The Single Particle Level Density Calculations for $^{232}_{90}Th$ Using Equidistant Space Model (ESM) and NON-ESM in Fermi Gas Model

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Abstract

The single-particle level densities for $^{232}_{90}Th$, at certain exciton number, are calculated in terms of Equidistant Space Model, ESM, and NON-ESM, of Fermi Gas Model. It is found that the single particle level density, g, has no longer a constant value and becomes an energy dependent on the contrary with NON-ESM. The finite depth of the nuclear well and pairing corrections are examined with behavior of the single level density for both models. The particle-hole state density has been calculated, by means of the energy dependence of excited particles and hole level densities, for one and two fermions systems and different exciton number in $^{232}_{90}Th$. The present results are compared between two models with and without the inclusion of the finite well depth correction. NON-ESM system has a major effect in the present calculations at high excitation energies.

Keywords: **S**ingle-particle level density, $^{232}_{90}Th$, Fermi Gas Model, ESM, NON-ESM, Finite depth well correction, Paring correction.

حسابات كثافة مستوي الجسيم المنفرد في $^{232}_{90}Th$ بأستخدام انموذج المسافات المتساوية وغير المتساوية في انموذج فيرمي الغازي

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الخلاصة

حسبت كثافات مستوي الجسيم المنفرد في $^{232}_{90}Th$ في عدد معين من الاكسايتونات (اكسايتون=جسيمة–فجوة) بدلالة انموذج المسافات المتساوية وغير المتساوية في انموذج فيرمي الغازي. وجد بان كثافة مستوي الجسيم المنفرد g لاتملك قيمة ثابتة ولكن تعتمد على الطاقة في حالة انموذج المستويات غير المتساوية. اختبرت تصحيحات العمق المحدد للبئرالنووي وتصحيح الازدواج مع سلوك كثافة المستوي المنفرد لكلا الانموذجين. تم حساب كثافة مستوي الجسيمة–الفجوة باستخدام اعتماد طاقة التهيج للجسيمات والفجوات لنظام ونظامين ولاعداد مختلفة من الاكسايتونات في $^{232}_{90}Th$. قورنت النتائج الحالية بين انموذجين، مع وبدون ادخال تصحيح عمق البئر المحدد. انموذج المستويات غير المتساوية له تأثير كبير في الحسابات الحالية عند الطاقات العالية

Introduction

The single particle level density g is an important quantity that needed in calculating the state density ω. The first model in calculating the state density is the exciton model which was formulated by Griffin [1]. The Griffin model was developed by several authors by adding multiple corrections to Griffin state density formula [2-5]. Some of these models suggested that the single-particle levels are equally spaced in energy [6] and the excitation energy will distributed among the particles above the Fermi energy and the holes below it, this model is called the equi-spacing model ESM. The models which do not distinguish between protons and neutrons particles are called the one component model [6], which divided the excions into particles and holes only. The real nucleus does not composed of one type of particles and holes but it consists of protons and neutrons particles and holes, the model which is deal with the nucleus as a system of protons and neutrons is called the two component model [7-9]. In 1985 Kalbach [10] made a large deviation from the equispacing model (ESM), she started from the energy dependent single particle level density and assumed that the level spacing are not equally separated but these spacing are change with the single particle excitation energy NON-ESM. What we have mentioned previously illustrates the importance of the single particle level density in calculating the state density, for this we will study the behavior of this parameter in equal spacing ESM and NON ESM Fermi gas model in details, and examine its effect on the behavior of the state density.

The Energy Independent Single Particle Level Density for One and Two Component Systems

Generally, the single particle level density can be expressed as [11, 12]

$$g = \omega(1,0,E_p) = \omega(0,1,E_h) \qquad (1)$$

In ESM of Fermi gas model g=3A/2F.

Where E_p, E_h are the single particle and single hole energy respectively, A is the mass number and F is the Fermi energy which was assumed to be 38 MeV..

