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Assessment of Transitional Orbits from a High Eccentricity LEO to a Circumlunar Orbit

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Abstract:

This study presents an evaluation of the achievability and execution of transitional orbits for spacecraft transitioning from a high eccentricity Low Earth Orbit (LEO) to a circumlunar orbit, with the point of distinguishing the foremost proficient and ideal trajectory while considering variables such as velocity and required fuel. Transitional orbits have a role in space exploration, and optimizing their characteristics can enormously advantage mission planning and spacecraft design. A numerical simulation model was developed to attain this objective, A gravity perturbations effect is included in the calculation of the transition. The model utilized progressed numerical integration strategies for exact trajectory analysis.

Three cases were examined to investigate the impacts of varying eccentricity and the argument of perigee on the ideal transition, In the first case, eccentricity values of [0.001, 0.01, 0.1] were inspected, whereas the argument of perigee was fixed at 80 degrees. Within the second case, the argument of perigee values of [80, 170, 260] was considered, with the eccentricity fixed at 0.1. The third case included at the same time varying the eccentricity and argument of perigee values. The results showed that the three cases agreed the most favorable transition occurred when eccentricity was set to 0.001 and the argument of perigee was set to 260 degrees. This produced a velocity increase of 2.195694 km/s, which is the lowest increase in ΔV , a metric used to measure fuel efficiency and power required. This finding also demonstrated the lowest change in eccentricity, the lowest change in inclination, and the lowest change in the ascending node, all of which are indicative of increased orbit stability.

Keywords: Transitional Orbits; Eccentricity; argument of perigee; LEO; lunar orbit

تقييم المدارات الانتقالية من المدار الأرضي المنخفض شديد الانحراف إلى مدار حول القمر

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الخلاصة

تقدم هذه الدراسة تقييماً لجدوى وأداء المدارات الانتقالية للمركبات الفضائية التي تنتقل من مدار الأرضي المنخفض شديد الانحراف إلى مدار حول القمر، بهدف تحديد المسار الأكثر كفاءة والأفضل مع مراعاة عوامل مثل السرعة واستهلاك الوقود. تلعب المدارات الانتقالية دوراً حاسماً في مهام استكشاف الفضاء المستقبلية، ويمكن أن تعود تحسين خصائصها بشكل كبير على تخطيط المهمة وتصميم المركبة الفضائية. لتحقيق هذا

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الهدف، تم تطوير نموذج محاكاة عددي يدمج نموذج واقعي لجاذبية القمر وتأثيرات جاذبية الأرض والاضطرابات. استخدم النموذج تقنيات التكامل العددي المتقدمة لتحليل المسار بدقة. تمت دراسة ثلاث حالات لاستكشاف تأثير تغير الانحراف وزاوية اتجاه الحضيض على الانتقال الأمثل. في الحالة الأولى، تم تغير قيم الانحراف [0.001، 0.01، 0.1]، في حين ثبتت زاوية اتجاه الحضيض عند 80 درجة. في الحالة الثانية، تم تغير قيم زاوية اتجاه الحضيض [80، 170، 260]، مع تثبيت قيمة الانحراف عند 0.1. شملت الحالة الثالثة تغيير قيم الانحراف وزاوية اتجاه الحضيض في الوقت نفسه. أظهرت النتائج أن الحالات الثلاث اتفقت على أن الانتقال الأكثر ملاءمة حدث عندما تم ضبط الانحراف على 0.001 وتم ضبط زاوية اتجاه الحضيض على 260 درجة. أنتج هذا زيادة في السرعة قدرها 2.195694 كم/ثانية، وهي أقل زيادة في ΔV ، وهو مقياس يستخدم لقياس كفاءة استهلاك الوقود والطاقة المطلوبة. أظهرت هذه النتيجة أيضاً أقل تغير في الانحراف المركزي، وأقل تغير في الميل، وأقل تغير في العقدة الصاعدة، وكلها مؤشرات على زيادة استقرار المدار.

1-Introduction:

This study presents an evaluation of the achievability and execution of transitional orbits for spacecraft transitioning from a high eccentricity Low Earth Orbit (LEO) to a circumlunar orbit, with the point of distinguishing the foremost proficient and ideal trajectory while considering variables such as velocity and required fuel. Transitional orbits have a role in space exploration, and optimizing their characteristics can enormously advantage mission planning and spacecraft design [1]. Research in space and the use of appropriate trajectories and mechanics have been human goals, and the Moon is a noteworthy focal point for visible space exploration [2].

To realize effective lunar missions, spacecraft must navigate from a high eccentricity Low Earth Orbit (LEO) to a circumlunar orbit. The trajectory plan amid this transitional stage plays a vital part in mission arranging, fuel proficiency, and spacecraft execution [3]. Transitional orbits function as basic pathways that encourage the transition from a high eccentricity LEO to a circumlunar orbit. These orbits take advantage of the gravitational forces applied by the Earth and the Moon to optimize the spacecraft's trajectory. Planning productive and optimal transitional orbits is basic to play down fuel utilization and the success of the space flight [4].

To assess the possibility and execution of transitional orbits, a numerical simulation model was developed. The model incorporates a realistic Moon's gravity, the impacts of Earth's gravity, and perturbations caused by different components such as solar radiation pressure and atmospheric drag. Progressed numerical integration strategies were utilized to precisely analyze the trajectory characteristics and execution [5].

The transitional orbits focused on examining the impacts of varying eccentricity and the argument of perigee. By changing these parameters individually and community, important bits of knowledge were gained about their impact on the trajectory. Understanding the relationship between eccentricity and the argument of perigee is pivotal for optimizing transitional orbits and accomplishing effective spacecraft transitions [6].

The investigation contributes to the field of lunar exploration by giving profitable information to mission organizers and spacecraft designers. By considering the impact of eccentricity and the argument of perigee, educated choices can be made to optimize trajectory planning and improve mission effectiveness. The discoveries of this consider laying the establishment for further analysis and optimization techniques, empowering the refinement of

transitional orbit trajectories, and progressing the successes of future space investigation endeavors [7].

