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Using the Nanogonal Membership Function and Fuzzy Parameters for The Exponential-Rayleigh Distribution of Coved-19

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Abstract

In this paper, we discuss estimating the fuzzy parameters of the Exponential-Rayleigh distribution using the Progressive Censored sample for the Rank Set method to become the proposed Rank Set Sampling estimation Method (RSSEM) in Censoring sample using the Newton-Raphson method to determine the parameter estimate values. Then utilizing Al Karkh General Hospital-Ministry of Health and Environment in Iraq for providing information about the actual data for COVID-19. To ascertain the Chi-square test was used. The distribution of the using sample is the Exponential-Rayleigh distribution. (ER). Then, we estimate the probability density function, hazard function, and survival function. To follow the estimation of the parameters and ER distribution's fuzzy parameters, we use the suggested approach. Moreover, to find the efficiency of the estimators, we use the mean square error of the survival function:

Keywords Exponential-Rayleigh distribution (ERD), Progressive censored Sample, COVID-19, RSSEM, Survival function.

المعلمات الضبابية لتوزيع رايلي– الاسي باستخدام دالة الانتماء الضبابية الموسعة لكوفيد 19

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الخلاصة

في هذا البحث ، نوقشت تقدير معلمتي توزيع رايلي–الاسي الضبابية بأستخدام طريقة المعاينة للمجموعة المرتبة المقترحة لعينة المراقبة. باستخدام طريقة نيوتن رافسن لا يجاد القيم المقدرة للمعلمات ثم حساب المعلمات الضبابية، استندت هذه الدراسة الى عينة حقيقية تم الحصول عليها من وزارة الصحة/ مستشفى الكرخ العام لمرضى فايروس كورونا.. تم استخدام اختبار كاي_سكوير لتوضيح توزيع العينة المستخدمة في الدراسة تتوزع توزيع رايلي_الاسي وبعد تقدير المعلمات الضبابة والاعتيادية للطريقة المقترحة كذلك تم حساب دالة البقاء ،الدالة المعولية، الدالة الاحتمالية والدالة التجميعية للتوزيع وايجاد كفاءة المقدرات باستخدام متوسط مربعات الخطى لدالة البقاء

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1-Introduction

Exponential-Rayleigh distribution (ERD) was proposed and introduced by the researchers L K. Hussein, I.H.Hussein, and H.A. Rasheed in 2021. It was mixed between the accumulation function of an exponential distribution with one parameter, which is a scale, and the Rayleigh distribution's cumulative function with one parameter also of scale type(1). [1]. The data gained from experiments is known as censoring data (2). Many life testing and reliability studies may not provide experimenters with complete information on experimental unit failure rates. [2]

There are three categories for censored samples: Samples that have been censored in the right, left, and interval directions Additionally, there are three branches to the right-censored sample: There was just one type (I) censoring sample and one type(II) censoring sample, and gradually censored sample. It also goes by the name "Randomly Censored Sample." [3] .This type is characterized by the different entering times in the period of the experiment. Most clinical and epidemiologic studies have a predetermined study duration. However some patients pass away before it does, of the study so the exact survival times are known, some of the patients are lost to follow-up, and some patients are still alive until the end of the experiment.[4]. Fuzzy number offers a natural approach to solving issues where imprecision and ambiguity are the root causes, and it has a wide range of applications in artificial intelligence, control systems, decision-making, expert systems, etc. Dubois, D., and Prade, H. define the idea of a fuzzy number as a fuzzy subset of the real line [5]. The researchers employ many membership functions of fizzy numbers, some of them use triangular membership, and others ale the trapezoidal membership. In this article, we consider the Nanogonal membership function for fuzzy numbers, which were used by Felix, Christopher, and Devadazs for the first time in the year (2015) [6]. This research aims to derive the estimated parameters of Exponential Rayleigh distribution for a progressive censored sample by using the Proposed Rank set sampling estimation method based on the new ton-Raphsan procedure to find the point estimate values of parameters. Then utilizing this point estimation to find the values of interval estimation. After that, the interval estimation is used to find the fuzzy numbers by emptying the Nonagnoal membership function. Finally, the suggested Ranking function is used to convert the fuzzily-numbered data to crisp data and then determine the survival function, hazard rate function, and death density function, and mean squares error for survival. function to find the efficiency of this estimator value for and after fuzzy numbers. In 2023 Suhaila N. Abdullah [7], introduced the Rayleigh distribution for estimating the two parameters via fuzzy theory with important applications, Suhaila N. Abdullah1 Iden Hasan Huessian2 in [[8]] , estimating fuzzy Weibull parameters.

