



Inelastic longitudinal electron scattering C2 form factors in ^{48}Ca nucleus by using sigma meson as a residual interaction.

Firas Z. Majeed , Sadeq S. Mashaan*

Department of Physics, College of Science, Baghdad University, Baghdad, Iraq.

Abstract

Inelastic longitudinal electron scattering C2 form factor in ^{48}Ca has been utilized to study the effects of fitting parameters on the sigma meson exchange type potentials as a residual interaction. By coupling the core particles with model space particle, where the latter used as an active part of residual interaction in the so called core polarization process, it is included as a correction with first order perturbation theory to the main calculation of model space, and the excitation energy has been carried out with $(2\hbar\omega)$. A model space wave vectors are generated in full fp shell model with FPD6 as effective interaction with mixing configuration technique and harmonic oscillator as a single particle wave function. Theoretical results are consisted with experimental data.

Keyword: sigma meson exchange, ^{48}Ca , electron scattering, fitting parameters

الاستطارة الالكترونية الطولية غير المرنة C2 في نواة ^{48}Ca باستخدام ميزون سگما Σ كبقية تفاعل.

فiras زهير مجيد و صادق سلطان مشعان*

قسم الفيزياء، كلية العلوم، جامعة بغداد، بغداد، العراق.

الخلاصة:

تم دراسة عوامل التشكل للاستطارة الالكترونية غير المرنة لمتعدد الأقطاب الطولي C2 في نواة ^{48}Ca بواسطة التبادل الميزوني Σ عن طريق ربط الجسيمات القلب مع جسيمات أنموذج الفضاء كجزء فعال في تفاعل البقية بعملية تسمى تأثير استقطاب القلب والتي تعتبر كتصحيح من المرتبة الأولى لنظرية الاضطراب وتجمع مع الحسابات الرئيسية لأنموذج الفضاء وبطاقة استثارته مقدارها $2\hbar\omega$ وقد تم اعتماد انموذج الفضاء fp-shell والتفاعل الفعال FPD6 لتولد الدوال الموجية مع تقنية مزج التشكيلات وكذلك اعتماد المتذبذب التوافقي كدالة موجية للجسيمة المفردة. النتائج النظرية تم مقارنتها مع الحسابات العملية .

Introduction

Electron scattering form factors give the most precise information about nuclear size and charge distribution [1].

The electromagnetic theory agrees with experimental observations, and Quantum Electrodynamics success in giving a well-defined description of the interaction deeply insight [2,3].

Mott first derived the scattering cross section which described the electron scattering and included the effect due to the electron spin. “Nuclear form factor” depends on the charge and magnetization distribution in the target nuclei and we can determine the form factor by the ratio of the measured cross section to the Mott cross section. The form factors can be found experimentally as a function of the momentum transfer \vec{q} by knowing the energies of the incident and scattered electron and the scattering angle [4].

*Email:sadeq278@ymail.com

Backward-angle resolution inelastic electron scattering on $^{40,42,44,48}\text{Ca}$ have been carried out by Steffen et al [5], where they observed a very strong magnetic dipole ground state transition in Ca isotopes.

Monopole transitions from the 1^+0 ground states to 2^+0 excited states at 3.353 MeV for ^{40}Ca , 1.837 MeV for ^{42}Ca , 1.884 MeV for ^{44}Ca and 4.272 MeV for ^{48}Ca had been investigated by Gräf et al [6] with high resolution inelastic electron scattering (FWHM ≈ 30 keV) at low momentum transfer ($0.29 < q < 0.53 \text{ fm}^{-1}$).

Zheag and Zamick [7] studied the relations between Polarized-Proton-Nucleus and unpolarized-transverse-electron-nucleus scattering and their application in ^{42}Ca .

The electron scattering form factors have been measured by Itoh et al [8], for 2^+ , 3^- and 5^- states up to 7 MeV excitation in ^{42}Ca and ^{44}Ca , the range of the incident electron energy were 62.5–250 MeV.

The derivation of Core Polarization (CP) effects with higher configuration in the first order perturbation theory and the two-body matrix elements of three parts of the realistic interaction: central, spin orbit and tensor force which are belong to M3Y interaction and the detection of meson exchange current for every channel of interaction in a separate pictures are introduced in the present work, harmonic oscillator single-particle basis has been used.

Theory

Shell-model theory showed that the true space might be divided into three separated spaces which are: model space, inert core and higher configurations. Higher orbits might be included or excluded according to the choice of the researcher and the model in use, but core orbits as have been proved [9], have an active contribution in the calculation of form factors. The main problem is that the inclusion of core orbits makes the space (Hilbert Space) very vast so, we must separate between the two spaces (a core part and a valence part) to express the interaction between the core and the valence particles, and that among the valence particles.

Through microscopic theory, the core polarization effect on the form factor combines shell-model wave functions and configurations with higher energy as first order perturbations; these are called "core-polarization effects".