Several studies have investigated orbital transitions, orbital elements, the Moon's orbit, and perturbations:

- Anas Salaman Taha (2002), investigated disruptions of satellites, which was a disorder affecting the orbits of satellites low-lying [8].
- Al-Ali (2011) computed the perturbation effects on orbital elements of the moon, which computed the perturbations, including atmospheric drag, non-spherical earth, solar radiation pressure, and a third-body attraction. These perturbations disrupted an object's orbit and were also found to cause changes in the moon's orbital elements with time [9].
- Taif A. Damin and Abdulrahman H. Salih (2016) investigated the solar attraction effect on orbital elements of the moon [10].
- Abouelmagd et al. (2016): Focus on the numerical integration of the relativistic two-body problem, using a multi-scale method to analyze relativistic effects on orbital dynamics [11].
- Doshi et al. (2023): This study explores periodic orbits in perturbed relative motion, highlighting the effects of disturbances on-orbit stability and dynamics [12].

As described in the old and new literatures, optimization of transitional orbitals is an important topic that has been mostly studied by advanced numerical modeling, integration techniques, and inclusion of perturbation effects. What makes this work unique is that it investigates the influence of orbital eccentricity, the argument of perigee how much these parameters affects the determination of the most energy-efficient and optimum orbital trajectory, while considering all the perturbations related to perturbed eccentricity. Its findings provide a great deal of information on minimizing costs for orbital transition while maximizing fuel savings as well as mission completion rates.

1.Types of orbits based on Eccentricity and the Argument of Perigee:

The optimization of transitional orbits plays a crucial part in spacecraft trajectory planning, fuel effectiveness, and mission success [13]. Two parameters that altogether affect transitional orbits are eccentricity and the argument of perigee [14]. This study highlights the significance of utilizing these components and investigates the reasons for their determination as factors for optimization within the transition from a high eccentricity Low Earth Orbit (LEO) to a circumlunar orbit [15].

Eccentricity could be a measure of how elliptical or stretched an orbit is, extending from 0 to 1 [16]. A circular orbit has an eccentricity of 0, whereas higher values show progressively stretched orbits. What interests us in this study is Elliptical Orbit ($0 < e < 1$): An elliptical orbit has an eccentricity between 0 and 1, causing the spacecraft's distance from the central body to differ all through the orbit [17]. The spacecraft moves quicker when closer to the central body (perigee) and slower when more distant and absent (apogee).

The argument of perigee decides the precise position of the perigee, which is the point in an orbit closest to the central body [14]. Low values demonstrate the perigee is found close to the reference direction or within the same plane, whereas high values position the perigee more distant from the reference direction or in a distinctive plane. Here's how the Argument of Perigee influences the orbit:

1. Low Argument of Perigee: A low Argument of Perigee shows that the perigee is found close to the reference direction or within the same plane. This results in an orbit where the spacecraft passes near to the central body close to the same locale amid each orbit [14].

2. High Argument of Perigee: A high Argument of Perigee implies that the perigee is situated distant from the reference direction or in a diverse plane. This results in an orbit where the spacecraft approaches the central body from diverse points amid each orbit [14].

By changing the eccentricity and the Argument of Perigee, distinctive combinations of orbit types can be accomplished, permitting a wide extent of trajectories and mission prerequisites [18].

The reasons why the Eccentricity and the argument of the Perigee values they are both chosen in this investigation:

By altering Eccentricity, you will successfully control the timing and positioning of the shuttle amid the transfer. This parameter permits you to alter the staging point between the spacecraft and the specified goal, which can affect the proficiency and achievability of the transfer [13].

By changing the argument of the Perigee, you can modify the orientation of the transfer orbit with respect to the Moon's orbit. This change can impact different components such as the encounter geometry, lunar capture elements, and fuel prerequisites. It permits you to investigate distinctive approaches for adjusting the spacecraft's trajectory with the required lunar orbit [19].

These two parameters were chosen for examination since they offer critical control over the transfer orbit characteristics and give experiences into how specific changes in these parameters influence the general transition process [15].

In this research, we chose values for eccentricity and the Argument of Perigee:

I. First Case:

In this case, you changed the eccentricity values to [0.001, 0.01, 0.1], whereas fixing the Argument of Perigee at 80 degrees. Let's look at the comparing orbits:

Eccentricity = 0.001: This value speaks to a nearly circular orbit. The spacecraft's path will closely take after a circle, with really little deviation from idealized symmetry [13].

Eccentricity = 0.01: With a somewhat higher eccentricity, the orbit gets to be more curved. The spacecraft will travel along a stretched path, where the distance between the spacecraft and the central body varies to a greater extent compared to a circular orbit [14].

Eccentricity = 0.1: A higher eccentricity value leads to an essentially more curved orbit. The spacecraft's path will have a particular elongation, with larger variations in the distance between the spacecraft and the central body during the orbit [14].

II. Second Case:

Within the second case, you changed the argument of the Perigee values to [80, 170, 260], while keeping the eccentricity fixed at 0.1. Within the second case, you changed the argument of the Perigee values to [80, 170, 260], while keeping the eccentricity fixed at 0.1.

III. Third Case:

By at the same time changing the eccentricity and the argument of the Perigee values, you'll get orbits that combine the characteristics mentioned above. For example, an eccentricity of 0.1 and an argument of the Perigee of 260 degrees would result in an elliptical orbit with a distinct elongation and a close approach to the central body from a different angle during each orbit [20].

These varieties in eccentricity and the argument of the Perigee permit for an assorted extent of orbits, each with it possess special characteristics. Understanding these characteristics is significant for planning and planning spacecraft trajectories that meet mission prerequisites and goals.

1. Calculating the coordinates of the moon

The calculation of the moon coordinates incorporates the calculation of the moon's position in three-dimensional space, and it requires information of the particular data and information for the required date and time to calculate the coordinates. Here are more point-by-point steps for calculating moon coordinates:

- a) The time reference: we must indicate the precise date and time for calculating the lunar coordinates. The adjusted Greenwich Mean Time System (UTC) is favored as the standard time reference in space [21].
- b) Calculating the Julian Day: utilize human dates and change over them to Julian days utilizing accessible equations [22].
- c) Mean Lunar Longitude: The Moon's mean equatorial angle speaks to the position of the Moon in relation to the Earth within the equatorial plane. You'll be able to calculate it utilizing the given formulas, which are based on the Julian day and time zone [23].
- d) Mean Lunar Motion: The mean lunar motion is the day-by-day increment within the lunar mean equatorial angle. We'll be able to calculate it utilizing accessible equations and given data [10].
- e) True Lunar Longitude: we must calculate the Moon's mean equatorial angle by taking into consideration extra components such as solar motion and Earth motion. The true angle of the moon can be calculated utilizing fitting equations and fundamental corrections [24].
- f) Position of the Moon: The position of the Moon can be calculated in three steps, as follows [9][25]:

- a) Compute elliptical coordinates [25]:

I. Lunar longitude (λ_M):

$$\lambda_M = 218.32^\circ + 481267.883^\circ T_2 + 6.29^\circ \sin(134.9^\circ + 477198.85^\circ T_2) - 1.27^\circ \sin(259.2^\circ - 413335.38^\circ T_2) + 0.66^\circ \sin(235.7^\circ + 890534.23^\circ T_2) + 0.21^\circ \sin(269.9^\circ + 954397.7^\circ T_2) - 0.19^\circ \sin(357.5^\circ + 35999.05^\circ T_2) - 0.11^\circ \sin(186.6^\circ + 966404.05^\circ T_2) \quad (1)$$

II. latitude of the Moon (β_M):

$$\beta_M = 5.13^\circ \sin(93.3^\circ + 483202.03^\circ T_2) + (0.28^\circ \sin(228.2^\circ + 960400.87^\circ T_2) - 0.28^\circ \sin(318.3^\circ + 6003.18^\circ T_2) - 0.17^\circ \sin(217.6^\circ - 407332.2^\circ T_2)) \quad (2)$$

III. The Moon's distance from the center of the Earth:

$$r_M = 385000 - 20905 \cos l - 3699 \cos(2D' - l) - 2956 \cos(2D') - 570 \cos(2l) + 246 \cos(2l - 2D') - 152 \cos(l + l' - 2D') \quad (3)$$

l : mean anomaly of the Moon.

l' : mean anomaly of the Sun.

D' : Difference in mean longitude of the sun and moon.

Above the three premasters are given as follows:

$$l = 134.96292^\circ + 477198.86753^\circ T_2 \quad (4)$$

$$l' = 357.52543^\circ + 35999.04944^\circ T_2 \quad (5)$$

$$D' = 297.85027^\circ + 445267.11135^\circ T_2 \quad (6)$$

- b) Convert elliptical coordinates to equatorial coordinates in terms of right ascension (α_M) and declination (δ_M) [10]:

$$\tan \alpha_M = \sin \lambda_M \cos \varepsilon - \tan \beta_M \sin \varepsilon / \cos \lambda_M \quad (7)$$

And

$$\sin \delta_M = \sin \beta_M \cos \varepsilon + \cos \beta_M \sin \varepsilon \sin \lambda_M \quad (8)$$

Finally, using equatorial coordinates, the position component in the Cartesian coordinate system can be computed as follows [10]:

$$\left. \begin{aligned} X_M &= r_M \cos \delta_M \cos \alpha_M \\ Y_M &= r_M \cos \delta_M \sin \alpha_M \\ Z_M &= r_M \sin \delta_M \end{aligned} \right\} \quad (9)$$

The distance between the moon and the center of the earth can be obtained by [14]:

$$R_M = \sqrt{X_M^2 + Y_M^2 + Z_M^2} \quad (10)$$

1. Perturbations of Transitional Orbits

Perturbations refer to external influences or forces acting on a spacecraft or celestial body that cause deviations from its expected trajectory. In the assessment of transitional orbits from a high eccentricity LEO to a Circumlunar Orbit, Perturbations near the Earth are detailed in Ref [25]. As for perturbations near the Moon, these perturbations can include:

1. Non-spherical Earth Gravity: The acceleration of the spacecraft without perturbations can be represented by the following equations[26]:

$$a_x = \frac{(-\mu x)}{r^3} \quad (11)$$

$$a_y = \frac{(-\mu y)}{r^3} \quad (12)$$

$$a_z = \frac{(-\mu z)}{r^3} \quad (13)$$

Where: -

μ : the gravitational parameter of the Earth.

x, y, z: the spacecraft's position coordinates.

r: the distance between the spacecraft and the Earth.

These equations calculate the acceleration components effected by non-spherical Earth gravity at x, y, and z axes.

The perturbation can be expressed as acceleration in the spacecraft's equations of motion [9]:

$$aj_{2x} = \left(\frac{\mu x}{r^3}\right) \left(\frac{3}{2} j_2 \left(\frac{R_m}{r}\right)^2\right) \left(5 \left(\frac{z^2}{r^2}\right) - 1\right) \quad (14)$$

$$aj_{2y} = \left(\frac{\mu y}{r^3}\right) \left(\frac{3}{2} j_2 \left(\frac{R_m}{r}\right)^2\right) \left(5 \left(\frac{z^2}{r^2}\right) - 1\right) \quad (15)$$

$$aj_{2z} = \left(\frac{\mu z}{r^3}\right) \left(\frac{3}{2} j_2 \left(\frac{R_m}{r}\right)^2\right) \left(5 \left(\frac{z^2}{r^2}\right) - 1\right) \quad (16)$$

Where: -

j_2 : a constant, $j_2 = 0.001082$.

R_m : The radius of the moon, $R_m = 1737.4$ km.

These equations calculate the acceleration components effected by the Moon's gravity at x, y, and z axes.

2. Third Body Attraction: The equation describing the moon-induced perturbed acceleration can be expressed as [1]:

$$\mathbf{a}_M = -\mu_M \left(\frac{\mathbf{r}_{M-Sat}}{(r_{M-Sat})^3} + \frac{\mathbf{r}_{E-M}}{(r_{E-M})^3} \right) \quad (17)$$

Where:

μ_{Moon} is the gravitational constant for the Moon and equals to 4902.794 km³/sec².

\mathbf{r}_{M-Sat} is the position vector from the Moon to satellite.

\mathbf{r}_{E-M} is the position vector from the Earth to the Moon.

1. Results and Discussion

The success of spacecraft design and mission planning relies heavily on Improving the properties of transition orbits. Especially for spacecraft that employ highly eccentric transition orbits to transfer from low-Earth orbit to lunar orbit. in this study, the focus was placed on

assessing the feasibility and performance of transition orbits in terms of mission trajectory optimization and efficiency, which considered factors like velocity and fuel consumption. Numerical integration techniques are utilized to develop sophisticated simulation models for mission trajectory analysis, which is vital for accomplishing this mission. The model includes lunar gravity, Earth gravity, and perturbations. We investigated three possible scenarios for the best transition examined by analyzing the effects of eccentricity and the perigee of argument. To gain a further understanding of potential routes and methods of implementing them, and the consequences of these parameters, both individually and when combined. The results for the three cases are shown in Table 1:

Table 1 : The change in velocity and time and orbital elements for each case tested.

