



Comparison of Pressure Losses Calculations by Approximate Methods in the Annulus (Laminar Flow)

Faleh H. M. Almahdawi*

Department of Petroleum, College of Engineering, University of Baghdad, Baghdad, Iraq.

Abstract

Most of the world's drilling companies use API method for calculating the pressure drop in drilling oil and gas wells. Iraqi Drilling Company also uses this method for drilling oil wells in all fields of Iraq. In Russia, the method is actively used Grodde for calculating the pressure drop in drilling oil and gas wells. This study includes a comparison of formulas API, Grodde and new techniques developed methods for the laminar flow of viscoplastic fluids in the annular space. The aim of this study is to determine the most accurate way to calculate the pressure loss in drilling oil wells. The comparison was focused on identifying changes in the values of β , using the above calculation methods.

Keywords: Drilling oil well, laminar flow, pressure drop calculations, annular space, petroleum engineering.

مقارنة حسابات فقدان الضغط بالطرق التقريبية في الفراغ الحلقي (الجريان الصفائحي)

فالح حسن محمد المهداوي*

قسم هندسة النفط، كلية الهندسة، جامعة بغداد، بغداد، العراق.

الخلاصة:

معظم شركات الحفر في العالم تستخدم الأسلوب API لحساب انخفاض الضغط في حفر آبار النفط والغاز. شركة الحفر العراقية كذلك تستخدم هذا الأسلوب لحفر آبار النفط في جميع حقول العراق. في روسيا، يستخدم أسلوب Grodde لحساب انخفاض الضغط في مجال حفر آبار النفط و الغاز. وتشمل هذه الدراسة المقارنة بين الطرق (API، Grodde) والتقنية الجديدة التي طورت لحساب هبوط الضغط لتدفق السوائل الصفائحي laminar flow of viscoplastic fluid في الفراغ الحلقي الهدف هو تحديد الطريقة الأكثر دقة لحساب فقدان الضغط في حفر آبار النفط. وتركزت المقارنة على تحديد التغيرات في قيم β ، وذلك باستخدام طرق الحساب أعلاه.

1.Introduction

Flow in concentric annular channels:

Find the relation between pressure drop $\Delta p = p_2 - p_1$ and consumption $Q = v F_k$, where v - average velocity in the annulus cross-sectional area $F_k = \pi(d_c^2 - d_H^2)/4$. When flows viscoplastic fluids in concentric annular space, formed the core, which has a hollow cylindrical shape with a cross section of $\pi(b^2 - a^2)$, the lateral surface of $2\pi(a + b)L$, and moves at a speed ω_0 Figure-1. This core is divides all flow in two gradient layers: I, where the derivative $\partial \omega / \partial r < 0$, and

*Email: Fhmetr@yahoo.com

II, where the derivative $\partial \omega / \partial r > 0$. In this connection it should solve the Eq.(1-4) and finding the velocity profile of ω for each layer separately, as the rheological equation (4) has the same form for each layer:

