engineering practice, particularly in chemical industries, investigations of such fluids are desirable. A number of industrially important fluids including molten plastics, polymers, pulps, foods and fossil fuels, which may saturate in underground beds, display non-Newtonian behavior. Examples, of such fluids, second grade fluid is the simplest subclass for which one can hope to gain an analytic solution. The MHD phenomenon is characterized by an interaction between the hydrodynamic and boundary layer electromagnetic field. The study of MHD flow in a channel also has application in many devices like MHD power generators, MHD pumps, accelerators, etc. Some recent contributions in the field may be mentioned in [1,2,3 and 4]. The effect of wall porosity on the two dimensional laminar flow of a viscous incompressible fluid in a parallel-walled was first studied theoretically by Berman[3].

Authors like Brady [5], Prodman [6], and many others have extended Berman's symmetric series solution. The aim of this paper is to investigate the heat transfer analysis for (MHD) flow in a porous channel. The second grade fluid fills the porous space inside the channel. In the next section we had present the equations which are used to describe the Magnetohydrodynamics (MHD) flow and heat transfer effects in the channel with porous medium. The third section deals with the analytical solutions for velocity and temperature fields by using powerful technique Homotopy analysis method (HAM), which was developed by Liao [7] is employed to solve the problems for velocity and temperature fields. The fourth section concerns with the convergence of the solutions. In section 5, we present the graphical results and discussion. In the last section, we give concluding remarks on the results.

2. Description of The Problem

We consider the unsteady, incompressible (MHD) flow of a second grade fluid in a channel of width H with porous medium. The x-axis is along the centerline of the channel, parallel to the channel surfaces and the y-axis is perpendicular to it. The porous surfaces are $y=\pm H/2$. The flow is symmetric about both x-and y-axes. The fluid is either injected into the channel or extracted out at a uniform velocity V/2 (the velocity $V>0$ corresponds to the suction and $V<0$ for injection). The temperature at the centerline (y = 0) and the upper wall (y=H/2) are T_w and T_H respectively. A constant magnetic field B^0 is applied perpendicular to the channel walls and the electric field is taken zero. The induced magnetic field is neglected for small magnetic Reynolds number. It is assume that the pressure gradient zero. Under these assumptions the governing equations for MHD boundary layer flow as the following:

Momentum Equation

$$u_t+uu_x+v\,u_y=\nu\,u_{yy}+\frac{\alpha_1}{\rho}\left[u_{xxt}+u_{yyt}+u_x u_{yy}+u\,u_{yyx}+u_y v_{yy}+v\,u_{yyy}\right]-\frac{\sigma\beta_0^2}{\rho}u-\frac{\mu\phi}{k}u \quad (1)$$

Energy Equation

$$\rho c_p(T_t+uT_x+vT_y)=k_1T_{yy}+\mu(u_y)^2+\alpha_1[u_{xt}u_x+u_x(u_y)^2+uu_yu_{yx}+(u_y)^2v_y+vu_yu_{yy}] \quad (2)$$

Where u and v are the velocities components in x- and y- directions respectively, ρ is the density, μ is the dynamic viscosity, σ is the electrical conductivity, ϕ is the porosity of the medium, k is the permeability of the medium, β_0 is a constant magnetic field, ν is the kinematic viscosity, α_1 material parameter of a second grade fluid. c_p is a specific heat, k_1 is the thermal conductivity.

We can write down the momentum and energy equations in non-dimensional form, through introducing the following new quantities:

$$\psi=v\,\zeta^{\frac{1}{2}}\,x\,f(\eta,\zeta)\ ,\ \eta=\frac{1}{H}\zeta^{-\frac{1}{2}}y\,,$$

$$\zeta=1-e^{-t^*},\ t^*=t\,\frac{v}{H}\ ,\ q(\eta,\zeta)=\frac{T-T_H}{T_w-T_H}$$

$$T=q(\eta,\zeta)\Delta\theta+T_H\,,\ \Delta\theta=T_w-T_H$$

substituting above quantities in Eqs. (1) and (2) respectively, we obtain:

$$\mathcal{R}e(1-\zeta)*\left[1/2\,\eta\zeta*\partial_{\eta,\eta}f+\zeta^2*\partial_{\eta,\zeta}f\right]+\zeta^2\mathcal{R}e*\left[(\partial_\eta f)^2-f\partial_{\eta,\eta}f\right]+\zeta*\partial_{\eta,\eta,\eta}f-\zeta^2\lambda*\partial_\eta f-\zeta^2M^2*\partial_\eta f-1/2\,\alpha(1-\zeta)\eta*\partial_{\eta,\eta,\eta,\eta}f-\alpha\zeta*\left[2\partial_\eta f\,\partial_{\eta,\eta,\eta}f-f\,\partial_{\eta,\eta,\eta,\eta}f-(\partial_{\eta,\eta}f)^2\right]=0 \quad (3)$$