This search deals with the Exponential Rayleigh distribution and its properties, as well as the method of parameters estimation. Moreover the description of data, and numerical results. Finally we submitted the conclusions of this work.

2-Exponential Rayleigh distribution (ERD) [9]

In 2021, the proposal and introduction of the exponential Rayleigh distribution, of Hussein LK, Hussein IH, and Rasheed HA employed the following combination of the cumulative of each distribution:

Let $T = max\ (v, w)$, if v and w represent two independent unknowns, then

$$F(t; \theta, \beta) = p_r(T \leq t)$$
$$F(t; \theta, \beta) = 1 - p_r(T > t)$$
$$F(t; \theta, \beta) = 1 - p_r(max\ (v, w) > t)$$

$$F(t;\theta,\beta) = 1 - [\, p_r(v > t).\, p_r(w > t)]$$
$$F(t;\theta,\beta) = 1 - [\left(\int_t^{\infty} f(v;\theta)dv\right).\left(\int_t^{\infty} f(w;\beta)dw\right)]$$
$$F(t;\theta,\beta) = 1 - [\left(\int_t^{\infty} \theta e^{-\theta v}dv\right).\left(\int_t^{\infty} \beta w e^{-\frac{\beta}{2}w^2}dw\right)$$

$F(t;\theta,\beta) = 1 - e^{-(\theta t+\frac{\beta}{2}t^2)}$ … (1)

The probability density function of ERD is:

$$f(t;\theta,\beta) = \begin{cases}(\theta+\beta t)e^{-\left(\theta t+\frac{\beta}{2}t^2\right)} & , \quad t>0 \\ 0 & ,otherwise\end{cases} \quad \ldots (2)$$

ERD has two scale parameters which are denoted by θ, β, the parameter space is as follows: $\Omega = \{(\theta,\beta); \theta > 0, \beta > 0\}$

The Survival function is given by:

$$S(t;\theta,\beta) = e^{-\left(\theta t+\frac{\beta}{2}t^2\right)} \quad \ldots(3)$$

The Hazard rate function is obtained by:

$$h(t;\theta,\beta) = \theta + \beta t \quad \ldots(4)$$

3-Parameters Estimation[10]

This part uses the (RSSEM) for the Progressively Censoring sample to display the derivative and estimate of the ER distribution's unmeasured properties, and estimate the fuzzy parameters of distribution. McIntyre (1952) suggested to use of rank-set sampling to calculate pasture yield in Australia. For a long time, this method was not used, but in the past 30 years, it has been extensively used in research, making it vital in many places. (4). A methodology for data collecting and analysis called Ranked Set Sampling has been the subject of extensive methodological study It has given rise to other related approaches, which are also active research fields. Moreover, it is now starting to go outside the agricultural context in which it was first introduced in the key paper by McIntyre. (5). In this paper, we add the Progressive Censored sample for the Rank Set method to become the proposed Rank Set Sampling Estimator Method (RSSEM) in the Censoring sample.

