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Investigating the Correlations between Physical Parameters of Elliptical and Lenticular Galaxies

Rabab Mozher J., Al Najm Mohammed Naji*

Department of Astronomy and Space, College of Science, University of Baghdad, Baghdad, Iraq

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Abstract

 This study aims to investigate the features of stellar-gaseous kinematics and dynamics mass using scaling coefficient relationships (such as the Faber–Jackson relation (FJR)) of two samples of elliptical and lenticular galaxies. These two samples of 80 ellipticals and 97 lenticulars were selected from previous literature works. The Statistical Package for Social Sciences (SPSS) and Matrix Laboratory (MATLAB) program were used to find out the associations of multiple factors under investigation such as main kinematic properties of the gaseous-stellar (effective radius R_e, surface density within the effective radius (Σ_{eff}), stellar mass in the blue band (M_{star} (B)), gas mass (M_{gas}), dynamic (M_{dyn}) and baryonic (M_{bar}), supermassive black holes masses (M_{BH}) in the elliptical and S0 galaxies. We concluded that the experimental relations between (Log M_{bar} (B) - Log M_{dyn} (B), Log M_{bar} (B) - Log M_{BH}) have a robust correlation of \sim 1, and the slope appears to be more linearly (Slope \geq 1) for both types of galaxies (Ellipticals and Lenticulars). This paper also noted that the empirical convergences between Log Σ_{eff} (B) and Log M_{dyn} have a strong high regression relationship of \sim 1. The slope appears to be approximately linear ~1 in the lenticular galaxies, but there is no relationship between Log Σ_{eff} (B) and Log M_{dyn} in the elliptical galaxies. Due to the strong impact of dynamical rotation on elliptical galaxies, it is interestingly believed that they are objects under turbulence and originate from interactions and mergers with galaxies in a spiral shape.

Keywords: Galaxies: Ellipticals and Lenticulars; Kinematics and dynamics galaxies; Structure – Evolution galaxies.

التحقق من العالقات االرتباطية بين المعلمات الفيزيائية للمجرات االهليلجية والعدسية

*** رباب مزهر جاسم ، محمد ناجي ال نجم**

قسم الفلك والفضاء، كلية العلوم، جامعة بغداد، بغداد، العراق

الخالصة

 تهدف هذه الدراسة الى التحقق من الخصائص الحركية الغازية- النجمية والكتلة الديناميكية باستخدام عالقات معامالت القياس)مثل عالقة فايبر-جاكسون)FJR))لعينتين من المجرات اإلهليلجية والعدسية. تم اختيار هاتين العينتين المكونتين من 08 مجرة اهليليجية و79 مجرة عدسية من االعمال االدبية السابقة. استخدمنا الحزمة اإلحصائية لبرنامج العلوم االجتماعية (SPSS (ومختبر المصفوفة)MATLAB)اليجاد ارتباطات العوامل المتعددة قيد البحث مثل الخصائص الحركية الغازية- النجمية الرئيسية)نصف القطر الفعال

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 $M_{\rm star}$)) كثافة السطوع السطحيه عند نصف القطر الفعال Σ eff ، الكتلة النجمية عند الحزمة الزرقاء ((B $\,$ (Re ، كتلة الغاز M_{gas} ، الكتل الديناميكية (M_{dyn}) والباريونية (B والثقب الاسود (M_{BH})) في المجرات اإلهليلجية و العدسية. (Log Mbar (B) - Log M_{bH} ،Log M_{bar} (B) - Log M_{dyn}) استنتجنا أن هناك علاقات تجريبية بين (لها معامل ارتباط قوي جدا ~ 1 ، ويبدو أن المنحدر أكثر خطيًا (الميل اكبر او يساوي الواحد) في كلا النوعين من المجرات (الاهليلجية والعدسية). اشارت هذه الورقة أيضًا أن التقاربات التجريبية بين log ∑_{eff} وB)و log M_{dyn} لها علاقة انحدار عالية قوية تبلغ الواحد. يبدو أن ميل المنحدر خطي تقريبًا مساوي للواحد في المجرات العدسية، لكن ال توجد عالقة بين (B (Σeff Log و Mdyn Log ا ً في المج ارت اإلهليلجية ، نظر ألن المجرات اإلهليلجية تتأثر بشدة بالدوران الديناميكي ، ومن المثير لالهتمام أنها اجرام تحت االضطراب وتنشأ من التفاعالت واالندماجات مع المجرات ذات الشكل الحلزوني.