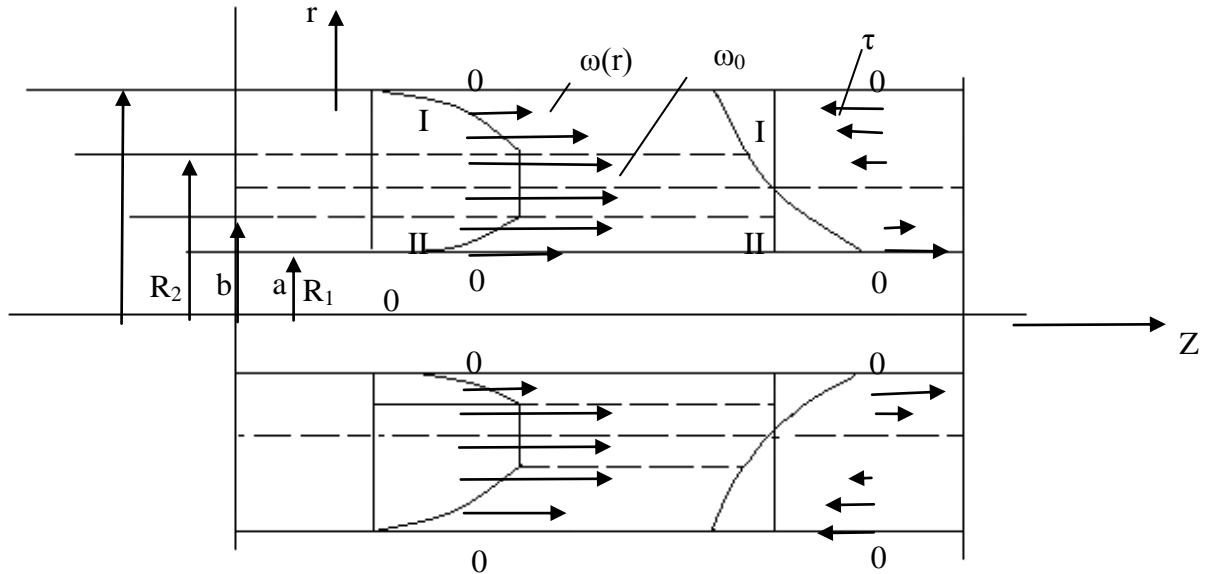


Figure.1- The distribution pattern of velocities and stresses(laminar flow) in an annular channel viscoplastic fluids.

Steady fluid flow in the elements of the circulation system of wells

1. Homogeneous steady flows equation of incompressible fluid.

Flow in annuli:equation of motion

$$\frac{\partial p}{\partial z} = -\frac{1}{r} \frac{\partial r \tau}{\partial r}, \quad (1)$$

The equation of mass conservation

$$\frac{\partial \omega}{\partial z} = 0 \quad (2)$$

equation of state

$$\rho = \text{const.} \quad (3)$$

The rheological equation for viscoplastic fluids in tubes with $\partial \omega / \partial r < 0$ has the form:

for layer I

$$\tau = -\tau_0 + \eta \frac{\partial \omega}{\partial r}$$

for layer II

$$\tau = \tau_0 + \eta \frac{\partial \omega}{\partial r}, \quad (4)$$

The condition of forces equilibrium acting on the core is described as follows:

$$\pi(b^2 - a^2)\Delta p = 2\pi\tau_0(a+b)L. \quad (5)$$

The boundary conditions in the absence of slip along the walls of the annular space have the form

$$\begin{aligned} \omega &= 0 & \text{at} & & r &= R_1 \\ \omega &= 0 & \text{at} & & r &= R_2. \end{aligned} \quad (6)$$

Since the core is moving with constant velocity ω_0 , the values of the velocity distributions in the gradient layers at the boundaries of the core will be

$$\omega = \omega_0 \quad \text{at} \quad a \leq r \leq b, \quad (7)$$

In addition, the condition

$$\frac{\partial \omega}{\partial r} = 0 \quad \text{at} \quad r = a \quad \text{and} \quad r = b. \quad (8)$$

Integration gives eq.(9)

Flow in the annular space

$$q = \frac{\pi R_2^4 \Delta p}{8\eta L} \left[1 - \delta^4 - 2 \frac{b}{R_2} \left(\frac{b}{R_2} - \frac{2\tau_0 L}{\Delta p R_2} \right) (1 - \delta^2) - \frac{4}{3} (1 + \delta^3) \frac{2\tau_0 L}{R_2 \Delta p} + \frac{1}{3} \left(2 \frac{b}{R_2} - \frac{2\tau_0 L}{\Delta p R_2} \right)^3 \frac{2\tau_0 L}{R_2 \Delta p} \right], \quad (9)$$

where b is the Volarovich-Gutkin equation [1-7].:

$$b \left(-\tau_0 + b \frac{\Delta p}{L} \right) \ln \frac{R_1 b}{R_2 \left(b - \frac{2\tau_0 L}{\Delta p} \right)} - \tau_0 (R_2 - R_1) + \frac{\tau_0}{2} \left(2b - \frac{2\tau_0 L}{\Delta p} \right) - \frac{1}{4} \frac{\Delta p}{L} (-R_2^2 + R_1^2) = 0.$$