In this paper, we add the Progressive Censored sample for the Rank Set method to become the proposed Rank Set Sampling Estimator Method (RSSEM) in Censoring sample

Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from continuous probability density function f(x) with provided $a < x < b$, let Y_1 be the smallest of these X_i , Y_2 be the next order to magnitude and Y_n be the largest of these X_i. That means $Y_1 < Y_2 < \cdots < Y_n$ represent $X_1, X_2, X_3, \ldots, X_n$ when the latter are arranged in a sending order of magnitude. Then $Y_1, Y_2, \ldots, Y_n$ are denoted of ith order statistic of random sample $X_1, X_2, X_3, \ldots, X_n$. Now easy to formulate the probability density function of any order statistic, called Y_i in term of f(x) and F(x) as follows:

$$g(y_i) = \begin{cases}\frac{m!}{(i-1)!\,(m-i)!}[F(y_i)]^{i-1}\,[1-F(y_i)]^{m-i}\, f(y_{(i)}) & a < y_i < b \\ 0 & otherwise\end{cases}$$

In the first step, apply the order statistic as following:

$$g(\,t_{(i)}) = \frac{m!}{(i-1)!(m-i)!}\left[F\left(t_{(i)}\right)\right]^{i-1}\left[1-F\left(t_{(i)}\right)\right]^{m-i} f(t_{(i)}) \quad \ldots (5)$$

Applying the formula of order statistic in Eq. (5) to the (ER) distribution as follows:

$$g(t_{(i)}) = \frac{m!}{(i-1)!(m-i)!}\left[1 - e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}\right]^{i-1} \left[e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}\right]^{m-i+1} \left(\theta + \beta t_{(i)}\right) \quad \ldots(6)$$

After that finding the likelihood function for Eq. (6) by utilizing Progressive Censoring formula as follows:

$$L = \frac{n!}{(n-m)!}\prod_{i=1}^{n}\{\left[g(t_{(i)}, \theta, \beta)\right]^{\delta_i}\left[s(t_{(i)})\right]^{1-\delta_i}\}$$

where $\delta_i = \begin{cases} 1 & if\ the\ patients\ die \\ 0 & if\ the\ patients\ alive \end{cases}$

$$L = \frac{n!}{(n-m)!}\prod_{i=1}^{n}\{\left[\frac{m!}{(i-1)!(m-i)!}\left(1 - e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}\right)^{i-1}\left(e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}\right)^{m-i+1}\left(\theta + \beta t_{(i)}\right)\right]^{\delta_i} \cdot \left[e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}\right]^{1-\delta_i}\}$$

$let\ \frac{n!}{(n-m)!} = c, \frac{m!}{(i-1)!(m-i)!} = d$ then we get:

$$L = c\prod_{i=1}^{n}\{\left[d\left(1 - e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}\right)^{i-1}\left(e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}\right)^{m-i+1}\left(\theta + \beta t_{(i)}\right)\right]^{\delta_i} \cdot \left[e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}\right]^{1-\delta_i}\}$$

$$L = c\, d^{n\delta_i}\prod_{i=1}^{n}\left[1 - e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}\right]^{\delta_i(i-1)}\prod_{i=1}^{n}\left[e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}\right]^{\delta_i(m-i+1)}$$

$$\prod_{i=1}^{n}\left[\theta + \beta t_{(i)}\right]^{\delta_i}\prod_{i=1}^{n}\left[e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}\right]^{1-\delta_i} \quad \ldots(7)$$

By taking the natural logarithm for each side of Eq. (7), we obtain

$$:LnL = lnc + n\delta_i lnd + \sum_{i=1}^{n}\delta_i(i-1)ln\left(1 - e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}\right) - \sum_{i=1}^{n}\delta_i(m-i+1)\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)$$

$$+\sum_{i=1}^{n}\delta_i ln\left(\theta + \beta t_{(i)}\right) - (1 - \delta_i)(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2) \quad \ldots(8)$$

Eq. (8) is partially derived with regard to, θ and β and then set it to zero.

Deriving the Eq.(8) putting it equal to zero.

$$\frac{\partial lnl}{\partial\theta} = \sum_{i=1}^{n}\frac{\delta_i(i-1)t_{(i)}e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}}{1 - e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}} - \sum_{i=1}^{n}\delta_i(m-i+1)t_{(i)} + \sum_{i=1}^{n}\frac{\delta_i}{(\theta + \beta t_{(i)})} - \sum_{i=1}^{n}(1 - \delta_i)t_{(i)} = 0$$

$$\frac{\partial lnl}{\partial\beta} = \frac{1}{2}\sum_{i=1}^{n}\frac{\delta_i(i-1)t_{(i)}^2 e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}}{1 - e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}} - \frac{1}{2}\sum_{i=1}^{n}\delta_i(m-i+1)t_{(i)}^2 + \sum_{i=1}^{n}\frac{\delta_i t_{(i)}}{(\theta + \beta t_{(i)})} - \frac{1}{2}\sum_{i=1}^{n}\delta_i t_{(i)}^2 = 0$$

