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Determination of the characteristics of the perturbed transition orbit to transfer from LEO to GEO

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Abstract

This paper aims to study, the process of satellite transition between different orbits and evaluate the characteristics of the turbulent transitional orbit of transferring a satellite from Low Earth Orbit (LEO) to geostationary orbit (GEO) (hp=35790km). The measurements in this research involve considering different LEO characteristics of perigee heights (h_p=200, 400, 600,800) and eccentricities (e=0.01, 0.05 and 0.1), whereas the inclination is fixed to be (i= 23.445 degree).Also, the transfer of the satellite was studied in two stages. The first stage was from the perigee of the first orbit at height 200 km, to the apogee of the second orbit at height 945 km with eccentricity (0.01). the second stage, was the transition the satellite from the perigee of the second orbit at height 800 km to the final orbit (GEO) with the same eccentricity. Two types of turbulence were considered which are atmospheric drag and J_2 perturbation (Geopotential acceleration The perturbing acceleration effects on satellite due to the Earth's gravity potential represent as function of J2 parameter). A program in MATLAB was designed to calculate the speed that for the transition, time and the ratio mass change, which represents the percentage of fuel as well as the extent of the effect of turbulence on the orbital elements of the transitional orbit.

The perturbed equation of motion was solved using Runge–Kutta method. The results showed that the increase altitude, the lower ΔV(velocity required to transition), where the ΔV at altitude of 800 km was the best results, reaching (2.244 km/sec). As for the transition in two stage, it may need a higher energy for the transition, as the difference between it and the direct phase in (ΔV= 0.011 km/sec). As the height of the initial orbit increases, the effect of the disturbance on the orbital elements of the transitional orbit decreases, and since the transition time is less than half a day (0.44 day), the effect of the disturbance is clear, so it cannot be neglected.

Keywords: Atmospheric drag , J2 perturbation, Low Earth orbit, Geo transfer orbit.

تحديد خصائص المدار الانتقالي المضطرب للانتقال من المدار الأرضي المنخفض (LEO) إلى المدار المتزامن الارضي (GEO)

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الخلاصة:

يهدف هذا البحث دراسة عملية انتقال القمر الصناعي بين مدارات مختلفة، وحساب خصائص مدار الانتقال المضطرب لنقل قمر صناعي من المدار الأرضي المنخفض إلى المدار الثابت بالنسبة للأرض (GEO) على ارتفاع (35790 كم). الحسابات في هذا البحث تتضمن قيماً مختلفة لخصائص المدار الارضي المنخفض، حيث ان قيم الارتفاع من الحضيض هي (200، 400، 800,600) كم، الانحراف المركزي (0.01، 0.05، 0.1) وقيمة ثابتة للميل (23,445 درجة). كما تمت دراسة نقل القمر الصناعي على مرحلتين. المرحلة الأولى كانت من حضيض المدار الأول على ارتفاع 200 كم إلى أوج المدار الثاني على ارتفاع 945 كم مع انحراف مركزي (0.01). أما المرحلة الثانية، فكانت انتقال القمر الصناعي من نقطة حضيض المدار الثاني على ارتفاع 800 كم إلى المدار النهائي (GEO) بنفس الانحراف المركزي. تم دراسة نوعين من الاضطراب وهما كبح الغلاف الجوي والاضطراب J2 (التسارع الأرضي تمثل تأثيرات التسارع المضطربة على القمر الصناعي نتيجة لجاذبية الأرض كدالة ل J2). تم تصميم برنامج في MATLAB لحساب سرعة الانتقال والزمن الانتقال ونسبة تغير الكتلة والتي تمثل نسبة الوقود وكذلك مدى تأثير الاضطراب على العناصر المدارية للمدار الانتقالي. تم حل معادلة الحركة المضطربة باستخدام طريقة Runge–Kutta. أظهرت النتائج أن زيادة الارتفاع، يحتاج أقل ΔV (السرعة المطلوبة للانتقال)، حيث كانت ΔV على ارتفاع 800 كم هي أفضل النتائج، إذ بلغت (2.244 كم/ثانية). أما الانتقال على مرحلتين فقد يحتاج إلى طاقة أعلى للانتقال، إذ يبلغ الفرق بينه وبين الانتقال المباشر ΔV (0.011 كم/ثانية). ومع زيادة ارتفاع المدار الأولي، يتناقص تأثير الاضطراب على العناصر المدارية للمدار الانتقالي، وبما أن زمن الانتقال أقل من نصف يوم (0.44 يوم)، فإن تأثير الاضطراب يكون واضحا، لذا فإنه لا يمكن إهمالها.