with boundary conditions

$$f(0,\zeta;p)=0,\ \partial_{\eta,\eta}f(0,\zeta;p)=0,$$

$$f\left(\tfrac{1}{2},\zeta;p\right)=\tfrac{1}{2},\ \partial_\eta f\left(\tfrac{1}{2},\zeta;p\right)=0$$

$$\alpha_1(1-\zeta)\left[1/2\,\eta\partial_\eta q+\zeta*\partial_\zeta q\right]+\partial_{\eta,\eta}q-\mathcal{R}e\Pr\zeta*(f\,\partial_\eta q)+$$

$\Pr \mathrm{Ec} * (\partial_{\eta,\eta} f)^2 - \alpha_2(1-\zeta) *$
$[\partial_\eta f \partial_{\eta,\eta} f * \eta + \partial_\eta f * \zeta * \partial_{\eta,\zeta} f] -$
$\alpha \Pr \mathrm{Ec} * (\partial_\eta f (\partial_{\eta,\eta} f)^2 - f\partial_{\eta,\eta} f \, \partial_{\eta,\eta,\eta} f)$
$=0$ (4)

with boundary conditions

$q(0,\zeta;p) = 1,\ q(\frac{1}{2},\zeta;p) = 0$

Where the respective values of Hartmann number M, the Reynold number $\mathcal{R}e$, Prandtl number Pr, Eckert number Ec, the parameter α, the porosity parameter λ are: $M^2 = \frac{\sigma H^2 \beta_0^2}{\mu}$, $\mathcal{R}e = \frac{HV}{v}$, $\Pr = \frac{c_p \mu}{k_1}$, $\mathrm{Ec} = \frac{V^2 x^2}{c_p H^2 \Delta\theta}$, $\alpha = \frac{\alpha_1 V}{H\mu}$, $\lambda = \frac{\phi H^2}{k}$, $\alpha_1 = \frac{\rho c_p HV}{k_1}$, $\alpha_2 = \frac{\alpha_1 V^3}{k_1 \Delta\theta H}$

3. Solution of The Problems

To solve momentum and energy equations, we choose initial guesses and linear operators in the following form:

$$f_0(\eta) = \eta(\frac{3}{2} - 2\eta^2) \quad (5)$$

$$q_0(\eta) = 1 - 2\eta \quad (6)$$

$$\mathcal{L}_1(f) = \partial_{\eta,\eta,\eta,\eta} f \quad (7)$$

$$\mathcal{L}_2(q) = \partial_{\eta,\eta} q \quad (8)$$

with

$$\mathcal{L}_1(c1\eta^4 + c2\eta^3 + c3\eta^2 + c4\eta) = 0 \quad (9)$$

$$\mathcal{L}_2(c1\eta + c2) = 0 \quad (10)$$

In which ci [i=1-4]are constants

Upon making use above definitions, we first construct the zeroth - order deformation problems:

$$(1-p)\mathcal{L}_1[f(\eta,\zeta;p) - f_0(\eta)] = p\hbar_1 \aleph_1[f(\eta,\zeta;p)] \quad (11)$$

$$f(0,\zeta;p) = 0,\ \partial_{\eta,\eta} f(0,\zeta;p) = 0,$$
$$f\left(\frac{1}{2},\zeta;p\right) = \frac{1}{2},\ \partial_\eta f\left(\frac{1}{2},\zeta;p\right) = 0 \quad (12)$$

$$(1-p)\mathcal{L}_2[q(\eta,\zeta;p) - q_0(\eta)] = p\hbar_2 \aleph_2[q(\eta,\zeta;p), f(\eta,\zeta;p)] \quad (13)$$

$$q(0,\zeta;p) = 0,\ q\left(\frac{1}{2},\zeta;p\right) = 0 \quad (14)$$

$\zeta^2 * \partial_{\eta,\zeta} f) +$
$\zeta * \quad \partial_{\eta,\eta,\eta} f - \zeta^2 \lambda * \partial_\eta f$
$+\zeta^2 \mathcal{R}e * \left[(\partial_\eta f)^2 -\right.$
$\left. f\partial_{\eta,\eta} f\right] - \zeta^2 M^2 *$
$\partial_\eta f - 1/2\ \alpha(1 -$
$\zeta)\eta\, \partial_{\eta,\eta,\eta,\eta} f - \alpha$
$\zeta * [2\partial_\eta f\, \partial_{\eta,\eta,\eta} f - f$
$\partial_{\eta,\eta,\eta,\eta} f - (\partial_{\eta,\eta} f)^2]$ (15)