1-Introduction

 Elliptical galaxies are often structurally uniform and do not involve distinguishing characteristics. Although they have a visible disc shape, lenticular or S0 galaxies share a similar smooth structure to that in elliptical galaxies [1]. The lenticulars are the morphological transition class between the elliptical and spiral galaxies [2]. The observed ellipticity of a galaxy is incorporated in the morphological name of Hubble's notation by adding a number $(10 \times (1 - \frac{b}{a}))$ $\frac{d}{dx}$)) after the letter E, where a and b are the observed major and minor axes of the galaxy. In light of this, E0 galaxies are spherical, whereas E7 galaxies have the most extreme ellipticities of all elliptical galaxies. Elliptical galaxies exhibit broad light patterns of distribution, nearly no structure, and an elliptical appearance in photographs [3, 4, 5]. Earlytype galaxies (E and S0) are where most of the stellar mass in the local universe is located [6]. Mergers of disk galaxies are the foundation of a popular theory explaining the genesis of early-type galaxies. In particular, due to the connection between effective radius R_e , central stellar speed dispersion, and mean central surface brightness, early-type galaxies adhere to a systematic set of scaling relations that connect their photometric and kinematic features [7- 9].

 The relationship between the stellar mass, size, absolute magnitude and velocity dispersion of elliptical galaxies is described by the Fundamental Plane (FP); its projection onto "mass, velocity" space is known as the Faber-Jackson relation (FJR) [10]. Therefore, it is exciting to investigate FP relations using a sample of stellar systems that spans the most conceivable range of luminosity, mass, and physical dimensions. The structural characteristics of distinct early-type stellar systems, often gas-poor ones, were examined. These hot stellar systems have a dynamic nature [11, 12].

 Based on Hubble's classification, galaxies can be categorised into elliptical (E), lenticular (S0), spiral or barred spiral (S, SB), or irregular (I). His first two-dimensional classification scheme Is well-known as"tuning fork". The various morphological types of galaxies could result in important astrophysical investigations. Such different morphologies must be included in every hypothesis of galaxy structure and evolution. The cosmological parameters adopted in this research are: $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_{\text{matter}} = 0.3$, and $\Omega_{\text{vacuum}} = 0.7$ [4] [13, 14].

 In a previous research (Nipoti et al., 2003) [15] , it was reported that dissipationless merger cannot reproduce the relation between black hole mass and dispersion velocity at the centre $(M_{BH-}\sigma_0)$. If the black hole masses add linearly, a black hole merging with substantial emission of gravitational waves reproduces the $M_{BH-}\sigma_0$ relation but fails to reproduce the Magorrian relation. Cappellari et al [16] investigated the relationships between the dynamical mass-to-light ratio (M/L) and various other global elliptical and lenticular galaxies observables. Lintott et al. (2006) [17] found a tight correlation between stellar mass and velocity dispersion, called the 'Baryonic Faber-Jackson relation'. They found a slope of M_{star} $(Re) $\propto \sigma^{2.33\pm0.18}$. The radius of the system and the velocity dispersion can be used to derive$ a dynamical mass in addition to the stellar mass. The two mass estimations show that dark matter is more prevalent than the cosmic baryon proportion in the center of these galaxies.

 Several studies have analyzed the basic scaling relationships of elliptical galaxies resulting from mergers by discovering that tilts in the fundamental plane (FP) relationship are significantly influenced by gas dissipation. The tilt of the resulting FP connection increases and the slope of the R_e -M relation steepens as the gas abundance of progenitor disk galaxies rises. Serra et al. 2008 [6] interpreted the results as gas-rich mergers play a significant role in E/S0 formation, especially at lower mean velocity dispersion σ . HI-rich galaxies with no significant age gradients could be formed in interactions characterised by high-angular momentum gas. A research that was published in 2011 [12] proved that there are relationships between effective radius, effective mass surface density, and effective radius of brightness and stellar mass plane. This could explain how stars might be distributed in the unrelaxed stellar groups investigated in this study if it agrees with the measured slopes of $R_{\text{eff}} \propto M^{3/5}$ or $R_{\text{eff}} \propto M^{4/5}$. Bahe et al. (2016) [18] displayed that HI discs are increasingly common in simulated galaxies of increasing M_{HI} and decreasing M_{star} . However, most discs appear vertically disturbed at $M_{HI} \ge 10^{9.4} M_{\odot}$. Many simulated galaxies contain large HI haloes, a factor of several larger than seen in observations. They are more common at high M_{HI} and low M_{star} but show no clear correlation with galaxies' specific star formation rate. Based on the analysis of every simulated merge occurrence carried out by Frigo and Balcells in 2017 [19], it was noticed that the development of velocity dispersion within the effective radius (σ_e) with a ratio M/L is related to the increase of Re with M/L in a particular way that is incompatible with homology: homologous evolution necessitates a more elevated than the observed reduction in σ_e as R_e grows. According to this study, the average velocity dispersion of elliptical galaxies equals 207.1667 km/s. However, when calculating the effective radius (R_e) , the authors found that elliptical galaxies are forced to have a complete dispersion velocity (rotators are slow) and have to overcome a deficit in the center. Al-Dahlaki (2023) [5] demonstrated that there is a high correlation between the mass of ellipticals and their effective radius ($M - R_e$), indicating the presence of active elements in elliptical galaxies.

