Elastic Electron Scattering form Factors for Odd-A *2s*-1*d* Shell Nuclei

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Abstract
The charge density distributions (CDD) and the elastic electron scattering form factors F(q) of the ground state for some odd mass nuclei in the $2s-1d$ shell, such as $^{19}K,\ ^{25}Mg,\ ^{27}Al,\ ^{29}Si$ and ^{31}P have been calculated based on the use of occupation numbers of the states and the single particle wave functions of the harmonic oscillator potential with size parameters chosen to reproduce the observed root mean square charge radii for all considered nuclei. It is found that introducing additional parameters, namely; $\alpha,\ \alpha_1$ and α_2, which reflect the difference of the occupation numbers of the states from the prediction of the simple shell model leads to very good agreement between the calculated and experimental results of the charge density distributions throughout the whole range of r. The experimental electron scattering form factors for $^{19}K,\ ^{25}Mg,\ ^{27}Al,\ ^{29}Si$ and ^{31}P nuclei are in reasonable agreement with the present calculations throughout all values of momentum transfer q.

Keywords: charge density distributions; elastic electron scattering form factors; odd- A $2s-1d$ shell nuclei; root mean square charge radii; occupation numbers of higher states for $^{19}K,\ ^{25}Mg,\ ^{27}Al,\ ^{29}Si$ and ^{31}P nuclei.

عوامل التشكل للاستطارة الالكترونية المرنة لنوى القشرة النووية $2s$–$1d$ الفردية

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الخلاصة:
حسب كل من توزيع كثافة الشحنة النووية و عوامل التشكل لاستطارة الالكترون المرنه للحاله الارضيه للنوى الفرديه ($^{19}K,\ ^{25}Mg,\ ^{27}Al,\ ^{29}Si$ ^{31}P ,) الواقعة ضمن القشرة النووية $2s-1d$ بالاعتماد على كل من اعداد الأشغال للحالات النووية والدوال الموجية لجهد المتذبذب التوافقي ذي أعلومات حجمية اختيرت لكي تعيد انتاج مربع متوسط انصاف اقطار الشحنة لجميع النوى تحت الدراسة. لقد وجد ان ادخال الأعلومات الأضافية (α و α_1 و α_2)، والتي تعكس الفرق بين اعداد الأشغال للحالات النووية قيد الدراسة وبين تلك التي يتنبأ بها أنموذج القشرة البسيط ، يؤدي الى توافق ممتاز بين النتائج المحسوبة والنتائج العملية لتوزيع كثافة الشحنة النووية ولكل قيم المسافة r . أظهرت هذه الدراسة بأن عوامل التشكل لاستطارة الالكترون المرنة النظرية تتفق مع النتائج العملية للنوى ($^{19}K,\ ^{25}Mg,\ ^{27}Al,\ ^{29}Si$ ^{31}P ,)

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Introduction

High-energy electron scattering is a precise tool for probing nuclear structure, in particular nuclear charge densities [1]. There are many advantages for elastic electron-nucleus scattering because the electromagnetic interaction between them is clearly understood and the nuclear properties will not be influenced because the electron is a lepton. A lot of studies in this field have been done and many valuable and precise data have been obtain on the nuclear electromagnetic properties [2,3]. Electron scattering has been considered by Antonov *et al*. [4] for both light and heavy nuclei. For the light mass systems, densities from large space shell models [5,6] were used to obtain the elastic longitudinal form factors. For the He isotopes they found variations of the charge densities, and so likewise for the form factors, for ^{4}He and ^{6}He but not a significant change in the form factor between ^{6}He and ^{8}He. They also found that the proton density extends far with increasing neutron number. However, no conclusion was made as to the role of the halo in ^{6}He. A phenomenological method, that is based on the natural orbital representation, was applied to calculate the ground state one body density matrix[7]. This method describes correctly both density and momentum distributions in closed shell nuclei ^{4}He, ^{16}O and ^{40}Ca. Here, the parameters of the matrix are fixed by a best fit to the experimental density distribution and to the correlated nucleon momentum distribution. An analytical expression was derived for the one and two body terms in the cluster expansion of CDD and elastic form factors of $1s-1p$ and $2s-1d$ shell nuclei [8, 9]. This expression was used for the systematic study of the effect of the short-range correlations on the CDD and F(q). Their study depends on the harmonic oscillator size parameter (b) and the correlation parameter β, where these parameters were determined by fitting the theoretical charge form factors to the experimental ones. Many theoretical works (with various assumptions) concerning the calculations of CDD, elastic form factors and nuclear momentum distribution have been carried out [10-14] for even-A and odd-A of $2s-1d$ shell nuclei. The $CDDs$ of $2s-1d$ shell nuclei [15] were calculated with the assumption that there is an inert core of filled $1s$ and $1p$ shells and the proton numbers in $2s$ and $1d$ shells are equal to $2-\alpha$ and $Z-10+\alpha$, respectively. Here, α represents the deviation of the shell charges from the prediction of the simple shell model and Z is the proton number (total charge of the nucleus). In general, the calculated $CDDs$ were in good agreement with those of experimental data for all considered $2s-1d$ shell nuclei.