In the shell-model, we are unable to solve Schrodinger equation in the full Hilbert space, because of the huge number of configurations, so we must truncate it to a smaller part of the configurations with finite dimensions Hilbert space. This is called the shell model-space or simply the model space. For this reason, we must use effective interactions and operators. The effective interaction is used to give the nuclear properties microscopically, starting with realistic NN interaction using quantum mechanical many-body theory. For light nuclei, there are different effective interactions such as the FPD6 [10],

For a selected operator T_{JT}^η the reduced matrix elements are written as the sum of the product of the one-body transition density matrix elements (OBDM) times the single-particle transition matrix elements:

$$\langle \Gamma_f \| \hat{T}_{JT}^\eta \| \Gamma_i \rangle = \sum_{\alpha, \beta} OBDM(\Gamma_i, \Gamma_f, \alpha, \beta) \langle \alpha \| \hat{T}_{JT}^\eta \| \beta \rangle \quad (1)$$

where $\Lambda = JT$ is the multi-polarity and the states $\Gamma_i = J_i J_i$ and $\Gamma_f = J_f J_f$ are initial and final states of the nucleus. While α and β denote the final and initial single-particle states, respectively (isospin is included).

The reduced matrix element of the electron scattering operator \hat{T}_Λ^η consist of two parts, one is for the "Model space" matrix elements, and the other is for the "Core-polarization" matrix elements are given by:

$$\langle \Gamma_f \| \hat{T}_\Lambda^\eta \| \Gamma_i \rangle = \langle \Gamma_f \| \hat{T}_\Lambda^\eta \| \Gamma_i \rangle_{MS} + \langle \Gamma_f \| \delta \hat{T}_\Lambda^\eta \| \Gamma_i \rangle_{CP} \quad (2)$$

where, $\langle \Gamma_f \| \hat{T}_\Lambda^\eta \| \Gamma_i \rangle_{MS}$ are the model-space matrix elements,

and, $\langle \Gamma_f \| \delta \hat{T}_\Lambda^\eta \| \Gamma_i \rangle_{CP}$ are the core-polarization matrix elements.

The $|\Gamma_i\rangle$ and $|\Gamma_f\rangle$ are described by the model-space wave functions.

The core-polarization matrix elements can be written as [9]:

$$\langle \Gamma_f || \delta \hat{T}_\Lambda^\eta || \Gamma_i \rangle_{CP} = \sum_{\alpha, \beta} OBDM(\alpha, \beta) \langle \alpha || \delta \hat{T}_\Lambda^\eta || \beta \rangle_{CP} \quad (3)$$

The first order perturbation theory says that the single-particle matrix element for the higher-energy configurations can be expressed as [11]:

$$\langle \alpha | \delta \hat{T}_J^\eta | \beta \rangle = \langle \alpha | V_{res} \frac{Q}{E-H^{(0)}} \hat{T}_J^\eta | \beta \rangle + \langle \alpha | \hat{T}_J^\eta \frac{Q}{E-H^{(0)}} V_{res} | \beta \rangle \quad (4)$$

The single-particle energies are calculated according to [11]:

$$e_{nlj} = (2n+l-\frac{1}{2})\hbar\omega + \begin{cases} -\frac{1}{2}(l+1)\langle f(r) \rangle_{nl} & \text{for } j=l-\frac{1}{2} \\ \frac{1}{2}l\langle f(r) \rangle_{nl} & \text{for } j=l+\frac{1}{2} \end{cases} \quad (5)$$

with:

$$\begin{aligned} \langle f(r) \rangle_{nl} &\approx -20 A^{-2/3} \text{ MeV} \\ \hbar\omega &= 45 A^{-1/3} - 25 A^{-2/3} \end{aligned} \quad (6)$$

A computer program written in FORTRAN 90 language was used by Radhi to include realistic interaction M3Y, is modified to receive new fitting parameters which calculate meson part of interaction.

For the two-body matrix elements of the residual interaction $\langle \alpha\alpha_1 | V_{res} | \beta\alpha_2 \rangle_\Gamma$ and $\langle \alpha\alpha_2 | V_{res} | \beta\alpha_1 \rangle_\Gamma$, which appear in equation (4), the Michigan sum of three range Yukawa potential (M3Y) interaction of Nakada [12] is adopted. This interaction is a modified version of M3Y interaction of Berstch et al [13]. This interaction containing terms like those were given in LS-coupling and tensor effects, and density dependence part which calculates the zero range term. A transformation of the wave function from jj to LS coupling must be done to get the relation between the two-body shell model matrix elements and the relative and centre of mass coordinates, using the harmonic oscillator radial wave functions with Talmi-Moshinsky transformation. The realistic M3Y effective NN interaction, which is used in electron scattering ($V_{res} = v_{12}$) is expressed as a sum of the central potential parts $v_{12}^{(c)}$ as follows:

$$\begin{aligned} v_{12}^{(c)} &= \sum_n t_n^{(SE)} P_{SE} f_n^{(c)}(r_{12}) \quad (7) \\ V_{12}(\delta) &= t_1^{(SE)} P_{SE} \frac{e^{-\frac{r_{12}}{R_{112}}}}{R_{112}} + t_2^{(SE)} P_{SE} \frac{e^{-\frac{r_{12}}{R_{212}}}}{R_{212}} + t_3^{(SE)} P_{SE} \frac{e^{-\frac{r_{12}}{R_{312}}}}{R_{312}} \quad (8) \end{aligned}$$

The values of the best fit to the potential parameters ($t_n^{(SE)}$) are shown in table (1) [14].

