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Stellar Thermonuclear Reaction Rates of Proton Radiative Capture by Closed Light Shell Isotopes

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Abstract

Light isotopes, especially closed shell nuclei, have significance in thermonuclear reactions of the Carbon-Nitrogen-Oxygen (CNO) cycle in stars. In this research, $^{12}\text{C}(p, \gamma) ^{13}\text{N}$ and $^{14}\text{N}(p, \gamma) ^{15}\text{O}$ reactions have been calculated by means of Matlab codes to find the reaction rate across a temperature range of 0.006 to 10 GK using non-resonant parts, as well as the astrophysical S- factor $S(E)$ at low energies. It was concluded that the high binding energy of ^{12}C and ^{14}N nuclei make the reaction less probable thus enabling other competitive processes to develop, which enhances the probability of other competitive proton reactions in the CNO cycle.

Keywords: Gamow Energy, Nuclear Reactions, Nucleosynthesis, Radiative Capture Reactions, S- Factor, Sommerfeld Parameter.

معدلات التفاعل النووي الحراري النجمي لالتقاط البروتون الاشعاعي بواسطة نظائر القشرة الخفيفة المغلقة

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الخلاصة

للنظائر الخفيفة، وخاصة نوى القشرة المغلقة، أهمية في التفاعلات النووية الحرارية في دورة الكربون والنيتروجين والأكسجين في النجوم. في هذا البحث، تم حساب تفاعلات $^{14}\text{N}(p, \gamma) ^{15}\text{O}$ و $^{12}\text{C}(p, \gamma) ^{13}\text{N}$ عن طريق كود الماتلاب للعثور على معدل التفاعل عبر نطاق درجة حرارة يتراوح من 0.006 إلى 10 غيغا كلفن باستخدام الجزء غير الرنان، بالإضافة إلى العامل الفيزيائي الفلكي $S(E)$ عند الطاقات المنخفضة. وقد تم التوصل إلى أن طاقة الارتباط العالية لنواة ^{12}C و ^{14}N تجعل التفاعل أقل احتمالاً وبالتالي تمكين العمليات التنافسية الأخرى من التطور، مما يعزز احتمال تفاعلات البروتون التنافسية الأخرى في دورة CNO.

1. General Introduction:

The $^{12}\text{C}(p, \gamma) ^{13}\text{N}$ reaction an important plays part of the Carbon-Nitrogen-Oxygen (CNO) cycle, which is active in the hydrogen-burning regions of stars on the main sequence, the red giant Branch (RGB), and the Asymptotic Giant Branch (AGB). It impacts the abundances of many nuclei in the cores of stars, for example, the ^{12}C is created by the

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$^{15}\text{N}(p,\alpha)^{12}\text{C}$ reaction and then consumed by proton capture [1,2]. The $^{12}\text{C}(p,\gamma)^{13}\text{N}$ reaction in the CNO reaction sequence is crucial in deciding the destiny of the ^{12}C isotope in an H-burning environment, as well as ^{13}N synthesizing. The purpose of a recent experiment at Laboratory for Underground Nuclear Astrophysics (LUNA) was to determine the $^{12}\text{C}(p,\gamma)^{13}\text{N}$ cross-section at astronomical energy. To assess and limit systematic experimental uncertainties, different experimental approaches (targets and detection setups) were used [3]. This reaction is a radiative capture mechanism that relies on electromagnetic interaction rather than the strong interaction. As a result, the cross section is expected to be significantly weaker. At low energies, however, the $^{12}\text{C}(p,\gamma)^{13}\text{N}$ reaction is distinguished by contributions from two strong resonances and a non-resonant direct capture mechanism, as well as potential interference between these reaction components [3]. The slowest reaction in the CNO cycle is the radiative proton capture of ^{14}N , hence the rate of this $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction determines the cycle's rate, and thus its efficiency and contribution to stellar energy generation. Recognizing its importance, various studies have been conducted to determine its cross section (a comprehensive list of references may be found in Ref. [4] and the three most recent sets of data were published in Refs. [5,6]). The temperature at which the CNO cycle is active and crucial varies depending on the astrophysical location and ranges between ~ 15 and 200 MK. This corresponds to center-of-mass energy ranges (the Gamow window) for the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction ranging from 20 to 200 keV. At lower energies, the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction is largely driven by the direct capture mechanism, with some help from broad resonances. The capture occurs to the $E_x = 6.79$ MeV state of excitement in ^{15}O dominates the overall cross section, while the capture to the ground state, as well as the $E_x = 6.17$ MeV excited state contribute considerably. When cross section data is available, transitions to other states including narrow and broad resonances, play important parts at higher energy [6].

