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Strong and Weak Forms of μ-Kc-Spaces

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Abstract
In this paper, we provide some types of μ-Kc-spaces, namely, μ-$K(\alpha c)$- (respectively, μ-$\alpha K(\alpha c)$- , μ-$\alpha K(c)$- and μ-$\theta K(c)$-) spaces for minimal structure spaces which are denoted by (m-spaces). Some properties and examples are given. The relationships between a number of types of μ-Kc-spaces and the other existing types of weaker and stronger forms of m-spaces are investigated. Finally, new types of open (respectively, closed) functions of m-spaces are introduced and some of their properties are studied.

Keywords: Kc-space, minimal structure spaces, μ-Kc-space, α-open , θ-open.

الصيغ القوية و الضعيفة للفضاء $\mu-$Kc

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الخلاصة
في هذا البحث قدمنا بعض الانواع من فضاءات$-\mu-Kc$ اي فضاءات $-\mu-\alpha K(c)$, $-\mu-\alpha K(\alpha c)$, $\mu-K(\alpha c)$ $(-\mu-\theta K(c)$ على التوالي) لفضاءات minimal structure والدي رمزنا له (فضاء–m). واعطيت بعض الخصائص والامثلة. العلاقات بين بعض الانواع من فضاءات $\mu-Kc$ والانواع الموجودة الاخرى من الصيغ الاضعف و الاقوى لفضاء–m حققت. اخيرا انواع جديدة من الدوال المفتوحة (المغلقة على التوالي) في فضاء–m قدمت ودرست بعض صفاتها.

1. Introduction

The concept of Kc-space was introduced by Wilansky [1], that is "A topological space $(\mathcal{X},\mathcal{T})$ is said to be Kc-space if every compact subset of $\mathcal{X}$ is closed". Also, many important properties were provided by that study, e.g., "Every Kc-space is T_1-space" and "every T_2-space is Kc-space". In 1996, Maki [2] introduced the minimal structure spaces , shortly m-spaces, that is " A sub collection μ of $P(\mathcal{X})$ is called the minimal structure of $\mathcal{X}$, if $\emptyset \in \mu$ and $\mathcal{X} \in \mu$, $(\mathcal{X},\ \mu)$ is said to be m-structure space". The elements of μ are called μ-open sets and their complements are μ-closed sets, which is a generalization of topological spaces. Popa and Noiri [3] studied the m-spaces and defined the notion of continuous functions between them. In 2015, Ali et al. [4] defined the concept of Kc-space with

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respect to the m-space to obtain a new space which they called the μ-Kc-space. A weaker and stronger form of open sets plays an important role in topological spaces. In 1965, Najasted [5] introduced the concept of α-open sets as a generalization of open sets. That is, let $(\mathcal{X},\mathcal{T})$ be a topological space and a nonempty subset $\mathcal{A}$ of $\mathcal{X}$ is said to be α-open set, if $\mathcal{A} \subseteq Int(Cl(\,Int(\mathcal{A})))$. In 2010, Min [6] generalized the concept of α-open sets to m-spaces. On the other hand, in 1968, Velicko [7] introduced the concept of θ-open sets. That is "Let $(\mathcal{X},\mathcal{T})$ be a topological space, $\mathcal{N} \subseteq \mathcal{X}$, a point $b \in X$ is said to be an $\theta\mu$-adherent point for a subset $\mathcal{N}$ of $\mathcal{X}$, if $\mathcal{N} \cap Cl(G) \neq \emptyset$ for any open set G of $\mathcal{X}$ and $b \in \mathcal{N}$. The set of θ-adherent point is said to be an θ-closure of $\mathcal{N}$ which is denoted by $\theta Cl(\mathcal{N})$. A subset $\mathcal{N}$ of $\mathcal{X}$ is called θ-closed set if every point to $\mathcal{N}$ is an θ-adherent point. Also, in 2018, Makki [8] defined θ-open sets in m-space. The aim of the present paper is to introduce and study new type of μ-Kc-spaces, namely, μ-$K(\alpha c)$- (resp. μ-$\alpha K(c)$- , μ-$\alpha K(\alpha c)$- and μ-$\theta K(c)$-) spaces by using the concept of α-open, respectively θ-open sets, with respect to the m-space. We study the basic properties of each space and give the relationships between them. Also, we introduce new kinds of continuous, open (respectively closed) functions on m-spaces and investigate their properties.

2.Preliminaries

Let us recall the following definitions, properties and theorems which we need in this work

Definition 2.1 [3] Let $\mathcal{X}$ be a non-empty set and $P(X)$ be the power set of $\mathcal{X}$. A sub collection μ of $P(\mathcal{X})$ is called the minimal structure of $\mathcal{X}$, if $\emptyset \in \mu$ and $\mathcal{X} \in \mu$, $(\mathcal{X},\mu)$ is said to be m-structure space (shortly, m-spaces). The elements of μ are called μ-open sets and their complements are μ-closed sets.
For a subset $\mathcal{B}$ in an m-space on $(\mathcal{X},\mu)$, the interior (respectively, closure) of $\mathcal{B}$ denoted by $\mu Int(\mathcal{B})$ (respectively, $\mu Cl(\mathcal{B})$) is defined as follows:
$\mu\mathrm{Int}(\mathcal{B}) = \cup\{U: U \subseteq \mathcal{B}, U \in \mu\}$ and $\mu Cl(\mathcal{B}) = \cap\{F: \mathcal{B} \subseteq \mathcal{F}, \mathcal{F}^c \in \mu\}$.

Remark 2.2 Note that according to a previous study [9], $\mu\mathrm{Int}(\mathcal{B})$ (respectively, $\mu Cl(\mathcal{B})$) is not necessarily μ-open (respectively, μ-closed), but if $\mathcal{B}$ is μ-open then $\mathcal{B} = \mu\mathrm{Int}(\mathcal{B})$, respectively, and if $\mathcal{B}$ is μ-closed, then
$\mathcal{B} = \mu Cl(\mathcal{B})$.

Definition 2.3 [10] an m-space $(\mathcal{X},\mu)$ has a property β (respectively ϒ) if the union (respectively intersection) of any family (respectively finite subsets) of μ also belongs to μ.

Definition 2.4 [6] A subset A of an m-space $(\mathcal{X},\mu)$ is said to be an $\alpha\mu$-open, if $A \subseteq \mu Int(\mu Cl(\mu Int(A)))$.The complement of $\alpha\mu$-open set is called $\alpha\mu$-closed set or, equivalently, $\mu Cl\left(\mu Int(\mu Cl(A))\right) \subseteq A$.

Definition 2.5 [6] An m-space $(\mathcal{X},\mu)$ has a property $\alpha\Upsilon$, if the intersection of finite $\alpha\mu$-open sets is an $\alpha\mu$-open set in $\mathcal{X}$.

Remark 2.6 [6] From Definition 2.4, it is clear that every μ-open (respectively μ-closed) set is an $\alpha\mu$-open (respectively $\alpha\mu$-closed) set.