 This research is organized as follows: The second section explicitly demonstrates the steps of collecting data in this paper, the standard model employed in mathematical analysis, and the technique used to determine the study's parameters. The third section presents the statistical analysis of the samples. The results are presented in section 4 and the conclusions are discussed in section 5.

Data collection

 This study involves collecting data of two samples of 97 lenticular galaxies and 80 elliptical galaxies. Galaxies' names were chosen from previously published articles [20-25]. Extraction of some parameters such as the morphological type of galaxies, apparent magnitude (m_{btc}) in blue-band corrected by galactic extinction, central velocity dispersion (σ_0) and surface brightness at effective radius (μ_0) were collected from the French website Lyon-Meudon Extra-Galactic Database (HyperLeda) [26], the NASA/IPAC Extra-Galactic Database (NED) [27, 28], and Set of Identifications, Measurements, and Bibliography for Astronomical Data (SIMBAD) [29]. For elliptical (E) and lenticular (S0) galaxies, these parameters for the first ten data points in each of these categories of galaxies in this paper are listed in Table (1).

Elliptical (E) galaxies sample						
No.	Name galaxy	Morphology	m_{htc} (mag)	σ_0 (km/s)	μ_e (mag arcsec ⁻²)	Ref.
1.	NGC 2768	E ₆	10.6	185	21.99	$\overline{2}$
2.	NGC 2974	E4	11.61	232.2	20.8	$\overline{2}$
3.	NGC 3379	E1	10.11	202.5	20.2	$\overline{2}$
4.	NGC 4105	E ₃	11.27	246.2	20.2	$\overline{2}$
5.	NGC 4472	E2	9.23	282	21.36	$\overline{2}$
6.	NGC 4636	$E0-1$	9.9	199.5	22.2	$\overline{2}$
7.	NGC 4936	E0	11.55	278.3	22.14	$\overline{2}$
8.	NGC 5018	E ₃	11.23	211.9	20.35	$\overline{2}$
9.	NGC 5638	E1	11.98	161.6	21.37	$\overline{2}$
10.	NGC 5796	$E0-1$	12.23	266.5	21.08	$\overline{2}$
Lenticular (S0) galaxies sample						
1.	NGC 3156	S ₀	12.9	73.7	20.8	12
2.	NGC 3414	S ₀	11.93	237.6	20.54	12
3.	NGC 4526	S ₀	10.45	224.6	20.82	12
4.	NGC 4550	S ₀	12.27	96.2	20.42	12
5.	NGC 1543	S ₀	11.35	149.4	21.22	10
6.	NGC 1553	S ₀	10.2	173.8	20.91	10
7.	NGC 1596	S ₀	11.94	166.1	20.13	10
8.	NGC 3316	S ₀	13.44	184.5	21.28	10
9.	NGC5128	S ₀	7.28	103.4	22.38	18
10.	NGC1596	S ₀	11.94	166.1	19.99	18

Table 1: Collected data of elliptical and lenticular galaxies study from the websites HyperLeda and NED and articles.

3- Characterization of Parameters

1- To calculate the distance of galaxies (D), one needs to calculate the absolute magnitudes of the galaxies (M_B). So, M_B represents the blue absolute magnitude of the galaxies at 0.44 μ m blue-band in the unit (mag) using the Faber–Jackson relationship to calculate for a sample elliptical and lenticular, the Faber-Jackson relation as M_B vs σ_0 is given by [30, 31].

$$
M_B = -10 \times log_{10} \sigma_0 + 2 \tag{1}
$$

The cosmological distance modulus method was used to compute the galaxy's distance scale in (Mpc), which had the following form [32]:

$$
D=10^{\frac{(m_{btc}-MB-25)}{5}}
$$
 (2)

2- L^B is the luminosity at visible wavelength (particularly at blue band) which is expressed in solar units and has been computed at wavelength 4400A°, using the relationship below [33, 34]

$$
L_B = 10^{(-0.4 \times (M_B - 5.47))}
$$
 (3)

3- M_{HI} is the amount of all the mass in units of solar mass that is made up of neutral hydrogen gas (HI), as determined by the standard method using the magnitude corrected (m_{21c}) . The HI line is also visually faint on galactic estimates; therefore, its strength depends on the mass [35, 36].

$$
M_{\text{HI}} = (2.36 \times 10^5 \times D^2 \times 10^{-0.4} \, \text{(m}_{21c} - 15.84)} + 0.626) \tag{4}
$$

4- \mathbf{R}_e is the effective radius whose relation with the mean effective surface brightness (μ_e) and central velocity dispersion (σ_0) of elliptical galaxies can be described employing FP [37].