Light radioactive nuclei, as we know, play a vital role in many astrophysical systems. Furthermore, the cross section of capture reactions as a function of energy is crucial for researching different astrophysical subjects such as the Universe's initial nucleosynthesis, key trend in stellar evolution, novae and supernovae explosions, X-ray bursts, and so on. Radiative capture processes, in which an atomic nucleus fuses with one or more nucleons or nuclei, emit electromagnetic radiation, are essential in astrophysics. Bethe & Critchfield [7] and Bethe [8] originally explained their role in 1938, the (pp-chain) and the CN cycle were used to power the stars. Furthermore, radiative capture processes play an important role in the explosive circumstances encountered in novae, X-ray bursts, and supernovae. They are highly relevant to the field of stellar astrophysics due to two characteristics. For starters, because radiative captures are the only proton – induced reactions with positive Q values for many nuclei, understanding their rates is crucial for discovering reaction routes and releasing energy. Second, when contrasted to strong contacts, radiative capture reactions are sluggish, as a result, they serve as rate-limiting phase in a number of chemical routes and cycles. As a result, they commonly impact the reaction flow and nucleosynthesis rate in a procedure. The 35 stable neutron deficient nuclei, which range from ^{74}Se to ^{196}Hg , are immune to neutron capture processes and need a distinct mode of creation. These are p-nuclei (which have more protons than other stable isotopes of the same element). As demonstrated by increases in the abundance of those nuclei with closed nuclear shells (^{92}Mo , $N = 50$; Sn isotopes, $Z = 50$; ^{144}Sm , $N = 82$), the abundance trend for these proton-rich nuclei is analogous to that of the neutron-rich isotopes [9]. Burbidge et al. [10] postulated a scenario in which (p, γ) or (γ ,n) reactions were responsible for the formation of many proton-rich isotopes. In this scenario, extreme temperatures of ($T_9=2-3$) and proton densities ($\geq 10^2\text{g/cm}^3$) are required. For nuclei with $Z \geq 54$, proton capture is low due to the growing Coulomb barrier for heavy nuclei [11]. Nucleosynthesis in heavier species is mostly accomplished by photodisintegration events, which provide highly (γ ,n), (γ , α) and (γ ,p) processes produce proton-rich nuclei. The (rp-

process), also known as the "rapid proton capture process" is characterized by consecutive proton captures on seed nuclei, with occasional beta⁺-decays or electron captures (rp-process) is vital for synthesizing many of the light p-nuclei in the nuclear valley of stability's proton-rich side, similar to the role performed by neutron capture processes (s-process and r-process) in creating stable and neutron rich nuclides [10].

The aim of the present work is to study the thermonuclear reaction rates at stellar temperatures from 10⁶ to 10⁹ K by focusing on specific samples of massive stellar nuclei and comparing the model calculations with data from available compilations and libraries. Furthermore, it is planned to find the best probability distribution functions that govern the rate of these reactions. A MATLAB computer code, has been used to perform the analysis and the results have been compared with the available experimental data of NACRE [European Nuclear Astrophysics Compilation of Reaction Rates].