1. Introduction

Orbital perturbation is the effect of an external force on a satellite's orbit .There are two types of perturbations that affect the position, velocity and lifetime of a satellite. These two kinds of perturbations are usually known as gravitational and nongravitational perturbation; gravitational perturbation occurs due to non-spherical shape of Earth, Earth Tide effect, ocean tide effect and effect of Moon and Sun attraction. Non-gravitational perturbation, on the other hand, happens due to the atmospheric drag force, solar radiation pressure (SRP), etc. Moraes (1991) studied the joint effects of direct atmospheric drag and (SRP) on the orbit of an Earth satellite and found that these perturbations affect the orbital elements of a satellite [1]. Where these perturbations affect the orbital elements of the satellite. The atmospheric drag perturbation affects low-altitude satellites, and its value is zero at an altitude of more than 1200 km. This type of perturbation depends on several parameters, such as the density (ρ) of the atmosphere, the cross-sectional area to mass ratio (A/m), the velocity of the satellite relative to the atmosphere (v), as well as the drag coefficient C_D [2,3,4,5]. Non-spherical gravitational field of the Earth: occurs due to the fact that the Earth is not spherical but instead is flattened at the poles. This leads to irregular gravity and generates extra disturbance force in the velocity and orbital normal direction on the satellite Orbital elements are elements that define the specifications of the orbit in space and are divided into two parts: dimension orbital elements and orientation orbital elements, these two types of elements are described below:

a. Dimension orbital elements that determine the dimensions of an orbit in space [6]:

Eccentricity (e) defines the orbit's shape. For example if e=0, the orbit is circle and if e=1 the orbit is ellipse. In general, if the eccentricity is between 0 and 1 (i.e. 0<e<1), then the orbit is elliptical while if e>1 the orbit is hyperbola. Semi-major axis (a) is define , on the other hand defines the orbit's size in space. Time after the perigee passage (t): It illustrates the link between the time and the satellite's position within orbit.

b. Orientation orbital elements: these orbital elements are called Euler angles, which determine the position of the orbit in space [6] :

Inclination (i): It is an angle measured in degrees, which represents the angle at which the satellite orbit is inclined from the celestial equator plane (the reference circle for satellites), is range (0-180) degree also divided in four type (a) equatorial if i=0 or 180 (b) polar if i=90 (c) prograde if 0<i<90 (b) retrograde if 90<i<180, as show in Figure (1).
Right ascension of the ascending node (Ω): This is the angle between the vernal equinox, which symbolizes the point where the equator and the ecliptic circle intersect, and the ascending node, which is where the satellite's orbit crosses the equator from south to north. This angle's value lies between 0 and 360 degrees.

Argument of perigee (ω): Its value ranges from 0 to 360 degrees and reflects the angle between the ascending node and the line connecting the Earth's center and the perigee.
Previous research and studies on this topic focused on geostationary orbit (GEO), medium Earth orbit (MEO), and low Earth orbit (LEO) satellites, such as GPS and GLONAS, which were studied by Sue in 2000 [6]. The effects of atmospheric drag and zonal Harmonics on the orbits of satellites in low Earth orbit, is study by Al-Burmani and S. Barron (2009) concluded that there is no apparent effect of the J2 spherical harmonics of the gravitational potential during the early stage of injection, and the dominant effect was the drag force in the atmosphere at the perigee, which leads to a decrease in the altitude of the satellite until its return after 1560 days in a dense atmosphere [7]. In (2015) Al-Mohammadi and Mutlaj studied the Modified Model to calculate (LEO) for a satellite with Atmospheric Drag and concluded that the satellite's low orbit was affected by different types of perturbations; the most important one is atmospheric drag. They also found that the semi-major axis and eccentricity (e) are reduced very slowly with larger values of r_p. As for the perturbation of solar radiation pressure, Saleh and Murad (2016) calculated the effect of solar and lunar attraction and SRP on the HEO of satellite. As for the transfer between orbits, Mark Linick (2016) studied the transfer process between orbits. [8]. And in the year (2017) Allam, Awad, and Amin calculated the Bi-Elliptic Hohmann transfer and one tangent burn transfer calculations using Monte Carlo simulation. In 2020, Mahdi, Saleh, and Jarad studied determination and published an article whose main objective was to evaluate the orbital transition methods between two elliptical earth orbits.

