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Study of Longitudinal Electron Scattering Form Factors with the Radial Wave functions of Transformed Harmonic-Oscillator for some Light Nuclei

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Abstract

The wave functions of converted harmonic-oscillator in local scaling transformations are employed to evaluate charge distributions and elastic charge electron scattering form structures for ^{6,7}Li, ^{9}Be, ^{14,15}N and ^{16}O nuclei. The nuclear shell-model was fulfilled using Warburton-Brown psd-shell (WBP) interaction with $(0+2+4)\hbar\omega$ truncation in $spsdpf$ model space. Very good agreements with the experimental data were obtained for the aforementioned quantities.

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Keywords: Transformed Harmonic-Oscillator Wave functions, Longitudinal Electron Scattering Form Factors, Charge Density Distributions, Size radii

دراسة عوامل التشكل للأستطارة الألكترونية الطولية مع الدوال الموجية القطرية للمتذبذب التوافقي المحور لبعض النوى الخفيفة

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الخلاصة

الدوال الموجية القطرية للمتذبذب التوافقي المحور بتحويلات المقياس الموضعي استخدمت لحساب توزيعات الكثافة الشحنية وعوامل التشكل للاستطارة الالكترونية المرنة للنوى ^{6,7}Li و ^{9}Be و ^{14,15}N و ^{16}O. انجزت حسابات انموذج القشرة النووي باستخدام تفاعل WBP مع قطع فضاء الحركة بمقدار $(0+2+4)\hbar\omega$ في انموذج فضاء القشرة $spsdpf$. استحصلت تطابقات جيدة جدا بالمقارنة مع القيم العملية للكميات المذكورة.

1. Introduction

The proper use of wave functions in nuclear physics is essential in studying nuclear bulk properties, such as size radii, density distributions, electromagnetic form factors, total binding energy, magnetic dipole, and electric quadrupole moments [1]. The use of bare oscillator wave functions is not acceptable because such potential has a Gaussian steep-slope performance at the asymptotic region; besides the potential goes to infinity for large r. A realistic potential or a modification must be adopted. Elton and Swift [2] used the radial wave functions of Woods-Saxon (WS) potential with great success for some light and medium nuclei. Noori and Ridha

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[3] used the WS to calculate Charge Density Distributions (CDDs) and elastic longitudinal electron scattering form structures with very good results for some stable and exotic nuclei. The Local-Scaling Transformation (LST) was used by Karataglidis and Amos [4] to study the halo nuclei with results similar to WS potential. Mohammed and Ridha [5] used the LST successfully to study stable and unstable boron isotopes. The effect of correlation among nucleons on the density distributions and Coulomb form factors are a good approach used in the works of Al-Rahmani et al. [6] and Hussein and Flaiyh [7] obtaining good results when compared with empirical data for some stable and exotic nuclei. The inclusion of occupation numbers was used by Raheem and Abdulhasan [8] to investigate some nuclei in 1f-2p shell with very good results. Mohammed and Fatah [9] obtained good results for studying the elastic and inelastic Coulomb transitions in ^{17}O using the theory of Hartree-Fock. Besides, one of the important mean fields which was used by Odah and Ridha [10] with excellent agreement with laboratory data for nitrogen isotopes is the Cosh potential. Finally, another treatment to amend the theoretical calculations was done by Khalaf and Ridha [11] by connecting the interior harmonic-oscillator part with the exterior modified Bessel part; such an approach gave very good results on stable and exotic fluorine isotopes.

In the current work, the technique of LST is adopted and applied to Harmonic Oscillator Wave Functions (HO WFs), then tested to evaluate CDDs and elastic form structures for the stable ^{6,7}Li, ^{9}Be, ^{14,15}N and ^{16}O nuclei using Warburton-Brown psd-shell (WBP) interaction in enlarged model with $(0+2+4)\hbar\omega$ truncation.