Table 1- The value of the best fit to the potential parameters [14].

Parameters	Unit	M3Y-P0	M3Y-P1	M3Y-P2	M3Y-P3	M3Y-P4	M3Y-P5	M3Y-E
$R_1^{(c)}$	fm	0.25	0.25	0.25	0.25	0.25	0.25	0.25
$t_1^{(SE)}$	MeV	11466	8599.5	8027	8027	8027	8027	9958
$R_2^{(c)}$	fm	0.40	0.40	0.40	0.40	0.40	0.40	0.40
$t_2^{(SE)}$	MeV	-3556	-3556	-2880	-2637	-2637	-2650	-3105
$R_3^{(c)}$	fm	1.414	1.414	1.414	1.414	1.414	1.414	1.414
$t_3^{(SE)}$	MeV	-10.463	-10.463	-10.463	-10.463	-10.463	-10.463	-10.463

In this part we will focus attention on Ca isotope, ^{48}Ca , where there is a closed core ^{40}Ca and eight neutrons freely distributed in fp-shell model space. We have chosen FPD6 as a model space effective interaction to generate the model space wave functions and OBDM.

The total C2 form factor is calculated, from the core contribution only, and indicated that the model space has no contribution, because the neutrons do not contribute to the charge form factor, because they are neutral particles, so the core protons will play this role. In general, the total results are in a good agreement with the experimental data. The experimental data are taken from Heisenberg et.al.[15].

Results and Discussion

The core-polarization effects have been included in order to account for the contribution of configurations from outside of the model space in the transition. The nucleus ⁴⁸Ca is the lightest doubly magic nucleus with a neutron excess. It is known to be a good shell-model nucleus and thus provides an excellent testing ground of nuclear models. In fact, the nucleus ⁴⁸Ca is more inert than ⁴⁰Ca, ⁴⁸Ni and ⁵⁶Ni because of the closed sub shell neutron 1f_{7/2} so that it is an interesting one in fp shell nuclei. We use the single particle wave functions of the harmonic oscillator (HO) with size parameter (b= 1.988 fm).

The interaction equation for sigma meson (δ) when T=0, S=0, which is expressed as (t^(SE)), which represents fitting parameter for nucleon interaction in this channel and be the interaction equation as follows[16]:

$$V_{12}(\delta) = \sum_{n=1}^3 t_n^{(SE)} P_{SE} \frac{e^{-\left(\frac{r_{12}}{R_{n12}}\right)}}{\frac{r_{12}}{R_{n12}}} \quad (9)$$

$$V_{12}(\delta) = t_1^{(SE)} P_{SE} \frac{e^{-\left(\frac{r_{12}}{R_{112}}\right)}}{\frac{r_{12}}{R_{112}}} + t_2^{(SE)} P_{SE} \frac{e^{-\left(\frac{r_{12}}{R_{212}}\right)}}{\frac{r_{12}}{R_{212}}} + t_3^{(SE)} P_{SE} \frac{e^{-\left(\frac{r_{12}}{R_{312}}\right)}}{\frac{r_{12}}{R_{312}}} \quad (10)$$

Note: The sum process of the fitting parameter (t₁^(SE) + t₂^(SE)) and negligence (t₃^(SE)) to their small value and equal to the value of all the interactions found at the time of the collection process (t₁^(SE) + t₂^(SE)) devolve Descending and digital output as follows:

P0=7910MeV>E=6853MeV>P3=P4=5390MeV>P5=5377MeV>P2=5147MeV>P1=5043.5MeV

The C2 Charge Form Factor For 2⁺state

1) Charge form factors for 2₁⁺state at Ex=7.319 MeV.

Inelastic longitudinal form factors (C2) was calculated by using M3Y (including the OBDM elements as table 2) as shown in figure 1-; in this figure the calculated form factors using M3Y(E,P0,P1,P2,P3,P4 and P5) shows the behavior of results as two peaks .

where: P0> P1> E> P3> P5> P4> P2 (the first peak)

P0> E> P4> P2> P1> P5 = P3 (second peak)

Table 2- The OBDM elements for the C2₁⁺ transition obtained by FPD6 interaction in ⁴⁸Ca .

