# On Generalized Leftly e ─ Core Transference

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**Abstract**
Richards in 1996 introduced the idea of leftly e ─ core transference by using many conditions, including that the difference between the colums (k) is greater than of weight. In this paper, we generalized this idea without the condition of Richards depending on the mathematical and computational solution.

**Keywords**: e – core and $\beta-$ numbers .

# حول التحويلات اليسارية المعممة للقلب – e

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**الخلاصة:**
في (1996) قدم ريتشارد لأول مرة فكرة التحويلات اليسارية للقلب – e بواسطة استعمال عدة شروط تضمنت الفرق بين الأعمدة (k) هو اكبر من وزن التجزئة . في هذا البحث عممنا فكرة التحويل بدون استخدام هذا الشرط لـ ريتشارد معتمدين الحل رياضياً وحاسوبياً .

## 1 Introduction

Let r be a non – negative integer. A composition $\mu = (\mu_1, \mu_2, \dots , \mu_n)$ of r is a sequence of non – negative integers such that $|\mu| = \sum_{j=1}^{n} \mu_j = r$ for all $j \geq 1 j \geq 1$, if $\mu_j \geq \mu_{j+1}$ the composition $\mu$ is called a partition. [1], defined $\beta-$numbers by: Fix $\mu$ is a partition of r choose an integer b greater than or equal to the number of parts of $\mu$ and define: $\beta_i = \mu_i + b - i$, $1 \leq i \leq b$ , the set of $\beta-$numbers for $\mu$ .

Let $e \geq 2$ and we can represent $\beta-$numbers in many of runners $(C_0, C_1, C_2, \dots, C_{e-1})$ as the following :

| $C_0$ | $C_1$ | … | $C_{e-1}$ | |
|---|---|---|---|---|
| 0 | 1 | … | e-1 | |
| e | e+1 | … | 2e-1 | |
| 2e | 2e+1 | … | 3e-1 | diag.(A) |

Where every $\beta$ will be represented by a bead which takes its location in diag. (A) .

For example; if $\mu=(10, 6 , 4 , 2^3)$ and if we take b = 10, then $\beta-$numbers are {19 , 14 , 11 , 8 , 7 , 6 , 3 , 2 , 1 , 0} and if we choose e = 3, then diag. (A) is

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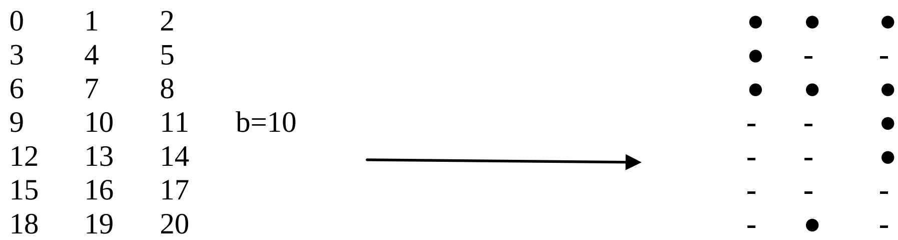
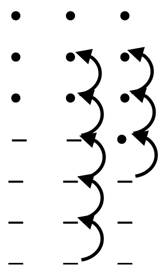
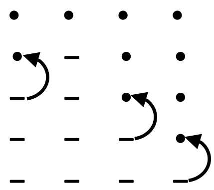



Given an abacus configuration for $\mu$. we created configuration by moving all beads as high as possible on each runner the partition, denoted by $\rho$, corresponding to this new abacus configuration is called the e − core of $\mu$ . note [2, 3]



3 − core

$\rho=(2)$



4 − core

$\rho=(6, 3^2, 1^2)$

**2 Generalized Leftly e − Core Transference (GLT)**

The concept of leftly e − core transference presented by Richards in [4] depending on the many conditions and the principal condition is the difference in the number of beads in every two adjacent columns is greater than of weight. In this section, we will try to generalize this idea without the condition of Richards, and then we will give the following definition:

**Definition (2.1) :** Let $\mu$ be a partition of r which is an e − core and suppose that is such a configuration for $\mu$ there are $C_i$ beads on the runner headed by position number i for each i = 0, 1, …, e – 1. If $C_{i+1}$ largest $C_i$ such that $C_{i+1} - C_i = k > 0$, Then

$$C_i' = \begin{cases} C_{i+1} & \text{if } k > 0 \\ C_i & \text{if } k \leq 0 \end{cases}$$

This transfer is called generalized leftly e − core transference and we denote $\bar{p}$ for any new partition after the application of definition (2.1) .

We start the biggest runner highest degrees of beads and compare it with the runner which is located to the left if he has held less than an exchange sites and symbolized for them $C_i'$ and the other $C_j'$ then continue in compared every time the largest equipped with beads, which is located on his left and each time we add $C_i^m$ to express that the runner $C_i$ move m of times .

**Rule (2.2) :** In the case of conversion to the left, the total of new partition parts will be less than of the total parts of major partition by the difference between the beads in the runners mutual .