In the present work, the method of Gul'karov *et al*. [15] is followed but with the inclusion of some higher shells into consideration with the aim of driving an analytical expressions for the CDD and F(q), based on the use of the single particle wave functions of the harmonic oscillator potential and the occupation numbers of the states, applicable throughout the whole region of $2s-1d$ shell. The derived expressions are then employed for determination of CDD and the elastic form factors for odd-A of $2s-1d$ shell nuclei ^{19}K, ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P. The calculated $CDDs$ and the elastic form factors demonstrate remarkable agreement with those of experimental results.

Theory

In the simple shell model, the CDD is evaluated in terms of the radial part of the harmonic oscillator wave functions $R_{nl}(r)$ as

$$\rho(r) = \frac{1}{4\pi}\sum_{nl} 2(2l+1)\left|R_{nl}(r)\right|^2 \qquad (1)$$

where the form of $R_{nl}(r)$ is very well known while n and l are the single particle principal and orbital angular momentum quantum numbers, respectively. In the simple shell model, the $2s-1d$ shell nuclei are considered as an inert core of filled $1s$ and $1p$ shells and the proton numbers in $2s$ and $1d$ shells are equal to 2 and $Z-10$, respectively. According to the assumption of the simple shell model of Eq. 1-, an analytical expression for the CDD of $2s-1d$ shell nuclei is obtained as [15, 16]

$$\rho(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2}b^3}\left\{5 + \left(\frac{4Z}{15} - \frac{4}{3}\right)\left(\frac{r}{b}\right)^4\right\} \qquad (2)$$

and the corresponding mean square charge radii (MSR) can be determined by [17]

$$\langle r^2 \rangle = \frac{4\pi}{Z}\int_0^\infty \rho(r) r^4 dr \qquad (3)$$

where the normalization condition of the CDD is given by [2]

$$Z = 4\pi \int_0^\infty \rho(r) r^2 dr \qquad (4)$$

Introducing Eq. 2- into Eq. 3- and integrating, the MSR of $2s-1d$ shell nuclei is obtained as

$$\langle r^2 \rangle = b^2\left(\frac{7}{2} - \frac{10}{Z}\right) \qquad (5)$$

In this study, the higher shells are included in the calculations of CDD by using the assumption that there is an inert core of filled $1s$ and $1p$ shells and the proton numbers in the shells $2s, 1d$ and $2p$ are equal to, respectively, $1-\alpha,\ Z-9+\alpha_1$ and α_2 (for ^{19}K), $2-\alpha,\ Z-10+\alpha_1$ and α_2 (for $^{25}Mg,\ ^{27}Al,\ ^{29}Si,\ ^{31}P$) and not to $2,\ Z-10$ and 0 as in the simple shell model of Eq. 2-. The parameters α, α_1 and α_2 (with $\alpha = \alpha_1 + \alpha_2$) represent the deviation of the shell charges $2s, 1d$ and $2p,$ respectively, from the prediction of the simple shell model. Using this assumption with the help of Eq. 1-, an analytical expression of the CDD for the $2s-1d$ shell nuclei is obtained as

$$\rho(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2}b^3}\left\{\frac{7}{2} - \frac{3\alpha}{2} + \left[2 + \frac{11\alpha}{3} - \frac{5\alpha_1}{3}\right]\left(\frac{r}{b}\right)^2 + \left(\frac{4Z}{15} - \frac{26}{15} - 2\alpha + \frac{24\alpha_1}{15}\right)\left(\frac{r}{b}\right)^4 + \frac{4\alpha_2}{15}\left(\frac{r}{b}\right)^6\right\} \qquad (6a)$$