2. Theoretical bases

The transition proton/neutron density distribution is given by [3]:

$$\rho_{J,t_z}(r) = \frac{1}{\sqrt{4\pi}} \frac{1}{\sqrt{2J_i+1}} \sum_{ab} X_{a,b,t_z}^{J_f,J_i,J} \langle j_b \| Y_J \| j_a \rangle R_{n_a l_a j_a, t_z}(r) R_{n_b l_b j_b, t_z}(r) \quad (1)$$

The terms, n, l, j and t_z $\left(t_z = \frac{1}{2}\ for\ protons\ and\ t_z = -\frac{1}{2}\ for\ neutrons\right)$ denote the principle, orbital, total spin and isospin's projection quantum numbers, respectively, for individual nucleon; J_i and J are the overall initial nucleus spin and multiplicity, respectively; a and b stand for nuclear states in the initial and final cases, respectively; $X_{a,b,t_z}^{J_f,J_i,J}$ is the single-nucleon transition matrix element obtained from shell model calculations for an opted effective interaction and model space; $\langle j_b \| Y_J \| j_a \rangle$ represents the spherical-harmonics matrix element as given by Brussard and Glademans [12]; and $R_{n_a l_a j_a, t_z}(r)$ is the radial wave function of an individual nucleon in a state a. In the LST, the transformed HO (THO) wave functions can be written as [7]:

$$R_{nl}^{THO}\left(r, b_{t_z}\right) = s(r) R_{nl}^{HO}\left(f(r), b_{t_z}\right) \quad (2)$$

Where: b is the HO size parameter and $s(r)$ is the wave function scale, which can be written as [7]:

$$s(r) = \frac{f(r)}{r} \sqrt{\frac{df(r)}{dr}} \quad (3)$$

The scalar function ($f(r)$) in Eqs. (2) and (3) is assumed to take the following form [7]:

$$f(r) = \left[\frac{1}{\left(\frac{1}{r}\right)^{m} + \left(\frac{1}{\gamma\sqrt{r}}\right)^{m}} \right]^{\frac{1}{m}} \tag{4}$$

The m and γ in Eq. (4) are numerical numbers governor how gradually the tail of wave function will be. The root-mean square radius (rms), $\langle r^2 \rangle_x^{1/2}$, can be written as [13]:

$$\langle r^2 \rangle_x^{1/2} = \sqrt{\frac{4\pi}{x} \int_0^{\infty} \rho_x(r) r^4 dr} \tag{5}$$

x stands for either N (number of neutrons), or Z (number of protons) or A (number of nucleons).

The Charge Density Distribution (CDD) is given by:

$$\rho_{ch}(r) = \rho_{ch,t_z=1/2}(r) + \rho_{ch,t_z=-1/2}(r) \tag{6}$$

The first and second parts in Eq. (6) represent the contribution to CDD from protons [13] and neutrons [14] respectively, both are given by:

$$\rho_{ch,t_z=1/2}(r) = \int \rho_{J=0,t_z=1/2}(r) \rho_{pr}(r - r') dr' \tag{7}$$

$$\rho_{ch,t_z=-1/2}(r) = \int \rho_{J=0,t_z=-1/2}(r) \rho_{neu}(r - r') dr' \tag{8}$$

$\rho_{pr}(\vec{r})$ and $\rho_{neu}(\vec{r})$ in Eqs. (7) and (8) are given by:

$$\rho_{pr}(r) = \frac{1}{\left(\sqrt{\pi} a_{pr}\right)^3} e^{\left(\frac{-r^2}{a_{pr}^2}\right)} \tag{9}$$

$$\rho_{neu}(r) = \frac{1}{(\pi r_i^2)^{\frac{3}{2}}} \sum_1^2 \theta_i \, e^{\frac{-r^2}{r_i^2}} \tag{10}$$

In Eq. (9), $a_{pr} = 0.65\, fm$ that rebreed the empirical charge rms of the a single proton; $\langle r^2 \rangle_{pr}^{1/2} = \left(\frac{3}{2}\right)^{1/2} a_{pr} \approx 0.8\, fm$. The constants θ_i and r_i are taken from the works of Elton [13], and Chandra and Sauer[14].

The squared coulomb longitudinal form structure in a Plane-Wave Born Approximation (PWBA) is given by [15]:

$$|F(q)|^2 = \sum_J \left| F_{J,ch}^C(q) \right|^2 \tag{11}$$

$F_{J,ch}^C(q)$ in Eq. (11) can be expressed as:

$$F_{J,ch}^C(q) = \frac{1}{Z} \sqrt{\frac{4\pi}{(2J_i+1)}} \sum_{t_z} \langle J_f \| O_J^C(q, t_z) \| J_i \rangle f_{t_z}(q) \tag{12}$$

and

$$F_J^C(q,t_z) = \frac{1}{Z}\sqrt{\frac{4\pi}{(2J_i+1)}}\langle J_f \| O_J^C(q,t_z) \| J_i \rangle \quad (13)$$