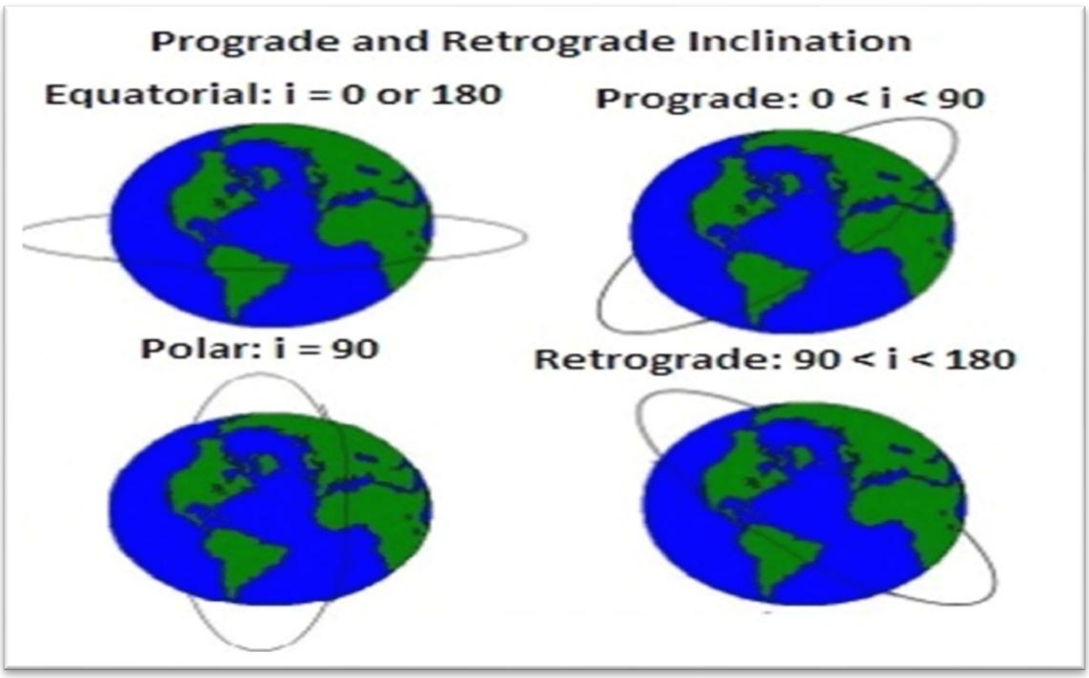

Figure 1: Inclination Classification [9].

2. Theory

According to Newton's law of gravity, every two masses in the universe have a gravitational force between them, and they rotate in an orbital motion according to Kepler's law, but the movement of the satellite is not Keplerian because there is an influence of an external force, which is called perturbation [10,11]. Hohmann transfers method between coaxial elliptical orbits to satellite transmission was used in this research. The transition from the original orbit to the target orbit occurs in one of two ways, either from perigee or from apogee, as shown in Figure 2. To determine which of the two transfers requires the least amount of energy, the individual total change of velocity needed for the transfer must be calculated.

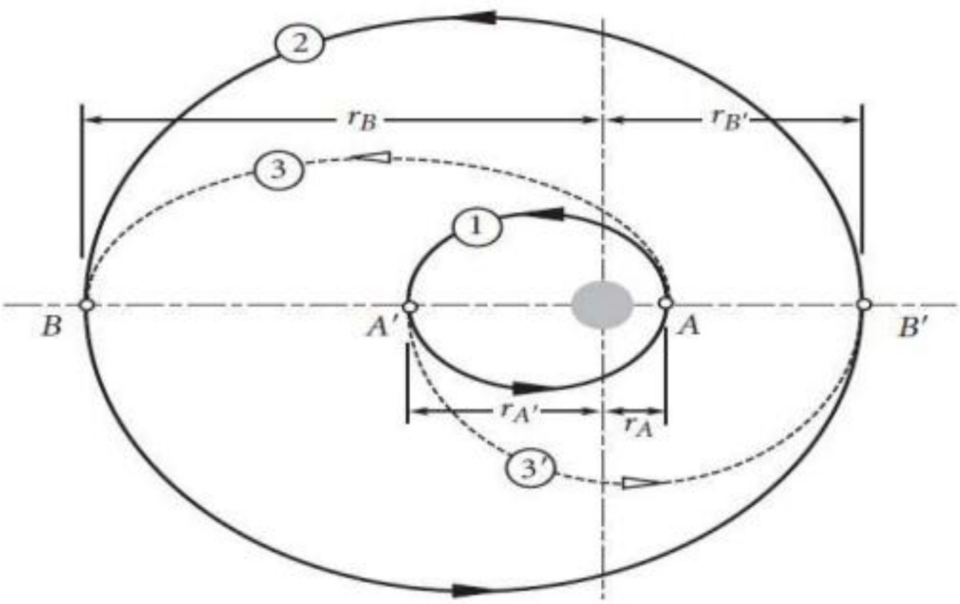

Figure 2: Hohmann transfers method between coaxial elliptical orbits [12].