In most practical calculations g was assumed to be g=A/13 or g=A/15 [4].

the formulae given above are refer to one fermions system, in case of two fermions system the value of g will be changed to refer to protons and neutrons and it given as [4-11]

$g_\pi = (Z/A)g$ or $Z/13$, $g_\nu = (N/A)\,g$ or N/13 or $g_\pi = g_\nu = g/2$

The Energy Dependent Single Particle Level Density

In order to take into account the energy dependent single particle level density, the average excitation energy for particles and holes are proposed to find the single particle level density in terms of the excitation energies for particles and holes [10]

$$g_p(u_p) = g\left(\frac{F+u_p}{F}\right)^{1/2} = g\ {}_2F_1\left(-\frac{1}{2},\beta;\beta;-\left(\frac{u_p}{F}\right)\right) \quad (2)$$

$$g_h(u_h) = g\left(\frac{F-u_h}{F}\right)^{1/2} = g\ {}_2F_1\left(-\frac{1}{2},\beta;\beta;\left(\frac{u_h}{F}\right)\right) \quad (3)$$

where ${}_2F_1$ is the hyper geometrical function.

In case of infinite potential well (or if excitation energy less than Fermi energy F) the single particle and hole excitation energy can be given as $u_p = u_h = E/n$, where n is the Exciton number. The u_p *and* u_h can be defined in terms of the ratio for potential well correction function (f_k^+ / f_k), which can identify the effective potential well for different Exciton configurations:

$$u_p = \frac{E}{n}\frac{f_k^+(p,h,E,F)}{f_k(p,h,E,F)} \qquad (4)$$

$$u_h = \frac{E - pu_p}{h} \qquad (5)$$

Where

$$f_k(p,h,E,F) = \sum_{i=0}^{p} (-1)^{i+j} C_p^i \left(\frac{E - A_k(p,h) - jF}{E}\right)^{n-1} \times \Theta(E - E_{thresh} - jF) \quad (6)$$

$$f_k^+(p,h,E,F) = \sum_{i=0}^{p} (-1)^{i+j} C_p^i \left(\frac{E - A_k(p,h) - jF}{E}\right)^{n} \times \Theta(E - E_{thresh} - jF) \quad (7)$$

With $C_p^i = \binom{i}{p}$ and A_k is the paring correction

$$A_k(p,h) = E_{thresh}(p,h) - \frac{p(p+1)+h(h+1)}{4g} + \frac{(p-1)^2+(h-1)^2}{gF(p,h)} \qquad (8)$$

The ESM state density equation with energy independent single particle level density for one and two component systems are given respectively by [13, 14]

$$\omega_1(n,E,A_k) = \frac{g^n(E-A_k(p,h))^{n-1}}{p!h!(n-1)!} \qquad (9)$$

$$\omega_2(n,E,A_k) = \frac{g_\pi^{n_\pi} g_\nu^{n_\nu}(E-A_k(p_\pi,h_\pi,p_\nu,h_\nu))^{n-1}}{p_\pi!h_\pi!p_\nu!h_\nu!(n-1)!} \qquad (10)$$

Entering the NON-ESM system the state density for one and two component systems becomes:

$$\omega_1(n,E,A_k) = \frac{[g_p(\varepsilon)]^p[g_h(\varepsilon)]^h[E-A_k(p,h)]^{n-1}}{p!h!(n-1)!} \times \sum_{i=0}^{h}(-1)^i\binom{h}{i}\left(\frac{E-A_k(p,h)-iF}{E}\right)^{n-1}\Theta(E-A_k(p,h)-iF) \qquad (11)$$

$$\omega_2(n,E,A_k) = \frac{[g_p(\varepsilon)]^{p_\pi}[g_p(\varepsilon)]^{p_\nu}[g_h(\varepsilon)]^{h_\pi}[g_h(\varepsilon)]^{h_\nu}}{p_\pi!h_\pi!p_\nu!h_\nu!(n-1)!} \times[E-A_k(p_\pi,h_\pi,p_\nu,h_\nu)]^{n-1}\times\sum_{i=0}^{h}(-1)^i\binom{h}{i}\left(\frac{E-A_k(p_\pi,h_\pi,p_\nu,h_\nu)-iF}{E}\right)^{n-1}\times\Theta(E-A_k(p_\pi,h_\pi,p_\nu,h_\nu)-iF) \qquad (12)$$