$\aleph_2\begin{bmatrix} q(\eta,\zeta;p), \\ f(\eta,\zeta;p) \end{bmatrix}$
$= \alpha_1(1-\zeta) * (1/2\,\eta *$
$\partial_\eta q_{m-1} + \zeta *$
$\partial_\zeta q_{m-1}) + \partial_{\eta,\eta} q_{m-1} -$
$\mathcal{R}e\Pr\zeta * (f\partial_\eta q) +$
$\Pr\mathrm{Ec} *$
$(\partial_{\eta,\eta} f)^2 - \alpha_2(1-\zeta) *$
$[(\partial_\eta f * \partial_{\eta,\eta} f * \eta) +$
$(\partial_\eta f * \zeta * \partial_{\eta,\zeta} f)] -$
$\alpha\Pr\mathrm{Ec} * \left(\partial_\eta f (\partial_{\eta,\eta} f)^2\right.$
$\left. - f\partial_{\eta,\eta} f \partial_{\eta,\eta,\eta} f\right)$ (16)

Where $p \in [0,1]$ is an embedding parameter, $\hbar_1$ and $\hbar_2$ are the auxiliary non zero parameters. Obviously for p=0 and p=1,we have:

$$f(\eta,\zeta;0) = f_0(\eta),$$
$$f(\eta,\zeta;1) = f(\eta,\zeta) \quad (17)$$
$$q(\eta,\zeta;0) = q_0(\eta),$$
$$q(\eta,\zeta;1) = q(\eta,\zeta) \quad (18)$$

Now as p increases from 0 to 1 then $f(\eta,\zeta;p)$ varies from $f_0(\eta)$ to $f(\eta,\zeta)$, $q(\eta,\zeta;p)$ so does varies from $q_0(\eta)$ to $q(\eta,\zeta)$. Using Taylor's theorem and Eqs. (17) and (18) we can write

$$f(\eta,\zeta;p) = f(\eta) + \sum_{m=1}^{\infty} f_m(\eta,\zeta)p^m \quad (19)$$
$$q(\eta,\zeta;p) = q(\eta) + \sum_{m=1}^{\infty} q_m(\eta,\zeta)p^m \quad (20)$$

Where

$$f_m(\eta,\zeta) = \frac{1}{m!}\frac{\partial^m f(\eta,\zeta;p)}{\partial p^m}\bigg|_{p=0}$$

$$q_m(\eta,\zeta) = \frac{1}{m!}\frac{\partial^m q(\eta,\zeta;p)}{\partial p^m}\bigg|_{p=0}$$

the convergence of two series is strongly dependent upon $\hbar_1$. Assume that $\hbar_1$ and $\hbar_2$. Assume that $\hbar_1$ and $\hbar_2$ are chosen that these series are convergent at p=1, we have from Eqs.(19) and (20) that:

$$f(\eta,\zeta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta,\zeta) \quad (21)$$

$$q(\eta,\zeta) = q_0(\eta) + \sum_{m=1}^{\infty} q_m(\eta,\zeta) \quad (22)$$

Differentiating Eq.(11) and (13) m times with respect to p, then setting p=0, and finally dividing (11) and (13) by m!,we obtain the following mth-order deformation problems:

$$\mathcal{L}_1[f_m(\eta,\zeta) - \chi_m f_{m-1}(\eta,\zeta)] = \hbar_1 \mathcal{R}_{1m}[f_{m-1}(\eta,\zeta)] \quad (23)$$

$$f_m(0,\zeta)=\partial_{\eta,\eta} f_m(0,\zeta)=f_m\left(\frac{1}{2},\zeta\right)=\partial_\eta f_m\left(\frac{1}{2},\zeta\right)=0 \quad (24)$$

$$\mathcal{L}_2[q_m(\eta,\zeta) - \chi_m q_{m-1}(\eta,\zeta)] = \hbar_2 \mathcal{R}_{2m}[q_{m-1}(\eta,\zeta)] \quad (25)$$