J _i	J _f	OBDM (ΔT=0)	OBDM (ΔT=1)
7/2	7/2	0.07960	0.05138
7/2	3/2	0.30835	0.19904
7/2	5/2	0.11637	0.07512
3/2	7/2	1.95301	1.26066
3/2	3/2	0.04680	0.03021
3/2	5/2	0.00256	0.00165
3/2	1/2	0.01778	0.01148
5/2	7/2	0.27266-	-0.17611
5/2	3/2	0.01412-	0.00911-
5/2	5/2	0.03051	0.01970
5/2	1/2	0.01765	0.01139
1/2	3/2	0.04228-	0.02729-
1/2	5/2	0.01848	0.01193

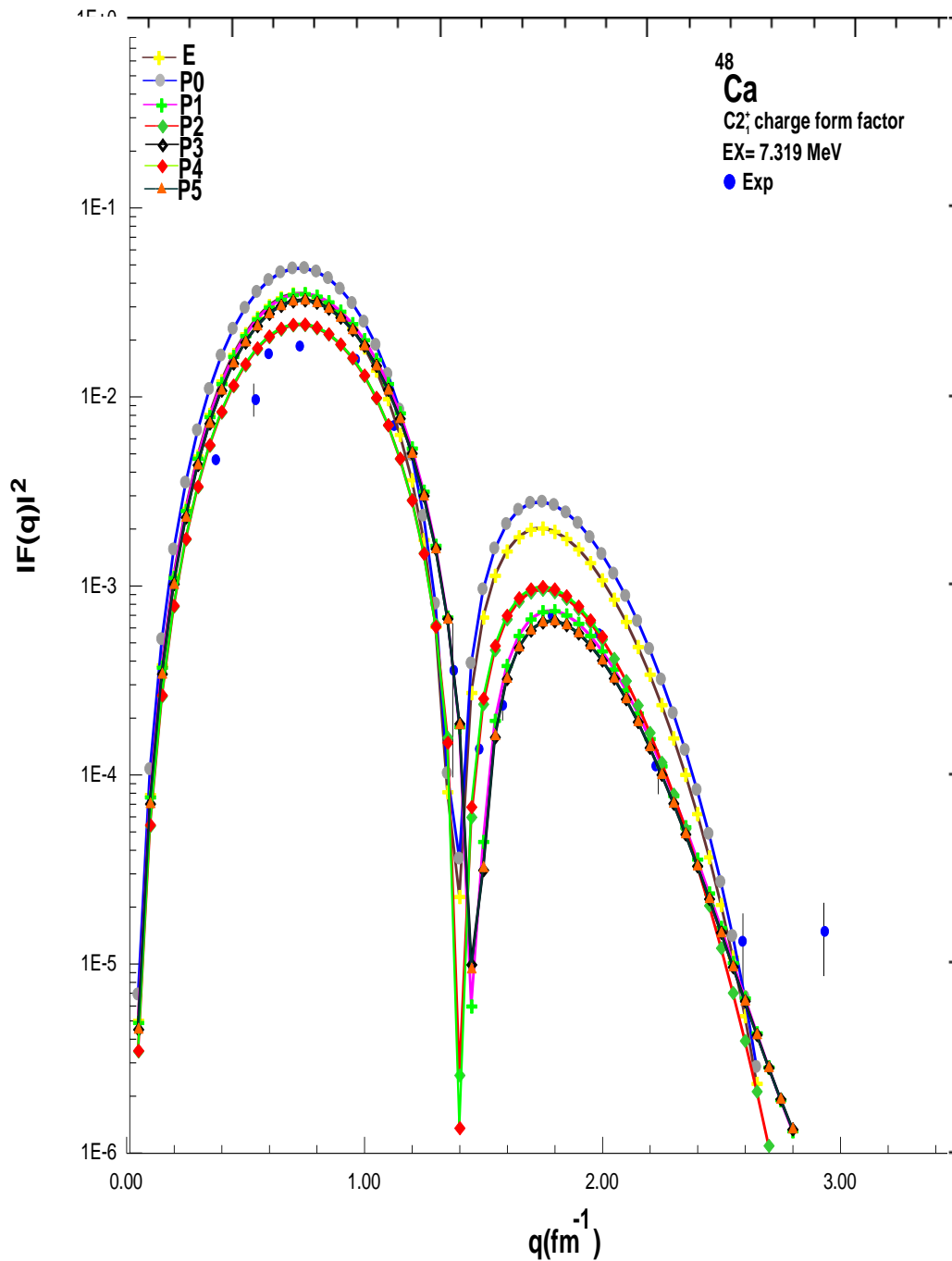


Figure 1- Charge form factor for the $C2_1^+$ state in ^{48}Ca using $E_x=7.319$ MeV.

2) Charge form factors for 2_1^+ state at $E_x=8.918$ MeV.

The quadruple $C2$ charge form factor for ^{48}Ca in $1f-2p$ shell model space is illustrated in figure 2-, as a residual interactions. Using M3Y (including the OBDM elements as table 3) gave good agreement in results, which shifted than it in all region of (q) .