No. Cases	ω °	e	Δa (Km)	Δi °	ΔV (km/sec)	$\Delta \Omega$ °	ΔV_p (km/sec)	ΔT min	Δf °
Case 1	80	0.1	198189.53	0.000001 4	2.231486	- 0.000017	-0.384	24980.1	-73.75 9
		0.01	208058.60	0.000001 6	2.198922	- 0.000019	-0.396	26996.1	-82.13 4
		0.001	209067.46	0.000001 6	2.195631	- 0.000019	-0.397	27208.6	-88.11 5
Case 2	80	0.1	198189.53	0.000001 4	2.231486	- 0.000017	-0.384	24980.1	-73.75 9
	170		198179.51	0.000007 9	2.231589	- 0.000003	-0.384	24980.4	-84.30 0
	260		198183.65	0.000000 9	2.231547	0.000011	-0.384	24980.6	-76.96 0
Case 3	80	0.1	198189.53	0.000001 4	2.231486	- 0.000017	-0.384	24980.1	-73.75 9
		0.01	208058.60	0.000001 6	2.198922	- 0.000019	-0.396	26996.1	-82.13 4
		0.001	209067.46	0.000001 6	2.195631	- 0.000019	-0.397	27208.6	-88.11 5
	170	0.1	198179.51	0.000007 9	2.231589	- 0.000003	-0.384	24980.4	-84.30 0
		0.01	208046.64	0.000009 1	2.199030	- 0.000003	-0.396	26995.3	-54.93 4
		0.001	209055.29	0.000009 2	2.195739	- 0.000004	-0.397	27207.7	-83.11 4
	260	0.1	198183.65	0.000000 9	2.231547	0.000011	-0.384	24980.6	-76.96 0
		0.01	208051.66	0.000001 0	2.198985	0.000012	-0.396	26995.6	-77.11 5
		0.001	209060.40	0.000001 0	2.195694	0.000012	-0.397	27208.1	-62.10 1

Now we discussion of the results for each case:

Case 1:

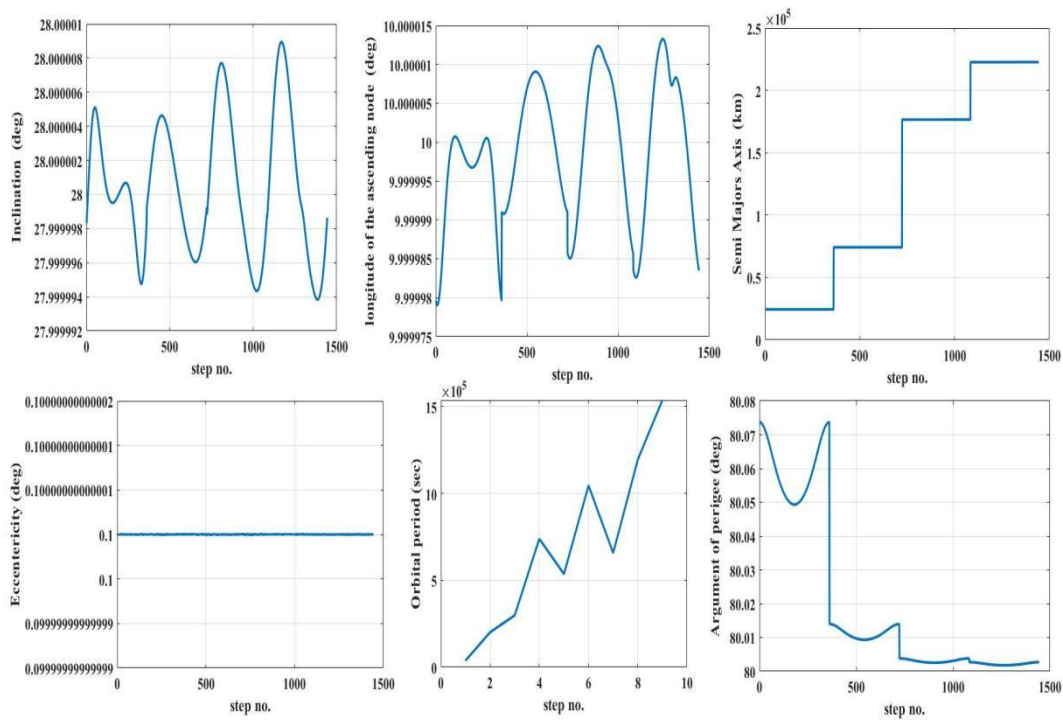


Figure 1: Orbital Element Variations for Lunar Transfer Trajectory when $e= 0.1$ and $\omega = 80$.

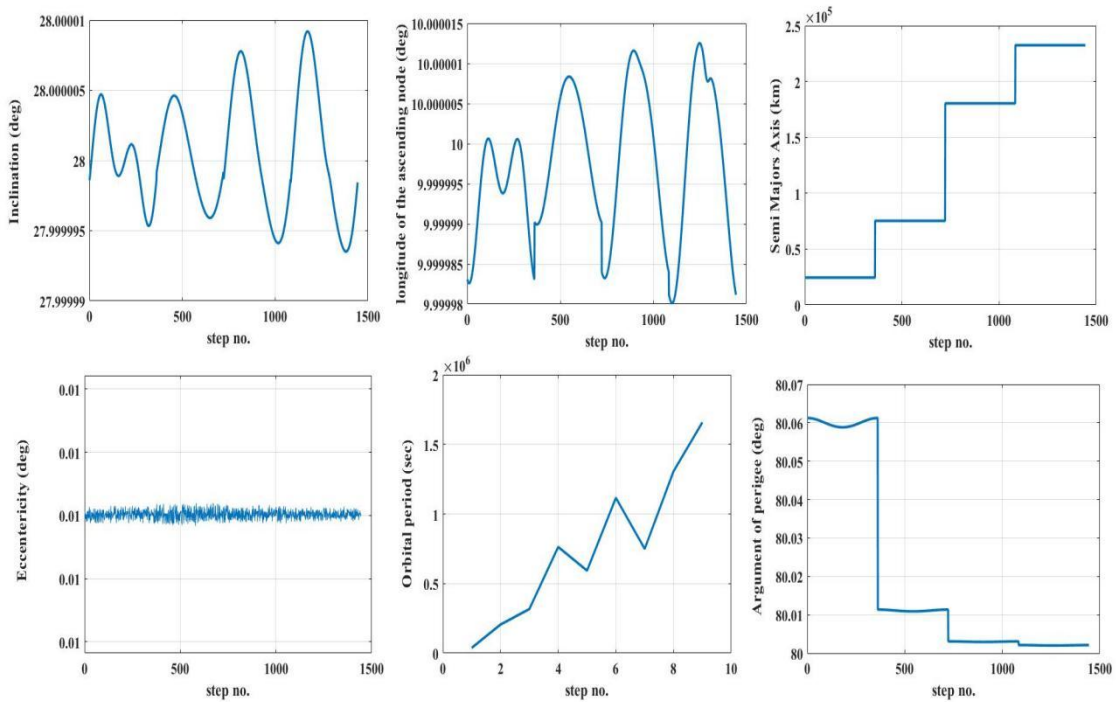


Figure 2: Orbital Element Variations for Lunar Transfer Trajectory when $e= 0.01$ and $\omega = 80$.