$$q_m(0,\zeta)=q_m\left(\frac{1}{2},\zeta\right)=0 \quad (26)$$

$$\mathcal{R}_{1m}[f_{m-1}(\eta,\zeta)] = \mathcal{R}e(1-\zeta)*(1/2\,\eta\zeta*\partial_{\eta,\eta}f_{m-1}+\zeta^2*\partial_{\eta,\zeta}f_{m-1})+\mathcal{R}e\zeta^2*\left(\sum_{i=0}^{m-1}\partial_\eta f_{m-1-i}\,\partial_\eta f_i-\sum_{i=0}^{m-1}f_{m-1-i}\;\partial_{\eta,\eta}f_i\right)+\zeta*\partial_{\eta,\eta,\eta}f_{m-1}-\mathrm{M}^2\zeta^2*\partial_\eta f_{m-1}-\lambda\zeta^2*\partial_\eta f_{m-1}-1/2\,\alpha(1-\zeta)\eta*\partial_{\eta,\eta,\eta,\eta}f_{m-1}-\alpha\zeta*\left(2\sum_{i=0}^{m-1}\partial_\eta f_{m-1-i}\;\partial_{\eta,\eta,\eta}f_i-\sum_{i=0}^{m-1}f_{m-1-i}\;\partial_{\eta,\eta,\eta,\eta}f_{m-1-i}-\sum_{i=0}^{m-1}\partial_{\eta,\eta}f_{m-1-i}\partial_{\eta,\eta}f_i\right) \quad (27)$$

$$\mathcal{R}_{2m}\begin{bmatrix} q_{m-1}(\eta,\zeta) \\ f_{m-1}(\eta,\zeta)\end{bmatrix} = \alpha_1(1-\zeta)(1/2\eta*\partial_\eta q_{m-1}+\zeta*\partial_\zeta q_{m-1})+\partial_{\eta,\eta}q_{m-1}+\sum_{i=0}^{m}(-\mathcal{R}e\Pr\zeta*f_{m-1-i}\partial_\eta q_i+\Pr\mathrm{Ec}*\partial_{\eta,\eta}f_{m-1-i}*\partial_{\eta,\eta}f_i-\alpha_2(1-\zeta)*\lfloor\partial_\eta f_{m-1-i}\partial_{\eta,\eta}f_i*\eta+\zeta*\partial_\eta f_{m-1-i}\partial_{\eta,\zeta}f_i\rfloor-\alpha\Pr\mathrm{Ec}*\sum_{k=0}^{i}(\partial_\eta f_{m-1-i}\,\partial_{\eta,\eta}f_{i-k}\;\partial_{\eta,\eta}f_k-f_{m-1-i}\;\partial_{\eta,\eta}f_{i-k}\;\partial_{\eta,\eta,\eta}f_k)) \quad (28)$$

Where χ_m is defined by $\begin{cases}0, & m\le 1\\ 1, & m>1\end{cases}$

Upon making use of MATHEMATICA, the solution of Eqs.(23) and (25) can be expressed in the form:

$$f_m(\eta,\zeta) = \sum_{k=0}^{3m+5}\sum_{i=0}^{2m} a_{m,i}^k\;\eta^k\zeta^i \quad (29)$$

$$q_m(\eta,\zeta) = \sum_{k=0}^{5m+1}\sum_{i=0}^{2m} e_{m,i}^k\;\eta^k\zeta^i \quad (30)$$

Where $a_{m,i}^k$ and $e_{m,i}^k$ is a coefficient for $m \ge 1$

We obtain in fact the following explicit, totally analytic solution of the momentum and energy eqs.

$$f(\eta,\zeta) = \sum_{m=0}^{\infty} f_m(\eta) = \lim_{M\to\infty}\sum_{m=0}^{M}\left(\sum_{k=0}^{3m+5}\sum_{i=0}^{2m} a_{m,i}^k\;\eta^k\zeta^i\right) \quad (31)$$

$$q(\eta,\zeta) = \sum_{m=0}^{\infty} q_m(\eta) = \lim_{M\to\infty}\sum_{m=0}^{M}\left(\sum_{k=0}^{5m+1}\sum_{i=0}^{2m} e_{m,i}^k\;\eta^k\zeta^i\right) \quad (32)$$

4. Convergence of The HAM Solutions

As pointed out by Liao [7], the convergence region and rate of approximations given by homotopy analysis method are strongly dependent upon $\hbar$. Figure 1,2 portray the $\hbar$-curves of the velocity and temperature profiles respectively. The range for admissible values of $\hbar$ for the velocity is $-10 \le \hbar_1 \le 0$ and for temperature it is $-5 \le \hbar_2 \le 5$. We see that the series given by Eqs. (31) and (32) converges in the whole region of η when $\hbar = -0.3$. This value of $\hbar$ lie in the admissible range of $\hbar$.