Now, we can put $\frac{\partial lnl}{\partial\theta}$ as a function $f(\theta)$ and put $\frac{\partial lnl}{\partial\beta}$ as a function $f(\beta)$

$$f(\theta) = \sum_{i=1}^{n}\frac{\delta_i(i-1)t_{(i)}e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}}{1 - e^{-\left(\theta t_{(i)} + \frac{\beta}{2} t_{(i)}^2\right)}} - \sum_{i=1}^{n}\delta_i(m-i+1)t_{(i)} + \sum_{i=1}^{n}\frac{\delta_i}{(\theta + \beta t_{(i)})} - \sum_{i=1}^{n}(1 - \delta_i)t_{(i)} \quad \ldots(9)$$

$$f(\beta)=\frac{1}{2}\sum_{i=1}^{n}\frac{\delta_i(i-1)t_{(i)}^2e^{-\left(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2\right)}}{1-e^{-\left(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2\right)}}-\frac{1}{2}\sum_{i=1}^{n}\delta_i(m-i+1)t_{(i)}^2+\sum_{i=1}^{n}\frac{\delta_i t_{(i)}}{(\theta+\beta t_{(i)})}-\frac{1}{2}\sum_{i=1}^{n}\delta_i t_{(i)}^2 \quad \ldots(10)$$

Noting that Eqs. (9) and (10) are challenging to be solved, one could use an iterative technique like the Newton-Raphson algorithm to use them to discover the value of $\hat{\theta}$ and $\hat{\beta}$ as follows:

$$\theta_{k+1}=\theta_k-\frac{f(\theta_k)}{f^{\backslash}(\theta_k)}$$

Where $f^{\backslash}(\theta_k)=-\sum_{i=1}^{n}\frac{\delta_i(i-1)t_{(i)}^2e^{-\left(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2\right)}}{\left(1-e^{-\left(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2\right)}\right)^2}-\sum_{i=1}^{n}\frac{\delta_i}{\left(\theta+\beta t_{(i)}\right)^2}$

And $\beta_{k+1}=\beta_k-\frac{f(\beta_k)}{f^{\backslash}(\beta_k)}$

Where $f^{\backslash}(\beta_k)=-\frac{1}{4}\sum_{i=1}^{n}\frac{\delta_i(i-1)t_{(i)}^4e^{-\left(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2\right)}}{\left(1-e^{-\left(\theta t_{(i)}+\frac{\beta}{2}t_{(i)}^2\right)}\right)^2}-\sum_{i=1}^{n}\frac{\delta_i t_{(i)}^2}{\left(\theta+\beta t_{(i)}\right)^2}$

The phrase that describes the error is:

$\epsilon_{k+1}(\theta)=\theta_{k+1}-\theta_k$

$\epsilon_{k+1}(\beta)=\beta_{k+1}-\beta_k$

Where θ_k and β_k are presumptive initial values.

4-Fuzzy Set Theory:

Any set that enables its members to have a range of memberships, known as membership functions, between [0,1] , is said to be fuzzy. Accordingly, a membership function $\mu A\ (x)$ is connected with a fuzzy set that translates each component of the X universe of discourse to the range [0,1] as an expansion of the traditional notion of set, which was independently introduced by Zadeh in 1965, where $A=\{(x,\mu\tilde{A}\ (x)), x\ \epsilon\ X\}$ is the definition of a fuzzy set.