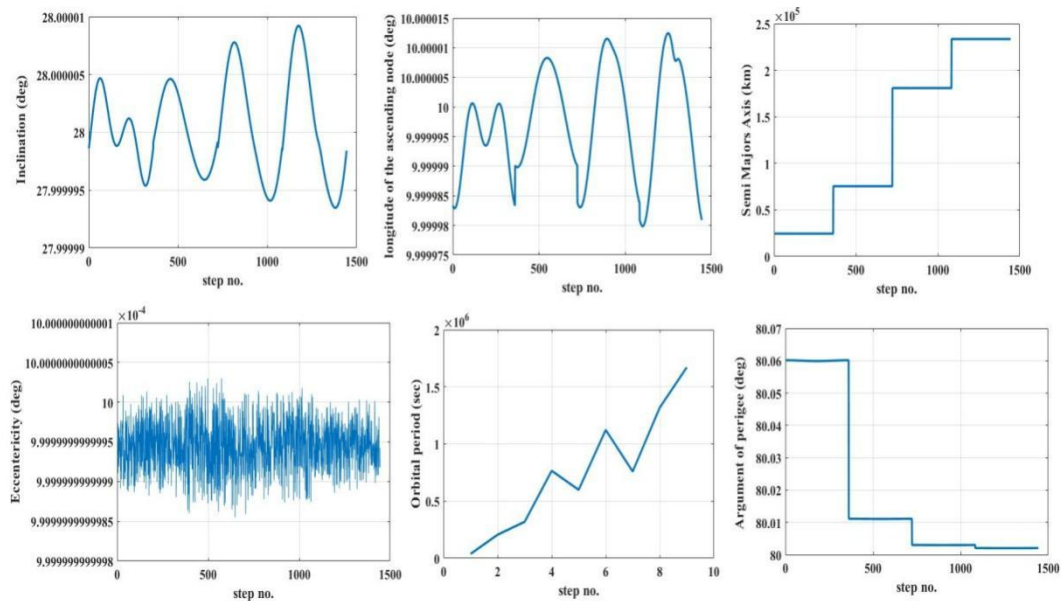


Figure 3: Orbital Element Variations for Lunar Transfer Trajectory when $e= 0.001$ and $\omega = 80$

The nature of the transition trajectory and its relationship to eccentricity (shown in Table 1) was examined in the first case of this study. With the Argument of Perigee fixed at 80 degrees, the effect of different eccentricity values was observed. This leads to interpretable findings.

By examining the relevant orbital elements and perturbations as shown in Figures (1), (2) (3), The figures show the change in orbital elements when eccentricity change with Note that one step in the figures is equivalent to one degree. The x-axis represents mean anomaly (0-360) degree, and we used the steps for the arrangement. we can understand how eccentricity affects transition orbits. The analysis shows that the transition from high eccentricity Low Earth Orbit (LEO) to lunar orbit is most efficient when the eccentricity is 0.001, with a velocity (v) of 1.3077835 km/s. The higher the eccentricity, the longer the orbit, and the greater the difference between perigee (the point closest to Earth) and apogee (the point farthest from Earth). This increased eccentricity impacts the energy efficiency during a spacecraft's transition to lunar orbit. Therefore, leveraging the varying gravitational pulls of the Earth and Moon could be more effective.

Our investigation focused on the role of eccentricity in orbital transitions, consistently setting the Argument of Perigee at 80 degrees. We discovered that an eccentricity level of 0.001 offers the most streamlined and energy-efficient path, minimizing velocity changes and thus reducing the energy required for transportation. This indicates that a higher-than-average eccentricity might facilitate an easier and more efficient shift from a high-eccentricity Low Earth Orbit (LEO) to a lunar trajectory.

Case 2:

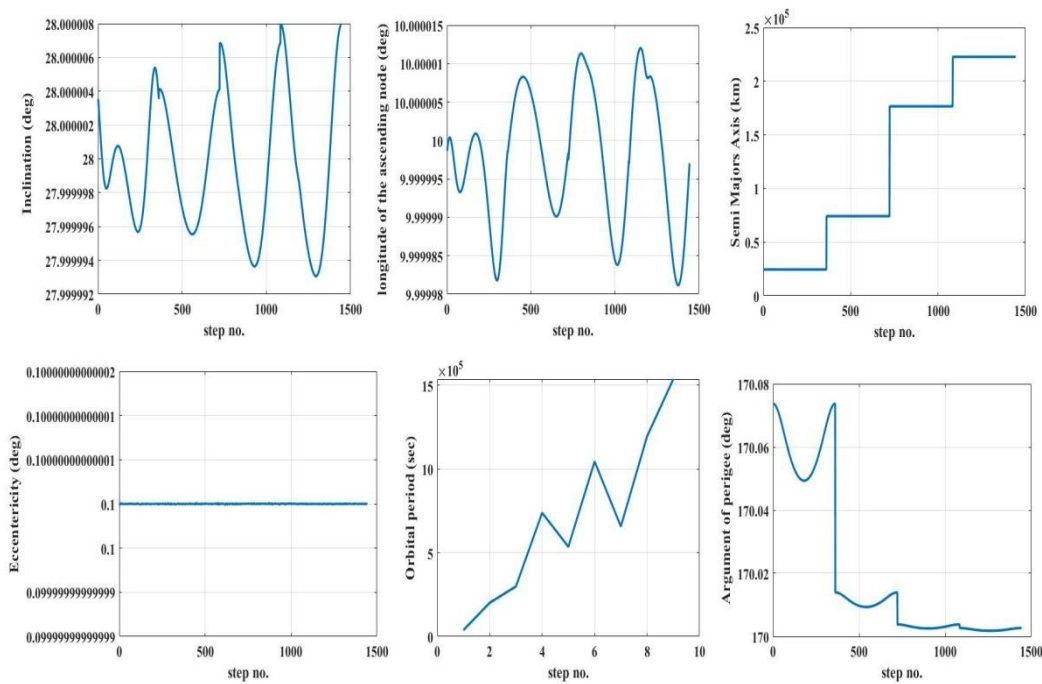


Figure 4: Orbital Element Variations for Lunar Transfer Trajectory when $e = 0.1$ and $\omega = 170$.