The effective radius R_e Is measured in units of Kpc, and is estimated using the following formula [11, 38].

$$
R_e = 10^{((1.25 \times \log_{10} \sigma_0) + (0.32 \times \mu_0) - 8.895)}
$$
(5)

5- M_{dyn} is the total dynamical mass which can be estimated using the formula below, that is derived from the virial theorem [39].

$$
\mathbf{M}_{\mathbf{dyn}} = \left(\frac{(\mathbf{K}_{V}(\mathbf{n}) \times \mathbf{R}_{e} \times \sigma_{0}^{2})}{(4.32 \times 10^{-6})}\right) \tag{6}
$$

The virial coefficient is K_v , and the value 4.32 \times 10⁻⁶ is the general gravitational constant in unit kps.km²/s².M_o. Note that even for early-type galaxies (ETGs) in virtual equilibrium [40], the parameter (K_V) depends on the Sersic index (n) because it is represented in equations (7) and (8) as an unknown parameter that can be guessed from the observed values of R_e , n, and σ_0 [15, 16, 39, 41].

$$
K_V(n) = \left(\frac{73.32}{(10.465 + (n - 0.94)^2)}\right) + 0.954\tag{7}
$$

$$
n = (1.90546 \times R_e^{0.52})
$$
 (8)

6- $M_{\text{Star}}(B)$ Is the stellar mass that is obtained by integrating to merge the stellar luminosity profile in blue band 440 nm. Stellar kinematics provides a powerful means of studying the mass profile of the core of galaxies. M_{star} (B) is calculated from the stellar mass-to-light ratio $\Upsilon^* \equiv (M/L)^*$. In this study, Υ^* is assumed to be constant and equal to 1.4 [42].

$$
M_{star}(B) = 1.4 \times L_B
$$
 (9)

7- Σ_{eff} (B) Is the number density of galaxies, commonly referred to as galaxies density, has been an important tool for analysing the relationship between the properties of galaxies and their surroundings. The relationship between surface density and effective radius is shown as follows [43].

$$
\Sigma_{\rm eff} \left(\rm B \right) = (\frac{(M_{\rm star} \left(\rm B \right))}{(2 \times 3.14 \times \rm R_{\rm e}^2)}) \tag{10}
$$

8- $M_{\text{bar}}(B)$ is the baryonic masses that are a mixture of star and gas masses, as given by the following formula [44].

$$
\mathbf{M}_{\text{bar}}(\mathbf{B}) = (\mathbf{M}_{\text{star}}(\mathbf{B}) + \mathbf{1} \cdot \mathbf{4} \times \mathbf{M}_{\text{HI}}) \tag{11}
$$

9- M_{BH} is the mass of the black hole. Analyses of the evolution of the central black hole (BH) and host galaxy throughout cosmic time depend critically on the scaling interactions between these variables, and provide an indirect means to measure of supermassive black hole (SMBH) masses in many galaxies. Therefore, the mass of the supermassive black hole M_{BH} is calculated by the formula below [45]:

$$
M_{BH} = 1.4 \times 10^8 \times \left(\frac{\sigma_0}{200 \text{ km/s}}\right)^{3.5}
$$
 (12)

4- Results and Discussion

 A polynomial is a straightforward formula for resolving difficult mathematical equations, and it is provided by equation (14).

$$
a_0 + a_1 x + a_2 x^2 + a_{n-1} x^{n-1} + a_n x^n \tag{14}
$$

Where a_1 , a_2 , a_{n-1} , and a_n , respectively, are constants and x is the indeterminate. The word "indeterminate" means that x represents no particular value [46, 47].

 The results of the statistical analysis in this paper were obtained using SPSS, a Windowsbased program that can perform data entry and analysis [30]. Suppose there is a brightness correlation between several bands. The best line regression determines the degree of correlation between the calculated physical properties and whether there is any significant association between the two parameters' characteristics. The linear partial correlation coefficient (R) values range from $[+1, -1]$. The two components are wholly connected if the regression coefficient equals 1. However, when the regression correlation (R) is between 0 and 1, the optimal result is near 1, which denotes that the fitting model accounts for both the variability and mean of the responding points [48, 49]. Elliptical and lenticular galaxies are analysed using statistical regression techniques.

Elliptical (E) and lenticular (S0) galaxies are two distinct types of galaxies that exhibit different characteristics and properties. In this section, we will compare these two types of galaxies based on various parameters, including partial correlation coefficient, slope and probability.