5-Nonagonal Fuzzy Number: [6]

A Nonagnoal of fuzzy number is symbolized as $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$, and the membership function defined as

$$\mu_{\tilde{A}}(x)=\begin{cases}\frac{1}{4}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1\leqslant x\leqslant a_2\\ \frac{1}{4}+\frac{1}{4}\left(\frac{x-a_2}{a_3-a_2}\right) & a_2\leqslant x\leqslant a_3\\ \frac{1}{2}+\frac{1}{4}\left(\frac{x-a_3}{a_4-a_3}\right) & a_3\leqslant x\leqslant a_4\\ \frac{3}{4}+\frac{1}{4}\left(\frac{x-a_4}{a_5-a_4}\right) & a_4\leqslant x\leqslant a_5\\ 1-\frac{1}{4}\left(\frac{x-a_5}{a_6-a_5}\right) & a_5\leqslant x\leqslant a_6\\ \frac{3}{4}-\frac{1}{4}\left(\frac{x-a_6}{a_7-a_6}\right) & a_6\leqslant x\leqslant a_7\\ \frac{1}{2}-\frac{1}{4}\left(\frac{x-a_7}{a_8-a_7}\right) & a_7\leqslant x\leqslant a_8\\ \frac{1}{4}\left(\frac{x-a_8}{a_9-a_8}\right) & a_8\leqslant x\leqslant a_9\\ 0 & \text{o.}w\end{cases}$$

To illustrate this Nonagnoal membership function see Figure 1

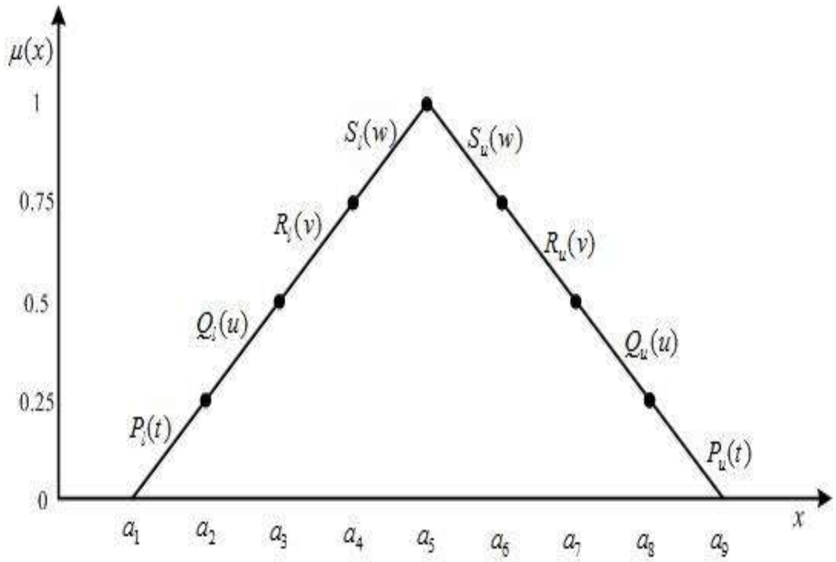

Figure 1: The Nonagnoal fuzzy number

Algorithm steps of methods

$C.I\ FOR\ \ \hat{\theta} = \hat{\theta} \mp t_{(n-p,1-\propto)}\sqrt{var(\hat{\theta})} = [\hat{\theta}_{lower}, \hat{\theta}_{upper}] \ldots (11)$

The Nonagnoal numbers is follows:

$$A = (\hat{\theta}_{lower}, \bar{x} - 3S,\ \bar{x} - 2S, \bar{x} - S,\ \bar{x}, \bar{x} + S, \bar{x} + 2S,\ \bar{x} + 3S,\ \hat{\theta}_{upper}) \quad \ldots(12)$$

Where

$$\bar{x} = \frac{\hat{\theta}_{lower} + \hat{\theta}_{upper}}{2}\ , S = \sqrt{\frac{(\hat{\theta}_{lower} - \bar{x})^2 + (\hat{\theta}_{upper} - \bar{x})^2}{2}} \quad \ldots (13)$$