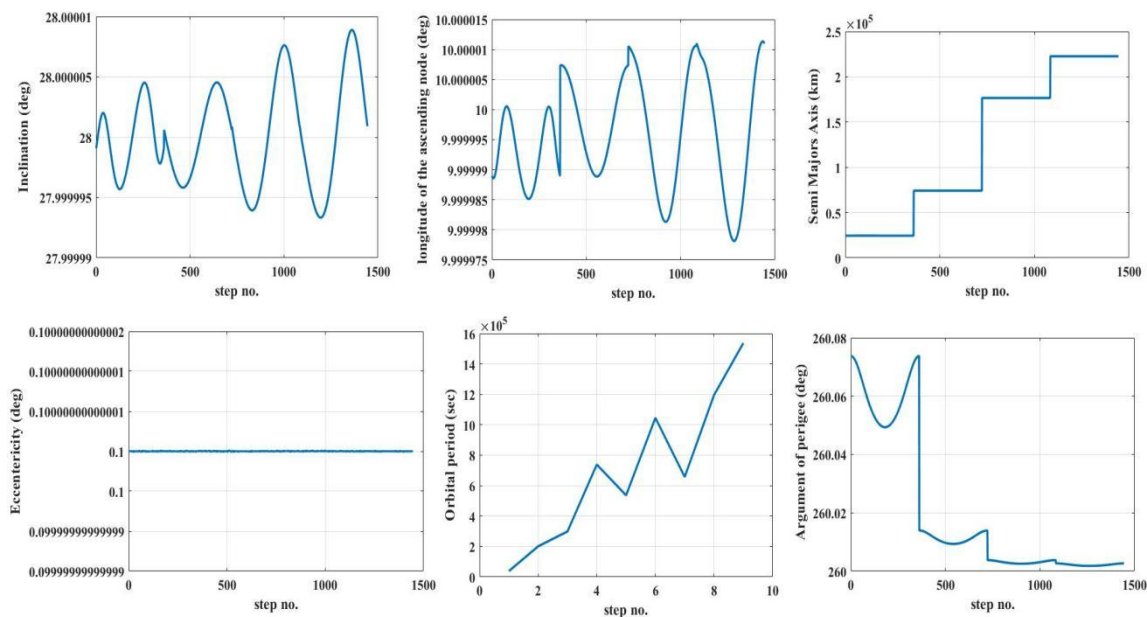


Figure 5: Orbital Element Variations for Lunar Transfer Trajectory when $e = 0.1$ and $\omega = 260$.

The second case of our research examined the impact of varying the Argument of Perigee while keeping the eccentricity constant at 0.1. We aimed to understand how the perigee's position influences the efficiency of transition orbits, taking into account other orbital elements and external disturbances. Our analysis, particularly of Figures (1), (4), and (5), revealed that the most effective transition is achieved with an Argument of Perigee of 80 degrees, correlating to a velocity of 1.4792253 km/s. The perigee's position being the nearest orbital point to Earth is crucially determined by its angle, making it an integral factor in orbit

design. Additionally, variations in the perigee's position significantly influence the timing and execution of crucial maneuvers, thereby affecting the overall orientation of the orbit in space. Different values of the argument of perigee parameter led to trajectory changes during the transition. A spacecraft may orbit the Moon most efficiently when the orbit is 80° from its closest point. During this transformation, energy expenditure and gravity are minimized - thus making the journey smoother and more successful. After much experimentation, we found that 80° is the best argument of perigee. This shows how important it is to get the correct perigee parameters. Therefore, choosing the correct perigee parameters is the key to a successful trajectory. This view could drastically change performance, stability, and fuel economy in the interim. Therefore, a careful analysis of the Argument of Perigee is very important when refining the transition orbital circulation.

Case 3:

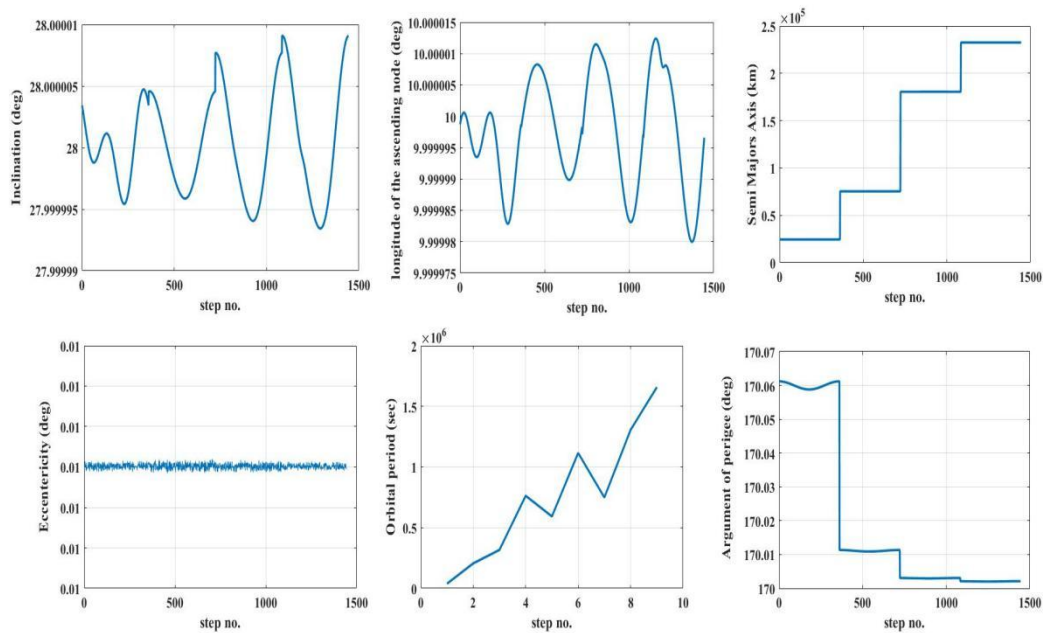


Figure 6: Orbital Element Variations for Lunar Transfer Trajectory when $e= 0.01$ and $\omega= 170$.