These two equations are transformed to the form

$$Se = \frac{8\beta}{\frac{1 + \delta^2}{(1 - \delta)^2} - \frac{2\xi}{1 - \delta} \left(\frac{\xi}{1 - \delta} - \beta \right) - \frac{4}{3} \frac{1 - \delta + \delta^2}{(1 - \delta)^2} \beta + \frac{1}{3} \left(\frac{2\xi}{1 - \delta} - \beta \right)^3 \beta \frac{1 - \delta}{1 + \delta}}, \quad (10)$$

$$\xi \beta (1 - \delta) + \xi (\xi - \beta) (1 - \delta) \ln \frac{\xi \beta}{\xi - \beta (1 - \delta)} + \frac{1 - 2\beta(1 - \delta) - (\beta(1 - \delta) + \delta)^2}{2} = 0, \quad (11)$$

where $\delta = 2R_1/2R_2 = d_H/d_c$ -ratio of the external diameter of the inner tube to the inner diameter of the outer tube; $\xi = 2b/d_c$ - the ratio of twice the distance, measured from the axis of the tube to the outer boundary of the core annular flow, the inner diameter of the outer tube, and $\delta + \beta (1 - \delta) \leq \xi \leq 1$; Se - setting the Saint-Venant;

$$Se = \frac{\tau_0 (d_c - d_H) F_k}{\eta q}, \quad (12)$$

$$\beta = \frac{4\tau_0 L}{(d_c - d_H) \Delta p}, \quad (13)$$

By Eq.(10) with Eq. (11) we can construct a graph of $Se = f(\beta, \delta)$.

Thus, using Eq. (13) can determine the pressure in the annulus[1-2].

2.Methods Description

2.1 - Grodde graphic-analytical method:

By Eq. (13), we determine the pressure drop Δp (friction loss) when moving viscoplastic fluids in the annular space. For this, by Eq. (12) it allows to calculate the number of (Se), and using its value in figure-2 to determine the value of(β)[6, 8-9]. Choosing a value of (Se) and to plan this value to the given figure-2 curve, we can get the appropriate value (β) on the vertical axis, and then by Eq. (13) the desired pressure drop Δp . The errors caused by inaccurate techniques Grodde method to determine the projection of the curve on the vertical and horizontal axis because they are determined by visual means.