In the region $q = (0-1.5)\text{fm}^{-1}$ $P0 > P1 > P3 = P5 > E > P4 > P2$

In the region $q = (1.5-3)\text{fm}^{-1}$ $P0 > E > P2 = P4 > P1 > P3 = P5$

Table 3- The OBDM elements for the $C2_2^+$ transition obtained by FPD6 interaction in ^{48}Ca .

J_i	J_f	OBDM ($\Delta T=0$)	OBDM ($\Delta T=1$)
7/2	7/2	-0.11819	-0.07629
7/2	3/2	0.04547	0.02935
7/2	5/2	-0.00837	-0.00540
3/2	7/2	0.29438	0.19002
3/2	3/2	-0.26976	-0.17413
3/2	5/2	-0.01156	-0.00746
3/2	1/2	-0.01532	-0.00989
5/2	7/2	-0.02310	-0.01491
5/2	3/2	0.01530	0.00988
5/2	5/2	-0.00743	-0.00480
5/2	1/2	-0.00094	-0.00060
1/2	3/2	0.06802	0.04391
1/2	5/2	-0.00374	-0.00242

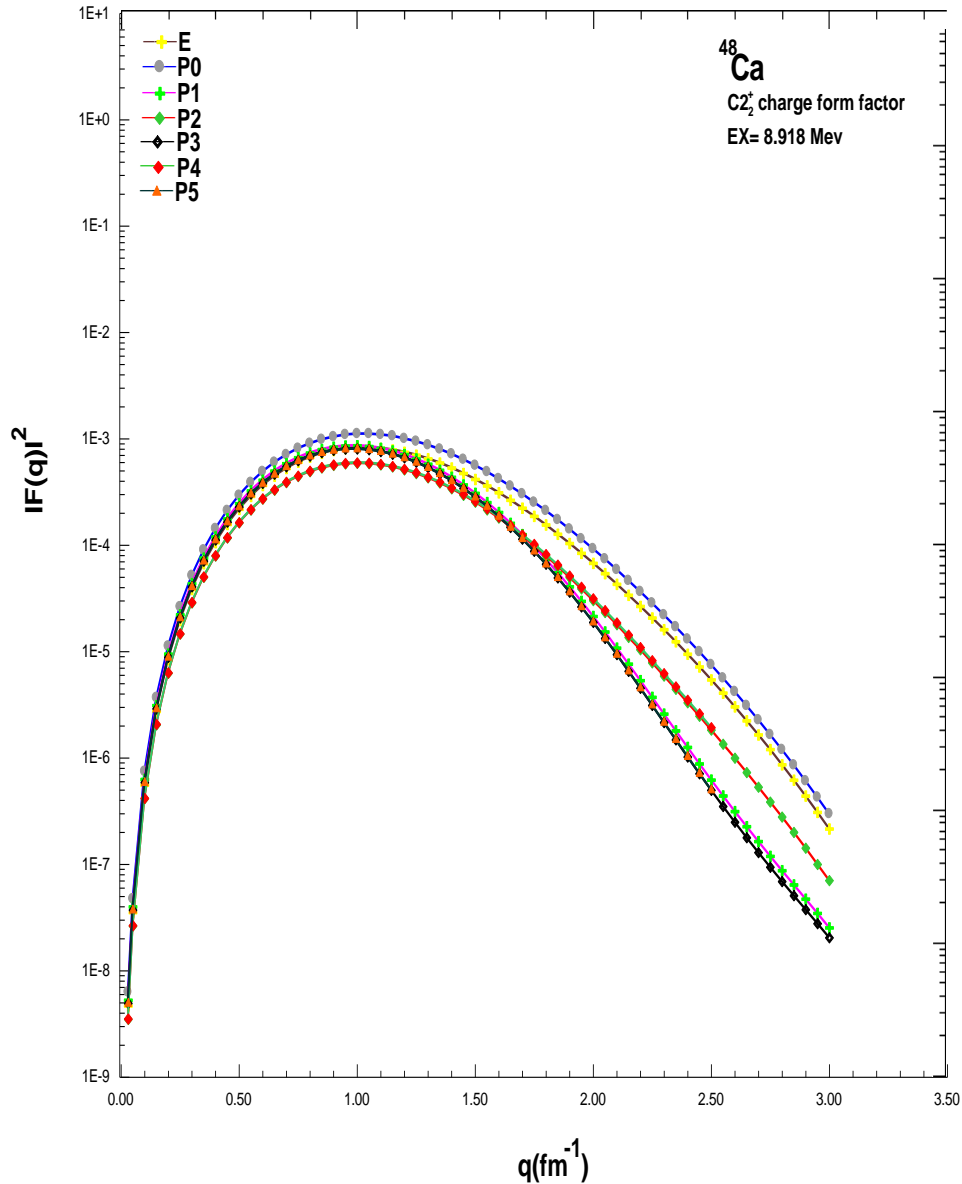


Figure.2- Charge form factor for the $C2_2^+$ state in ^{48}Ca using $E_x=8.918$ MeV.