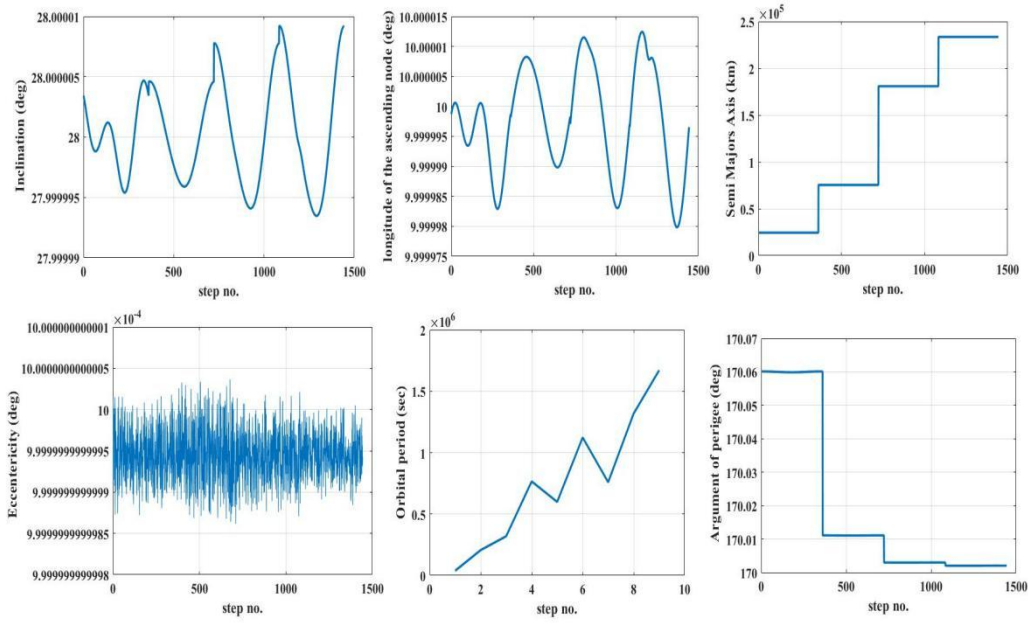


Figure 7: Orbital Element Variations for Lunar Transfer Trajectory when $e= 0.001$ and $\omega= 170$.

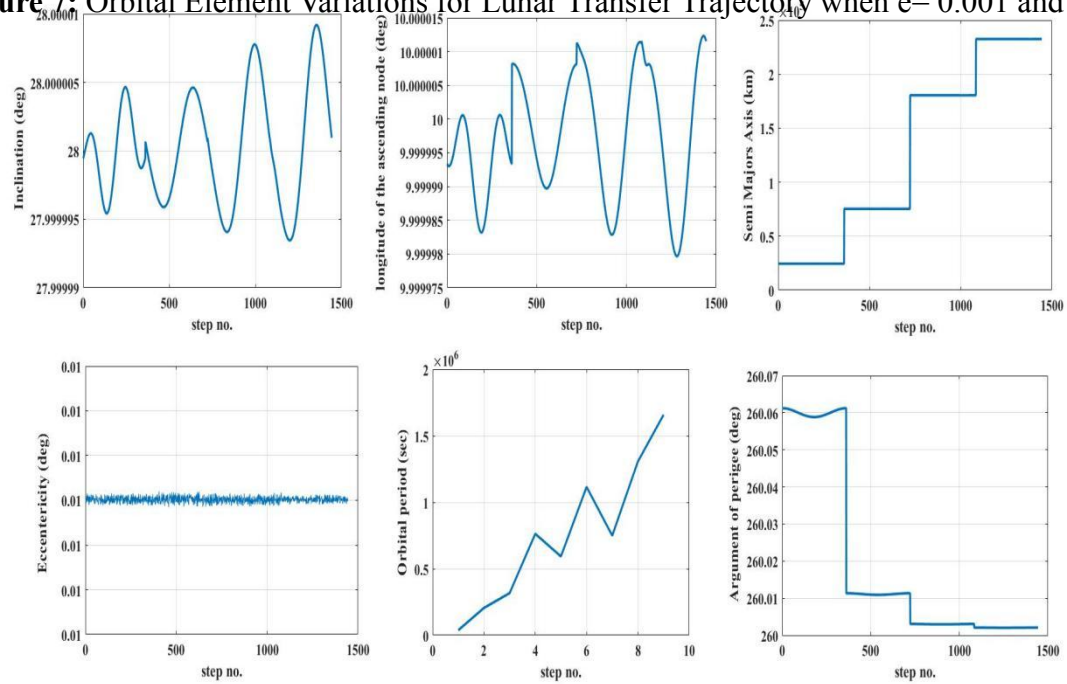


Figure 8: Orbital Element Variations for Lunar Transfer Trajectory when $e= 0.01$ and $\omega= 260$.

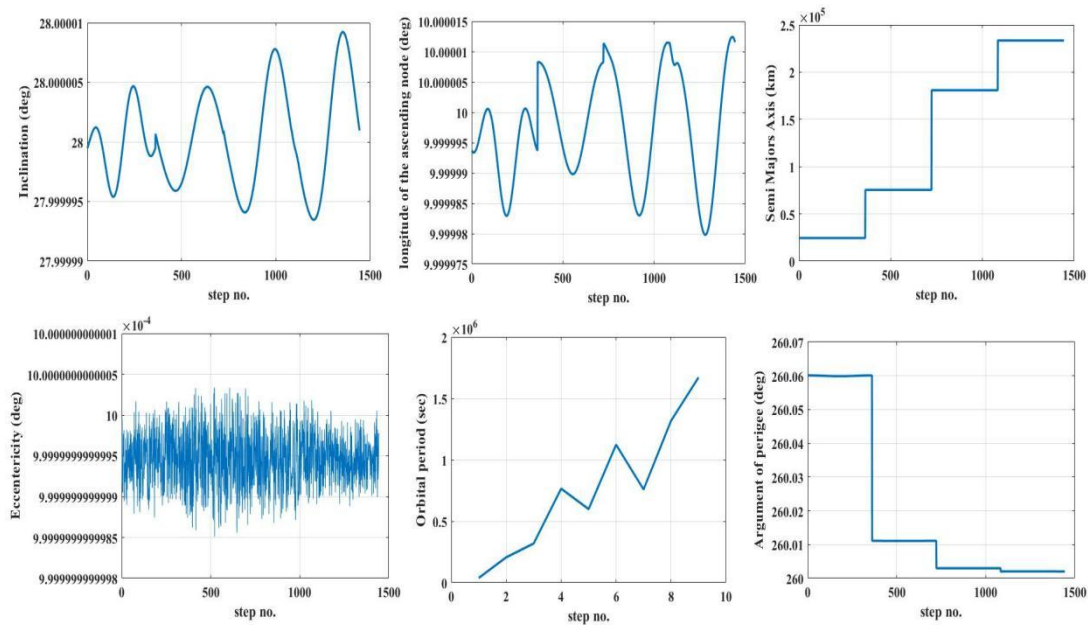


Figure 9: Orbital Element Variations for Lunar Transfer Trajectory when $e = 0.001$ and $\omega = 260$.