2. Theoretical Background of Reaction Rate

A Maxwell-Boltzmann distribution effectively approximates nuclear velocities under sunny circumstances. Therefore, The Maxwell distribution also applies to the relative velocity distribution. The constant of Boltzmann, which is regulated by the decreased mass of colliding nuclei, is estimated from [12]

$$\langle \sigma v \rangle = \left(\frac{8}{\pi \mu} \right)^{\frac{1}{2}} (kT)^{-\frac{3}{2}} \int_0^{\infty} E \sigma(E) e^{-\frac{E}{kT}} dE \quad (1)$$

Where μ is the reduced mass of the entrance channel, k is Boltzmann's constant, σ is the reaction cross section and E is the center-of-mass energy between the reacting particles. The cross-section, that in most cases has a combined contribution from both non-resonant and resonance components. In this work non-resonant contribution will be determined as follows: Considering charged-particle induced reactions, the cross section $\sigma(E)$ can be expressed as [13]

$$\sigma(E) = S(E) / E e^{-2\pi\eta} \quad (2)$$

where $S(E)$ is the astrophysical S-factor and is determined by this equation.

$$\eta(E) = \frac{Z_1 Z_2 e^2}{\hbar v} = 0.1575 Z_1 Z_2 \left(\frac{\mu}{E} \right)^{\frac{1}{2}} \quad (3)$$

is $\eta(E)$ the Sommerfeld parameter, Z_1 and Z_2 are the interacting nuclei's charge numbers, and \hbar is the reduced Planck constant. In its absence of resonances, the S-factor is a significantly smoother function of energy than the cross section [13], and it corresponds to a simple quantum tunneling model and the actual cross section of the reaction. This is accomplished by accounting for all of the contributions of the internal structure of the reacting nuclei to the $\sigma(E)$.

The total rate of reaction for a pair of projectile and target nuclei becomes [14]:

$$\langle \sigma v \rangle = \left(\frac{8}{\pi \mu} \right)^{\frac{1}{2}} \left(\frac{1}{kT} \right)^{\frac{3}{2}} \int_0^{\infty} S(E) \exp \left(-\frac{E}{kT} - 2\pi\eta(E) \right) dE \quad (4)$$

The integral part of the reaction rate equation, Eq. (4), is governed by a combined effect of two exponential terms: the first represents the Maxwell Boltzmann (M.B.) distribution ($e^{-E/kT}$) and the second reflects the Gamow factor ($e^{-1/\sqrt{E}}$), and each of them is interestingly energy-dependent $e^{-E/kT}$ inversely with E and $e^{-1/\sqrt{E}}$ proportionally with E . As a result, the

integral's largest contribution must originate from energies where the product of both elements is near maximal. This area is known as the Gamow window, and it reflects the relatively small energy range in plasma of a star where most of the non-resonant thermonuclear events occur [14]. This window's effective width is [15],

$$\Delta = 0.2368 (Z_1^2 Z_2^2 \mu T_9^5)^{\frac{1}{6}} \text{ MeV} \quad (5)$$

Centered around an energy E_0 ,

$$E_0 = 0.122 (z_1^2 z_2^2 \mu T_9^2)^{1/3} \text{ MeV} \quad (6)$$

That gives the effective mean energy for thermonuclear reaction at a given temperature T .

In most cases, it is more convenient to describe theoretical or the experimental S -factor as the first three terms of a Taylor series centered on zero E but far from nuclear resonance.

$$S(E) = S(0) + \dot{S}(0)E + \frac{1}{2}\ddot{S}(0)E^2 + \dots \quad (7)$$

where the dot indicates differentiation with respect to energy. By substituting this expansion and Eq. (2) into Eq. (1), we get [16],

$$N_A \langle \sigma v \rangle_{\text{Non.R.}} = \frac{4.339 \times 10^8}{z_0 z_1} \frac{m_0 + m_1}{m_0 m_1} S_{\text{eff}} e^{-\tau} \tau^2 \quad (8)$$

with N_A is the Avogadro's number and

$$S_{\text{eff}} = S(0) \left[1 + \frac{5}{12\tau} + \frac{\dot{S}(0)}{S(0)} \left(E_0 + \frac{35}{36} kT \right) + \frac{1}{2} \frac{\ddot{S}(0)}{S(0)} \left(E_0^2 + \frac{89}{36} E_0 kT \right) \right] \text{ (MeV.b)} \quad (9)$$

where S_{eff} is the effective S -factor that takes the above corrections into account

$$\tau = \frac{3E_0}{kT} = 4.2487 (Z_1^2 Z_2^2 \mu / T_9)^{\frac{1}{3}} \quad (10)$$