3) Charge form factors for 2_{3}^{+} state at $E_x=9.121$ MeV

The quadruple C2 charge form factor for ^{48}Ca in 1f-2p shell model space is illustrated in figure 3-, as a residual interactions. Using M3Y (including the OBDM elements as table 4) gave good agreement in results, which shifted than it in all region of (q).

In the region $q = (0-0.6) \text{ fm}^{-1}$ $P1 > P3 = P5 > P0 > E > P2 = P4$

In the region $q = (0.6-1.6) \text{ fm}^{-1}$ $P0 > E > P1 > P3 = P5 > P4 > P2$

In the region $q = (1.6-3) \text{ fm}^{-1}$ $P0 > E > P4 > P2 > P1 > P3 = P5$

Table 4- The OBDM elements for the $C2_{3}^{+}$ transition obtained by FPD6 interaction in ^{48}Ca .

J_i	J_f	OBDM ($\Delta T=0$)	OBDM ($\Delta T=1$)
7/2	7/2	0.23143	0.14939
7/2	3/2	0.01477	0.00953
7/2	5/2	0.02670	0.01723
3/2	7/2	0.06763	0.04365
3/2	3/2	-0.39544	-0.25526
3/2	5/2	0.00317	0.00204
3/2	1/2	-0.04782	-0.03087
5/2	7/2	-0.20087	-0.12966
5/2	3/2	0.00749	0.00484
5/2	5/2	-0.01038	-0.00670
5/2	1/2	-0.01217	-0.00785
1/2	3/2	0.02408	0.01554
1/2	5/2	-0.00453	-0.00292

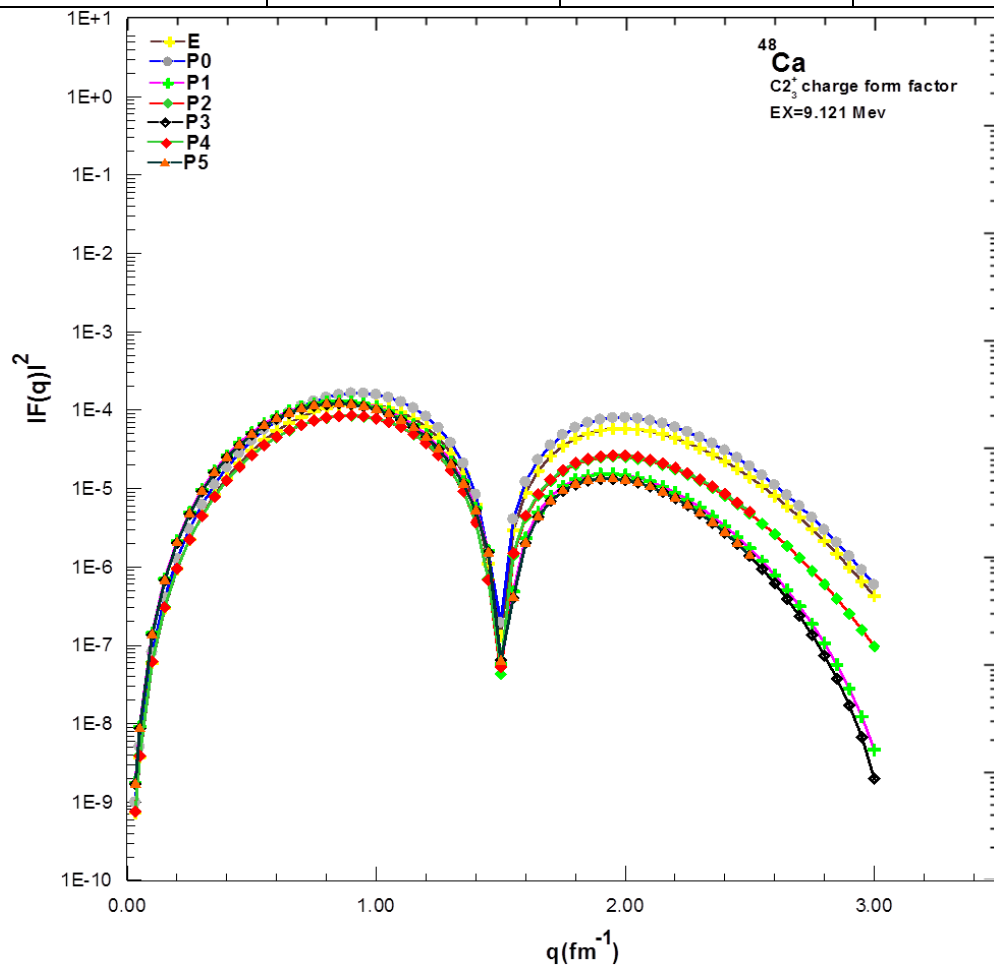


Figure 3- Charge form factor for the $C2_{3}^{+}$ state in ^{48}Ca using $E_x=9.121$ MeV.