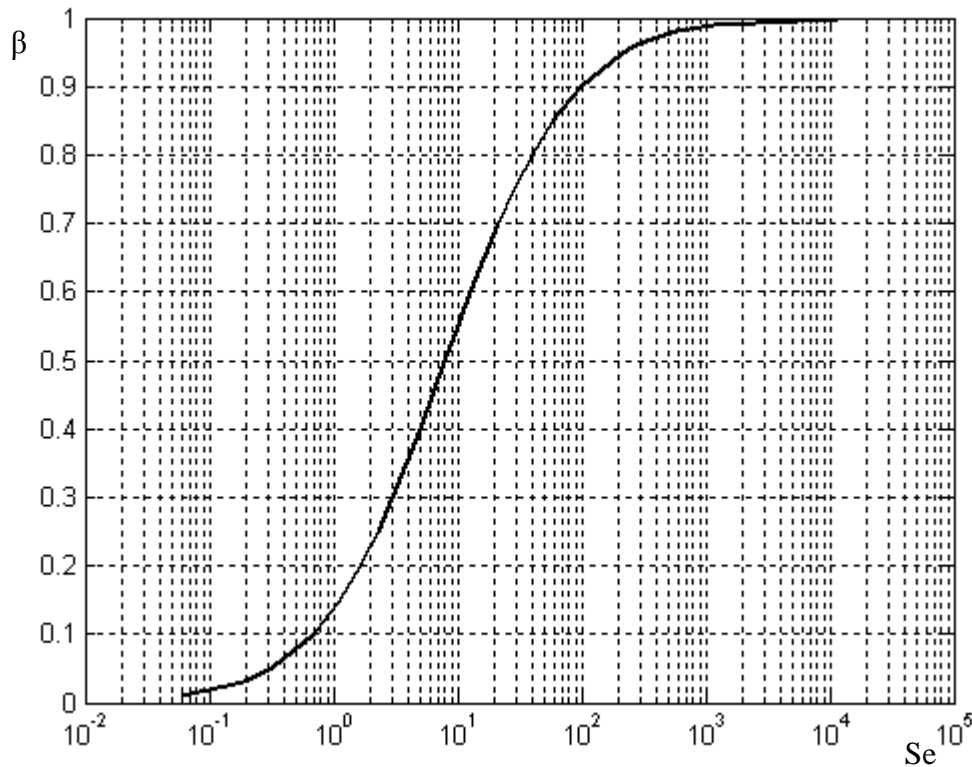


Figure 2- Graph of the function $\beta = f(Se)$ for the annular space (using the formula 15).

For example, for cause Se value, we can prevent some errors, which lead us uncorrected value (β) on the vertical axis, and ultimately affect the value of Δp .

finding the coefficient β in figure-2, pre-computing the number of Se from the known flow rate, rheological τ_0 , η and geometric dc , d_H data.

2.2 . API method:

By API method, figure-3 as a result of integration, we obtain the formula Buckingham, in the following form:

$$\frac{6u}{g_c W} = \frac{\tau_\omega}{\eta} \left[1 - \frac{3}{2} \left(\frac{\tau_0}{\tau_\omega} \right) + \frac{1}{2} \left(\frac{\tau_0}{\tau_\omega} \right)^2 \right], \quad (14)$$

$$q = \frac{\pi W^4 \Delta p}{6 \eta L} \left[1 - \frac{3}{2} \frac{2\tau_0 L}{W \Delta p} + \frac{1}{2} \left(\frac{2\tau_0 L}{W \Delta p} \right)^2 \right],$$

Where

$$u = q / F; \quad \tau_\omega = \frac{W \Delta p}{2L},$$

τ_0 -dynamic shear stress, lb/ft²; η -plastic viscosity lbm / ft sec; d_H - outer diameter of the pipe, in; dc - diameter borehole, in; L -depth, ft; u - velocity, ft / sec; g_c -conversion units. $32.7 \frac{lbm \cdot ft}{lbf \cdot sec^2}$ [3-5,10].

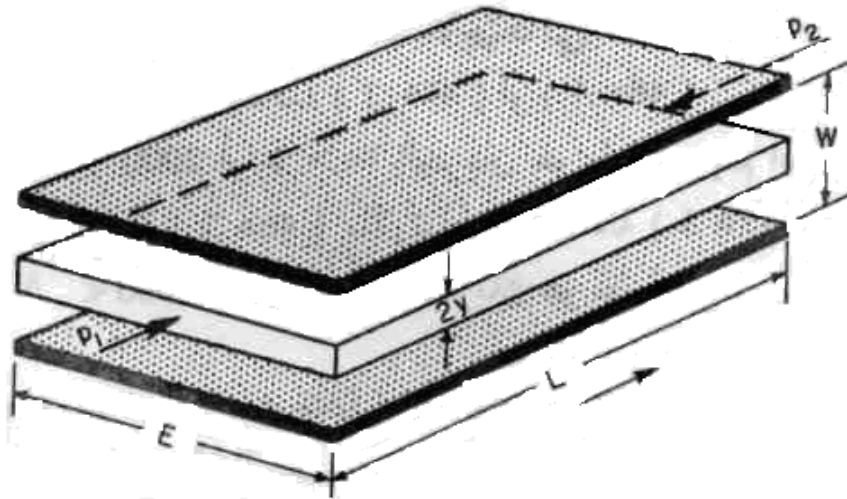


Figure 3- Direct flow between two parallel plates [10].

where:

W-distance between two parallel plates; L- length ; E-width; 2y-thickness core; P₁ and P₂-pressure. This formula(formula Buckingham) we present to the form

$$Se = \frac{6\beta}{1 - \frac{3}{2}\beta + \frac{1}{2}\beta^2}, \quad (15)$$

where Se and β , as in Formula (12) and Eq. (13).