Then using the same procedure algorithm to find the fuzzy numbers for β

The fuzzy number for Nonagnoal membership function is as follows:

$$(x) = \begin{cases} inf_1\tilde{A}(\propto) = 4 \propto (a_2 - a_1) + a_1 & a_1 \leqslant x \leqslant a_2 \\ inf_2\tilde{A}(\propto) = (4 \propto -1)(a_3 - a_2) + a_2 & a_2 < x \leqslant a_3 \\ inf_3\tilde{A}(\propto) = (4 \propto -2)(a_4 - a_3) + a_3 & a_3 < x \leqslant a_4 \\ inf_4\tilde{A}(\propto) = (4 \propto -3)(a_5 - a_4) + a_4 & a_4 < x \leqslant a_5 \\ sup_1\tilde{A}(\propto) = (4 - 4 \propto)(a_6 - a_5) + a_5 & a_5 < x \leqslant a_6 \\ sup_2\tilde{A}(\propto) = (3 - 4 \propto)(a_7 - a_6) + a_6 & a_6 < x \leqslant a_7 \\ sup_3\tilde{A}(\propto) = (2 - 4 \propto)(a_8 - a_7) + a_7 & a_7 < x \leqslant a_8 \\ sup_4\tilde{A}(\propto) = (4 \propto)(a_9 - a_8) + a_8 & a_8 < x \leqslant a_9 \\ 0 & \text{o}.w \end{cases} \quad \ldots(14)$$

Where $\propto is\ \propto -cut$ in fuzzy number

The fuzzy number is converted to a crisp number by the ranging function

$Rf(\tilde{A}) = \frac{1}{2}\int_0^1 [inf\ \tilde{A}(\propto) + sup\tilde{A}(\propto)]d \propto \ldots(15)$

6-Description of data

This work was done on real sample data brought from the Al-Karkh General Hospital. About Covid-19. The study took a period of (120) days. The number of patients in the study was (1058). excluded six cases from the study, the number of prisoners (26), the number of people whose swab results were negative (48), and the number of patients whose exit status from the study was unknown (29). As for the number of patients who escaped from the hospital (2), the number of patients transferred to other hospitals (35), and the number of patients who were discharged at their own responsibility (133). Accordingly, the number of patients under observation became (785), who (88) died during the study period. In this work, we will use the chi-square test to see whether the group data has a Rayleigh exponential distribution or not.

H_0 : Dates distributed as part of ER distribution

H_1 : The distribution of the dates is not done using ER. T provides the chi-square test formula.

$$x^2 = \sum_{i=1}^{k} \frac{(\theta i - E_i)^2}{E_i}$$

With degree of freedom (9) , significance level (0.01), the estimated value (15.32008) is smaller than the table value (21.67); hence, H_0 is accepted.

7-Numerical Results

In this section, the variables of the essential parameters were estimated using Matlab programming (version 2021): $\hat{\theta} = 0.00964$, $\tilde{\beta} = 0.00558$ when the initial values are $\theta_0 = 0.0058$, $\beta_0 = 0.0058$..

There were (15) patients who passed away on a single day while organizing the data, noticing that many patients passed away at the same time and expressing the data succinctly to minimize repetition in the table., Patients who passed away two days later included (7), three days later included (5), four days later included (9) and five days later included (4)., (17) patients passed away six days later, (5) passed away seven days later, and (4) passed away eight days later, (2) patients who passed away after ten days, (3) patients who passed away after fifteen days, (4) patients who passed away after eighteen days, and (6) patients who passed away after twelve days. Substitute the estimated parameter and survival values in Eqs. 1,2, and 3. In Eq. 4, we derive the $f(t), F(t), S(t)$, and h(t). $\hat{f}$(t), $\hat{F}$(t), $\hat{S}$(t), and $\hat{h}$(t) in Table 1.