4) Charge form factors for 2_4^+ state at $E_x=9.469$ MeV.

The quadruple C2 charge form factor for ^{48}Ca in 1f-2p shell model space is illustrated in figure 4-, as a residual interactions. Using M3Y (including the OBDM elements as table 5) gave good agreement in results, which shifted than it in all region of (q).

In the region $q = (0-0.6) \text{ fm}^{-1}$ $P_0 > P_1 > E > P_3 = P_5 > P_4 > P_2$ (first peak)

In the region $q = (0.6-2) \text{ fm}^{-1}$ $P_0 > E > P_1 > P_2 = P_3 = P_4 = P_5$ (first peak)

In the region $q = (2-2.8) \text{ fm}^{-1}$ $P_0 > P_1 > E > P_3 = P_5 > P_4 > P_2$ (second peak)

Table 5- The OBDM elements for the $C2_4^+$ transition obtained by FPD6 interaction in ^{48}Ca .

J_i	J_f	OBDM ($\Delta T=0$)	OBDM ($\Delta T=1$)
7/2	7/2	0.06266	0.04045
7/2	3/2	-0.05720	-0.03693
7/2	5/2	-0.08568	-0.05531
3/2	7/2	0.24400	0.15750
3/2	3/2	-0.02814	-0.01817
3/2	5/2	-0.03463	-0.02236
3/2	1/2	-0.03560	-0.02298
5/2	7/2	1.91335	1.23506
5/2	3/2	0.03926	0.02534
5/2	5/2	-0.03004	-0.01939
5/2	1/2	-0.00592	-0.00382
1/2	3/2	0.05899	0.03808
1/2	5/2	-0.00137	-0.00088

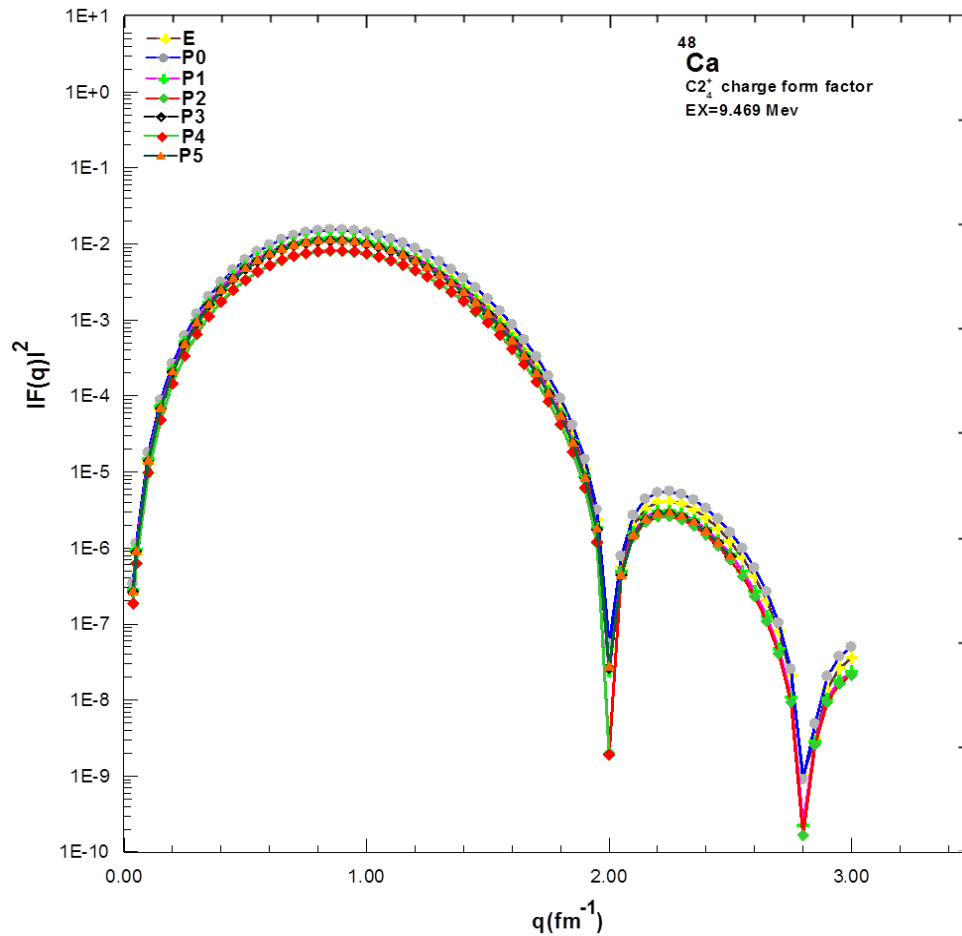


Figure 4- Charge form factor for the $C2_4^+$ state in ^{48}Ca using $E_x=9.469$ MeV.