Results, Discussion and Conclusions

In the ESM model the single particle level density, g, is constant and does not depends on the excitation energy for particles and holes. It gives a constant value at all values of excitation energy and it just changed according to the mass number of the nuclei. By introducing the NON-ESM model g is no longer constant and becomes energy dependent as shown in Figure 1. One can see the energy variation of the Fermi gas single particle and single hole level density for ${}^{232}_{90}Th$, for exciton number n=3 and without the finite well depth correction. The importance of the finite depth of the nuclear potential well and the paring correction on the behavior of the single particle level density is shown in Figure2, where the value of the ESM single particle level density is seen as a constant line in all excitation energy ranges, while for NON-ESM model the value of g_p will be always at values higher than the value of the ESM single particle level density, g . Where g_h is always lower than g and it reached saturation at energies higher than 70 MeV.

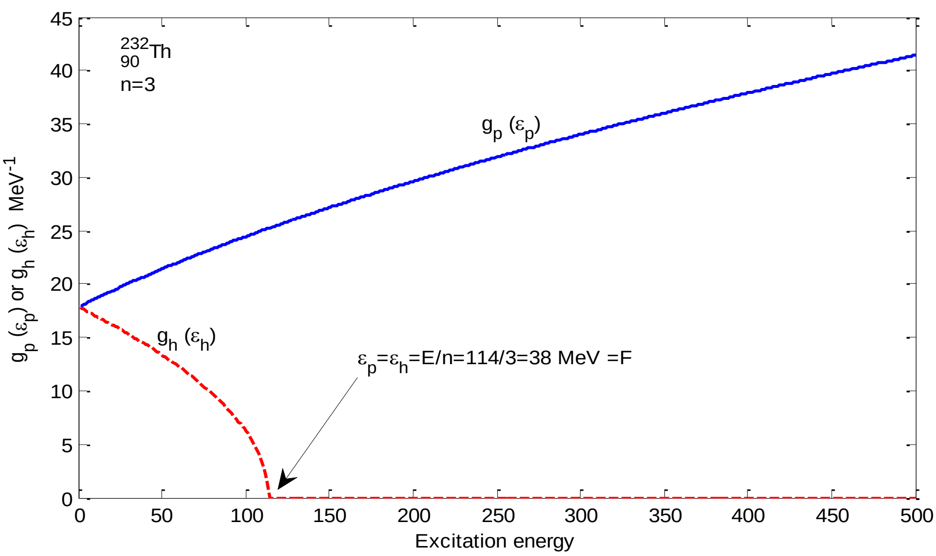

Figure 1- The single particle (or hole) level density in ${}^{232}_{90}Th$ as a function of single particle (or hole) excitation energy in case of infinite potential well using equations (2) and (3).

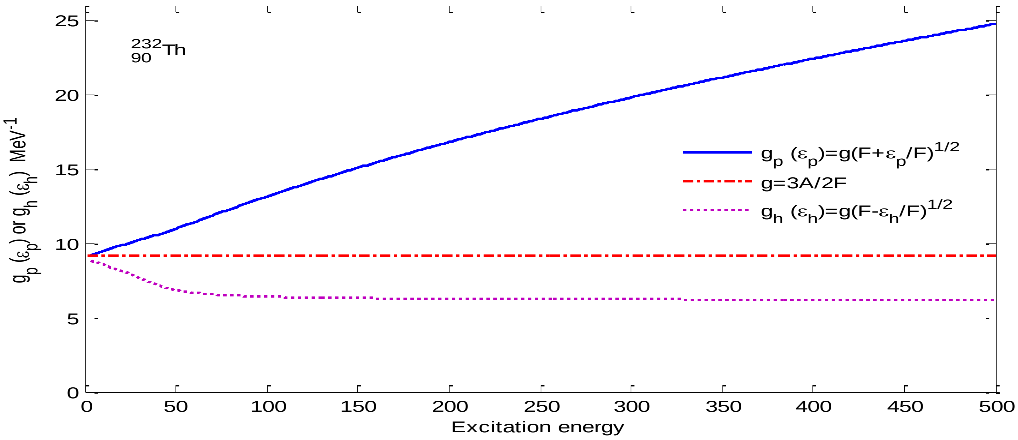

Figure 2- The energy dependent single particle level density in $^{232}_{90}Th$, where the calculations using the finite depth and the paring corrections. The dotted dashed line gives the value of g =3A/2F MeV^{-1}only.