By conducting the third case of the study, we analyzed the impact of varying the arguments of perigee values and eccentricities on transitional orbits. Our investigation sought to understand how these parameters interact and influence the efficiency of transitioning from a high eccentricity Low Earth Orbit (LEO) to a circumlunar orbit.

Through analysis Figures (1) (2) (3) (4) (5) (6) (7) (8) (9), which included having an eccentricity set to 0.001 and an argument of perigee set to 80 degrees, resulting in a velocity (v) of 1.3077835 km/s. This configuration was recognized as the most effective trajectory for transitioning from a high eccentricity LEO to a circumlunar orbit.

By selecting eccentricity and argument of perigee values, one can maximize gravitational assists while minimizing perturbations. This combination is likely to be the most optimal and achieve a balanced result. Regarding perturbations, the perturbations are small deviations from the idealized Kepler orbit caused by influential external. These perturbations can have a significant impact on the behavior of transition orbits. It was noticed during our analysis of the three cases that the main effects occur near the perigee and apogee because the gravity and non-sphericity of both the moon and the Earth together affect the trajectory of the spacecraft in addition to a group of other side perturbations. Noting that the effect at the perihelion is higher than at the apogee, it still affects the path and should therefore be taken into consideration.

By precisely examining the combined effect of these factors, planners and designers of space missions can uncover paths that minimize disruptions while maximizing the utilization of gravitational forces, boosting the efficiency of transitioning orbits. Selecting an eccentricity of 0.001 permits sustained elongation, optimizing the spacecraft's engagement with the Earth and the Moon's gravitational fields. In addition, aligning the argument of perigee at 80 degrees leads to an ideal configuration for enhanced interaction. Consequently, future missions can enhance their planning and spacecraft design to ensure a more precise and dependable shift from a high eccentricity LEO to a circumlunar orbit.

2. Conclusion

1- The three cases agreed on examining the effect of altering eccentricity and the argument of perigee values, it was determined that the most favorable transition occurred when

eccentricity was set to 0.001 and the argument of perigee was set to 80 degrees, with a velocity increase of 1.307784 km/s, which is the lowest increase in ΔV , a metric used to measure fuel efficiency and power required. This finding also demonstrated the lowest change in eccentricity, the lowest change in inclination, and the lowest change in the ascending node, all of which are indicative of increased orbit stability. This specific combination struck a balance between increasing gravitational assists and decreasing perturbations, resulting in a more efficient trajectory.

2- That the better 0.001 eccentricity leads to the most efficient transmission, as higher deviation values allow a higher change of speed during transmission. The higher the eccentricity, the longer the orbit, and the greater the difference between perigee (the point closest to Earth) and apogee (the point farthest from Earth). This increased eccentricity impacts the energy efficiency during a spacecraft's transition to lunar orbit. Therefore, it makes the greatest use of the differential gravitational forces of the Earth and the Moon,

3- At 80° a spacecraft's orbit is 80° away from the nearest point to the Moon, it can reach a circumlunar orbit most effectively. The time and location of critical maneuvers are impacted by the varying argument of perigee, resulting in the change of the orbit's orientation in space, hence Energy used, and gravitational forces are minimized making for a much smoother and more successful journey.

4- Most perturbations occur at perigee and apogee due to the gravitational influence of the moon and the earth together. As shown in the figure (10)

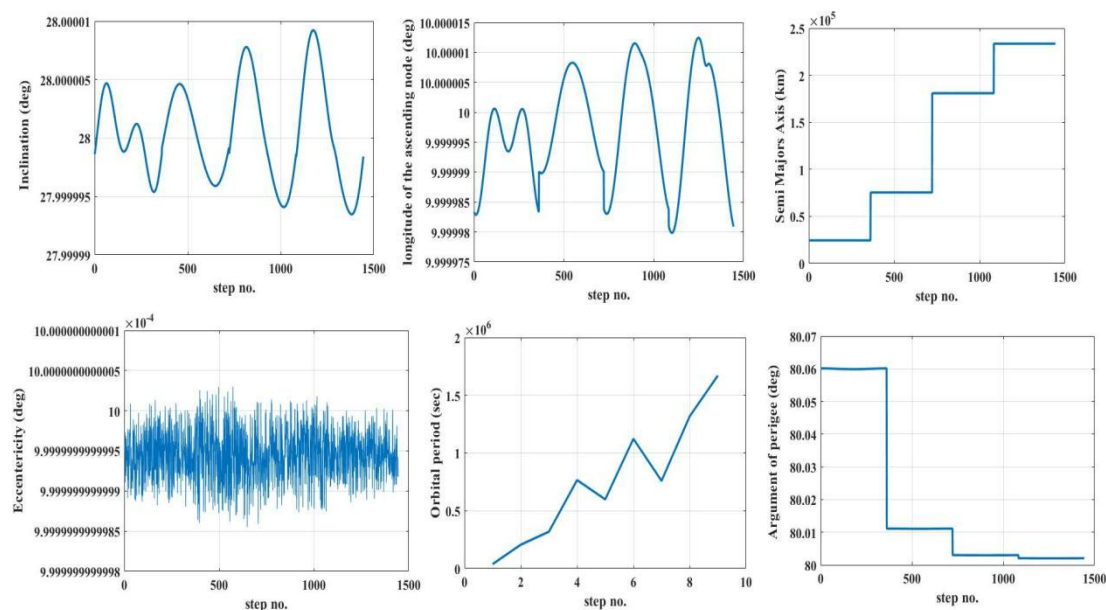


Figure 10: Orbital Element Variations for the foremost proficient and ideal trajectory for three cases.

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