5) Charge form factors for 2^+_{52} state at $E_x=9.792$ MeV.

The quadruple C2 charge form factor for ^{48}Ca in 1f-2p shell model space is illustrated in figure 5-, as a residual interactions. Using M3Y (including the OBDM elements as table 6) gave good agreement in results, which shifted than it in all region of (q).

In the region $q = (0-0.8) \text{ fm}^{-1}$ $P1 > P3 = P5 > P0 > P4 = P2 > E$ (first peak)

In the region $q = (0.8-2) \text{ fm}^{-1}$ $P0 > E > P1 > P3 = P5 > P4 = P2$

In the region $q = (2-3) \text{ fm}^{-1}$ $P0 > E > P4 = P2 > P1 > P3 = P5$ (second peak)

Table 6- The OBDM elements for the $C2^+_{52}$ transition obtained by FPD6 interaction in ^{48}Ca .

J_i	J_f	OBDM ($\Delta T=0$)	OBDM ($\Delta T=1$)
7/2	7/2	-0.34331	-0.22160
7/2	3/2	0.02567	0.01657
7/2	5/2	-0.01734	-0.01119
3/2	7/2	0.39721	0.25640
3/2	3/2	0.07856	0.05071
3/2	5/2	0.07061	0.04558
3/2	1/2	0.00427	0.00276
5/2	7/2	0.15140	0.09773
5/2	3/2	0.00767	0.00495
5/2	5/2	-0.01432	-0.00925
5/2	1/2	0.01000	0.0064
1/2	3/2	-0.03347	-0.02160
1/2	5/2	0.01138	0.00735

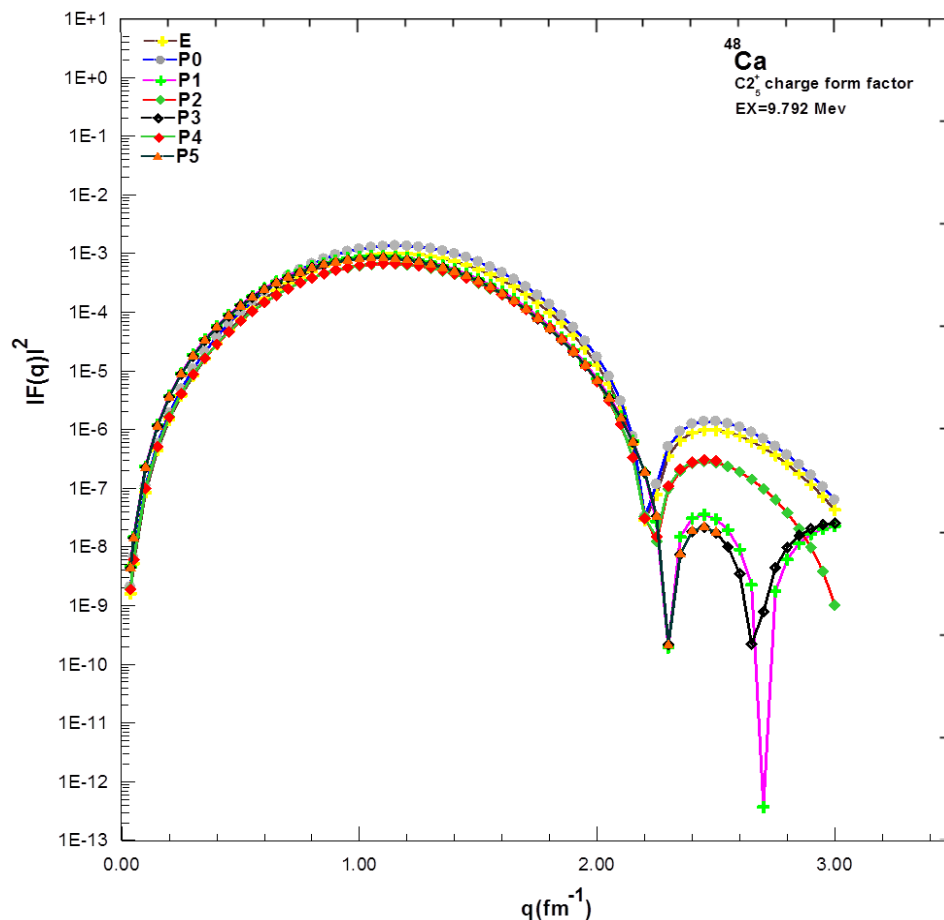


Figure 5- Charge form factor for the 2^+_{52} state in ^{48}Ca using $E_x=9.792$ MeV.

Conclusions:-

The amplitude of any channel for two body matrix elements coupling core particles to model space one is not the main factor responsible of the amplitudes of electron scattering form factors as a results of quantum tail of interactions, especially in the regions of momentum transfer (q larger than 2.1 fm^{-1}) as a results of repulsive force of short range meson sigma type and attractive force of medium range of the same meson type with a different mass and fitting parameters values.

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