The particle-hole state density has been obtained by means of the energy dependence of excited particles and holes level densities for one and two-component systems and different exciton number in ^{232}Th. The results are compared with the results of the state density for ESM system, with and without the inclusion of the finite well depth correction. In case of infinite potential well and for n=2, the state densities for one and two fermions systems reached a certain value of excitation energy, then it will droped down to zero, as seen in Figures 3,4. This behavior is due to the fact that the single particle and hole excitation energy reached to 38 MeV, which is the Fermi energy. So it is preferred to include the finite well depth correction which gives good results, especially at high excitation energy. The inclusion of the energy-dependent level density, which treats the system as particles with excitation energies higher than the Fermi energy and holes with excitation energies, does not exceed the Fermi energy. It is concluded that the results are obtained from NON-ESM is much better than those for ESM system, where the single hole energy does not reach zero at a certain value of excitation energy. Figures 5,6 illustrate a comparison between the results of the state density calculated for $^{232}_{90}Th$ and $^{58}_{26}Fe$ and for one and two-component systems; in these figures, one can see the effect of the mass number on the value of the state density and the number of states for ESM and NON-ESM systems, which increased as the mass number increases

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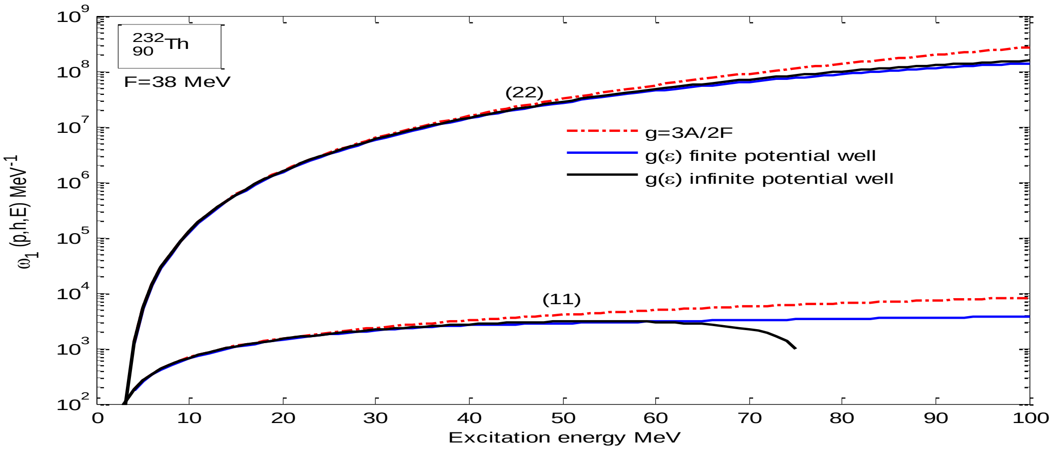

Figure 3- The state density for one fermion system as a function of excitation energy for1p1h and 2p2h configurations in $^{232}_{90}Th$. The solid curves give the state density for NON-ESM system while the dashed dotted curve is for ESM system.