We neglect the last term on the right side of equation (14), because it is a very small amount:

$$\frac{1}{2} \left(\frac{\tau_0}{\tau_w} \right)^2 \approx 0$$

Formula (14) to the form

$$\frac{6u}{g_c W} = \frac{1}{\eta} \left(\tau_w - \frac{3}{2} \tau_0 \right), \quad (16)$$

solving (16) with respect to Δp , we obtain

$$\Delta p = \frac{12\eta Lu}{g_c W^2} + \frac{3\tau_0 L}{W}, \quad (17)$$

where: $W = \frac{1}{2}(d_c - d_n),$ (18)

$$\Delta p = \frac{48\eta Lu}{g_c (d_c - d_n)^2} + \frac{6\tau_y L}{(d_c - d_n)}, \quad (19)$$

equation (19) is reduced to a convenient SI units system , is as follows:

$$\Delta p = \frac{L}{(d_c - d_H)} \left[\frac{47.8\eta u}{(d_c - d_H)} + 5.33\tau_0 \right]. \quad (20)$$

where:

τ_0 -dynamic shear stress, Pa; η -plastic viscosity, Pa . c; u - velocity, m / c; Δp - pressure, Pa; d -tube diameter, m; L -depth, m.

Formula (19) to the form

$$\beta = \frac{Se}{6 + \frac{3}{2}Se} \quad (21)$$

2.3. New approximate formulas:

$$\beta = 1 - \frac{7.3}{Se} \left(\sqrt[4]{1.2 + 0.5Se} - 1 \right) \quad \text{at } Se \geq 15 \quad (22)$$

$$\beta = \frac{Se}{6.5 + 1.2Se}, \quad \text{at } Se < 15 \quad (23)$$

3.Example:

Dynamic shear stress $\tau_0 = 7.18 \text{ Pa} = 15 \text{ lb}/100\text{ft}^2$; plastic viscosity $\eta = 0.03 \text{ Pa}\cdot\text{sec} = 30 \text{ cP}$; pipe outside diameter $d_H = 0.168 \text{ m} = 6.625 \text{ in}$; hole diameter $d_c = 0.444 \text{ m} = 17.5 \text{ in}$; depth $L = 1000 \text{ m} = 3280\text{ft}$; flow rate $q = 0.0167 \text{ m}^3 / \text{sec} = 265 \text{ gal} / \text{min}$; density $\rho = 10 \text{ lb} / \text{gal} = 1200 \text{ gr}/\text{cm}^3$.

1- Determination Δp using the formula Buckingham:

$$q = \frac{\pi W^4 \Delta p}{6\eta L} \left[1 - \frac{3}{2} \frac{2\tau_0 L}{W\Delta p} + \frac{1}{2} \left(\frac{2\tau_0 L}{W\Delta p} \right)^2 \right]$$

$$q = \frac{\pi (d_c - d_H)^4 \Delta p}{96\eta L} \left[1 - \frac{6\tau_0 L}{(d_c - d_H)\Delta p} + 8 \cdot \left(\frac{\tau_0 L}{(d_c - d_H)\Delta p} \right)^2 \right]$$

Where : $W = \frac{1}{2}(d_c - d_H)$

$$q = \frac{\pi (0.444 - 0.168)^4 \Delta p}{96 \cdot 0.03 \cdot 1000} \left[1 - \frac{6 \cdot 7.18 \cdot 1000}{(0.444 - 0.168)\Delta p} + 8 \cdot \left(\frac{7.18 \cdot 1000}{(0.444 - 0.168)\Delta p} \right)^2 \right]$$

$$q = 0.01673$$

$$q \approx 0.0167.$$

$$\Delta p = 109080 \text{ Pa}.$$

2 - Determination ΔP on the method API:

Find the average speed

$$u = \frac{q}{2.45(d_c - d_H)^2} = \frac{265}{2.45(17.5 - 6.625)^2} = 0.914\text{ft}/\text{sec} = 0.278 \text{ m}/\text{sec}.$$

We find a Hedstrom number (He) . It is a dimensionless quantity used to determine the nature of fluid flow.