Definition 2.7 [10] Let $(\mathcal{X},\mu)$ be an m-space. A point $x \in \mathcal{X}$ is called an $\alpha\mu$-adherent point of a set $A \subseteq \mathcal{X}$ if and only if $G \cap A \neq \emptyset$ for all $G \in \mu$ such that $x \in G$ .The set of all $\alpha\mu$-adherent points of a set A is denoted by $\alpha\mu ICl(A)$, where $\alpha\mu Cl(A) = \cap\{F: A \subseteq F, F\ is\ \alpha\mu\text{-closed set}\}$.

Proposition 2.8 [6] A subset F of m-space $\mathcal{X}$ is αμ-closed set in $\mathcal{X}$ iff $\mathrm{F} = \alpha\mu Cl(F)$.

Definition 2.9 [7] Let $(\mathcal{X},\mu)$ be an m-space, $\mathcal{A} \subseteq \mathcal{X}$. Then $a \in \mathcal{X}$ is said to be $\alpha\mu$-interior point to $\mathcal{A}$ iff $\in U \subseteq \mathcal{A}$, for some $\alpha\mu$-open set U and $x \in U$. The $\alpha\mu$ -interior point of a set $\mathcal{A}$ is all $\alpha\mu$-interior point to $\mathcal{A}$ and denoted by $\alpha\mu Int(\mathcal{A})$, where $\alpha\mu Int(\mathcal{A}) = \bigcup\{U: U \subseteq \mathcal{A}, U\ is\ \alpha\mu\text{-open set}\}$.

Proposition 2.10 [6] any subset of m-space $\mathcal{X}$ is αμ-open set iff every point in it is αμ -interior point.

Remark 2.11 [6] If $(\mathcal{X},\mu)$ is an m-space, then:

1. The union of any family of $\alpha\mu$-open sets is $\alpha\mu$-open set.
2. The intersection of any two $\alpha\mu$-open sets may be not $\alpha\mu$-open set.

Definition 2.12 [12] An m-space, $(\mathcal{X},\mu)$ is called μ-compact if any μ-open cover of $\mathcal{X}$ has a finite subcover. A subset $\mathcal{H}$ of an m-space is said to be μ-compact in $\mathcal{X}$, if for any cover by μ-open of $\mathcal{X}$, there is a finite subcover of $\mathcal{H}$.

Proposition 2.13 [11] Every μ-closed set in μ-compact space is an μ-compact set.

Definition 2.14 [6] An m-space $(\mathcal{X}, \mu)$ is said to be $\alpha\mu$-compact space if any $\alpha\mu$-open cover of $\mathcal{X}$ has a finite subcover. A subset $\mathcal{B}$ of m-space $\mathcal{X}$ is called $\alpha\mu$-compact, if any $\alpha\mu$-open set of $\mathcal{X}$ which covers $\mathcal{B}$ has a finite subcover of $\mathcal{B}$.
Remark 2.15 Any $\alpha\mu$-compact is μ-compact set. However the converse is not necessarily true as shown by the following example.
Example 2.16 Let $\mathcal{R}$ be the set of real numbers and $\mathcal{X}$ be a non-empty set such that $\mathcal{X}=\{x\} \cup \{r: r \in \mathcal{R}\}$, where $x \in \mathcal{X}$. Also $\mu=\{\phi, \mathcal{X}, \{x\}\}$, then $\mathbb{C}=\{\{x, r\}: r \in \mathcal{R}\}$ is an $\alpha\mu$-open cover to $\mathcal{X}$. Since$\{x, r\} \subseteq \mu Int\left(\mu Cl\left(\mu Int(\{x,r\})\right)\right) = \mathcal{X}$, so $\{x, r\}$ is an $\alpha\mu$-open set. Now, $\mathbb{C}$ is an $\alpha\mu$-open cover to $\mathcal{X}$, but it has no finite subcover to $\mathcal{X}$, since, if we remove {x, 50} then the reminder is not cover $\mathcal{X}$ (cover all $\mathcal{X}$ except 50), and it is infinite cover. Hence, $\mathcal{X}$ is not $\alpha\mu$-compact space and it is clear that $\mathcal{X}$ is μ-compact space, since the only μ-open cover of $\mathcal{X}$ is $\mathcal{X}$ itself, which is one set, that is, a finite open cover to $\mathcal{X}$.
Definition 2.17 [10] An m-space is called an μ-T_1-space, if for any two points a, b in $\mathcal{X}$, $a \neq b$ there is two μ-open sets N, M such that $a \in$ N, but $b \notin$ N and $b \in$ M but $a \notin$ M.
Proposition 2.18 [4] An m-space is μ-T_1-space if and only if every singleton set is μ-closed set, whenever $\mathcal{X}$ has β property.
Definition 2.19 [10] An m-space is said to be $\alpha\mu$-T_1-space, if for every two t points c, d in $\mathcal{X}$, there are two $\alpha\mu$-open sets $\mathcal{K}, \mathcal{H}$ with $c \in \mathcal{K}$, but $c \notin \mathcal{H}$ and $d \in \mathcal{H}$ but $d \notin \mathcal{K}$.
Remark 2.20 [10] Every μ-T_1-space is $\alpha\mu$-T_1-space.
Definition 2.21 [10] An m-space $(\mathcal{X},\ )$ is called μ-T_2-space (respectively $\alpha\mu$-T_2-space), if for any two distinct points x, y in $\mathcal{X}$, there are two μ-open (respectively $\alpha\mu$-open) U, V, such that $x \in U, y \in V$, and $U \cap V = \emptyset$.
Definition 2.22 [4] An m-space $(\mathcal{X}, \mu)$ is said to be μ-Kc-space if any μ-compact subset of $\mathcal{X}$ is μ-closed set.
Example 2.23 Let $\mathcal{R}$ be the real numbers, $(\mathcal{R}\ , \mu_{\mathrm{U}})$ is the usual μ-space which is μ-Kc-space.
Proposition 2.24 [12] Every μ-compact set in μ-T_2-space, that has the property β and Υ, is μ-closed set.
Remark 2.25 [4]
1. Every μ-Kc space is μ-T_1-space.
2. Every μ-T_2-space with the property β and Υ is μ-Kc-space.

Definition 2.26 Let $f: (\mathcal{X}, \mu) \longrightarrow (\mathcal{Y}, \mu')$ be a function. Then f is called:
1. m-continuous **[15]** iff for any μ'-open $\mathcal{N}$ in , the inverse image $f^{-1}(\mathcal{N})$ is an μ-open set in $\mathcal{X}$.
2. αm-continuous **[6]** iff for any μ'-open set $\mathcal{M}$ in $\mathcal{Y}$, the inverse image $f^{-1}(\mathcal{M})$ is an αμ-open set in $\mathcal{X}$.