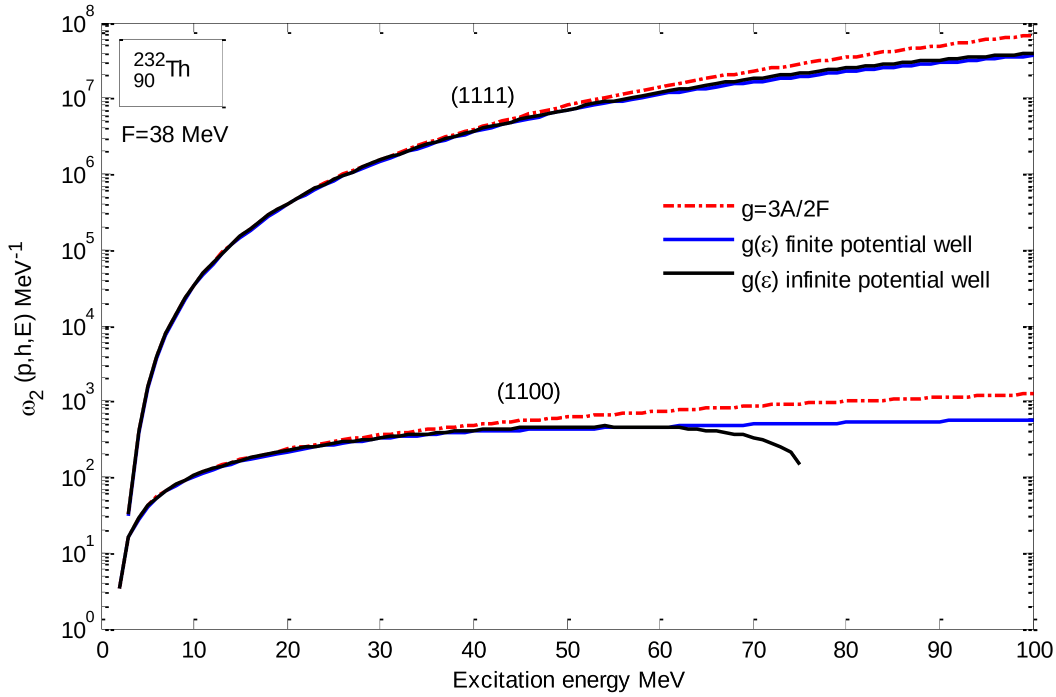

Figure 4- The state density for two fermions systems as a function of excitation energy for 1100 and 1111 configurations in ${}^{232}_{90}Th$. The solid curves give the state density for NON-ESM system while the dashed dotted curve is for ESM system.

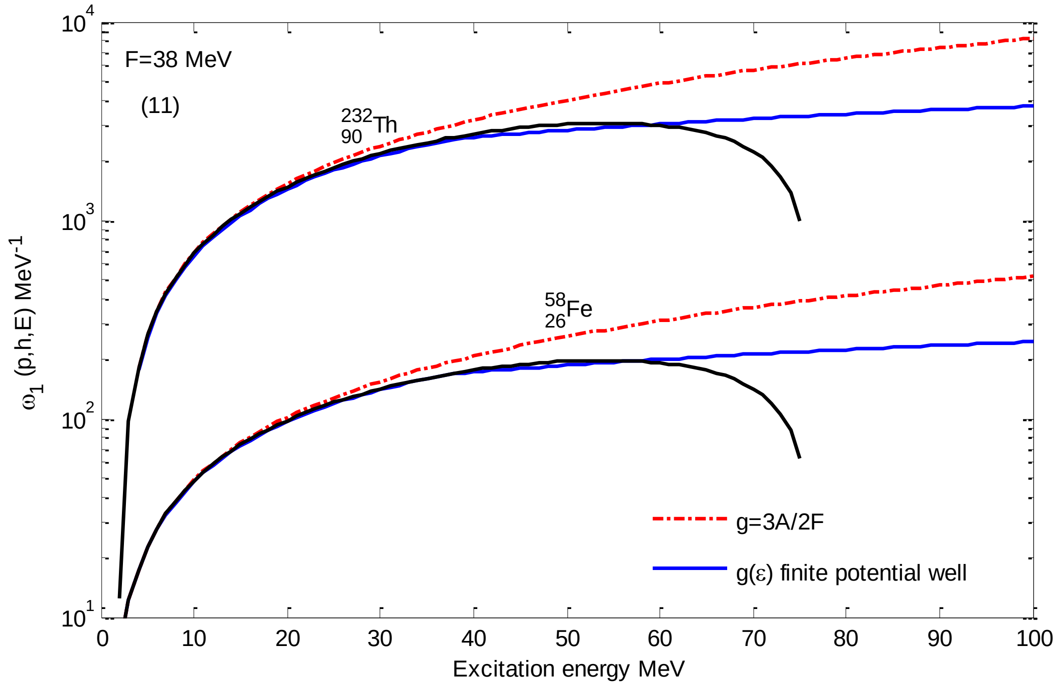

Figure 5- A comparison between the state densities for one fermion system as a function of excitation energy for 1p1h configuration in ${}^{232}_{90}Th$ and ${}^{58}_{26}Fe$. The solid curves give the state density for NON-ESM system while the dashed dotted curve is for ESM system.