The perturbed equation of motion on an elliptical orbit with perturbation is shown as follows [13].

$$\ddot{r} = -\frac{\mu}{r^3}\vec{r} + \ddot{r_p} \quad (1)$$

$\ddot{r} = \frac{d^2r}{dt^2}$ is the satellite acceleration at time (t) in unit of km/s^2.

Where $\ddot{r}$ = $\frac{d^2r}{dt^2}$ is the satellite acceleration at time (t) in units of km/s^2, μ= G*Me=398603.4416 at r in km, G is the gravitational constant, Me is the Earth mass, $\vec{r}$ is the position vector of the satellite, r is the distance between the Earth's center and satellite at time (t) and $\ddot{r_p}$ is the perturbation acceleration which can be written as follows:

$$\ddot{r_p} = \ddot{r}_E + \ddot{r}_s + \ddot{r}_M + \ddot{r}_{sp} + \ddot{r}_A \quad (2)$$

Where $\ddot{r}_E$ is the non-spherically and inhomogeneous mass distribution within Earth (central body), $\ddot{r}_s$ and $\ddot{r}_M$ is the Sun and the Moon acceleration attraction on the satellite , $\ddot{r}_{sp}$ acceleration due to Solar radiation pressure and $\ddot{r}_A$ acceleration due to atmospheric drag

Two types of perturbations were used in this study (atmospheric drag and non-spherical Earth orbit J2) [14].

The solution of equation (1) gives the satellite distance as follows [16]:

$$r = \frac{h^2}{\mu} \frac{1}{1+ecos(f)} \tag{3}$$

where h is the angular momentum per unit mass and f is the true anomaly angle (0,360°). From equation (3), the perigee and apogee distances r_p , r_a are calculated at f= 0 and 180 degree using the following formulas.

$$r_a = a\,(1+e) \tag{4A}$$

$$r_p = a\,(1-e) \tag{4B}$$

Where a is the semi-major axis of the elliptical orbit and e is the eccentricity of orbit that can be calculated using the following formula.

The eccentricity of the orbit can be calculated by [14,15]:

$$e = \frac{r_a - r_p}{r_a + r_p} \tag{5}$$

The angular momentum is determined using the following equation [14]:

$$h = \sqrt{2\mu}\sqrt{\frac{r_a - r_p}{r_a + r_p}} \tag{6}$$

The equation below is used to calculate the velocity of an elliptical orbit [14].

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \tag{7}$$

The semi major axis (a) is calculated from equation (4) [14].

$$a = rp/(1-e) \tag{8}$$

For a circular orbit (r=a) , equation (7) can be rewritten as follows [14]:

$$v^2 = \frac{\mu}{r} \tag{9}$$

The following equation may be used to determine the velocity needed to transfer the satellite from its original orbit to its final orbit through the transition orbit. The change in speed required for a transfer is denoted by the ∆V [16].

Equation (9) is used if the transfer is from a circular orbit to another circular orbit. , whereas equation (7) is utilized to describe the transition from one elliptical orbit to another.

Then the translation time was evaluated using the following formula:

$$T_{tran} = \frac{1}{2}\left(\frac{2\pi}{\sqrt{\mu}} a^{\frac{3}{2}}\right) \tag{10}$$

where μ=398603.4415 and a in km for half period in sec.

The mass of satellite is not constant through transition orbit because mass burn. This can be written as follows [14,17]:

$$\frac{\Delta m}{m} = 1 - e^{-\frac{\Delta v}{Isp\,go}} \tag{11}$$

Where:

∆ m is the consume mass for propellant, g_o is the gravity standard of acceleration and Isp is the impulsive specific of the propellants.