For E and S0 galaxies selected ($N_E = 80$, $N_{S0} = 97$), the correlation coefficient measures the strength and direction of the linear relationship between pairs of variables. In this case, the correlation coefficient reflects the relationship between the logarithmic stellar mass in B-band and logarithmic surface density within the effective radius $\Sigma_{\text{eff}}(B)$. Elliptical galaxies exhibit a correlation coefficient of 0.52, while lenticular galaxies showed a significantly higher correlation coefficient of 0.88. These values indicate a moderate positive correlation between stellar mass and mass density in both types of galaxies, with the lenticular galaxy displaying a slightly stronger correlation and a robust likelihood of ($P \le 10^{-7}$), with the slope being linearly in elliptical and lenticular (0.76 , 0.86) respectively. As shown in Figure (1). This correlation has a very robust probability of ($P \leq 10^{-7}$) in lenticular, but a high probability of $(P \leq 1.7x10^{-6})$ in ellipticalls. Lenticular galaxies have more stellar mass than elliptical galaxies, and hence if the stellar mass increases, the surface density increases and becomes visible. Therefore, the results of this research results showed a more tight relationship between stellar mass and surface density in lenticular galaxies than in elliptical galaxies.

Figure 1: The correlation between Log M_{star} (B) and Log Σ_{eff} (B) for galaxies using linear regression, where (a) is for elliptical galaxies and (b) is for lenticular galaxies.

Our work revealed that there is a clear relation between Log M_{star} (B) and Log R_e with a positive correlation coefficient for both E and S0 ($RM_{Star(B)-E}$, $RM_{Star(B)-SO} = 0.263262$, 0.537013 respectively). Similarly, this correlation has a good probability of ($P \leq 3.8 \times 10^{-3}$) in lenticular but ($P \leq 1.2 \times 10^{-6}$) in elliptical, and the analysis of the results shows that the slope is about non-linear (0.269554, 0.536088), (see Figure 2). Both types of galaxies have been not linearly. Lenticular galaxies have a higher correlation coefficient than elliptical galaxies. This is due to the complicated relationship between a galaxy's mass and size, which is influenced by several variables including the galaxy's creation, type, and redshift. larger galaxies typically have larger sizes, however, distinct power law slopes can be used to characterize the link between mass and size for various kinds of galaxies, with star-forming galaxies having a shallower slope than quiescent galaxies. Larger galaxies are often more massive, though the precise link varies according to the number of galaxies under study. This is because, in contrast to disk galaxies, huge elliptical galaxies typically do not have rotationsupported systems. Furthermore, there is a relationship between the effective radius (R_e) and the turnover radius (R_t) , whereby galaxies with smaller R_e typically have smaller R_t . Additionally, the rotation curve's outer slope changes with galaxy mass and morphological type, becoming steeper for lower-mass late-type galaxies and lowering for giant elliptical galaxies as M_{star} lowers. The inner slope of the rotation curve also tends to mildly increase with M_{star} for certain galaxy types.

Figure 2: The correlation between the Log $M_{star}(B)$ and Log R_e for galaxies using linear regression, where (a) is for elliptical and (b) is for lenticular galaxies.

The study indicated that there is a good relationship between Log (M_{HI}) and Log $M_{star}(B)$ with a positive and strong correlation coefficient for E and S0 ($RM_{HI-E}, RM_{HI-S0} = 0.600696$, 0.565925 respectively) (see Figure 3). Similarly, these correlations have significantly high probability of ($P \leq 10^{-7}$) for both types of galaxies, and the analysis of the results shows that the slope is about linear (0.674168, 0.523255), Elliptical galaxies have been more linearly and the correlation coefficient than lenticular galaxies. Their different structures, star populations, and evolutionary paths—which are shaped by their formation mechanisms—are the reasons behind the disparity in the linear connection. Impacted by elements such as the amount of gas present, the rate of formation of stars, and the mechanisms of galaxy evolution. To fully comprehend the dynamics of galaxy creation and evolution, it is imperative to comprehend this link.

Figure 3: The correlation between the Log M_{star} (B) and Log (M_{HI}) for galaxies using linear regression, where (a) is for elliptical and (b) is for lenticular galaxies.

In Figure (4), there is a relation between Log $\Sigma_{\text{eff}}(B)$ and Log M_{dyn} with a positive and very strong correlation coefficient for only S0 with a lack of this relation for E galaxies RM_{dyn-S0} = 0.980625 respectively). Similarly, this correlation has an extremely strong probability of (P $\leq 10^{-7}$) in lenticular, and the analysis of the results shows that the slope is linear 0.981830). There is no result with sufficient surface brightness $\Sigma_{\text{eff}}(B)$ and mass dynamical for ellipticals. For elliptical galaxy formation scenarios, there is a problem with the lack of a correlation between Σ_{eff} (B) and M_{dyn} due to being considerably affected by dynamical evolution; interestingly, they are suspected to be objects out of dynamical equilibrium and close to disruption.