5. Results and Discussion

Figures 3–16 have plotted in order to see the effects of $\mathcal{R}e$, M, Pr, Ec, α, α_1, α_2, λ, and ζ on the velocity components f and temperature components q. Fiqs. 3-7 are sketched in order to see the effects of $\mathcal{R}e$, M, α, λ and ζ on the velocity component f. Figure 3, give the effect of Reynolds number $\mathcal{R}e$ on the velocity component f. It is found that f decreases when $\mathcal{R}e$ increases. In Figure 4, it is found that f increases when M increases. Figure 5 depict the effect of α on f. It is found that f decreases as α increases. In Figure 6, it is found that f increases when λ increases. Figure 7, depict the effect of ζ on f. It is found that f initially decreases but it increases as ζ increases

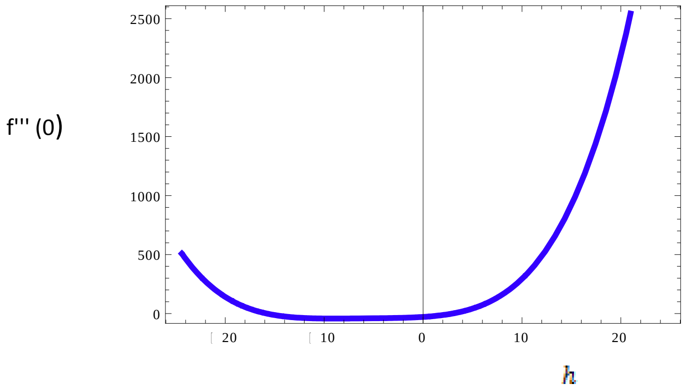

Figure 1- $\hbar_1$ - curve for velocity at fourth-order approximation

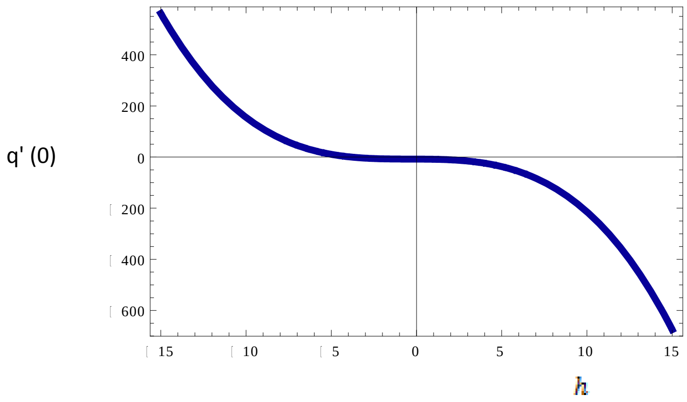

Figure. 2- $\hbar_2$ - curve for temperature at fourth-order approximation

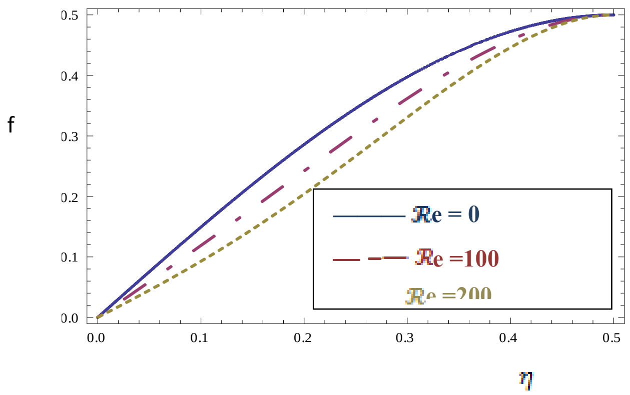

Figure 3- Effect of $\mathcal{R}e$ on fourth approximation for
M = 1, α = 0.2, λ = 0.2, ζ= π/4 , $\hbar$ = -0.3

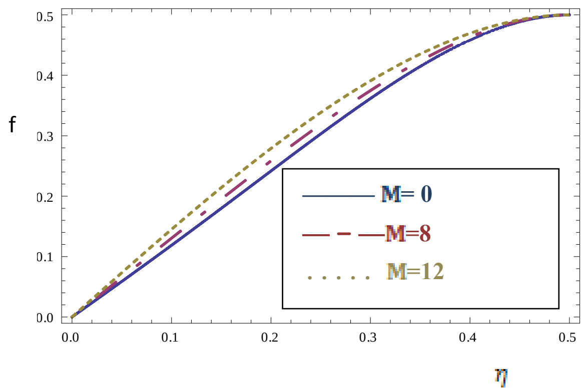

Figure 4- Effect of M on fourth approximation for
$\mathcal{R}e$ = 100, α = 0.2,λ = 0.2, ζ= π/4,$\hbar$ = -0.3