The atmospheric drag perturbation can be calculated using the following equation [2,3,4,5].

$$F_D = -\frac{1}{2}\frac{C_D\,\rho\,A_a\,V^2}{m} \tag{12}$$

The vector perturbation acceleration on a satellite is given by the following equation [19,20]:

$$a_{J2} = \frac{-3\mu R_e^2 J_2}{2r^7}\begin{bmatrix} x(x^2+y^2-4z^2) & . \\ y(x^2+y^2-4z^2) & . \\ z(3x^2+3y^2-2z^2). & \end{bmatrix} \tag{13}$$

Where (R_e) is the radius of the Earth (6372 km) and (x, y, and z) are the components of the satellite position (r).

A program was designed in MATLAB program to calculate the orbital elements of the primary orbit as well as the transitional orbit and the effect of disturbance on the orbital elements of the transitional orbit, and the perturbed equation was solved using Runge–Kutta method which is one of the integration methods to calculate velocity from acceleration or distance from velocity. It uses six values of the variable that are predicted by exchanging the function in the previous period. [11,13,19].

3. Results and Discussion

This results gated by equations (1,4,5,6,7,8,10,11,13), were written in the table (1 and 2) note when the eccentricity of the initial orbit changes and note that the values of the orbital elements of the transitional orbit remain the same, only the required velocity $\Delta v1$ is changed.
When comparing the values in Table 2 to the change in the orbital elements, one can notice that the values change with the height of the orbit and the effects on the elements of the transitional orbit are less when increasing altitude, because due to the fact that atmospheric drag is less than the beginning of the transitional orbit.
Since the time required by the satellite to transfer is half a cycle and the values listed in the Table (1) are for a full cycle, so the effect on the satellite's transitional orbit is half of these values. The results were compared with two previously published works [20] to ensure the accuracy of the program and it was found to be completely identical (The eccentricity and angular momentum were calculated, as well as the velocity required for the transition. The equations were programmed in the MATLAB program, and the variables in the example were used. The program showed exactly the same results as in the example).
In addition, one can notice from Table (1) that increasing of altitude of the initial orbit, leads to a lower energy required for transition For example,, when the height is 200 km, $\Delta V1$ at e=0.01 is found to be 2.408 km/s, while at the height of 800 km, $\Delta V1$ at e=0.01 is found to be 2.252 km/sec. However,transition from an altitude of 200 km to 800 km also requires energy.
Therefore, launch process considered in this work consists of two stages:
First stage represents transition from 200 km to 945 km with eccentricity of 0.01 which requires speed changes $\Delta V1$ and $\Delta V2$ of 0.167 km/sec and 0.200 km/sec, respectively.
Second stage represents transition from 800 km to 35790 km with eccentricity of 0.01 which requires speed changes $\Delta V1$ and $\Delta V2$ of 2.252 km/sec and 1.416 km/sec, respectively..
While the transition from 200 km to 35790 km with eccentricity of 0.01 requires speed changes $\Delta V1$ and $\Delta V2$ of 2.408 km/sec and 1.475 km/sec, respectively.
The sum of total velocity required for a transition in two stages, is more than that of a direct transition 0.367+3.669= 4.036 km/sec. Comparing the velocity value of the two stages with that of a direct transition, and can notice a difference of 0.153 km/sec.
The orbital elements are mostly affected by perturbation are: semi-major axis and the values are almost stable in mean anomaly from 50 to 300 degree as shown in Figure (3). Perturbation has less impact on other orbital elements as shown in Figures (4, 5, 6 and 7) because the results at a height of 800 km and when to increase height the perturbation low and the atmospheric drag is zero to height 1200 km.

Table 1: The results of two stages process.

Altitude (km)	ΔV 1 (km/sec)	ΔV 2 (km/sec)	ΔV total (km/sec)	Δ T transition (Minute)
200 to 945	0.167	0.200	0.367	96.113
800 to 35790	2.252	1.416	3.669	642.825
200 to 35790	2.408	1.475	3.883	631.136

Table 2: The orbital element of transitional orbit with different values of altitude and eccentricity of an initial orbit.