Then, Eq. (11) can be reduced to:

$$F_{J,ch}^C(q) = \sum_{t_z} \quad F_J^C(q,t_z) f_{t_z}(q) \quad (14)$$

$$F_{J,ch}^C(q) = \sum_{t_z} \quad F_{J,ch}^C(q,t_z) \quad (15)$$

and

$$F_{J,ch}^C(q,t_z) = F_J^C(q,t_z) f_{t_z}(q) \quad (16)$$

Where: q represents momentum transferred to nucleus from electron in scattering process, $|J_i\rangle$ and $|J_f\rangle$ are the initial and final wave functions of the nucleus, respectively, $f_{t_z}(q)$ represents the charge form structure of an individual proton or neutron, and $O_J^C(q,t_z)$ is the longitudinal multipole operator of electron scattering [15,16].

In Eq. (12), the many-particle matrix elements is written in terms of the individual-nucleon matrix elements by the relation:

$$\langle J_f \| O_J^C(q,t_z) \| J_i \rangle = \sum_{ab} \quad X_{a,b,p/n}^{J_i J_f J} \langle b,t_z \| O_J^C(q,r,t_z) \| a,t_z \rangle \quad (17)$$

since,

$$O_J^C(q,r,t_z) = e_{t_z} j_J(qr) Y_J(\Omega_r) \quad (18)$$

At photon point($q = k = \frac{E_x}{\hbar c}$), E_x is the excitation energy, where $qr \ll 1$:

$$\left|F_{J,ch}^C(q=k)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)}\left(\frac{q^J}{(2J+1)!!}\right)^2 \left|\int_0^\infty \quad \rho_{ch,J}(r)\, r^{J+2} dr\right|^2 \quad (19)$$

The reduced transition probability, $B\left(CJ, J_i^{\pi_i} \to J_f^{\pi_f}\right)$, is associated to electric multipole operator by:

$$B\left(CJ, J_i \to J_f\right) = \frac{\left|\langle J_f \| O_{J=2}(\vec{r}) \| J_i \rangle\right|^2}{(2J_i+1)} \quad (20)$$

Since, quadrupole moment is expressed as [15,16]:

$$\because \; Q = \sqrt{\frac{16\pi}{5}} (J\ 2\ J\ -J\ 0\ J) \langle J_f \| O_{J=2}(\vec{r}) \| J_i \rangle \quad (21)$$

$$\because \; \langle J_f \| O_{J=2}(\vec{r}) \| J_i \rangle = \int_0^\infty \quad \rho_{ch,J}(r)\, r^{J+2} dr \quad (22)$$

$$\therefore \; Q = \sqrt{\frac{16\pi}{5}} (J\ 2\ J\ -J\ 0\ J) \int_0^\infty \quad \rho_{ch,J}(r)\, r^{J+2} dr \quad (23)$$

$$\therefore \; Q = \sqrt{\frac{16\pi}{5}} (J\ 2\ J\ -J\ 0\ J) \sqrt{(2J_i+1) B\left(EJ, J_i \to J_f\right)} \quad (24)$$

In this work, the inclusion of Core-Polarization (CP) effect was needed to improve the calculated components of the charge form factors. In Eq. (19), the transition distribution was accounted from the influence from Model-Space (MS) and CP which is given by [16]:

$$\rho_{ch,J}(r) = \rho^{CP}_{ch,J}(r) + \rho^{MS}_{ch,J}(r) \tag{25}$$

Two mathematical forms of CP were chosen, the Tassie [17] and Bohr-Mottelson [18] models are given respectively by:

$$\rho^{CP}_{ch,J}(r) = N_T r^{J-1} \frac{d}{dr}\rho_{ch}(r) \tag{26}$$

$$\rho^{CP}_{ch,J}(r) = N_{BM} \frac{d}{dr}\rho_{ch}(r) \tag{27}$$

the contribution from MS is given by:

$$\rho^{MS}_{J,t_z}(r) = \frac{1}{\sqrt{4\pi}}\frac{1}{\sqrt{2J_i+1}}\sum_{ab} \quad X^{J_f\ J_i\ J}_{a,b,t_z} \langle j_b \rangle R_{n_a l_a j_a, t_z}(r) R_{n_b l_b j_b, t_z}(r) \tag{28}$$