Proposition 2.27 [14] The m-continuous image of μ-compact is μ'-compact.
Definition 2.28 [4] A function $f: (\mathcal{X}, \mu) \longrightarrow (\mathcal{Y}, \mu')$ is said to be m-homeomorphism, if f is injective, surjective, continuous and f^{-1} continuous. If there exists an m-homeomorphism between $(\mathcal{X}, \mu)$ and $(\mathcal{Y}, \mu')$ then we say that $(\mathcal{X}, \mu)$ m-homeomorphic to $(\mathcal{Y}, \mu')$.
Definition 2.29 [13] Let $(\mathcal{X}, \mu)$ be m-space, $\mathcal{F}$ be a subset of $\mathcal{X}$ and $x \in \mathcal{X}$. A point x is called an $\theta\mu$-interior point of $\mathcal{F}$ if there is $\mathcal{C} \in \mu$ such that $x \in \mathcal{C}$ and $x \in \mu Cl(\mathcal{C}) \subseteq \mathcal{F}$. And $\theta\mu$-interior set which is denoted by $\theta\mu Int(\mathcal{F})$ is the set of all $\theta\mu$-interior points. A subset $\mathcal{F}$ of $\mathcal{X}$ is called an $\theta\mu$-open set if every point of $\mathcal{F}$ is an $\theta\mu$- interior point.
Definition 2.30 [13] Let $(\mathcal{X}, \mu)$ be m-space, $H \subseteq \mathcal{X}$, a point $b \in X$ is said to be an $\theta\mu$-adherent point for a subset H of $\mathcal{X}$, if $H \cap \mu Cl(G) \neq \emptyset$ for any μ–open set G of $\mathcal{X}$ and $b \in H$. The set of $\theta\mu$-adherent point is said to be an $\theta\mu$-closure of H, which is denoted by $\theta\mu Cl(H)$. A subset H of $\mathcal{X}$ is called $\theta\mu$-closed set if every point to H is an $\theta\mu$-adherent point.
Example 2.31 Any subset of a discrete m-space $(\mathcal{R}, \mu_D)$ on a real number $\mathcal{R}$ is $\theta\mu$-closed set and $\theta\mu$-open set.
Definition 2.32 [8] An m-space $(\mathcal{X}, \mu)$ is said to have the property $\theta\Upsilon$ (respectively $\theta\beta$) if the intersection (respectively union) of any finite number (respectively family) of $\theta\mu$-open sets is an $\theta\mu$-open set.
Remark 2.33 [8] If an m-space $(\mathcal{X}, \mu)$ has $\theta\Upsilon$ property, then every $\theta\mu$-closed is an μ-closed.

Definition 2.34 [8] Let $(\mathcal{X},\mu)$ be m-space**,** $\mathcal{X}$ is said to be $\theta\mu$-compact if any $\theta\mu$-open cover of X has a finite subcover. A subset A of an m-space $(\mathcal{X},\mu)$ is said to be $\theta\mu$-compact if for any $\theta\mu$-open cover $\{V_\alpha : \alpha \in I\}$ of $\mathcal{X}$ and cover A then there is a finite subset $\{\alpha_0,\alpha_1,\alpha_2,\dots,\alpha_n\}$ such that $A \subseteq \bigcup_{i=1}^{n} V_{\alpha_i}$.
Example 2.35 Let $(\mathcal{R}, \mu_{ind})$ be an m-space where μ_{ind} be indiscrete m-space on a real number $\mathcal{R}$, so is $\theta\mu$-compact.
Remark 2.36 [8] Every μ-compact with the property $\theta\beta$ is $\theta\mu$-compact.
Definition 2.37 [8] An m-space $(\mathcal{X},\mu)$ is called $\theta\mu$-T_2 -space, if for every two points a,b that belong to $\mathcal{X}$, $a \neq b$, there is $\theta\mu$-open sets M and N containing a and b, respectively, such that $M \cap N = \emptyset$.
Definition 2.38 [8] Let $(\mathcal{X}, \mu)$ and $(\mathcal{Y},\mu')$ be two m-spaces and $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ be a function. Then f is called:

1. θm-continuous function iff for any μ'-closed (μ'-open) subset $\mathcal{K}$of $\mathcal{Y}$, the inverse image $f^{-1}(\mathcal{K})$ is $\theta\mu$-closed ($\theta\mu$-open) set in $\mathcal{X}$.
2. $\theta^{*}m$-continuous function iff for every $\theta\mu'$-closed ($\theta\mu'$-open) $\mathcal{M}$ subset of $\mathcal{Y}$, the inverse image $f^{-1}(\mathcal{M})$ is μ-closed (μ-open) set in $\mathcal{X}$.
3. $\theta^{**}m$-continuous function iff for any$\mathcal{N}$ $\theta\mu'$-closed ($\theta\mu'$-open) $\mathcal{N}$ subset of $\mathcal{Y}$, the inverse image $f^{-1}(\mathcal{N})$ is $\theta\mu$-closed ($\theta\mu$-open) set in $\mathcal{X}$.
4. θm-closed function if $f(F)$ is $\theta\mu'$-closed set in $\mathcal{Y}$ for each μ-closed subset F of $\mathcal{X}$.
5. $\theta^{*}m$-closed function if $f(F)$ is μ'-closed set in $\mathcal{Y}$ for each $\theta\mu$-closed subset F of X.

Proposition 2.39 [8] The $\theta^{**}m$-continuous image of $\theta\mu$-compact is $\theta\mu'$-compact.
Proposition 2.40[8] If $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ is an m-homeomorphism and $\mathcal{B}$ is an $\theta\mu'$-compact set in $\mathcal{Y}$ then $f^{-1}(\mathcal{B})$ is an $\theta\mu$-compact set in $\mathcal{X}$, with $\mathcal{X}$ has the property $\theta\mathcal{B}$.

3. Strong and weak forms of μ-Kc-spaces

In this section, we provide some weak forms of μ-Kc-space, namely μ-$K(\alpha c)$-space, μ -$\alpha K(c)$-space and μ-$\alpha K(\alpha c)$-space. In addition, we introduce μ-$\theta K(c)$-space as a strong form of μ-Kc-space.
Definition 3.1 An m-space $(\mathcal{X},\mu)$ is said to be μ-$K(\alpha c)$-space if every μ-compact set in $\mathcal{X}$ is an $\alpha\mu$-closed set.
Now, we give some examples to explain the concept of μ-$K(\alpha c)$-space.
Example 3.2 The discrete m-space $(\mathcal{X},\mu_D)$ is μ-$K(\alpha c)$-space.
Example 3.3 Let $\mathcal{X} = \{1,2,3\}$ and let $\mu = \{\emptyset,\ \mathcal{X},\{1\}\}$. Then $(\mathcal{X},\mu)$ is not μ-$K(\alpha c)$-space, since there exists an μ-compact set $\{1,2\}$ in $\mathcal{X}$ but it is not $\alpha\mu$-closed.
To show that Definition 3.1 is well defined, we give the following example to illustrate that there is no relation between the concepts of μ-compact set and $\alpha\mu$-closed set.
Example 3.4

1. In the discrete m-space $(\mathcal{R},\mu_{\boldsymbol{D}})$ where $\mathcal{R}$ is a real number, $\mathbb{Q}$ is the rational numbers subset of $\mathcal{R}$, $\mathbb{Q}$ is $\alpha\mu$-closed but not μ-compact set.
2. In the indiscrete m-space $(\mathcal{R}, \mu_{ind})$, $\mathbb{Q}$ is μ-compact but not $\alpha\mu$-closed set.