Because of the cross-section's fundamental significance for calculating reaction rates for many astrophysical applications, especially at energies below the Coulomb barrier, several statistical models, involving Weisskopf [17], Hauser [18], and quantum mechanical models [19,20], have been created to compute the $S(E)$ via Eq. (2).

3. Results and discussion:

This research focused on calculating the contribution of the non-resonant component. These are called radiative proton capture that provides the necessary proton for p -process nucleosynthesis. The cross section, which in most circumstances contains a cumulative contribution from both non-resonant and resonant components, provides the foundation for establishing any stellar reaction rate. Instead of "This research looked at of the component non-resonant," clarify what aspect of the non-resonant component was examined or analyzed and the results are presented in Tables 1 and 2. Those two tables outline the strong dependence of the non-resonant contribution to charge particle-induced reactions on temperature and the charge of the target-projectile. It is obvious that, at a constant temperature, the entrance channel with a higher atomic number has a greater Coulomb potential ($U_0 \propto Z_1 Z_2$). As a result, as in the instance of $^{12}\text{C} (p, \gamma) ^{13}\text{N}$ and $^{14}\text{N} (p, \gamma) ^{15}\text{O}$ the reaction probability shifts to a higher temperature area to give the necessary energy for the projectile to cross the Coulomb barrier and produce a reaction. This explains why light nuclei provide the majority of nuclear energy in star plasma rather than heavy ones" could be clarified to specify how the higher Coulomb potential affects the energy distribution and reaction probabilities of light versus heavy nuclei [21].

Table 1: The numerical values of $^{12}\text{C}(p,\gamma)^{13}\text{N}$ reaction rate from European Nuclear Astrophysics Compilation of Reaction Rates ($N_A\langle\sigma v\rangle$ EXP) [from NACRE-II] [22] and calculated by Matlab ($N_A\langle\sigma v\rangle$ THE). T_9 values are the temperature given in (GK = $1 \times 10^9\text{K}$), and $N_A\langle\sigma v\rangle$ are in $\text{cm}^3\text{mol}^{-1}\text{sec}^{-1}$. The Error represents calculated percentage error ($\left| \frac{N_A\langle\sigma v\rangle \text{ THE} - N_A\langle\sigma v\rangle \text{ EXP}}{N_A\langle\sigma v\rangle \text{ THE}} \right| \times 100\%$)