For example, let e = 4 and b = 10, the diagram (A) for $\rho=(6, 3^2, 1^2)$ is

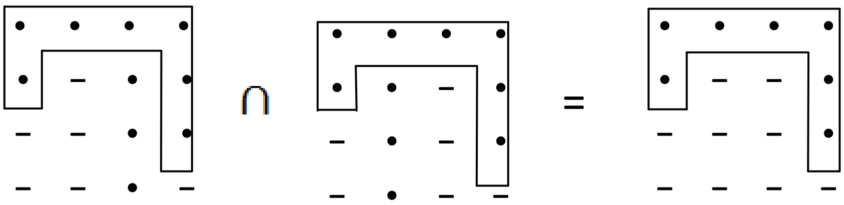
| $C_0$ | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| • | • | • | • |
| • | – | • | • |
| – | – | • | • |
| – | – | – | • |
| – | – | – | – |

| $GLT_1(C_3$ by $C_2)$ | $GLT_2(C_3'$ by $C_1)$ | $GLT_3(C_3''$ by $C_0)$ | $GLT_4(C_2'$ by $C_1')$ | $GLT_5(C_2''$ by $C_0')$ |
|---|---|---|---|---|
| $C_0\ C_1\ C_3'$<br>$C_2'$<br>• • • •<br>• − • •<br>− − • • | $C_0\ C_3''\ C_1'$<br>$C_2'$<br>• • • •<br>• • − •<br>− • − • | $C_3'''\ C_0'\ C_1'$<br>$C_2'$<br>• • • •<br>• • − •<br>• − − • | $C_3'''\ C_0'\ C_2''$<br>$C_1''$<br>• • • •<br>• • • −<br>• − • − | $C_3'''\ C_2'''\ C_0''$<br>$C_1''$<br>• • • •<br>• • • −<br>• • − − |
| $\overline{\rho_1} = (5, 3^2, 1^2)$<br>$r_1 = 13$ | $\overline{\rho_2} = (4, 3, 2, 1)$<br>$r_2 = 10$ | $\overline{\rho_3} = (3^2, 1^2)$<br>$r_3 = 8$ | $\overline{\rho_4} = (3, 2, 1)$<br>$r_4 = 6$ | $\overline{\rho_5} = (3, 1^2)$<br>and $r_5 = 13$ |

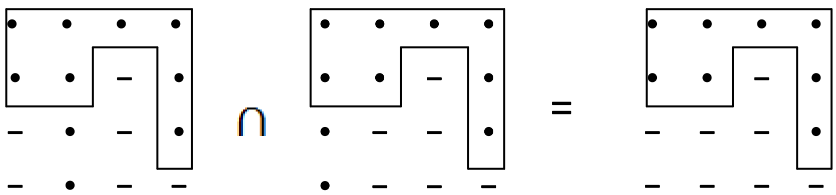
Mahmood in [5] introduced the intersection of any two diagrams. In this work we use the same (idea) of the intersection as following:

**Rule (2.3) :** The intersection of any $((GLT_i)$ and $(GLT_{i+1}), \forall i \geq 1)$ is equal to (b – k), where b is the sum of all beads in $(GLT_i)$ and k the difference of beads in $(GLT_{i+1})$.

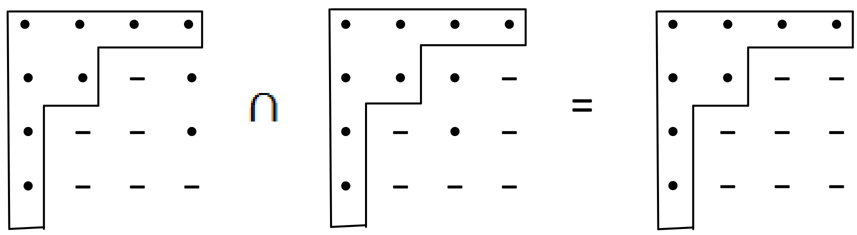
From the above example,

$GLT_1 \cap GLT_2 =$ 

where k=3

$GLT_2 \cap GLT_3 =$ 

where k=2

$GLT_3 \cap GLT_4 =$ 

where k=2

**Theorem (2.4):** Let e = 2, A and B be the number of beads in $C_0$ and $C_1$ respectively. Then

$$\cap\ GLT_i = \begin{cases} \#(A+B) & \text{if } A \le B \text{ or } A > B, \\ \#A & \text{if B is empty,} \\ \#B & \text{if A is empty,} \\ \emptyset & \text{otherwise.} \end{cases}$$

**proof :**

**<u>Case (1) if $A \le B$</u>**

Then the number of beads in $C_0$ is less than or equal the number of beads in $C_1$ and by using the definition (2.1) there exist a difference between the beads columns, and is very clear the conversion between $C_1$ and $C_0$ holds if $A < B$ only and no conversion if A = B.

Therefore the $\cap\ GLT_i = \#(A+B),\ i \ge 1$ .

**<u>and or $A > B$</u>**

If the number of beads in $C_0$ more than $C_1$ also there are no conversion and the intersection is the as the total partition beads runner is $\cap\ GLT_i = \#(A+B),\ i \ge 1$ .