Remark 3.5

1. Every μ-Kc space is μ-$K(\alpha c)$-space.
2. In discrete m-space, the two definitions of μ-Kc-space and μ-$K(\alpha c)$-paces are satisfied**.**

The following example indicates that the converse of Remark 3.5 part (1) is not necessarily hold.
Example 3.6 Let $(\mathcal{X},\mu)$ be an m-space, $\mathcal{X} = \{a,b,c\}$, $\mu = \{\emptyset,\mathcal{X},\{a\}\}$, so $\{c\}$ is μ-compact since $\{c\}$ is finite set. Also it is $\alpha\mu$-closed set since $\mu Cl(\mu Int(\mu Cl\{c\})) = \emptyset \subseteq \{c\}$, so $\mathcal{X}$ is μ-$K(\alpha c)$-space, but not μ-Kc-space since $\{c\}$ is not μ-closed set.
Proposition 3.7 An αμ-compact subset of αμ-T_2-space is αμ-closed, whenever $\mathcal{X}$ has $\alpha\Upsilon$ property.
Proof: Let $\mathcal{B}$ be αμ-compact in αμ-T_2-space. To show that $\mathcal{B}$ is αμ-closed, let $p \in \mathcal{B}^c$, since $\mathcal{X}$ is αμ-T_2-space. So for every $q \in \mathcal{B}$, $p{\neq}q$, there exist αμ-open sets G,H with $p \in H$, $q \in G$, such that $G \cap H{=}\emptyset$,. Now the collection $\{G_{q_i}{:}\, q_i \in \mathcal{B}, i \in I\}$ is αμ-open cover of $\mathcal{B}$.Since $\mathcal{B}$ is αμ -compact set, then there is a finite subcover of $\mathcal{B}$, so $\mathcal{B}{\subseteq} \bigcup_{i=1}^{n} G_{q_i}$. Let $H^* = \bigcap_{i=1}^{m} H_{q_i}(p)$ and $G^* = \bigcup_{i=1}^{m} G_{q_i}$, then H^* is an αμ-open set $p \in H^*$ (since $\mathcal{X}$ has property $\alpha\Upsilon$). Claim that $G^* \cap H^* = \emptyset$, let $x \in G^*$, then $x \in G_{q_i}$, for some $q_{i,}$ and suppose that $x \in H^*$, $\mathcal{B} \cap H^* \neq \emptyset$. This is a contradiction, then $p \in H^* \subseteq \mathcal{B}^c$, so $\mathcal{B}^c$ is αμ-open set in $\mathcal{X}$, hence $\mathcal{B}$ is αμ-closed set.
Theorem 3.8 Every αμ-closed set in αμ-compact space is αμ-compact set.

Proof: Let $(\mathcal{X}, \mu)$ be αμ-compact , A is αμ-closed set in $\mathcal{X}$, and $\{V_\alpha\}_{\alpha\in I}$ is an αμ-open cover of A, that is $A \subseteq \bigcup_{\alpha\in I} V_\alpha$, where V_α is αμ-open in $\mathcal{X}$. $\forall \alpha \in I$, since $\mathcal{X} = A \cup A^c \subseteq \bigcup_{\alpha\in\wedge} V_\alpha \cup A^c$, also A^c is αμ -open (since A is αμ-closed set in $\mathcal{X}$). So $\bigcup_{\alpha\in I} V_\alpha \cup A^c$ is αμ -open cover for $\mathcal{X}$ which is αμ-compact space, then there exists $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ such that $\mathcal{X} \subseteq \bigcup_{i=1}^{n} V_{\alpha_i} \cup A^c$, so $A \subseteq \bigcup_{i=1}^{n} V_{\alpha_i}$. Then $\bigcup_{i=1}^{n} V_{\alpha_i}$, $i = 1,2,3, \dots, n$ is a finite subcover of A. Therefore ,A is αμ-compact set.

Remark 3.9 In the above theorem, if we replace the αμ-compact by μ-compact, the theorem will not be true.

Now, we introduce the weak form of μ-$K(\alpha c)$-space which was introduced in Definition 3.1.

Definition 3.10 A space $\mathcal{X}$ is said to be μ-$\alpha K(\alpha c)$-space if any αμ-compact subset of $\mathcal{X}$ is αμ-closed set.

Example 3.11 Let $(\mathcal{R}, \mu_D)$ be a discrete m-space where $\mathcal{R}$ is a real number. Let $\mathbb{Q}$ is αμ-compact subset of $\mathcal{R}$, then $\mathbb{Q}$ is μ-compact in $\mathcal{R}$ from Remark 2.15, and $\mathbb{Q}$ is μ-closed so it is $\alpha\mu$-closed by Remark 2.6. Hence $(\mathcal{R}, \mu_D)$ is μ-$\alpha K(\alpha c)$-space.

Proposition 3.12 Every μ-$\alpha K(c)$-space is μ-$\alpha K(\alpha c)$-space**.**

Proof: Let $(\mathcal{X}, \mu)$ be m-space and $\mathcal{K}$ be αμ-compact subset of $\mathcal{X}$, which is μ-$\alpha K(c)$-space, so $\mathcal{K}$ is μ-closed subset of $\mathcal{X}$ and, by Remark 2.6, $\mathcal{K}$ is αμ-closed set. Hence $\mathcal{X}$ is μ-$\alpha K(\alpha c)$-space**.**

Theorem 3.13 $(\mathcal{X}, \mu)$ is αμ-T_1-space iff $\{x\}$ is αμ-closed subset of $\mathcal{X}$ for all $x \in \mathcal{X}$.

Proof: Let $\{x\}$ be αμ-closed set $\forall x \in \mathcal{X}$, let $a, d \in \mathcal{X}$ with $a \neq d$, and $\{a\}$ and $\{d\}$ are αμ-closed sets, then $\{a\}^c$ is αμ-open subset of $\mathcal{X}$, with $d \in \{a\}^c$ and $a \notin \{a\}^c$. Also $\{d\}^c$ is αμ-open subset of $\mathcal{X}$, with $a \in \{d\}^c$ and $d \notin \{d\}^c$, so $\mathcal{X}$ is αμ-T_1-space.

Conversely, we must prove that $\{x\}$ is αμ-closed subset of $\mathcal{X}$, that is αμCl$(\{x\}) = \{x\}$, since $\{x\} \subseteq$ αμCl$(\{x\})$ ... (1). Let $y \in$ αμCl$(\{x\})$ and $y \notin \{x\}$, so $x \neq y$, but $\mathcal{X}$ is αμ- T_1-space, so there exist two αμ-open sets U_x and V_y containing x and y, respectively, with $y \notin U_x$ and $x \notin V_y$. Then V_y containing y, so y is not αμ-adherent point to $\{x\}$, that is $y \notin$ αμCl$(\{x\})$, and this is contradiction. Therefore, $y \in \{x\}$ and αμCl$(\{x\}) \subseteq \{x\}$ ... (2), so by (1) and (2) we get αμCl$(\{x\}) = \{x\}$, and by Proposition 2.8, $\{x\}$ is αμ-closed subset of $\mathcal{X}$.