$$He = \frac{37100\tau_0 (d_c - d_H)^2 \rho}{\eta^2} = \frac{37100 \cdot 15 \cdot (17.5 - 6.625)^2 \cdot 10}{(30)^2} = 7.3 \cdot 10^5.$$

We find the Reynolds number Re - is another dimensionless parameter characterizing the fluid flow.

$$Re = \frac{928\rho u(d_c - d_H)}{\eta} = \frac{928 \cdot 10 \cdot 0.914(17.5 - 6.625)}{30} = 3074.7.$$

We find the critical Reynolds number. The critical Reynolds number allows to determine the moment of transition from laminar to turbulent flow. The correspondence between critical Reynolds and Hedstrom numbers shown in figure-5.

From this figure. Rec=15000.

laminar flow (Re < critical value, Rec)

$$\Delta p = \frac{L}{(d_c - d_H)} \left[\frac{47.8\eta u}{(d_c - d_H)} + 5.33\tau_0 \right] = \frac{1000}{(0.444 - 0.168)} \left[\frac{47.8 \cdot 0.03 \cdot 0.287}{(0.444 - 0.168)} + 5.33 \cdot 7.18 \right]$$

$$\Delta p = 144060 \text{ Pa}.$$

3 - Determination ΔP on the graph-analytical method Grodde

$$u = \frac{4 \cdot q}{\pi \cdot (d_c - d_H)^2} = \frac{4 \cdot 0.0167}{3.14 \cdot (0.444 - 0.168)^2} = 0.287 \text{ m/sec};$$

$$Re = \frac{\rho u (d_c - d_H)}{\eta} = \frac{1200 \cdot 0.287 \cdot (0.444 - 0.168)}{0.03} = 3168.5;$$

$$He = \frac{\tau_0 (d_c - d_H)^2 \rho}{\eta^2} = \frac{7.18 \cdot (0.444 - 0.168)^2 \cdot 1200}{0.03^2} = 7.3 \cdot 10^5;$$

$$Rec = 2100 + 7.3 \cdot He^{0.58} = 2100 + 7.3(7.3 \cdot 10^5)^{0.58} = 20456.7.$$

laminar flow (Re < critical value Rec)

$$Se = \frac{\tau_0 (d_c - d_H) F}{\eta q} = \frac{\tau_0 (d_c - d_H)}{\eta v} = \frac{7.18 \cdot (0.444 - 0.168)}{0.03 \cdot 0.287} = 230.$$

From figure-2 $\beta = 0.94$

$$\Delta p = \frac{4\tau_0 L}{(d_c - d_H)\beta} = \frac{4 \cdot 7.18 \cdot 1000}{(0.444 - 0.168) \cdot 0.94} = 110691 \text{ Pa}.$$

3 - Determination ΔP by New Approximate Formulas:

$$\beta = 1 - \frac{7.3}{Se} \left(\sqrt[4]{1.2 + 0.5Se} - 1 \right), \quad \text{at} \quad Se \geq 15;$$

$$\beta = 1 - \frac{7.3}{230} \left(\sqrt[4]{1.2 + 0.5 \cdot 230} - 1 \right) = 0.927,$$

$$\Delta p = \frac{4\tau_0 L}{(d_c - d_H)\beta} = \frac{4 \cdot 7.18 \cdot 1000}{(0.444 - 0.168) \cdot 0.927} = 112251.7 \text{ Pa}.$$

Δp Buckingham = 109 080 Pa;

Δp API = 144 060 Pa

Δp Grodde = 110691 Pa.

Δp New Approximate Formulas = 112251.7 Pa.

% API= 32

% Grodde = 1.47.

% of new formulas = 2.9.