T_9	$N_A\langle\sigma v\rangle$ EXP	$N_A\langle\sigma v\rangle$ THE	Error %
0.006	1.21E-24	1.52E-24	20.39
0.007	4.75E-23	5.68E-23	16.37
0.008	9.85E-22	9.29E-22	6.027
0.009	1.28E-20	1.06E-20	20.75
0.01	1.16E-19	1.13E-19	2.654
0.011	7.99E-19	7.33E-19	9.004
0.012	4.4E-18	4.24E-18	3.773
0.013	2.02E-17	2.5E-17	19.2
0.014	8.01E-17	8.11E-17	1.233
0.015	2.79E-16	2.15E-16	29.767
0.016	8.75E-16	7.75E-16	12.903
0.018	6.6E-15	6.57E-15	0.4566
0.02	3.76E-14	3.53E-14	6.515
0.025	1.23E-12	1.52E-12	19.078
0.03	1.74E-11	1.31E-11	32.824
0.04	8.35E-10	8.39E-10	0.4767
0.05	1.31E-08	1.96E-08	33.163
0.06	1.07E-07	1.46E-07	26.712
0.07	5.7E-07	5.2E-07	9.615
0.08	2.28E-06	2.67E-06	14.606
0.09	7.36E-06	8.08E-06	8.910
0.1	2.03E-05	2.09E-05	2.870
0.11	4.92E-05	4.77E-05	3.144
0.12	1.08E-04	1.98E-04	45.454
0.13	2.19E-04	1.90E-04	15.263
0.14	4.16E-04	3.414E-04	21.851
0.15	7.46E-04	5.807E-04	28.465
0.16	1.28E-03	1.42E-03	99.872
0.18	3.35E-03	3.21E-03	4.361
0.2	7.81E-03	6.09E-03	28.243
0.25	4.78E-02	4.19E-02	14.081
0.3	2.33E-01	2.9E-01	19.655
0.35	9.55E-01	9.42E-01	1.380
0.4	3.18E+00	2.9E+00	9.655
0.45	8.62E+00	8.3E+00	3.855
0.5	1.96E+01	1.80E+01	8.888
0.6	6.79E+01	6.63E+01	2.4132

0.7	1.64E+02	2.07E+02	20.772
0.8	3.13E+02	3.93E+02	20.356
0.9	5.10E+02	5.741E+02	11.165
1	7.43E+02	6.54E+02	13.608
1.25	1.41E+03	1.80E+03	21.666
1.5	2.06E+03	2.9E+03	28.965
1.75	2.62E+03	3.41E+03	23.167
2	3.06E+03	3.09E+03	0.9708
2.5	3.640E+03	2.4E+03	51.66
3	3.94E+03	3.7E+03	6.486
3.5	4.08E+03	4.2E+03	2.857
4	4.13E+03	4.31E+03	4.1763
5	4.12E+03	4.2E+03	1.904
6	4.06E+03	4.092E+03	0.7820
7	3.97E+03	3.76E+03	5.5851
8	3.88E+03	3.42E+03	13.450
9	3.78E+03	3.28E+03	15.243
10	3.68E+03	3.13E+03	17.571

Table 2: The numerical values of $^{14}\text{N}(\text{p},\gamma)^{15}\text{O}$ reaction rate from European Nuclear Astrophysics Compilation of Reaction Rates ($N_A\langle\sigma v\rangle$ EXP) [from NACRE-II] [23-25] and calculated by Matlab ($N_A\langle\sigma v\rangle$ THE). T_9 values are the temperature given in (GK = $1 \times 10^9\text{K}$), $N_A\langle\sigma v\rangle$ are in $\text{cm}^3\text{mol}^{-1}\text{sec}^{-1}$. The Error represents calculated percentage error ($\left| \frac{N_A\langle\sigma v\rangle \text{THE} - N_A\langle\sigma v\rangle \text{EXP}}{N_A\langle\sigma v\rangle \text{THE}} \right| \times 100\%$)

T_9	$N_A\langle\sigma v\rangle$ EXP	$N_A\langle\sigma v\rangle$ THE	Error %
0.008	5.84E-25	4.07E-25	43.488
0.009	1.01E-23	1.75E-23	42.285
0.01	1.18E-22	1.56E-22	24.358
0.011	1.01E-21	1.24E-21	18.548
0.012	6.74E-21	6.04E-21	11.589
0.013	3.68E-20	3.14E-20	17.197
0.014	1.7E-19	1.63E-19	4.294
0.015	6.82E-19	6.77E-19	0.738
0.016	2.43E-18	2.54E-18	4.3307
0.018	2.3E-17	1.91E-17	20.418
0.02	1.59E-16	1.41E-16	12.766
0.025	7.63E-15	7.52E-15	1.4627
0.03	1.45E-13	1.3E-13	11.538
0.04	1.06E-11	1.15E-11	7.8260
0.05	2.21E-10	2.06E-10	7.281
0.06	2.24E-09	2.67E-09	16.104
0.07	1.42E-08	1.61E-08	11.801
0.08	6.5E-08	6.01E-08	8.153
0.09	2.36E-07	2.97E-07	20.538