Proposition 3.14 Every μ-$\alpha K(\alpha c)$-space is αμ-T_1-space.

Proof: Let $x \in \mathcal{X}$ and let $\{x\}$ be αμ-compact set in $\mathcal{X}$, since $\mathcal{X}$ is μ-$\alpha K(\alpha c)$-space, hence $\{x\}$ is αμ-closed set, so $\mathcal{X}$ is αμ-T_1-space by Theorem 2.18.

The next example shows that the converse of Proposition 3.14 is not true.

Example 3.15 Let $(\mathcal{R}, \mu_{cof})$ be a co-finite m-space on a real number $\mathcal{R}$ which is αμ-T_1-space, if we take $\mathbb{Q} \subseteq \mathcal{R}$ as αμ-compact (since there exists one αμ-open cover of $\mathbb{Q}$ which is R), but $\mathbb{Q}$ is not αμ-closed in $\mathcal{R}$ (since $\mu Cl(\mu Int(\mu Cl(\mathbb{Q}))) = \mathcal{R} \not\subset \mathbb{Q}$.

Proposition 3.16 Every αμ-T_2-space is μ-$\alpha K(\alpha c)$-space, whenever $\mathcal{X}$ has $\alpha\Upsilon$ property.

Proof: Let $(\mathcal{X}, \mu)$ be an m-space and $\mathcal{P}$ be an αμ-compact subset in $\mathcal{X}$. Also $\mathcal{X}$ is αμ-T_2-space, so $\mathcal{P}$ is an αμ-closed set from Proposition 3.7. Therefore, $\mathcal{X}$ is μ-$\alpha K(\alpha c)$-space**.**

The converse of Proposition 3.16 may not be hold. The following example explains that.

Example 3.17

Let $(\mathcal{R}, \mu_{coc})$ be a co-countable m-space on a real number $\mathcal{R}$, which is μ-$\alpha K(\alpha c)$-space, but not αμ-T_2-space, since the μ-compact set in it are just the finite set, if we μ-compact set then it is finite, so it is countable, then it is μ-closed since in μ_{coc} the closed take sets are $\emptyset, \mathcal{R}$ and countable sets. Now suppose that it is αμ-T_2-space, $\forall\, x, y \in \mathcal{R}$, $x \neq y$, there are U_x, V_y as two αμ-open sets such that $x \in U_x$, $y \in V_y$ and $U_x \cap V_y = \emptyset$, $(U_x \cap V_y)^c = \emptyset^c$,$(U_x)^c \cup (V_y)^c = \mathcal{R}$, but this is a contradiction. Since U_x and V_y are countable, the union also countable, but $\mathcal{R}$ is not countable so it is not αμ-T_2-space. Therefore $(\mathcal{R}, \mu_{coc})$ are μ-Kc-, μ-$K(\alpha c)$- and μ-$\alpha K(\alpha c)$-spaces.

Proposition 3.18 A subset $\mathcal{F}$ of an m-space $\mathcal{X}$ is αμ-closed set in $\mathcal{X}$ if and only if there exists an μ-closed set M such that $\mu Cl(\mu Int(M)) \subseteq \mathcal{F} \subseteq M$.

Proof: Suppose that $\mathcal{F}$ is αμ-closed set in $\mathcal{X}$, so $\mu Cl\left(\mu Int(\mu Cl(\mathcal{F}))\right) \subseteq \mathcal{F}$, by Definition 2.3, and $\mathcal{F} \subseteq (\mu Cl(\mathcal{F})$, then $\mu Cl\left(\mu Int(\mu Cl(\mathcal{F}))\right) \subseteq \mathcal{F} \subseteq \mu Cl(\mathcal{F})$, put $\mu Cl(\mathcal{F}) = M$, so $\mu Cl(\mu Int(M)) \subseteq \mathcal{F} \subseteq M$.

Conversely, suppose that $\mu Cl(\mu Int(M)) \subseteq \mathcal{F} \subseteq M$. To prove that $\mathcal{F}$ is αμ-closed set whenever M is μ-closed set, $\mu Cl\left(\mu Cl(\mu Int(M))\right) \subseteq \mu Cl(\mathcal{F}) \subseteq \mu Cl(M) = M$, then $\mu Cl(\mu Int(M)) \subseteq \mu Cl(\mathcal{F}) \subseteq M$, and $\mu Int(\mu Cl(\mu Int(M))) \subseteq \mu Int(\mu Cl(\mathcal{F})) \subseteq \mu Int(M)$, by hypothesis $\mu Cl(\mu Int(M)) \subseteq \mathcal{F} \subseteq M$, we get $\mu Cl\left(\mu Int\left(\mu Cl(\mathcal{F})\right)\right) \subseteq \mathcal{F}$ .Therefore $\mathcal{F}$ is αμ-closed set.

Definition 3.19 An m-space $\mathcal{X}$ is called μ-$\alpha K(c)$-space if any αμ-compact subset in $\mathcal{X}$ is μ-closed set.

Example 3.20 Let ($, \mu_D$) be a discrete m-space on any space $\mathcal{X}$, it is μ-$\alpha K(c)$-space.

Remark 3.21

1. Every μ-Kc-space is μ-$\alpha K(c)$-space**.**
2. Every μ-$\alpha K(c)$-space is μ-$\alpha K(\alpha c)$-space.
3. Every μ-T_2-space is μ-$\alpha K(c)$-space.
4. Every μ-$\alpha K(c)$-space is αμ-T_1-space.

Now, we define a strong form of μ-Kc-space which is μ-$\theta K(c)$-space.

Definition 3.22 An m-space $(\mathcal{X}, \mu)$ is called μ-$\theta K(c)$-space, if every θμ-compact of $\mathcal{X}$ is μ-closed set.

Example 3.23 Let $(\mathcal{R}, \mu_{cof})$ be a co-finite m-space on a real line $\mathcal{R}$. Then $(\mathcal{R}, \mu_{cof})$ is an μ-$\theta K(c)$-space.

Proposition 3.24 Every $\theta\mu$-compact subset of θμ-T_2-space is $\theta\mu$-closed, whenever that space has $\theta\Upsilon$ property.