(for ^{19}K)

$$\rho(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2}b^3}\left\{5 - \frac{3\alpha}{2} + \left[\frac{11\alpha}{3} - \frac{5\alpha_1}{3}\right]\left(\frac{r}{b}\right)^2 + \left(\frac{4Z}{15} - \frac{4}{3} - 2\alpha + \frac{8\alpha_1}{5}\right)\left(\frac{r}{b}\right)^4 + \frac{4\alpha_2}{15}\left(\frac{r}{b}\right)^6\right\} \qquad (6b)$$

(for $^{25}Mg,\ ^{27}Al,\ ^{29}Si,\ ^{31}P$)

and the corresponding MSR is

$$\langle r^2 \rangle = b^2\left(\frac{7}{2} - \frac{10}{Z} + \frac{\alpha_2}{Z}\right) \qquad (7)$$

The central $CDD,$ i.e. at $r = 0,$ is determined from Eq. 6- as

$$\rho(0) = \frac{1}{\pi^{3/2}b^3}\left\{\frac{7}{2} - \frac{3\alpha}{2}\right\} \qquad (\text{for } ^{19}K) \qquad (8a)$$

$$\rho(0)=\frac{1}{\pi^{3/2}b^{3}}\left\{5-\frac{3\alpha}{2}\right\} \qquad (\text{ for } {}^{25}Mg,\ {}^{27}Al,\ {}^{29}Si,\ {}^{31}P) \qquad (8b)$$

The elastic electron scattering form factors from spin zero nuclei is determined by the ground state CDD and defined as

$$F(q)=\frac{4\pi}{Z}\int_{0}^{\infty} j_{0}(qr)\rho(r)r^{2}dr \qquad (9)$$

Where

$$j_{0}(qr)=\frac{\sin(qr)}{qr} \qquad (10)$$

is the zeroth order spherical Bessel function, q is the momentum transfer from the incident electron to the target nucleus and the $\rho(r)$ is the CDD of the ground state. In the limit $q \to 0,$ the target will be -represented as a point particle, and from Eq. 9- with the help of Eq. 4-, the form factor of this target is equal to unity, i.e. F(q = 0) =1.

An analytical expression for elastic electron scattering form factor F(q) can be obtained by introducing the form of the CDD of Eq. 6a- (for ${}^{19}K$) and Eq. 6b- (for ${}^{25}Mg,\ {}^{27}Al,\ {}^{29}Si,\ {}^{31}P$) into Eq. 9-, and performing the integration, i.e.,

$$F(q)=\frac{e^{-q^{2}b^{2}/4}}{Z}[Z+(\frac{5-Z}{3}-\frac{\alpha_{2}}{6})q^{2}b^{2}+(\frac{3\alpha_{2}-\alpha}{40}+\frac{2Z-13}{120})q^{4}b^{4}-\frac{\alpha_{2}}{240}q^{6}b^{6}] \qquad (11a)$$

$$F(q)=\frac{e^{-q^{2}b^{2}/4}}{Z}[Z+(\frac{5-Z}{3}-\frac{\alpha_{2}}{6})q^{2}b^{2}+(\frac{3\alpha_{2}-\alpha}{40}+\frac{Z-5}{60})q^{4}b^{4}-\frac{\alpha_{2}}{240}q^{6}b^{6}] \qquad (11b)$$

The form factor given in Eqs. 11a and 11b must be corrected to convert it into a representation appropriate for comparison with the experimental form factors. Therefore, Eq. 11- must be multiplied by the correction due to the finite nucleon size $F_{fs}(q)$ and the center of mass correction $F_{cm}(q)$ [18]

$$F_{fs}(q)=\exp[-\frac{0.43q^{2}}{4}] \qquad (12)$$

$$F_{cm}(q)=\exp[\frac{q^{2}b^{2}}{4A}] \qquad (13)$$

where A is the mass number of the nucleus.