The constants in Eqs. (26) and (27) are chosen to reproduce the experimental quadrupole moments calculated by Eqs. (23) and (24) and are given below:

$$N_T = \frac{\sqrt{(2J_i+1)B(EJ, J_i \to J_f)} - \int_0^\infty \quad r^{J+2}\rho^{MS}_{J,ch}(r)dr}{\int_0^\infty \quad r^{2J+1}\frac{d\rho_{ch}(r)}{dr}dr} \tag{29}$$

$$N_{BM} = \frac{\sqrt{(2J_i+1)B(EJ, J_i \to J_f)} - \int_0^\infty \quad r^{J+2}\rho^{MS}_{J,ch}(r)dr}{\int_0^\infty \quad r^{J+1}\frac{d\rho_{ch}(r)}{dr}dr} \tag{30}$$

3. Outcomes and discussion

The nuclear shell model was fulfilled using Nushell code [19] with WBP interaction [20] and $spsdpf$ model space. The $(0+2+4)\hbar\omega$ truncation for ^{6,7}Li, ^{9}Be, ^{14,15}N and ^{16}O nuclei was adopted. In Table 1, the total spin, total isospin and the parameters of THO were tabulated. The constants of THO were chosen to rebreed the available size radii. The evaluated size radii are given in Table 2.

In Figure 1, the evaluated CDDs are depicted and compared with the empirical data. The solid curves stand for the theoretical computations using THO wave functions. The dotted curves represent the empirical data. In general, very good agreement with empirical data was noted. In Figure 2, the evaluated charge form structures (solid curves) are presented and compared with the available empirical data (dotted curves). It was clear from calculations that the inclusion of the CP effect was highly improved with the Bohr-Mottelson than with the Tassie model. The constants in Eqs. (26) and (27) were adjusted so as to reproduce the empirical quadrupole moments of ^{6}Li ($-0.082 \pm 0.002\ e.fm^2$) [21], ^{7}Li ($-4.06 \pm 0.08\ e.fm^2$) [21] and ^{9}Be ($5.29 \pm 0.04\ e.fm^2$) [21].

Table 1: The total spin, isospin and the parameters of THO

ZAX_N	$J^{\pi}T$ [22]	$b_n(fm)$	$b_p(fm)$	m	$\gamma_n(fm^{-1})$	$\gamma_p(fm^{-1})$
$36Li_3$	1^+0	1.627	1.6	10	1.784	1.737
$37Li_4$	$\frac{3}{2}^- \frac{1}{2}$	1.68	1.66	14	2.624	2.182
$49Be_5$	$\frac{3}{2}^- \frac{1}{2}$	1.75	1.69	20	2.678	2.544
$714N_7$	1^+0	1.664	1.62	24	2.804	2.650
$715N_8$	$\frac{1}{2}^- \frac{1}{2}$	1.7	1.699	16	2.945	2.691
$816O_8$	0^+0	1.733	1.736	18	3.005	2.846

Table 2: The evaluated rms proton, neutron, charge and matter radii in fm unit for ^{6,7}Li, ^{9}Be, ^{14,15}N and ^{16}O nuclei

ZAX_N	$\langle r_{ch}^2\rangle^{1/2}$	Exp. $\langle r_{ch}^2\rangle^{1/2}$ [23]	$\langle r_p^2\rangle^{1/2}$	$\langle r_n^2\rangle^{1/2}$	$\langle r_m^2\rangle^{1/2}$	Exp. $\langle r_m^2\rangle^{1/2}$ [24]
$36Li_3$	2.560	2.56(5)	2.457	2.463	2.460	2.46 ± 0.21
$37Li_4$	2.390	2.39(3)	2.288	2.396	2.350	2.33 ± 0.02
$49Be_5$	2.509	2.50(9)	2.410	2.553	2.490	2.53 ± 0.072
$714N_7$	2.541	2.524(23)	2.437	2.503	2.470	2.47 ± 0.03
$715N_8$	2.650	2.650	2.554	2.575	2.565	2.42 ± 0.1
$816O_8$	2.730	2.730(25)	2.633	2.629	2.631	2.631 ± 0.061