Proof: Let A be an $\theta\mu$-compact set in $\mathcal{X}$. Let $p \not\subseteq \notin A$, so for each $q \in A$ then $p \neq q$. But $\mathcal{X}$ is θμ-T_2-space, so there exist two $\theta\mu$-open sets U and V containing q and p, respectively, then $A = \bigcup_{\alpha \in I}\{U_{q_\alpha}\}$. But A is $\theta\mu$-compact, so $A = \bigcup_{i=1}^{n}\{U_{q_{\alpha_i}}\} = U^*$ and $V^* = \bigcap_{i=1}^{n} V_{q_i}(p)$ is $\theta\mu$-open (since X has $\theta\Upsilon$ property). Claim that $U^* \cap V^* = \emptyset$, and suppose that $U^* \cap V^* \neq \emptyset$, since $p \in V^*$, let $p \in U^*$, that is $p \in A$, but this is a contradiction. So $U^* \cap V^* = \emptyset$ and then there exists V^*containing p and $V^* \subseteq A^c$, that is $p \in \mu Int(A^c)$, then A^c is $\theta\mu$-open, by Proposition 2.10, so A is $\theta\mu$-closed.

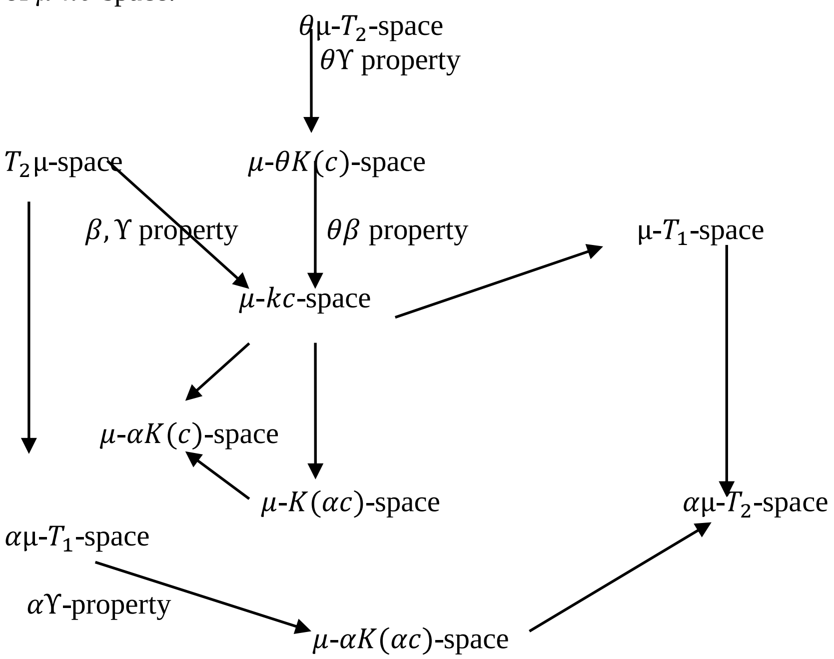
Proposition 3.25 If an μ-space has $\theta\Upsilon$ property, then every θμ-T_2-space is μ-$\theta K(c)$-space.

Proof: Let H be an $\theta\mu$-compact subset of $\mathcal{X}$. To prove that H is μ-closed set, since $\mathcal{X}$ is θμ-T_2-space, so by proposition 3.24, we get H is $\theta\mu$-closed set and by Remark 2.33, we get H is μ-closed, hence $\mathcal{X}$ is μ-$\theta K(c)$-space.

Proposition 3.26 If an μ-space has $\theta\beta$ property, then every μ-$\theta K(c)$-space is μ-kc-space.

Proof: Let $(\mathcal{X}, \mu)$ be m-space, A be μ-compact of $\mathcal{X}$ by Remark 2.36, A is $\theta\mu$-compact and since $\mathcal{X}$ is μ-$\theta K(c)$-space, so A is μ-closed subset of $\mathcal{X}$, hence $\mathcal{X}$ is μ-kc-space.

Remark 3.27 The following diagram shows the relationships between the stronger and weaker forms of μ-kc-space.



4-Some types of continuous, open (closed) function on m-spaces.

Definition 4.1 Let $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ be a function, then f is called:

1. m-open (respectively m-closed) function [2], if $f(\mathcal{H})$ is an μ'-open respectively μ'-closed set in $\mathcal{Y}$ for any μ-open (respectively μ-closed) $\mathcal{H}$ in $\mathcal{X}$.
2. αm-open (respectively αm-closed) function [6], if $f(A)$ is an $\alpha\mu'$-open (respectively $\alpha\mu'$-closed) set in $\mathcal{Y}$ for every μ-open (respectively μ-closed) A in $\mathcal{X}$.
3. $\alpha^{*}m$-open (respectively $\alpha^{*}m$-closed) function, if $f(\mathcal{K})$ is an μ'-open (respectively μ'-closed) set in $\mathcal{Y}$ for any αμ-open (respectively αμ-closed) subset $\mathcal{K}$ of $\mathcal{X}$.
4. $\alpha^{**}m$-open (respectively$\alpha^{**}m$-closed) function, if $f(\mathcal{N})$ is an $\alpha\mu'$-open (respectively $\alpha\mu'$-closed) subset of $\mathcal{Y}$ for any αμ-open (respectively αμ-closed) set$\mathcal{N}$ in $\mathcal{X}$.
5. $\alpha^{*}m$-continuous iff for any $\alpha\mu'$-open set $\mathcal{A}$ in $\mathcal{Y}$, the inverse image $f^{-1}(\mathcal{A})$ is μ-open set in $\mathcal{X}$.
6. $\alpha^{**}m$-continuous iff for every $\alpha\mu'$-open set $\mathcal{B}$ in $\mathcal{Y}$, the inverse image $f^{-1}(\mathcal{B})$ is αμ-open set in $\mathcal{X}$.

Example 4.2 Let $\mathcal{X}=\mathcal{Y}=\{a,b,c\}$, $\mu=\mu'=\{\emptyset,\mathcal{X},\{a\}\}$ and $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ defined by $f(a)=f(b)=a$ and $f(c)=c$. Then f is μ-open, αμ -open and α^{**}μ-open but it is not α^{*}μ-open function (where αμ-open set in μ and μ' are $\{\phi,\mathcal{X},\{a\},\{a,b\},\{a,c\}\}$.

Next, we introduce a proposition about α^{**}μ-closed function. But before that we need to introduce the following proposition:

Proposition 4.3 Let $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ be a function. Then for every subset A of $\mathcal{X}$:

1. f is m-homeomorphism iff $\mu Cl(f(A)) = f(\mu Cl(A))$.
2. f is m-homeomorphism iff $\mu Int(f(A)) = f(\mu Int(A))$.

Proof: The proof follows directly from the Definition 2.26 part (1) and Definition 4.1 part (1).

Theorem 4.4 If $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ is m-homeomorphism, then f is α^{**}μ-closed function.

Proof: Let $\mathcal{F}$ be αμ-closed subset of $\mathcal{X}$, by Proposition 3.18, there exists μ-closed set M such that $\mu Cl(\mu Int(M)) \subseteq \mathcal{F} \subseteq M$. Now, by taking the image, we get $f(\mu Cl(\mu Int(M))) \subseteq f(\mathcal{F}) \subseteq f(M)$. But f is m-homeomorphism, so

$$f(\mu Cl\left((\mu Int(M))\right) \subseteq f(\mathcal{F}) \subseteq f(M) \ldots (1).$$

Also from Proposition 4.3 $f(\mu Int(M)) = \mu Int(f(M))$, hence

$$\mu Cl\left(f(\mu Int(M))\right) = \mu Cl\left(\mu Int(f(M))\right) \ldots (2).$$

Now, from (1) and (2) we have, $\mu Cl(\mu Int(f(M))) \subseteq f(\mathcal{F}) \subseteq f(M)$. Therefore, $f(\mathcal{F})$ is αμ-closed subset of $\mathcal{Y}$.