0.1	7.2E-07	7.03E-07	2.418
0.11	1.97E-06	1.14E-06	72.807
0.12	5.21E-06	5.7E-06	8.5961
0.13	1.41E-05	1.87E-05	24.598
0.14	4.02E-05	4.72E-05	14.830
0.15	1.14 E-04	1.55 E-04	26.451
0.16	3.11 E-04	4.76 E-04	34.663
0.18	1.83 E-03	2.86 E-03	36.013
0.2	7.85 E-03	8.83 E-03	11.098
0.25	1.09 E-01	1.57 E-01	30.573
0.3	6.05 E-01	5.91 E-01	2.368
0.35	2 E+00	3 E+00	33.333
0.4	4.77 E+00	5.99 E+00	20.367
0.45	9.2 E+00	8.95 E+00	2.793
0.5	1.53 E+01	1.33 E+01	15.0376
0.6	3.18 E+01	3.88 E+01	18.041
0.7	5.18 E+01	5.87 E+01	11.754
0.8	7.31 E+01	7.403 E+01	1.256
0.9	9.39 E+01	9.82 E+01	4.3788
1	1.14 E+02	1.19 E+02	4.201
1.25	1.58 E+02	1.87 E+02	15.508
1.5	2.00 E+02	2.20 E+02	9.0909
1.75	2.50 E+02	2.60 E+02	3.8461
2	3.11 E+02	3.95 E+02	21.265
2.5	4.81 E+02	307 E+02	56.677
3	7.21 E+02	738 E+02	2.3035
3.5	1.030 E+03	1.040 E+03	0.9615
4	1.400 E+03	1.467 E+03	4.5671
5	2.310E+03	2091 E+03	10.473
6	3.370E+03	3569 E+03	5.5757
7	4.520E+03	4052 E+03	11.549
8	5.690E+03	5646 E+03	0.779
9	6.840E+03	6608 E+03	3.5109
10	7.930E+03	6023 E+03	31.662

As shown in Figures (1 and 2), the relation between the reaction rate of $^{12}\text{C}(p, \gamma)^{13}\text{N}$ and $^{14}\text{N}(p, \gamma)^{15}\text{O}$ compared with present theoretical calculations using Matlab as a function of temperature T_9 . The results clearly showed that with increasing temperature, the reaction rate of the target nuclei tends to increase more rapidly compared to the latter. Furthermore, many poison reactions will compete with the $^{12}\text{C}(p, \gamma)^{13}\text{N}$ channel, making $^{14}\text{N}(p, \gamma)^{15}\text{O}$ the proton production source, as seen in massive stars and they demonstrated that in the region with $T=0.09\text{GK}$, The shielding effect has a significant impact on the reaction rate value, particularly for targets with a higher atomic number Z .

However, when the temperature rises, the projectile energy rises, and it passes through the target at such a high velocity that the electrons are unable to shield it from the repulsive Coulomb potential. As a result, the effective screening potential becomes almost constant and has no influence on the response rate value. It was concluded that the increased binding energy of these isotopes makes the reaction less likely, allowing other competitive processes to arise and increasing the likelihood of other competitive proton reactions in the CNO cycle.