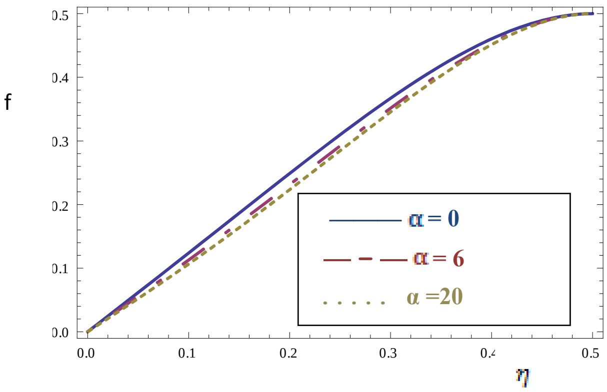

Figure 5- Effect of α on fourth approximation for $\mathcal{R}e = 100$, M=1, $\lambda = 0.2$, $\zeta = \pi/4$, $h = -0.3$

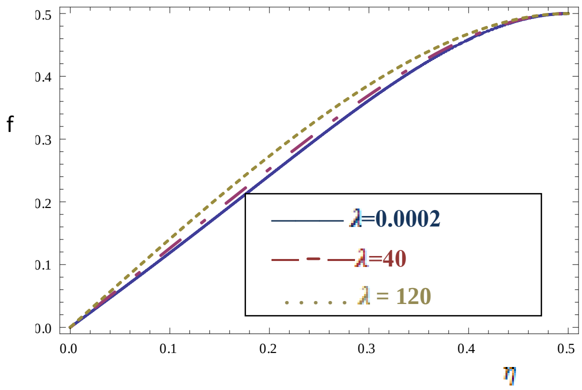

Figure 6- Effect of λ on fourth approximation for $\mathcal{R}e = 100$, M=1,$\alpha = 0.2$, $\zeta = \pi/4$, $h = -0.3$

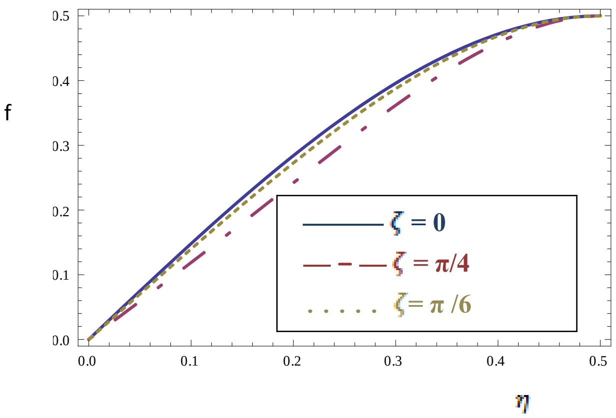

Figure 7- Effect of ζ on fourth approximation for $\mathcal{R}e = 100$, M=1, $\alpha = 0.2$, $\lambda = 0.2$, $h = -0.3$

Fiqures 8-16 are sketched in order to see the effects of $\mathcal{R}e$, M, Pr, Ec, α, α_1 , α_2, λ , and ζ on the temperature component q. Figure 8, give the effect of Reynolds number $\mathcal{R}e$ on the temperature component q. It is found that q increases when $\mathcal{R}e$ increases. In Figure 9, it is found that q increases when M increases. Figures 10,11,12 have the same effect of Ec, Pr and α on q when compared with Figure.9. In Figure 13, It is found that q is constant when α_1 increases. Figures 14,15 have the same effect of $\alpha_2$and λ on q when compared with Figure 9. Figure 16, depict the effect of ζon q. It is found that q initially increases but it decreases as ζ increases.