Corollary 4.5 If $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ is m-homeomorphism, then f is α^{**}μ-open function.

Proof: Let K be an αμ-open set in $\mathcal{X}$. To prove that $f(K)$ is αμ-open set in $\mathcal{Y}$. Now, K^{c} is αμ-closed set in $\mathcal{X}$, and since f is m-homeomorphism. From Theorem 4.4, $f(K^{c})$ is αμ-closed set in $\mathcal{Y}$. But f is surjective, so $f(K^{c})$ =$(f(K))^{c}$, which means that $f(K)$ is αμ-open set in $\mathcal{Y}$. Hence f is α^{**}μ-open function.

Theorem 4.6 Let $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ be $\alpha^{**}m$-continuous. Then $f(\mathcal{M})$ is αμ-compact in $\mathcal{Y}$, whenever $\mathcal{M}$ is αμ-compact in $\mathcal{X}$.

Proof: Let $\mathcal{M}$ be an αμ-compact in $\mathcal{X}$. To prove that $f(\mathcal{M})$ is αμ-compact in $\mathcal{Y}$, let $\{V_{\alpha}:\alpha\epsilon I\}$ be a family of αμ-open cover of $f(\mathcal{M})$. That is $(M) \subseteq \bigcup_{\alpha\in I} V_{\alpha}$, so $f^{-1}(V_{\alpha})$ is αμ-open cover of $\mathcal{M}$,$\forall\alpha\epsilon I$. Also, since $\mathcal{M}$ is αμ-compact in $\mathcal{X}$, then there exist $\alpha_1,\alpha_2,\alpha_3 \ldots,\alpha_n$ such that $\mathcal{M} \subseteq \bigcup_{i=1}^{n} f^{-1}(V_{\alpha_i})$, then $f(\mathcal{M}) \subseteq f(\bigcup_{i=1}^{n} f^{-1}(V_{\alpha_i})) = \bigcup_{i=1}^{n} V_{\alpha_i}$. Therefore, $f(\mathcal{M})$ is αμ-compact in $\mathcal{Y}$.

Theorem 4.7 Let $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ be α^{*}μ-continuous function. Then $f(\mathcal{N})$ is μ-compact in $\mathcal{Y}$, whenever $\mathcal{N}$ is αμ-compact in $\mathcal{X}$.

Proof: Let $\mathcal{N}$ be an αμ-compact in $\mathcal{X}$. To prove that $f(\mathcal{N})$ is μ-compact in $\mathcal{Y}$, let $\{V_{\alpha}:\alpha\epsilon I\}$ be a family of μ-open cover of $f(\mathcal{N})$. That is $(\mathcal{N}) \subseteq \bigcup_{\alpha\in I} V_{\alpha}$, so $f^{-1}(V_{\alpha})$ is an αμ-open cover of $\mathcal{N}$,$\forall\alpha\epsilon I$. Also, since $\mathcal{N}$ is αμ-compact in $\mathcal{X}$, then $\mathcal{N} \subseteq \bigcup_{i=1}^{m} f^{-1}(V_{\alpha_i})$. This implies that $f(\mathcal{N}) \subseteq f(\bigcup_{i=1}^{m} f^{-1}(V_{\alpha_i})) = \bigcup_{i=1}^{m} V_{\alpha_i}$. Therefore, $f(\mathcal{N})$ is μ-compact in $\mathcal{Y}$.

Theorem 4.8 Let $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ be α^{**}μ-continuous function. If a space $\mathcal{X}$ is αμ-compact and a space $\mathcal{Y}$ is αμ-T_2, then the function f is α^{**}μ-closed, whenever $\mathcal{X}$ has $\alpha\Upsilon$ property.

Proof: Let $\mathcal{H}$ be an αμ-closed set in $\mathcal{X}$. Since $\mathcal{X}$ is αμ-compact, then $\mathcal{H}$ is αμ-compact in $\mathcal{X}$ by Theorem 3.8 and the function f is α^{**}μ-continuous. Then $f(\mathcal{H})$ is $\alpha\mu'$-compact subset of $\mathcal{Y}$ from Theorem 4.6, and since $\mathcal{Y}$ is αμ-T_2-space, so $f(\mathcal{H})$ is $\alpha\mu'$-closed set of $\mathcal{Y}$ by proposition 3.7. Therefore f is α^{**}μ -closed function.

Theorem 4.9 Let $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ be a α^*μ-continuous function, from αμ-compact space $\mathcal{X}$ into μ-Kc-space $\mathcal{Y}$, then f is α^*μ-closed function.

Proof: Let $\mathcal{B}$ be αμ-closed set in $\mathcal{X}$ which is αμ-compact, so $\mathcal{B}$ is αμ-compact in $\mathcal{X}$ from Theorem 3.8. Also, from the hypotheses, f is α^*μ-continuous, then $f(\mathcal{B})$ is μ-compact in $\mathcal{Y}$ by Theorem 4.7. But $\mathcal{Y}$ is μ-Kc-space, hence $f(\mathcal{B})$ is μ'-closed set of $\mathcal{Y}$. Therefore, f is $\alpha\mu^*$-closed function.

Proposition 4.10 Let the function $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ be m-continuous. If $(\mathcal{X},\mu)$ is μ-compact and $(\mathcal{Y},\mu')$ is μ-$K(\alpha c)$-space, then f is αμ-closed function.

Proof: Let $\mathcal{S}$ be an μ-closed set in $\mathcal{X}$, also $\mathcal{X}$ is μ-compact, then $\mathcal{S}$ is μ-compact subset of $\mathcal{X}$ from Proposition 2.13, and f is m-continuous function, then $f(\mathcal{S})$ is μ-compact set in $\mathcal{Y}$ from Proposition 2.27. Also $\mathcal{Y}$ is μ-$K(\alpha c)$-space , so $f(\mathcal{S})$ is αμ-closed in $\mathcal{Y}$, therefore f is αμ-closed .

Proposition 4.11 If the function $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ is $\alpha^{**}m$-continuous, $(\mathcal{X},\mu)$ is αμ-compact and $(\mathcal{Y},\mu')$ is μ-$\alpha K(\alpha c)$-space, then f is $\alpha^{**}m$-closed function.

Proof: Let F be an αμ-closed set of $\mathcal{X}$, since $\mathcal{X}$ is αm-compact, so by Theorem 3.8, F is αμ-compact in $\mathcal{X}$ and f is $\alpha^{**}m$-continuous. Then $f(F)$ is αμ-compact in $\mathcal{Y}$. Also by Theorem 4.6, $\mathcal{Y}$ is μ-$\alpha K(\alpha c)$-space, hence $f(F)$ is αμ-closed in $\mathcal{Y}$. Therefore, f is $\alpha^{**}m$-closed .