hp	e	a (km)	Δv1 (km/sec)	Δv2 (km/sec)	Δv total (km/sec)	Δ m/m	Δ a (km)	Δi degree	ΔΩ degree	Δω degree	ΔT (min)
200	0.01	24373	2.4082	1.4750	3.8833	0.5589	36.1683	0.0006	0.0111	22.173	1.4053
	0.05	24373	2.2548	1.4750	3.7299	0.5353	36.1683	0.0006	0.0111	22.173	1.4053
	0.1	24373	2.0671	1.4750	3.5421	0.5047	36.1683	0.0006	0.0111	22.173	1.4053
400	0.01	24473	2.3499	1.4557	3.8056	0.5500	45.8546	0.0005	0.0103	22.1497	1.7855
	0.05	24473	2.1987	1.4557	3.6545	0.5263	45.8546	0.0005	0.0103	22.1497	1.7855
	0.1	24473	2.0138	1.4557	3.4696	0.4956	45.8546	0.0005	0.0103	22.1497	1.7855
600	0.01	24573	2.2963	1.4350	3.7313	0.5180	43.4049	0.0005	0.0098	22.1283	1.6936
	0.05	24573	2.1473	1.4350	3.5824	0.5180	43.4049	0.0005	0.0098	22.1283	1.6936
	0.1	24573	1.9651	1.4350	3.4001	0.4872	43.4049	0.0005	0.0098	22.1283	1.69363
800	0.01	24673	2.2442	1.4152	3.6594	0.5336	44.1899	0.0004	0.0092	22.1083	1.7277
	0.05	24673	2.0973	1.4152	3.51254	0.5097	44.1899	0.0004	0.0092	22.1083	1.7277
	0.1	24673	1.9176	1.4152	3.3328	0.4788	44.1899	0.00041	0.0092	22.1083	1.7277

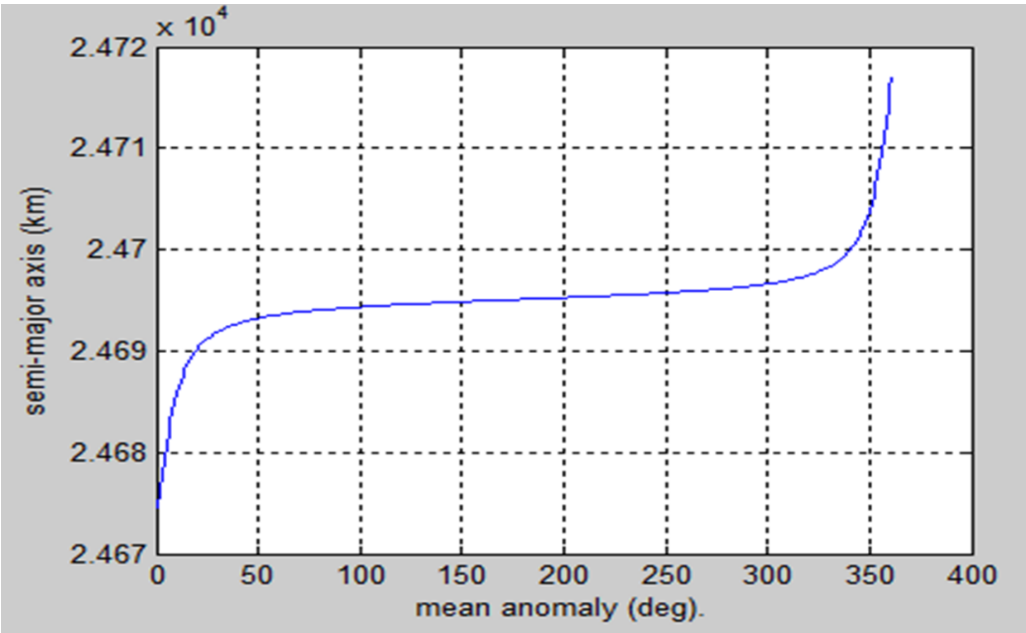

Figure 3: The variation of semi-major axis for transfer orbit with perturbation with mean anomaly at h_p=800km and eccentricity (e=0.01, 0.05, 0.1).