In this study, we compare the calculated CDD of considered odd-A nuclei with those of two parameter Fermi model $2PF$ and three parameter Fermi model $3PF$, which are extracted from the analysis of elastic electron-nuclei scattering experiments, and are given by [17]

$$\rho_{3PF}(r)=\rho_{0}\left(1+\frac{\omega r^{2}}{c^{2}}\right)\Bigg/\left(1+e^{\frac{r-c}{z}}\right) \qquad (14)$$

$$\rho_{2PF}(r)=\rho_{0}\Bigg/\left(1+e^{\frac{r-c}{z}}\right) \qquad (15)$$

where the constant ρ_{0} is obtained from the normalization condition of the charge density distribution of Eq. 4-.

Results and Discussion

The analytical expression of Eq. 6- has been used to study the CDD's for odd-A of $2s-1d$ shell nuclei ^{19}K, ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei. The harmonic oscillator size parameter b is chosen in such a way to reproduce the experimental root mean square charge radii $\langle r^2\rangle^{1/2}_{\exp}$ of the considered nuclei, the parameter α is determined by introducing the experimental $\rho_{\exp}(r=0)$ into Eq. 8-, the parameter α_2 is determined by introducing the experimental MSR into Eq. 7- and the parameter α_1 is determined from the relation $\alpha_1=\alpha-\alpha_2$. It is important to remark that when $\alpha=\alpha_1=\alpha_2=0$, Eq. 6 and 7- coincide with those of Eqs. 2 and 5-, respectively. The calculated $CDDs$ of considered odd-A nuclei are compared with those of $2PF$, $3PF$ models [17].

In table-1, we present the parameters ω, c and z required by the experimental CDD of $2PF$, $3PF$ models together with their root mean square charge radii $\langle r^2\rangle^{1/2}_{\exp}$ and central charge densities $\rho_{\exp}(r=0)$ for ^{19}K, ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei.

Table 1-Values of various parameters required by the CDD of $2PF$ and $3PF$ models

Nucleus	Type of CDD [13]	ω	c	z	$\rho_{\exp}(0)$ [17] (fm^{-3})	$<r^2>^{1/2}_{\exp}$ [17] (fm)
^{19}F	$2PF$		2.590	0.564	0.08334142	2.900
^{25}Mg	$3PF$	-0.236	3.220	0.580	0.08591833	3.003
^{27}Al	$2PF$		3.070	0.519	0.08338845	3.060
^{29}Si	$2PF$		3.170	0.520	0.08267015	3.130

Table-2 displays all parameters needed for calculating $\rho(r)$ of Eq. 6-, such as the harmonic oscillator size parameter b and the calculated parameters of α, α_1 and α_2 for considered nuclei.

Table 2- Calculated parameters used in Eq. 6- for the calculations of the CDD

Nucleus	Z	b	α	α_1	α_2
^{19}F	9	1.861	0.3057534	0.1839982	0.1217553
^{25}Mg	12	1.838	1.351724	1.3184830	0.03324072
^{27}Al	13	1.851	1.370566	1.3231510	0.04741479
^{29}Si	14	1.871	1.322084	1.141659	0.18042520
^{31}P	15	1.890	1.142182	0.9105916	0.23159020

Table-3 demonstrates the calculated occupation numbers for $2s, 1d$ and $2p$ shells and the calculated root mean square charge radii $\langle r^2\rangle^{1/2}_{cal}$ obtained by using Eqs. 5 and 7-.

Table 3-Calculated occupation numbers of $2s, 1d$ and $2p$ shells and the calculated $\langle r^2\rangle^{1/2}_{cal}$

Nucleus	Z	Occupation No. of $2s$	Occupation No. of $1d$	Occupation No. of $2p$	$\langle r^2\rangle^{1/2}_{cal}$ obtained from Eq.5	$\langle r^2\rangle^{1/2}_{cal}$ obtained form Eq.7
^{19}F	9	0.694246	0.183998	0.121755	2.891824	2.90000
^{25}Mg	12	0.648276	3.318483	0.033240	3.001441	3.00300
^{27}Al	13	0.629434	4.323151	0.047414	3.057959	3.06000
^{29}Si	14	0.677916	5.141659	0.180425	3.122785	3.13000
^{31}P	15	0.857818	5.910591	0.231590	3.181344	3.19000