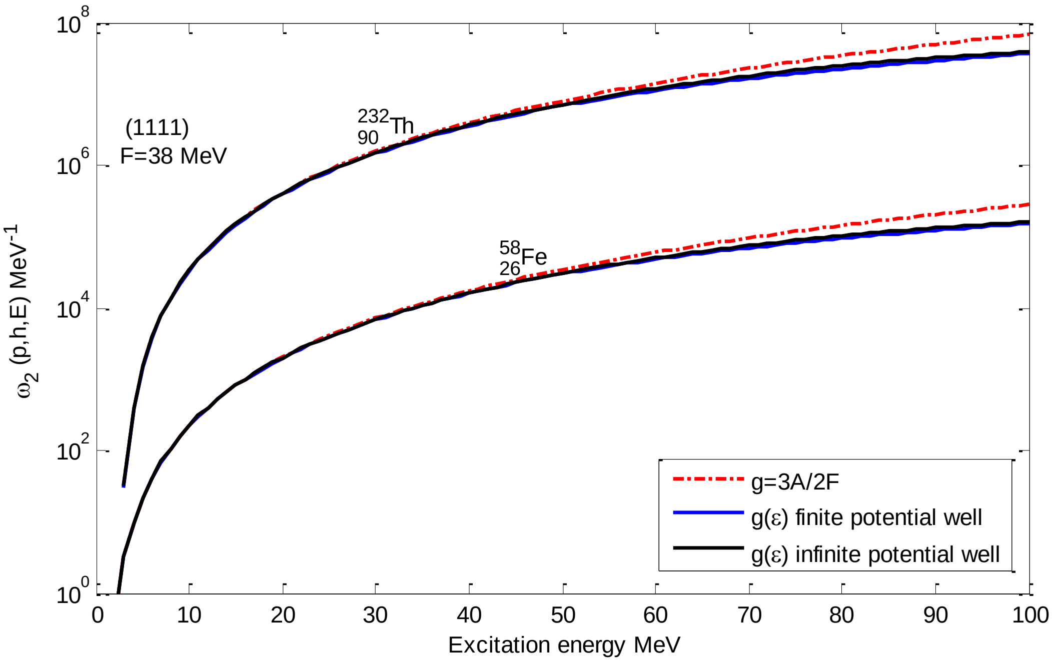

Figure 6- A comparison between the state densities for two fermion system as a function of excitation energy in $^{232}_{90}Th$ and $^{58}_{26}Fe$. The solid curves give the state density for NON-ESM system while the dashed dotted curve is for ESM system.

References

1. Griffin,J.J., **1966,** Phys.Rev.Lett.17, pp . 478.
2. Koning,A.J. and**,** Duivestijn M. C., **2004** , Nucl. Phys. A **744** :pp. 15-76.
3. Kalbach, C.,**2005**,Phys.Rev. C71, : pp . 034606.
4. Kalbach, C. **2006**, Phys. Rev. C73:pp. 024614.
5. Mocelj D., Rauscher T., Mart´ınez-Pinedo G. , Langanke K. , Pacearescu L., Faessler A., Thielemann F.-K., and Alhassid Y. ,**2007**,Phys. Rev. C75:pp. 045805.
6. Ericson T. , **1960**, Adv. Phys. **9**: pp.425.
7. Nuclear research and consultancy group, **2004**,westerduinweg,**3**,NL-1755, Petten, Netherlands,
8. Duijvestijn M. C. and Koning A. J., **2004**, AIP Conf. Proc. 769: pp. 1150-1153.
9. Fu C. Y., Nucl, **1984**, Sci. Eng. **86** :pp. 344.
10. Kalbach C.,**1985**, Phys. Rev. C32, 1157.
11. Běták E. and Hodgson P. E., **1998,** particle hole state density in pre-equilibrium nuclear reactions" university of oxford, report reference OUNP-98-02.
12. Kalbach C., **1986**, Phys. Rev. C33, 818.
13. Kalbach C.Nucl, **1987,** Sci. Eng . 95 : pp .70.
14. Harangozo A., Şteţcu I., Avrigeanu M., and Avrigeanu V., **1998**, Phys. Rev.C58: pp.295.