Theorem 4.12 If $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ is m-closed, $\alpha^{**}m$-open bijective function and $(\mathcal{X},\mu)$ is μ-$\alpha K(c)$-space, then (Y,μ') is μ-$\alpha K(c)$-space.

Proof: Let $\mathcal{K}$ be αμ-compact in $\mathcal{Y}$ and $\{V_\alpha : \alpha \in I\}$ be an αμ-open cover of $f^{-1}(\mathcal{K})$ in $\mathcal{X}$, that is $f^{-1}(\mathcal{K}) \subseteq \bigcup_{\alpha\in I} V_\alpha$. Since f is bijective, so $\mathcal{K} = f\big(f^{-1}(\mathcal{K})\big) \subseteq f(\bigcup_{\alpha\in I} V_\alpha) = \bigcup_{\alpha\in\Lambda} f(V_\alpha)$. And f is $\alpha^{**}m$-open function, so $\bigcup_{\alpha\in I} f(V_\alpha)$ is $\alpha\mu'$-open in $\mathcal{Y}$, for each $\alpha\in I$. Also, $\mathcal{K}$ is $\alpha\mu'$-compact in $\mathcal{Y}$, so $\mathcal{K} \subseteq \bigcup_{i=1}^{n} f(V_{\alpha_i})$. This implies that $f^{-1}(\mathcal{K}) \subseteq f^{-1}\big(\bigcup_{i=1}^{n} f(V_{\alpha_i})\big) = \bigcup_{i=1}^{n} f^{-1}(f(V_{\alpha_i})) = \bigcup_{i=1}^{n} V_{\alpha_i}$, so $f^{-1}(\mathcal{K})$ is αμ-compact in $\mathcal{X}$, which is μ-$K(\alpha c)$-space, so $f^{-1}(\mathcal{K})$ is μ-closed. Also, since f is m-closed function, therefore $f(f^{-1}(\mathcal{K})) = \mathcal{K}$ is μ'-closed in $\mathcal{Y}$. Hence $\mathcal{Y}$ is μ-$K(\alpha c)$-space.

Theorem 4.13 Let the injective function $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ be m-continuous and $\alpha^{**}m$-continuous. Then $(\mathcal{X},\mu)$ is μ-$K(\alpha c)$-space whenever $(\mathcal{Y},\mu')$ is μ-$K(\alpha c)$-space.

Proof: Let K be μ-compact in $\mathcal{X}$. To prove that K is αμ-closed, let $\{V_\alpha : \alpha \in I\}$ be an μ-open cover to $f(K)$ in $\mathcal{Y}$, that is $f(K) \subseteq \bigcup_{\alpha\in I} V_\alpha$. But f is m-continuous function, so by Proposition 2.27, $f(K)$ is μ-compact in $\mathcal{Y}$, hence $f(K) \subseteq \bigcup_{i=1}^{n} V_{\alpha_i}$. Also f is injective function, so $K = f^{-1}(f(K) \subseteq f^{-1}(\bigcup_{i=1}^{n} V_{\alpha_i}) = \bigcup_{i=1}^{n} f^{-1}(V_{\alpha_i})$. Also, f is m-continuous, hence $f^{-1}(V_{\alpha_i})$ is μ-open in $\mathcal{X}$, $\forall\, i = 1,2,3,\dots,n$. This implies that $f(K) \subseteq \bigcup_{i=1}^{n} V_{\alpha_i}$, hence $f(K)$ is μ-compact set of $\mathcal{Y}$ which is μ-$K(\alpha c)$-space, that is $f(K)$ is αμ-closed subset of $\mathcal{Y}$. But f is $\alpha^{**}m$-continuous and $f^{-1}\big(f(K)\big) = K$, so K is αμ-closed set in $\mathcal{X}$. Therefore $\mathcal{X}$ is μ-$K(\alpha c)$-space.

Theorem 4.14 Let a bijective function $f:(\mathcal{X},\mu) \to (\mathcal{Y},\mu')$ be $\alpha^{**}m$-continuous. If $\mathcal{Y}$ is μ-$\alpha K(\alpha c)$-space, then $\mathcal{X}$ is μ-$\alpha K(\alpha c)$-space.

Proof: Let A be αμ-compact in $\mathcal{X}$, so $f(A)$ is αμ-compact in $\mathcal{Y}$ by Theorem 4.6. And since $\mathcal{Y}$ is μ-$\alpha K(\alpha c)$-space, so that $f(A)$ is $\alpha\mu'$-closed set of $\mathcal{Y}$ and $f^{-1}(f(A)) = A$ (f is injective), so A is αμ-closed subset in $\mathcal{X}$ since f is $\alpha^{**}m$ -continuous function. Therefore, $\mathcal{X}$ is μ-$\alpha K(\alpha c)$-space.

Proposition 4.15 If $f:(\mathcal{X},\mu) \longrightarrow (\mathcal{Y},\mu')$ is m-continuous function, $\mathcal{X}$ is μ-compact space and $\mathcal{Y}$ is μ-$\theta k(c)$-space, then f is $\theta\mu^*$-closed function, whenever $\mathcal{X}$ has $\theta\Upsilon$ property.

Proof: Let $\mathcal{N}$ be $\theta\mu$-closed subset of $\mathcal{X}$, so that $\mathcal{N}$ is μ-closed in $\mathcal{X}$ by Remark 2.33. And since $\mathcal{X}$ is μ-compact, then $\mathcal{N}$ is μ-compact by Proposition 2.13. Also f is m-continuous function, so by Proposition 2.27, $f(\mathcal{N})$ is μ-compact, hence from Remark 2.36, $f(\mathcal{N})$ is $\theta\mu$-compact in $\mathcal{Y}$ which is μ-$\theta k(c)$-space. Therefore $f(\mathcal{N})$ is μ'-closed. That is f is θ^*m-closed function.

Proposition 4.16 Let $f:(\mathcal{X},\mu) \to (\mathcal{Y},\mu')$ be m-homeomorphsim function. Then $(\mathcal{Y},\mu')$ is μ-$\theta k(c)$-space, whenever $(\mathcal{X},\mu)$ is μ-$\theta k(c)$-space which has $\theta\beta$ property.

Proof: Let $\mathcal{H}$ be an $\theta\mu$-compact set in $\mathcal{Y}$, by Proposition 2.40, $f^{-1}(\mathcal{H})$ is $\theta\mu$-compact in $\mathcal{X}$ which is μ-$\theta k(c)$-space. So $f^{-1}(\mathcal{H})$ is μ-closed set in $\mathcal{X}$ and $f(f^{-1}(\mathcal{H})) = \mathcal{H}$ is μ'-closed set in $\mathcal{Y}$. Therefore, $(\mathcal{Y}, \mu')$ is μ-$\theta k(c)$-space.

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