**<u>Case (2) let B is empty</u>**

In this case there are no conversion and the result is beads $C_1$ which is equal to #A.

**<u>Case (3) let A is empty</u>**

In this case there is one conversion and the result of the intersection is the same beads in $C_1$ which became after conversion in $C_0$ then $\cap\ GLT_i = \#B$ ■

**Theorem (2.5) :**

let A,B and C be the number of beads in $C_0$ , $C_1$ and $C_2$ respectively where e = 3, Then

$$\cap\ GLT_i = \begin{cases} \#3A & \text{if } A \le B < C, \\ \#(2A+B) & \text{if } A \le C \le B, \\ \#(A+B+C) & \text{if } A = B > C \text{ or } A \ge C \ge B, \\ \emptyset & \text{otherwise.} \end{cases}$$

**Proof:**

**<u>Case (1) if $A \le B < C$</u>**

In this case three runners shall be filled with beads, if the number of beads in $C_0$ less than or equal to the number of beads in $C_1$ and $C_1$ is less than of $C_2$. There are three transfers to this case and that our symbol with A multiplied by 3 then

$\cap\ GLT_i = \#3A\ ,\quad i \ge 1.$

**<u>Case (2) let $A \le C \le B$</u>**

In this case three runners shall be filled with beads, if the number of beads in $C_0$ less than or equal to the number of beads in $C_2$ and $C_2$ is less than or equal from $C_1$ .

There are two transfers to this case and the intersection of these transfers will be equal to $C_0$ multiplied by 2 and plus the number of beads in $C_1$ then:

$\cap\ GLT_i = 2A + B\ ,\quad i \ge 1.$

**<u>Case (3) if $A = B > C$</u>**

In this case the runner $C_0$ and $C_1$ are equals of in the number of beads and larger than $C_2$. The intersection of all transfers equal to $\cap\ GLT_i = \#(2A+C),\ i \ge 1$ .

Is very clear if $C_2$ is empty then the intersection of this transfers is equal to

$\cap\ GLT_i = \#\,2A, \forall i \ge 1$

**<u>or $A \ge C \ge B$</u>**

Then we have one conversion ($C_1$ by $C_2$) and clearly

$\cap GLT_i = \#(A + B + C)$ , $i \geq 1$ ■

Now, for any $e \geq 2$, we design the following program for finding $\cap GLT_i$ :

```
#include<iostream.h>
#include<conio.h>
int x[100][100],m=−1 ;

void print(int a[], int n){
        for(int i=0;i<n;i++)
                cout<<a[i]<<"  ";
         cout<<endl ;
}

int max(int a[], int n, int i){
         int big=a[i], l=i ;
         for(int j=i;j<n;j++)
                 if(a[j]>big){ big=a[j]; l=j ; }
   return(l) ;
}

void swap(int a[], int n, int i){
            int l,j,t,k ;
            l=max(a,n,i) ;
           for(j=l;j>i;j--){
                  if(a[j]>a[j-1]){ t=a[j] ;
                                         a[j]=a[j-1] ;
                                         a[j-1]=t ; }
                  m++ ;
                  for(k=0;k<n;k++)
                                x[m][k]=a[k] ;
                  print(a,n);
           }
}

void main (){
         char *z="ABCDEFGHIJKLMNOPQRSTUVWXYZ" ;
          int n,i,j,a[100],aa[100],sm,ll ;
          clrscr() ;
          cin>>n ;
          for(j=0;j<n;j++){
                  cin>>a[j] ;
                  aa[j]=a[j] ;
                  x[m][j]=a[j] ;
  }
  clrscr() ;
  print(a,n) ;
  for(i=0;i<n-1;i++)
            swap(a,n,i) ;
  cout<<endl<<"=================="<<endl;
  for(j=0;j<n;j++){
           sm=x[0][j] ;
           for(i=1;i<=m;i++)
                  if(x[i][j]<sm) sm=x[i][j] ;
            if(sm>0) { for(int p=0;p<n;p++)
                                 if(sm==aa[p]){ ll=p; break ; }
```

```
                        cout<<z[ll] <<"+" ;
                       }
        }
        cout<<endl<<endl<<"Press any key to continue" ;
        getch() ;
}
```

To apply this program, the first number means the number of runners is any e — core diagram, the $2^{nd}$ number is the number of beads in first runner, … and so on.

```
3
1
5
4_
```

```
1 5 4

5 1 4
5 4 1

=================
B+A+A+

Press any key to continue_
```

```
4
2
0
3
5_
```

```
2 0 3 5

2 0 5 3
2 5 0 3
5 2 0 3
5 2 3 0
5 3 2 0

=================
A+

Press any key to continue_
```

```
6
8
4
1
6
8
4_
```

```
8 4 1 6 8 4

8 4 1 8 6 4
8 4 8 1 6 4
8 8 4 1 6 4
8 8 4 6 1 4
8 8 6 4 1 4
8 8 6 4 4 1

=================
A+B+C+C+C+C+

Press any key to continue_
```

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