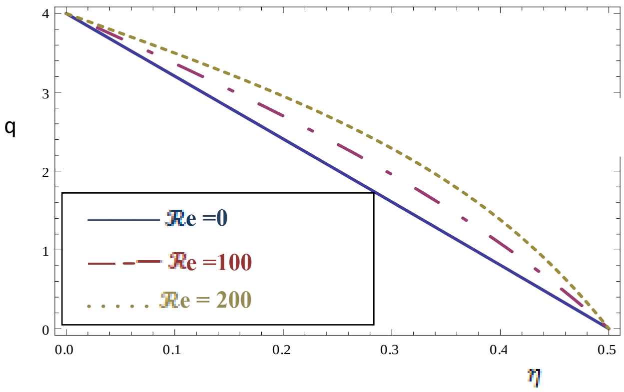

Figure 8- Effect of $\mathcal{R}e$ on fourth approximation for M = 1, Ec = 0.3, Pr = 0.3, $\alpha = 0.2$, $\alpha_1 = 0.4$, $\alpha_2 = 0.4$, $\lambda = 0.2$, $\zeta = \pi/4$, $h = -0.3$

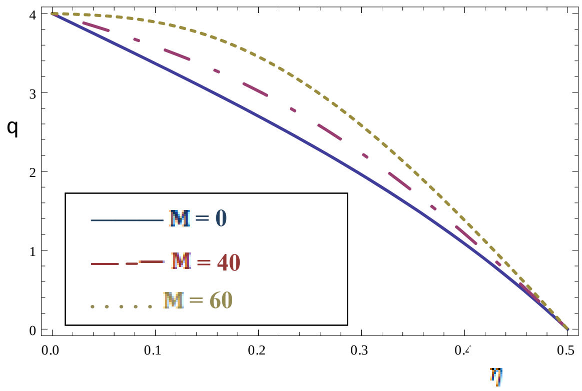

Figure 9 -Effect of M on fourth approximation for $\mathcal{R}e = 100$, Ec = 0.3, Pr = 0.3, $\alpha = 0.2$,$\alpha_1 =$ 0.4,$\alpha_2 = 0.4$, $\lambda = 0.2$, $\zeta = \pi/4$, $h = -0.3$

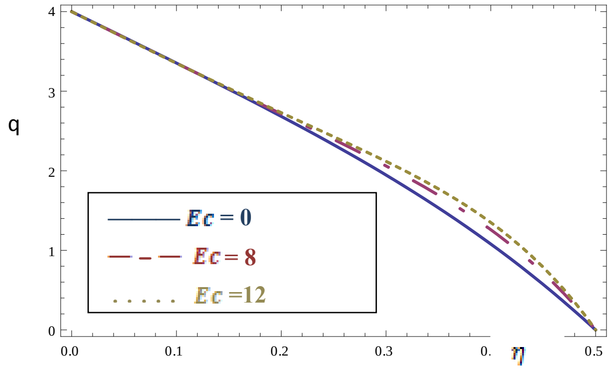

Figure 10- Effect of Ec on fourth approximation for Re= 100, M = 1, Pr = 0.3, α = 0.2,α_1= 0.4, α_2 = 0.4, λ = 0.2, ζ= π/4 , h = -0.3

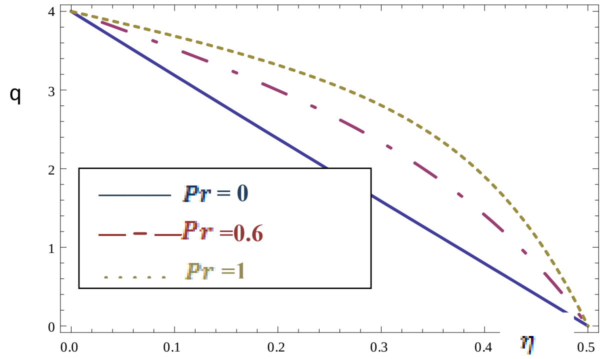

Figure 11- Effect of Pr on fourth approximation for Re = 100, M = 1, Ec = 0.3, α = 0.2,α_1= 0.4, α_2= 0.4, λ = 0.2, ζ= π/4 , h = -0.3

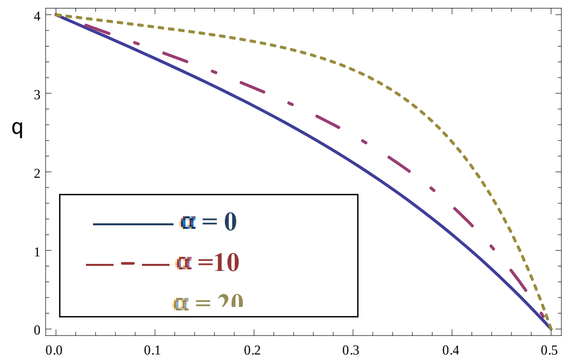

Figure 12- Effect of α on fourth pproxima for Re = 100, M=1, Ec = 0.3, Pr =0.3 α_1= 0.4, α_2 = 0.4, λ = 0.2, ζ= π/4,h = -0.3