The dependence of the CDD (in units of fm^{-3}) on r (in unit of fm) is shown in figure-1 for ^{19}K, ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei. The solid circles (●) are the experimental data of $2PF$, $3PF$ models [17]. The dashed symbol distributions are the calculated $CDDs$ obtained either from Eq. 2- or from Eq. 6- using the values $\alpha = \alpha_1 = \alpha_2 = 0$. The solid distributions are the calculated CDD when the higher shells are included in the calculations and obtained by Eq. 6- using the values of α, α_1 and α_2 given in table -2. It is obvious that the form of the $CDDs$ represented by Eq. 2 or 6- behaves as an exponentially decreasing function as seen by the plus symbol distributions or solid distributions for all considered nuclei of figure.-1. This figure shows that the probability of finding a proton near the central region ($0 \le r \le 2$ fm) of $\rho_{ch}(r)$ is larger than the tail region ($r > 2$ fm). presented in table -2, into Eq. 6- leads to reducing significantly the central region of $\rho_{ch}(r)$ and increasing slightly the tail region of $\rho_{ch}(r)$ as seen by the solid distributions. This means that the effect of inclusion of higher shells in our calculations tends to increase the probability of transferring the protons from the central region of the nucleus towards its surface and tends to increase the root mean square charge radius $< r^2 >^{1/2}$ of the nucleus (see table -3) and then makes the nucleus to be less rigid than the case when there is no such effect. Figure-1 also illustrates that the dashed symbol distributions in all considered nuclei are not in good accordance with those of $2PF$, $3PF$ models especially at the central region of $\rho_{ch}(r)$, but once the higher shells are considered in the calculations due to the introduction the calculated values of α, α_1 and α_2 given in table-2 into Eq. 6-, the results for the CDD become in astonishing accordance with those of $2PF$, $3PF$ throughout the whole values of r.

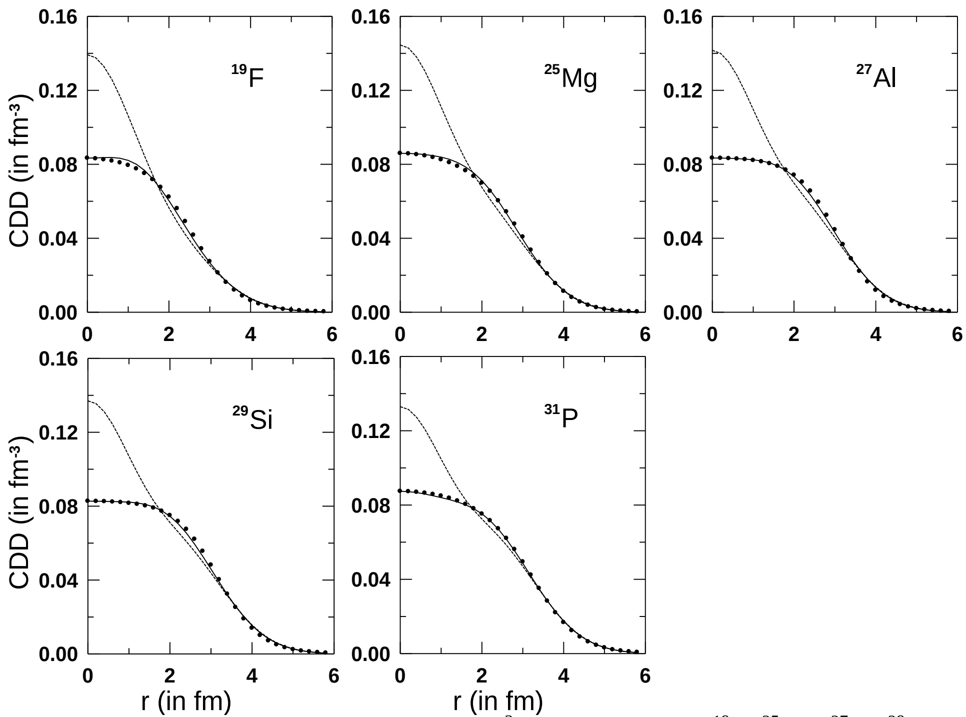

Figure 1- the dependence of the CDD (in fm^{-3}) on r (in fm) for ^{19}F, ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei. The solid circles are the experimental data [17] of *2PF* (for ^{19}F, ^{27}Al, ^{29}Si) *and 3PF* (for ^{25}Mg, ^{31}P) nuclei. The solid and dashed curves are the calculated CDD obtained by Eq 6- using the values $a \neq a_1 \neq a_2 \neq 0$ [as displayed in Table (2)] and $a = a_1 = a_2 = 0$, respectively.