Table 1:

$\hat{f}$(t)	$\hat{F}$(t)	$\hat{S}$(t)	$\hat{h}$(t)
.01504	.01236	.98764	.01523
.02018	.02999	.97001	.02082
.02499	.05262	.94738	.02639
.02942	.07986	.92014	.03197
.03337	.11129	.88871	.03755
.03681	.14643	.85357	.04313
.03971	.18474	.81526	.04871
.04204	.22567	.77433	.05429
.04379	.26863	.73137	.05987
.04496	.31305	.68695	.06545
.04558	.35837	.64163	.07104
.04566	.40403	.59597	.07662
.04525	.44953	.55048	.08219
.04438	.4943	.50570	.08776
.04312	.53816	.46185	.09336
.041505	.58049	.41951	.09894
.03961	.62107	.37893	.10452
.03748	.65963	.34037	.1101

It should be noted that until$t = 12$, the values of $\hat{f}$(t) are increasing; nevertheless, starting at $t = 13$, they start to decrease. As failure times climb, $\hat{F}$(t) increases, $\hat{S}$(t) decreases, and $\hat{h}$(t) rises, respectively. The (M.S.E) of survival function is 0.031229.
Initial values are $\theta_0 = 0.0058$, $\beta_0 = 0.0058$
Estimate values are $\tilde{\theta} = 0.00964$, $\hat{\beta} = 0.00558$
$(n - p\ , 1{-}\propto) = (88 - 2\ ,1 - 0.05) = t(86{,}0.095) = 1.664$
where $n = 88$ is the sample size which the patients died, and $P = 2$ is the numbers of parameters, and we find interval estimation by using Confidence Interval (C.I)
$var(\tilde{\theta}) = 0.0000189$. Then we find the C.I for $(\tilde{\theta})$, by Eq (10), it follows that
$[\tilde{\theta}_{\mathrm{L}}, \tilde{\theta}_u] = [0.00241{,}0.01687]$, and use Eq (12), we have $\bar{X}_{\tilde{\theta}} = 0.00964$, $S_{\tilde{\theta}} = 0.0073$
Also, from Eq (11), get, [$\tilde{\theta}_1$=0.00241, $\tilde{\theta}_2 = -0{,}01226, \tilde{\theta}_3 = -0.00496, \tilde{\theta}_4 = 0{,}00234, \tilde{\theta}_5 = 0.00964, \tilde{\theta}_6 = 0.01694, \tilde{\theta}_7 = 0.02424, \tilde{\theta}_8 = 0.03154, \tilde{\theta}_9 = 0.01687$] , as well as by Eq (13), one have the nonagon numbers by the following

$$(x) = \begin{cases} inf_1\, \tilde{\theta} = -0.05868 \propto +0.00241 & \tilde{\theta}_1 \leqslant x \leqslant \tilde{\theta}_2 \\ inf_2\, \tilde{\theta} = 0.0292 \propto -0.01226 & \tilde{\theta}_2 < x \leqslant \tilde{\theta}_3 \\ inf_3\, \tilde{\theta} = 0.0292 \propto -0.01956 & \tilde{\theta}_3 < x \leqslant \tilde{\theta}_4 \\ inf_4\, \tilde{\theta} = 0.0292 \propto -0.01956 & \tilde{\theta}_4 < x \leqslant \tilde{\theta}_5 \\ sup_1\, \tilde{\theta} = -0.0292 \propto +0.03884 & \tilde{\theta}_5 < x \leqslant \tilde{\theta}_6 \\ sup_2\, \tilde{\theta} = -0.0292 \propto +0.03884 & \tilde{\theta}_6 < x \leqslant \tilde{\theta}_7 \\ sup_3\, \tilde{\theta} = -0.0292 \propto +0.03884 & \tilde{\theta}_7 < x \leqslant \tilde{\theta}_8 \\ sup_4\, \tilde{\theta} = -0.05868 \propto +0.03154 & \tilde{\theta}_8 < x \leqslant \tilde{\theta}_9 \\ 0 & \mathrm{o}.w \end{cases}$$