Figure 4: The correlation between the Log $\Sigma_{\text{eff}}(B)$ and Log M_{dyn} for galaxies using linear regression, where (a) is for elliptical and (b) is for lenticular galaxies.

It also demonstrated that there is a strong relation between Log $M_{\text{star}}(B)$ and Log M_{dyn} (B) with a positive and very strong correlation coefficient for E and S0 ($RM_{Star(B)}$ -E, $RM_{Star(B)}$ $s_0 = 0.896939$, 0.944560 respectively, see Figure 5). Similarly, these correlations have a very high likelihood of ($P \leq 10^{-7}$) for both galaxies, and the analysis of the results shows that the slope is linear (0.848845, 0.917703), lenticular galaxies have been more linearly and correlation coefficient than elliptical galaxies. Due to their older star clusters and low levels of current star production, lenticular galaxies exhibit this phenomenon. On the other hand, elliptical or spherical galaxies are more likely to have older, sparser stars with minimal gas and dust, and their stars orbit the galaxy's core in random paths. Galaxy clusters are home to them. Lenticular galaxies are known for their special characteristics that blend elements of elliptical and spiral galaxies, producing a pattern in the relationship between dynamical mass and stellar mass that is consistent with the linear relationship seen in these galaxies.

Figure 5: The correlation between the Log M_{star} (B) and Log M_{dyn} (B) for galaxies using linear regression, where (a) is for elliptical and (b) is for lenticular galaxies.

In Figure (6), the association between Log M_{bar} (B) and Log (M_{HI}) is a positive and good correlation coefficient for E and S0 ($RM_{HI-E}, RM_{HI-S0} = 0.640284, 0.665935$, respectively). Similarly, these correlations have a very high likelihood of ($P \leq 10^{-7}$) for both galaxies. The analysis of the results shows that the slope is linear (0.711177, 0.612296), elliptical galaxies have been more linear, and correlation coefficients are higher than lenticular galaxies. Our findings strongly agree with those of Lintott et al. [17], which found that elliptical galaxies contain baryon-dominated interior regions. Due to the distinct evolutionary history of elliptical galaxies, their formation is thought to have occurred more gradually, with gas continuously accumulating and turning into stars over an extended period. A more dispersed distribution in the Log M_{bar} (B)-Log M_{HI} (B) plane may follow from this more complicated star population, which has a greater variety of ages and metallicities. Star formation is impeded in lenticular galaxies due to their low molecular gas content and lack of substantial hydrogen α or 21-cm emission. Nevertheless, the absence of characteristics such as dust lanes, spiral arms, or bars in elliptical galaxies indicates that their star populations are older. They are generally seen in big clusters and range in absolute visual magnitude from $M_V \sim -23$ to $M_V \sim -16$ mag. A galaxy's luminosity, which can be determined from observations made at various wavelengths including optical, infrared, or ultraviolet, is usually used to compute the galaxies baryonic mass (M_{bar}) . One common metric for estimating the size and structure of galaxies is the bulge-to-total baryonic mass ratio (B/T). In contrast, studies of 21-cm emission lines from hydrogen gas are used to calculate the neutral atomic hydrogen.

Figure 6: The correlation between the Log M_{bar} (B) and Log (M_{HI}) for galaxies using linear regression, where (a) is for elliptical and (b) is for lenticular galaxies.

The study also shows a strong relation between Log σ_0 and Log M_{dyn} with an extreme correlation coefficient for E and S0 ($R\sigma_0$ -E, $R\sigma_0$ -S0= 0.896939, 0.944560 respectively) (see Figure 7). Again, these correlations have a very high probability of ($P \leq 10^{-7}$) for both types of galaxies, and the analysis of the results shows that the slope is about linear (0.848845, 0.917703), lenticular galaxies have been more linearly and correlation coefficient than elliptical galaxies. The baryonic matter depends on the stellar and gas mass present in galaxies. Therefore, the velocity dispersion tends to be larger based on baryonic matter, i.e. the dynamic mass increases with the velocity dispersion of early-type galaxies (ETGs). Therefore, our results showed a very close relationship between dynamical mass and dispersion velocity in both types, but it is larger in lenticular galaxies because they contain more baryonic [17]. Our results are in agreement with previous studies [50, 51] with respect to elliptical galaxies.

Figure7: The correlation between the Log σ_0 and Log M_{dyn} for galaxies using linear regression, where (a) is for elliptical and (b) is for lenticular galaxies.