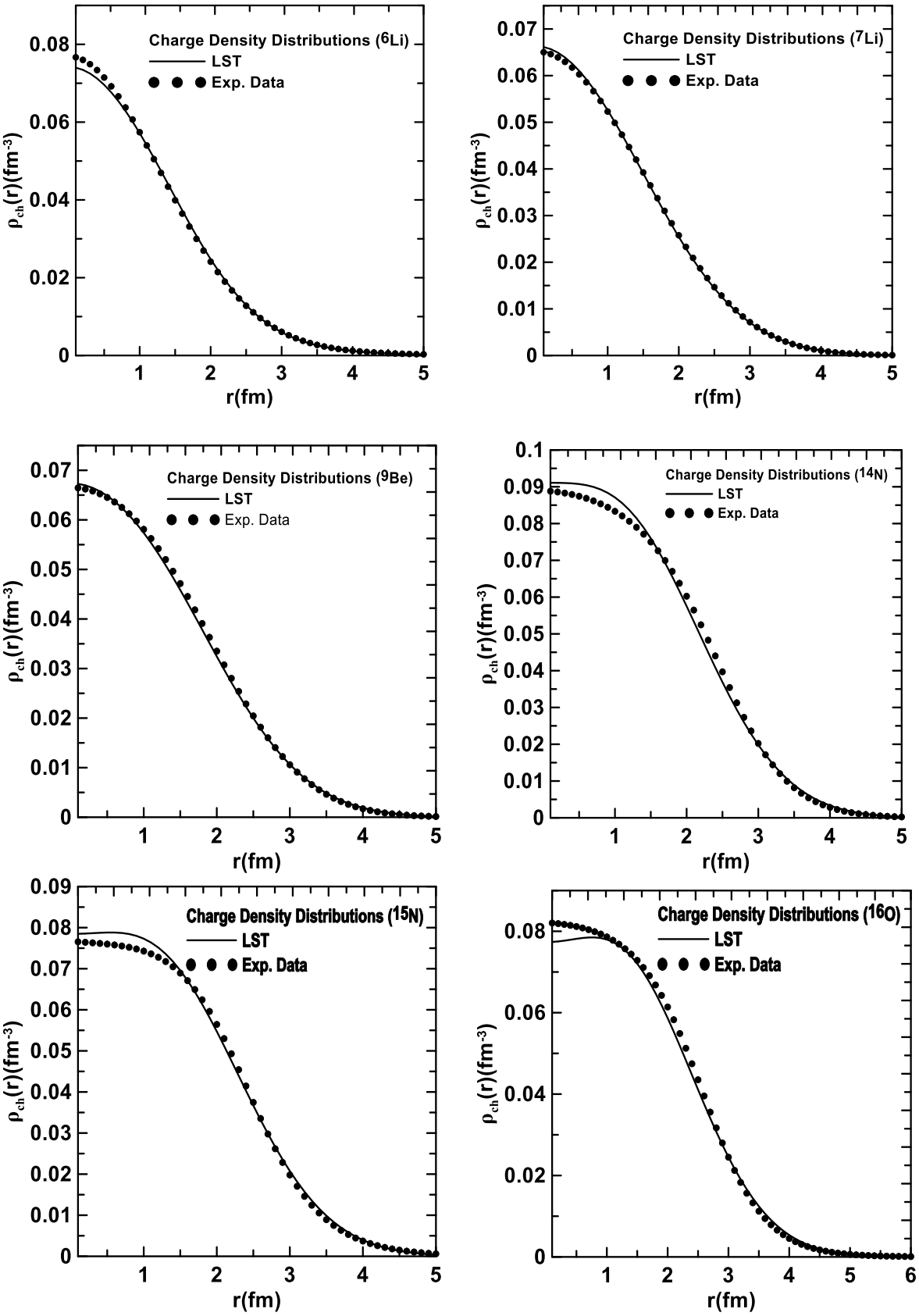

Figure 1: CDDs for ^{6,7}Li, ^{9}Be, ^{14,15}N and ^{16}O nuclei. The dotted curves are The experimental data [23].