So by using Eq (14), we can have crisp number, $R(\tilde{\theta}) = 0.007926$
In the same way, we can find fuzzy number crisp number of $\hat{\beta}$
$var(\tilde{\beta}) = 0.00000074$
$[\tilde{\beta}_{\mathrm{L}}, \tilde{\beta}_u] = [0.00105{,}0.01011]$
$\bar{X}_{\tilde{\beta}} = 0.00554, S_{\tilde{\beta}} = 0.00447$
[$\tilde{\beta}_1$=0.00105, $\tilde{\beta}_2 = -0.001783$, $\tilde{\beta}_3 = -0.00336$, $\tilde{\beta}_4 = 0{,}00111$, $\tilde{\beta}_5 = 0.00558$, $\tilde{\beta}_6 = 0.01005$, $\tilde{\beta}_7 = 0.01452$, $\tilde{\beta}_8 = 0.01899$, $\tilde{\beta}_9 = 0.01011$]

$$(x) = \begin{cases} inf_1\, \tilde{\beta} = -0.03552 \propto +0.00105 & \tilde{\beta}_1 \leqslant x \leqslant \tilde{\beta}_2 \\ inf_2\, \tilde{\beta} = 0.01788 \propto -0.0123 & \tilde{\beta}_2 < x \leqslant \tilde{\beta}_3 \\ inf_3\tilde{\beta} = 0.01788 \propto -0.0123 & \tilde{\beta}_3 < x \leqslant \tilde{\beta}_4 \\ inf_4\, \tilde{\beta} = 0.01788 \propto -0.0123 & \tilde{\beta}_4 < x \leqslant \tilde{\beta}_5 \\ sup_1\, \tilde{\beta} = -0.01788 \propto +0.02346 & \tilde{\beta}_5 < x \leqslant \tilde{\beta}_6 \\ sup_2\tilde{\beta} = -0.01788 \propto +0.02346 & \tilde{\beta}_6 < x \leqslant \tilde{\beta}_7 \\ sup_3\tilde{\beta} = -0.01788 \propto +0.02346 & \tilde{\beta}_7 < x \leqslant \tilde{\beta}_8 \\ sup_4\, \tilde{\beta} = -0.03552 \propto +0.01899 & \tilde{\beta}_8 < x \leqslant \tilde{\beta}_9 \\ 0 & \mathrm{o}.w \end{cases}$$

Then the crisp number
$R(\tilde{\beta}) = 0.002916$
By replacing the lifetime values in Equations 1, 2, and 3 with the estimated values of the parameters. And Eq.4 In Table 2, we find the $\hat{f}$(t), $\hat{F}$(t), $\hat{S}$(t) , and $\hat{h}$(t):
.

Table 2:

$\hat{f}$(t)	$\hat{F}$(t)	$\hat{S}$(t)	$\hat{h}$(t)
.00943	.00865	.99135	.00952
.0128	.02008	. 97992	.01306
.01542	.03421	.96579	.01597
.01793	.05089	.94911	.01889
.0203	.07001	.92999	.02181
.02246	.09139	.9086	.02472
.02446	.11487	.88513	.02764
.02627	.14026	.85974	.03055
.02793	.16734	.83266	.0335
.02926	.19592	.80408	.03639
.03043	.22578	.77422	.03930
.03138	.25671	.74329	.04222
.03211	.28847	.71153	.04513
.03267	.32086	.67914	.0481
.03294	.35367	.64633	.05097
.03305	.38667	.61333	.05388
.03296	.41969	.58031	. 0568
.03269	.45254	.54746	.05971

Now, we calculate the MSE of survival function by using the following function:

$$MSE[S(t)] = \sum_{i=1}^{n} \frac{[\tilde{S}(t) - \hat{S}(\mathrm{t})]^2}{n-p} = 0.003400$$

8-Conclusion

This investigation may yield the Exponential-Rayleigh distribution parameters for the Progressive Censored Samples. The scale parameters' estimated values can be found by utilizing the (RSSEM) which employs a deductive iterative technique like the Newton-Raphson method. Then we find the Interval Estimation for these parameters after that applying the Nanogonal membership to find the fuzzy numbers. Therefore applying the Ranking function to transform fuzzy numbers to crisp numbers find the survival function and Compare between them by using the main square error procedure to find the survival function after fuzzy and before fuzzy numbers. Based on actual Coronavirus data. It should be observed that the hazard rate function and failure times are directly related, with $\hat{h}$(t) increasing as failure times' increase. Due to the finding that the survival function and failure time s have an antagonistic connection, $\hat{S}$(t) is decreasing as failure times' rise.

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