4. Result and discussion :

Table 1-The values of $\beta = \beta (Se)$ and the error value between methods and the exact value.

no	Se exact	β Buckingham	β New Eqs.	β API	Se Grodde	B Grodde	% New Eqs.	% Grodde	% API
1	2	3	4	5	6	7	8	9	10
1	0.061	0.01	0,0093	0,01	0,065	0,011	7,0	10,0	0,0
2	0.3238	0.05	0,047	0,0499	0,3	0,047	6,0	6,0	0,2
3	0.7017	0.1	0,0956	0,0995	0,63	0,09	4,4	10,0	0,5
4	1.1447	0.15	0,1454	0,1483	1,3	0,16	3,06	9,33	1,13
5	1.6667	0.2	0,1961	0,1961	1,55	0,19	1,95	5,0	1,95
6	2.2857	0.25	0,2473	0,2424	1,97	0,23	1,08	8,0	3,04
7	3.025	0.3	0,2986	0,2871	2,56	0,27	0,56	10,0	4,3
8	3.9161	0.35	0,3497	0,3298	3,55	0,33	0,08	5,71	5,77
9	5.0	0.4	0,4	0,3704	4,55	0,38	0,0	5,0	7,4
10	6.3343	0.45	0,4492	0,4086	5,8	0,43	0,17	4,44	9,2
11	8.0	0.5	0,4969	0,4444	6,3	0,47	0,62	6,0	11,12
12	10.1149	0.55	0,5427	0,4777	10,6	0,56	1,32	1,81	13,14
13	12.857	0.6	0,5863	0,5085	11,7	0,58	2,28	3,33	15,25
14	16.508	0.65	0,6668	0,5366	15,0	0,63	2,6	3,07	17,44
15	21.538	0.7	0,7085	0,5622	20,0	0,68	1,21	2,86	19,68
16	28.8	0.75	0,7497	0,5854	25,5	0,73	0,04	2,66	21,94
17	40.0	0.8	0,7909	0,6061	40,0	0,8	1,13	0,0	24,23
18	59.13	0.85	0,8327	0,6244	61,8	0,86	2,03	0,6	26,5
19	98.182	0.9	0,8764	0,6406	92,5	0,89	2,62	1,11	28,8
20	217.14	0.95	0,9248	0,6546	210	0,94	2,65	1,05	31,09
21	11976.0	0.999	0,9952	0,6664	11976,0	0,999	0,38	0,0	33,29
22	119967	0.9999	0,9991	0,6666	119967	0,9999	0,08	0,0	33,33

$$\% \text{ Error value} = (|\beta_{cal.} - \beta_{exact}| / \beta_{exact}) 100\%. \quad (24)$$

The maximum error in the calculation $\beta_{new.}$ on new formulas (22) and (23) respectively is $\delta = \max\delta_{new.} \leq 7.0\%$ and $\delta = \max\delta_{new.} \leq 2.65\%$ (see column 8 in table-1). The results of calculations $\beta = \beta (Se)$ and the error δ in the formula given in table-1, for clarity, are shown in figure-3. Comparison of the curves in figure-3, we see that formulas (22) and (23) give a much smaller deviation (average $\delta_{new.} = 1.87\%$) compared with calculations by the method Grodde (average $\delta_{Gr.} = 4.36\%$ and $\max\delta_{Gr.} \leq 10,0\%$), the formula API (average $\delta_{API} = 14.06\%$ and $\max\delta_{API} \leq 33.33\%$). Thus, as a result of numerical experiments we have found that when calculating the pressure loss new formula gives smaller deviations from the exact values.