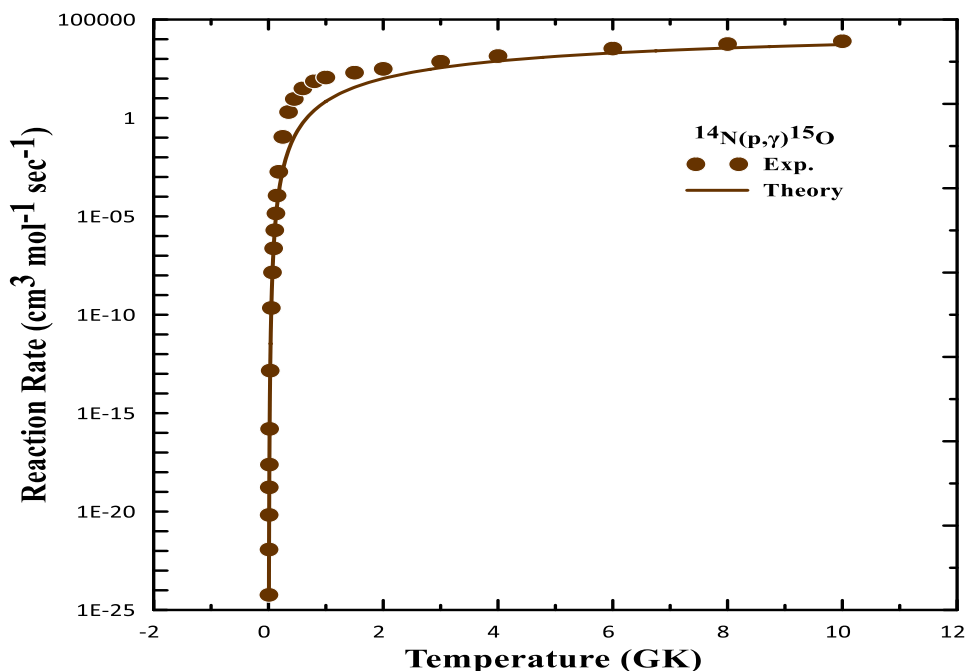


Figure1: The reaction rate (Exp.) of $^{12}\text{C} (p,\gamma)^{13}\text{N}$ from [22] compared with present theoretical (Theory) calculating using Matlab as a function of Temperature T_9 .

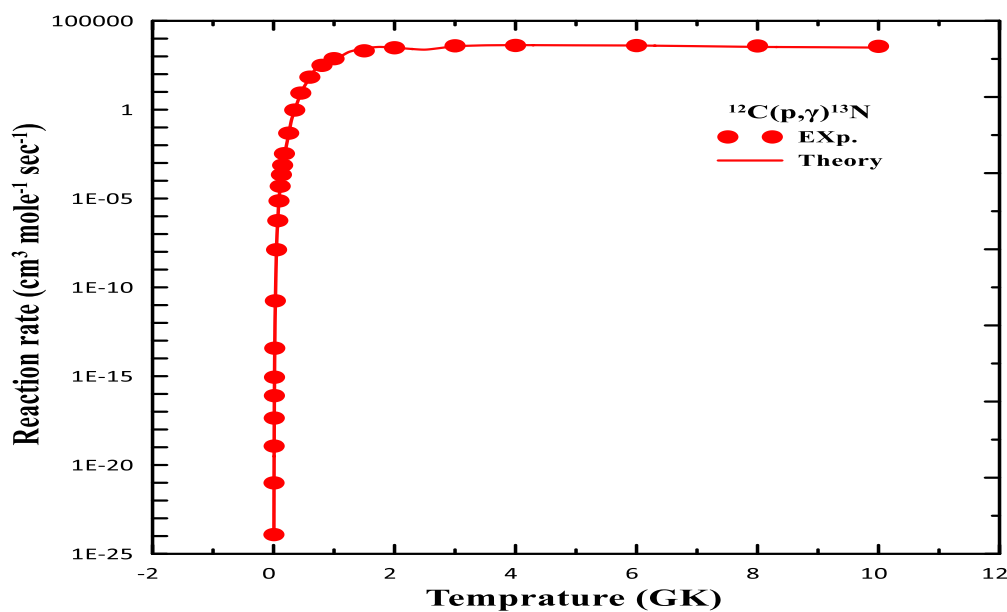


Figure 2: The reaction rate (Exp.) of $^{14}\text{N} (p,\gamma)^{15}\text{O}$ from [23-25] compared with present theoretical (Theory) calculating using Matlab as a function of Temperature T_9 .