There is a relation between Log (M_{HI}) and Log L_B with a positive and strong correlation coefficient for E and S0 $(RM_{HLE}, RM_{HLS0}= 0.600696, 0.565925, respectively)$. Similarly, these correlations have a very high likelihood of ($P \leq 10^{-7}$) for both galaxies, and the analysis of the results shows that the slope is about linear (0.535232, 0.612075), (see Figure 8). Lenticular galaxies have been more linearly and correlation coefficient than elliptical galaxies. This is because a main sequence star's brightness increases by about 11 times when its mass doubles. Because stars with larger masses exhibit higher luminosities due to faster rates of energy generation, the link between mass and luminosity is essential to understanding energy generation and balance within stars. Because stars with higher masses have shorter lives than stars with lower masses, this relationship is crucial for determining the ages of stars.

Figure 8: The correlation between the Log (M_{HI}) and Log L_B for galaxies using linear regression, where (a) is for elliptical galaxy, and (b) is for lenticular galaxy.

The connection between Log $\Sigma_{\text{eff}}(B)$ and Log L_B is a positive and robust correlation coefficient for lenticular S0 compared to ellipticals $(R\Sigma_{eff (B)-E,} R\Sigma_{eff (B)-S0} = 0.542175$, 0.879815 respectively) (see Figure 9). This correlation has an extremely strong probability of $(P \leq 10^{-7})$ in lenticular but $(P \leq 0.5 \times 10^{-6})$ in elliptical. The analysis of the results shows that the slope is about linear (0.647523, 0.904452), lenticular galaxies have been more linear, and correlation coefficients are higher than elliptical galaxies. This is because lenticular galaxies suffer from high bulge-to-disk ratios, which complicate matters when it comes to

precisely calculating rotation velocities; also, their intricate structure precludes the presence of cold gas necessary for conventional kinematic studies. Knowing this relationship sheds light on the processes of lenticular galaxies' creation and evolution by revealing information about the internal dynamics and mass distribution inside them.

Figure 9: The correlation between the Log $\Sigma_{\text{eff}}(B)$ and and Log L_B for galaxies using linear regression, where (a) is for elliptical galaxies, and (b) is for the lenticular galaxies.

Figure (10) shows a robust relation between Log M_{bar} (B) and Log M_{dyn} (B) with a positive and very strong correlation coefficient for E and S0 (RM_{dyn-E} , $RM_{dyn-S0}= 0.894662$, 0.942931respectively). Similarly, these correlations have a significantly high probability of (P $\leq 10^{-7}$) for both galaxies, and the analysis of the results shows that the slope is about linear (0.938152, 0.973564), lenticular galaxies have been more linearly and correlation coefficient than elliptical galaxies. The reason for this is that, in comparison to elliptical galaxies, lenticular galaxies have bigger rotation versus dispersion velocity (v/σ) ratios because of their larger rotational support resulting from the disk content, since their baryonic mass is directly proportional to their stellar mass.

Figure10: The correlation between the Log $M_{bar}(B)$ and Log M_{dyn} for galaxies using linear regression, where (a) is for elliptical galaxies, and (b) is for the lenticular galaxies.

Figure (11) revealed an association between Log M_{HI} and Log M_{BH} with a positive and good correlation coefficient for E and S0 $(RM_{HI-E}, RM_{HI-S0} = 0.600696, 0.565925$ respectively). Likewise, these correlations have a very high likelihood of ($P \leq 10^{-7}$) for both types of galaxies, and the results showed that the slope is about linear (0.674168, 0.523255), elliptical galaxies have been more linearly and correlation coefficient than lenticular galaxies. This correlation holds importance in comprehending the attributes of elliptical galaxies, including their structural features and dark matter composition. This is significant in the development and evolution of elliptical galaxies, emphasizing the intricate process of black

hole merger and its consequences for galactic architecture. This suggests that lenticular galaxies may have a more homogeneous stellar population distribution and a star formation history compared to elliptical galaxies. Large numbers of globular clusters encircle elliptical galaxies, which are primarily made up of older, low-mass stars with minimal gas and dust. The changes in the relationship between Log (M_{HI}) and Log M_{BH} that have been observed can be attributed to the structure of these galaxies, as well as their composition, and star formation activity that differ between elliptical and lenticular galaxies.

Figure 11: The correlation between the Log (M_{HI}) and Log M_{BH} for galaxies using linear regression, where (a) is for elliptical galaxy, and (b) is for lenticular galaxies.