In figure-2, we present our results for elastic form factors as a function of the momentum-transfer q for ^{19}K, ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei, respectively. The circles in this figure are the experimental results of refs.[18,19] , the solid and dashed curves are our calculated results. The experimental data for ^{19}K, ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei are very well described by the calculated ones up to q=3 fm^{-1}.

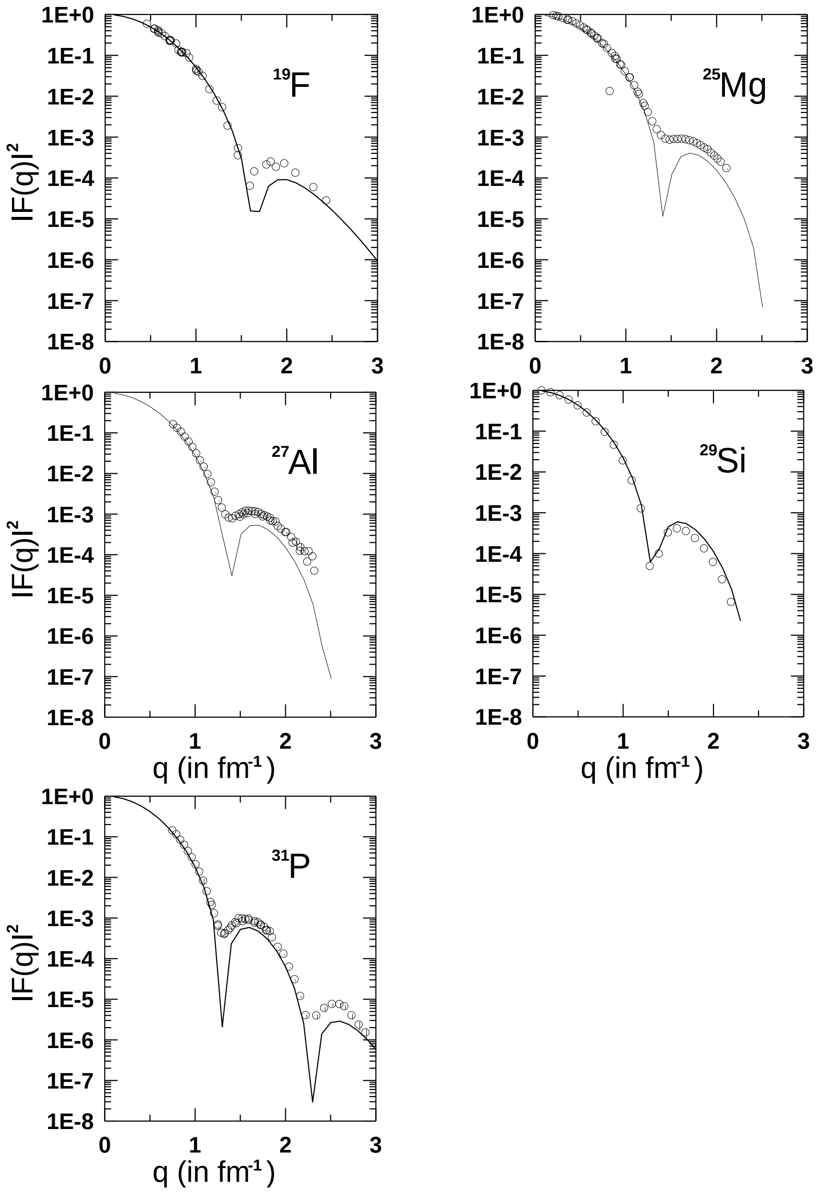

Figure 2- The dependence of the form factors on the q (in fm^{-1}) for ^{19}F, ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei. The open circle symbols are the experimental data [19]. The solid curves are the calculated form factors of Eq. 11- with including the corrections $F_{fs}(q)$ [Eq. 12] and $F_{cm}(q)$ [Eq. 13].

Conclusions

This study leads to the conclusion that the introduction of additional parameters α, α_1 and α_2, that reflect the difference of the occupation numbers of the states from the prediction of the simple shell model gives very good agreement between the calculated and experimental results of the charge density distributions throughout the whole range of r. The experimental elastic electron scattering form factors from ^{19}K, ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei are in reasonable agreement with the present calculations throughout all values of q.

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