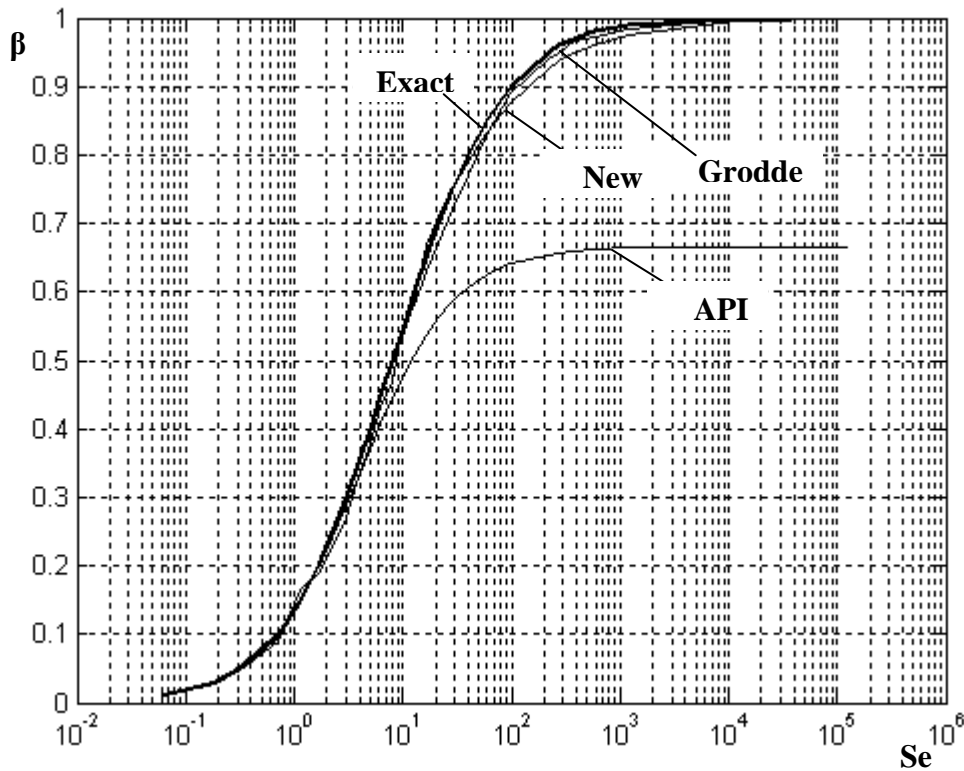


Figure 3- Graph of the function $\beta = f(Se)$ for the annular space, all methods.

5. Conclusion

1. As a result of numerical experiments it has been shown that new formula gives smaller deviations from the exact values.
2. The developed computational techniques and the mathematical relations can be recommended to use in engineering calculations.
3. Comparison of the curves, we see that new formulas give a much smaller deviation (average $\delta_{new} = 1.87\%$) compared with calculations by the Grodde method (average $\delta_{Gr} = 4.36\%$ and $\max \delta_{Gr} \leq 10,0\%$), the formula API (average $\delta_{API} = 14.06\%$ and $\max \delta_{API} \leq 33.33\%$).
4. The developed computational techniques and the resulting mathematical relations can be recommended for use in engineering calculations, when planning to wells construction and Rheology - hydraulic drilling programs to depths of 5000 m in the fields of Iraq.

References

1. Leonov, Isaev E.G. **1978**, *V.I. Hydraulic calculations of wishing boreholes while drilling M, Gubkin.* (in Russian)
2. Leonov Isaev E.G., Fisher V.I., **1984**, *V.A. Hydraulic calculation of the circulation system during drilling. M., Gubkin,* (in Russian)
3. Rabia H. **1989**, *Technology of drilling oil wells, the translation from English.M., Nedra,* (in Russian)
4. Getlin K. **1963**, *Drilling and completion, translation from English., Moscow .* (in Russian)
5. Adam T. Bourgoyne Jr., Keith k. Millheim, Martin E. Chenevert. **1991**, *Applied drilling engineering ,SPE, Richardson, TX,*
6. Grodde K.H. Rheology of colloidal suspension, in particular the Bohrspulungen. -*Petroleum and coal*, 13(1), pp:11-18, No.2, pp:79-83, 1960. (in German)
7. Buckingham E. **1921**, *On plastic flow through capillary tubes. proc.amer. sos. testing materials,* pp:1154 – 1161,.
8. Leonov E.G., Mohammed F.H. **2009**, "Comparison of pressure losses calculations results in pipes in laminar flow of viscoplastic fluids on approximate methods and exact formula Buckingham

- "STJ"Construction of oil and gas wells on land and at sea"M.:JSC "VNIOENG", №5 pp: 16 - 21,.(in Russian)
9. Mohammed F.H. **2010**."Comparison of calculation results of pressure losses for a typical well in the North Rumaila oil field in Iraq by washing prior to cementation". STJ "Construction of oil and gas wells on land and at sea" - M.: JSC "VNIOENG", - № 4. – pp: 22 - 23,.(in Russian).
 10. Craft, B.C.,Holden, W.R., Grave, E.D. **1987**, Well design (drilling and production).