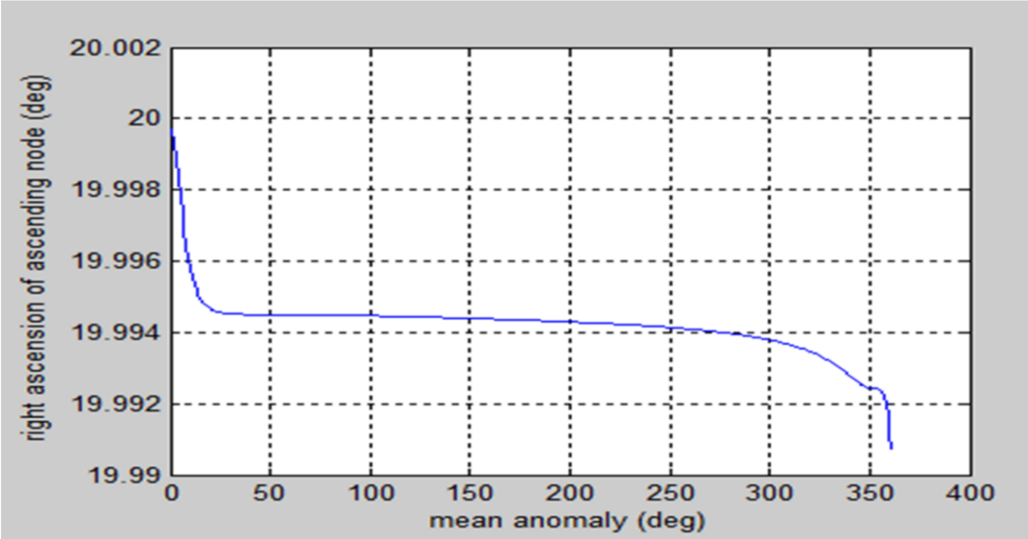

Figure 4: The variation of right ascension of ascending node for transfer orbit with perturbation with mean anomaly at h_p=800km and eccentricity (e=0.01, 0.05, 0.1).

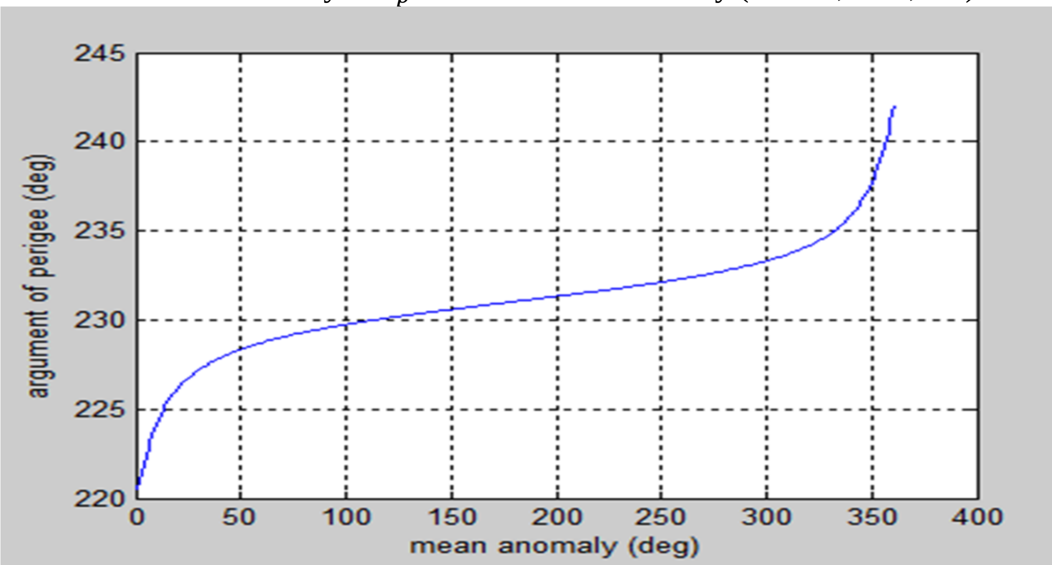

Figure 5: The variation of argument of perigee for transfer orbit with perturbation with mean anomaly at h_p=800km and eccentricity of initial orbit (e=0.01, 0.05, 0.1).