Figure 3 shows the relation between S_{eff} and temperature T_9 . The figure obviously shows that S_{eff} has a considerable impact on adjusting the rate value by several orders of magnitude in the low-temperature zone. However, when the temperature approaches 4GK for the $^{12}\text{C}(p, \gamma)^{13}\text{N}$ reaction and 3GK for the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction, the impact begins to fade. The negative sign of the $\dot{S}(0)$ and $\dot{S}(0)S$ -factor coefficients of $^{12}\text{C}(p, \gamma)^{13}\text{N}$ reaction, whose effects on the rate value rises with increasing temperature, was attributed to the shallow reduction of the rate value at $T \geq 10$ GK.

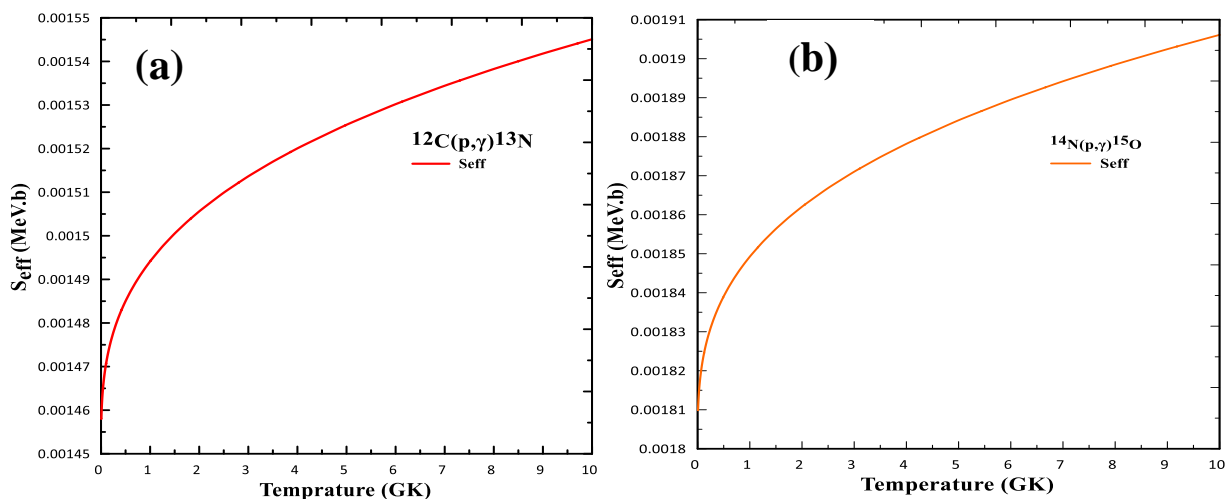


Figure 3: (a) the S_{eff} of $^{12}\text{C}(p, \gamma)^{13}\text{N}$ as a function of temperature T_9 (GK). (b) the S_{eff} of $^{14}\text{N}(p, \gamma)^{15}\text{O}$ as a function of temperature T_9 (GK).

Conclusion:

- 1- This study determined the astrophysical S-factor and thermonuclear reaction rates. The results agree with those of previously published articles.
- 2- S-factors are commonly employed in astrophysics applications for extrapolations to the astrophysical relevant Gamow energy region's very low energies. It will be able to derive cross-sectional values for low energy zone (E from 0.07 to 2.3 MeV) using the semi-empirical formulas for the S-factors. Non-resonance is an approximation of realistic nuclear reactions. In each reaction energy, there is an effective percentage of nuclear resonance due to the correspondence of part of the incident particle energy with the levels of the compound nucleus resulting from the reaction. This allows for more up-to-date data to be obtained.
- 3- The thermonuclear reaction rates decreased with rising atomic number Z of target nuclei at constant T_9 due to the increase in the Coulomb barrier with atomic number Z .
- 4- At $T < 0.15$ GK, the non-resonance reaction rate was found to have the higher values for total (p, γ) reaction. However, once the temperatures exceed 0.15 GK, the resonance rate took part in the total rate value.

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