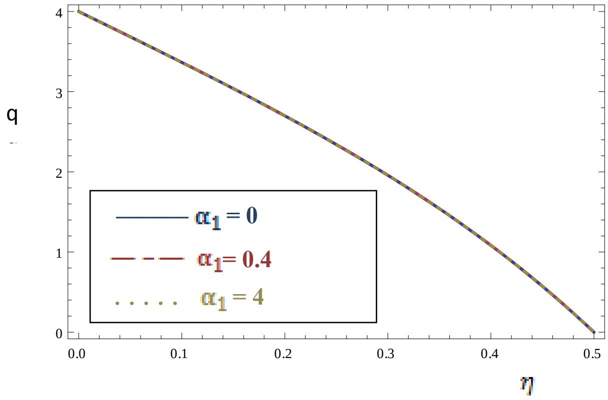

Figure 13- Effect of $\alpha_1$on fourth approximation for Re = 100, M = 1, Ec = 0.3,Pr = .3,α = 0.2, α_2= 0.4, λ = 0.2, ζ= π/4 , h = -0.3

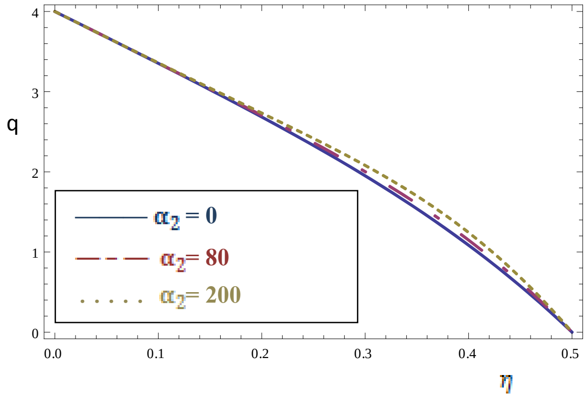

Figure 14- Effect of $\alpha_1$on fourth approximation for Re = 100, M = 1, Ec = 0.3,Pr =0.3 , α_2 = 0.4, λ = 0.2, ζ= π/4,h = -0.3

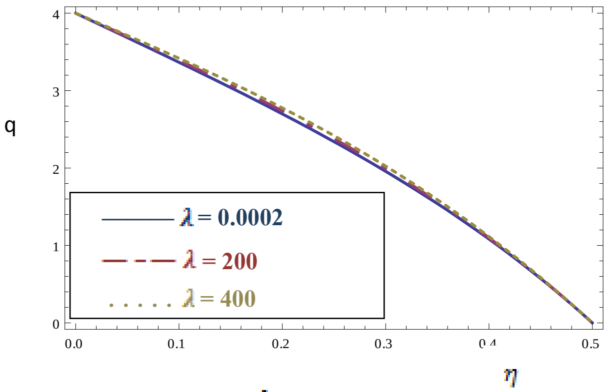

Figure 15- Effect of λ on fourth approximatio Re= 100, M = 1, Ec = 0.3,Pr=0.3, α_1 = 0.4, α_2 = 0.4, ζ= π/4,h = -0.3

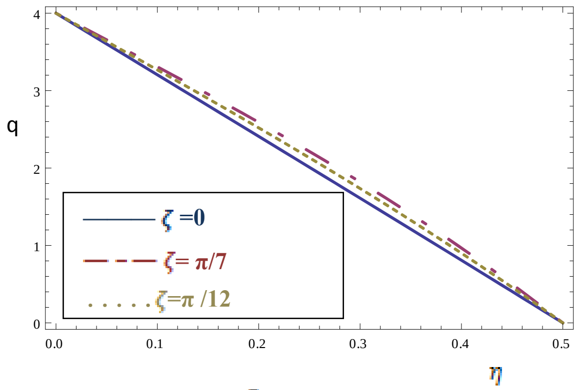

Figure16- Effect of ζ on fourth approximation for Re = 100, M =1, Ec = 0.3,Pr =0.3 $\alpha = 0.2$, α_4=0.4, α_2= 0.4, λ = 0.2,h=0.3

6. Concluding Remarks

In this article, the unsteady heat transfer is analyzed for magnetohydrodynamic (MHD) flow of a second grade fluid in a channel. The governing non-linear are solved by using HAM. The effect of each physical parameter upon the velocity and temperature distributions are analyzed and are shown graphically. The results have been summarized as the following:

I. The variation of $\mathcal{R}e$ on velocity and temperature distributions is opposite.

II. The effects of M and λ on velocity and temperature distributions are similar.

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