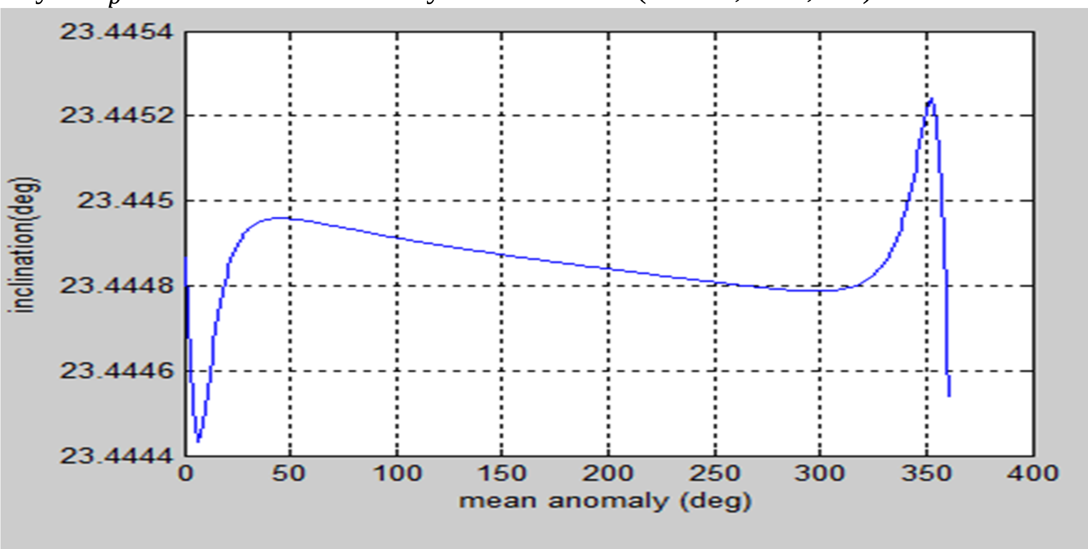

Figure 6: The variation of inclination for transfer orbit with perturbation with mean anomaly at h_p=800km and eccentricity of initial orbit (e=0.01, 0.05, 0.1).

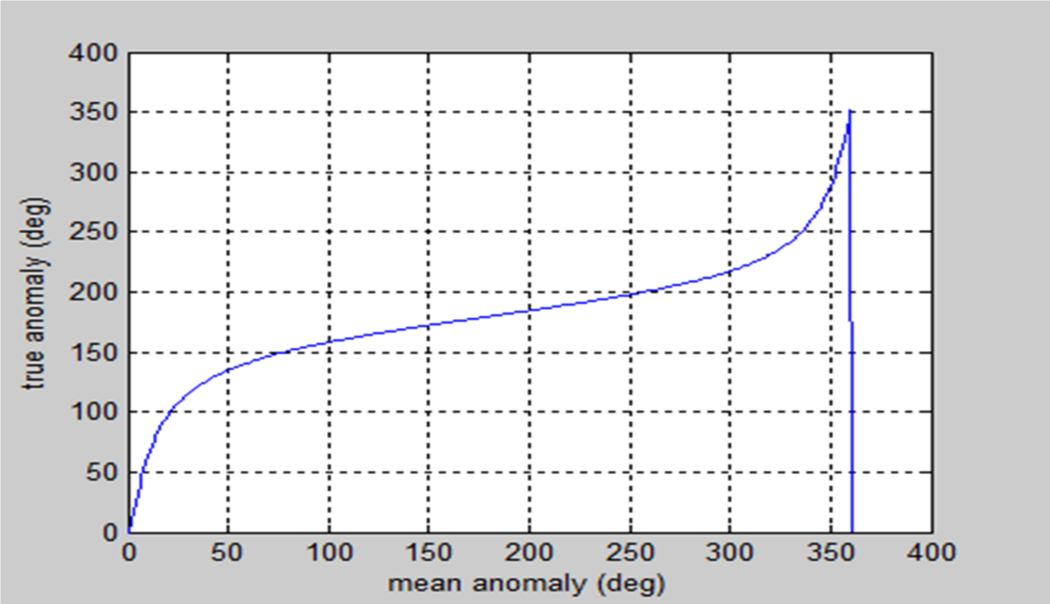

Figure 7: The variation of true anomaly for transfer orbit with perturbation with mean anomaly at h_p=800km and eccentricity of initial orbit (e=0.01, 0.05, 0.1).

4. Conclusions

1. It was concluded that the best orbit to transfer from an initial orbit to the GEO orbit is at an altitude of 800 km, because it needs a low velocity, that corresponds to, less energy, and the higher the altitude, the less energy is required for a transition.
2- The perturbation effect on the transition orbit is low but it cannot be neglected. The reason for that is the cumulative perturbation, so if there is more than one cycle before the transition, its impact will be higher.
3- The change in the values of the orbital elements of the transitional orbit due to the change in the altitude, leads to a conclusion that the higher altitude, the less effect of turbulence on the orbital elements, because the effect of atmospheric drag will be less at the beginning of the transitional orbit when increasing altitude.
4- Increasing the value of eccentricity of the initial orbit reduces the energy required for the transition, but it does not affect the change of the elements of the transition orbit due to perturbations.
5- The velocity at apogee of the transition orbit is affected by the altitude of the initial orbit and is affected by the perturbation.
6- The direct transition is better than two-stages transition because the required speed, energy and time transition of a direct method are less than those of a two-stages method.

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