Figure (12) shows a robust relation between Log R_e and Log M_{dyn} with a positive and good correlation coefficient for E and S0 $(RM_{dyn-F}$, $RM_{dyn-S0} = 0.655732$, 0.774317, respectively). Likewise, these correlations have a significantly high probability of ($P \leq 10^{-7}$) for both types of galaxies, and the results showed that the slope is about linear (0.606086, 0.753598), lenticular galaxies have been more linearly and correlation coefficient than elliptical galaxies. The relationship between Log R_e and Log M_{dyn} appears to have a clear correlation coefficient, with a linear slope for ellipticals. The results presented here agree with those of previous research [5]. Understanding the mass and structural characteristics of these galaxies is made possible by this correlation. It draws attention to the significance of comprehending the relationships between variations in dynamical mass, effective radius, and velocity dispersion in these galaxies. In contrast to elliptical galaxies, lenticular galaxies have a greater correlation coefficient and a more linear color-magnitude relation (CMR), according to research by Jaffe et al. (1994) [52]. In contrast to elliptical galaxies, this implies that lenticular galaxies might have a more uniform distribution of stellar populations and a star formation history.

Figure12: The correlation between the $Log R_e$ and $log M_{dyn}$ for galaxies using linear regression, where (a) is for elliptical galaxies, and (b) is for the lenticular galaxies. Finally, this research proved that there is a robust relation between Log M_{bar} (B) and Log M_{BH}

with a positive and extreme correlation coefficient for E and S0 (RM_{BH-E} , RM_{BH-S0}) = 0.997007, 0.985470, respectively) (see Figure 13). Also, these correlations have a very high likelihood of ($P \leq 10^{-7}$) for both galaxies, and the analysis of the results shows that the slope is linearly (0.986713, 0.979983), and it shows that the lenticular and elliptical galaxies have been more linear. The similarity in the evolutionary processes of galaxies is indicated by the positive slope of their relationship. We know of the effect of black holes on galaxy formation and evolution, which is reliant on galactic mass.

Figure13: The correlation between the Log $M_{bar}(B)$ and Log M_{BH} for galaxies using linear regression, where (a) is for elliptical galaxies, and (b) is for the lenticular galaxies.

5. Conclusions

 The study successfully demonstrated the estimation of stellar-gaseous kinematics and dynamics mass using scaling coefficient relationships, specifically the Faber–Jackson Relation (FJR), for a selected sample of elliptical and S0-type galaxies. Statistical analyses were performed using SPSS and MATLAB to investigate associations among various factors, including kinematic properties, effective radius (R_e) , surface brightness density at effective radius (Σ_{eff}), stellar mass (M_{sun}), gas mass (M_{gas}), dynamic and baryonic masses. These results exhibited the following:

The empirical relations between Log M_{star} (B) and Log Σ_{eff} (B) have a strong regression relationship of 0.9, and the slope appears to be almost linear (slope 0.8) in the lenticular galaxy. We found a weak relationship in the elliptical galaxies because lenticular galaxies have more stellar mass, and as the star mass increases, the surface density increases, making it more observable. As a result, our findings indicated that lenticular galaxies have a tighter stellar surface density relationship than elliptical galaxies.

1- The experimental relations between (Log M_{bar} (B) - Log M_{dyn} , Log M_{bar} (B) - Log M_{BH}) have a very strong correlation of \sim 1, and the slope appears to be more linearly \sim 1 in both types of galaxies

2- The empirical relations between Log Σ_{eff} (B) and Log M_{dyn} have a strong high regression relationship of \sim 1, and the slope appears to be approximately linear \sim 1 in the lenticular galaxy. Still, there is no relationship in the elliptical galaxies because elliptical galaxies are so impacted by dynamic rotation that, interestingly, it is thought that they are things that are close to disorder and outside of dynamic.

3- This work indicated that the slope appears to be more linearly 0.9 in both types of galaxies and that the experimental convergences between (Log M_{star} (B) and Log M_{dyn} , Log σ_0 and Log M_{dyn}) show a very significant correlation of 0.9. Strong connections have been noticed between the luminosity, central velocity dispersion, and the relationship between the half-light surface brightness and the effective radius of elliptical and lenticular galaxies. The formation and growth of early-type galaxies (lenticulars and ellipticals) are primarily concerned with merger occurrences, regardless of size. To measure the influence of the central bulge on merger dynamics and residual construction, disc-halo mergers are juxtaposed with bulgeless disc-halo mergers. Larger tidal tails, oblate ultimate fundamental forms, steeper rotation curves, stronger Sérsic index surface brightness profiles, and oblate-rotator dynamics within are all caused by the existence of central bulges. This result is consistent with recent forecasts of a dynamical mass increase driven by small mergers in galaxy-building simulations, where the dark matter percentage within the half-light radius rises with cosmic rotation.

4- The mass of gas and stars in galaxies determines the baryonic material. As a result, the speed dispersion of early-type galaxies appears to be bigger due to baryonic matter; that is, the dynamic mass grows with the velocity dispersion.

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