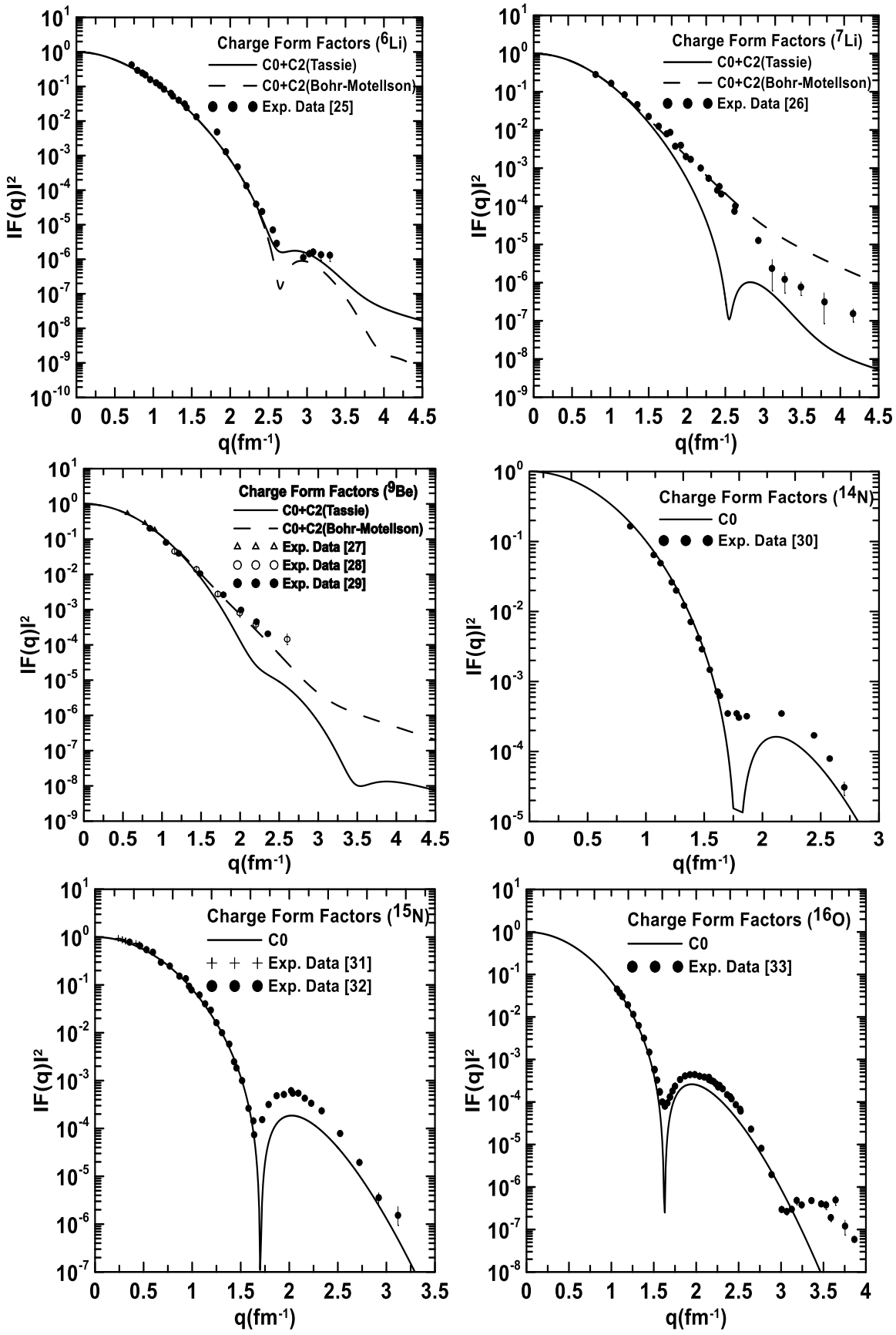

Figure 2: Charge form factors for ^{6,7}Li, ^{9}Be, ^{14,15}N and ^{16}O nuclei.

4. Conclusions

The converted harmonic-oscillator wave functions in the local-scaling transformation (LST) were harnessed to evaluate the charge density distributions (CDDs) and elastic form structures of electron scattering. The constants of THO were adjusted to rebreed the size radii for ^{6,7}Li, ^{9}Be, ^{14,15}N and ^{16}O nuclei. These nuclei were studied in the shell model using WBP interaction with $(0+2+4)\hbar\omega$ truncation in $spsdpf$ model space. Such extended model space, alongside using the radial wave functions of THO improved the calculated CDDs and form factors. The use of the core-polarization (CP) effect has highly improved the evaluated charge form structures, especially when using the Bohr-Mottelson model; in the Tassie model, the calculations were rather good.

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