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Certain Types of Soft Sets in the Games Theory

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Abstract

Some topological games with soft set theory became important in real life techniques, especially, in economics. Throughout the present work, we introduce new types of soft sets, say, soft-ῐ-g-open sets and some of their investigations are studied. In addition, some new kinds of soft separation axioms will be defined, and their implications are studied. Finally, numerous sorts of soft topological games that depend on soft separation axioms are investigated. We prove the important conditions that the first and second players win the proposed games.

Keywords: Soft set, soft separation axioms, soft-ῐ-g-open sets and soft games.

أنواع معينة من المجموعات الناعمة في نظرية المباريات

رنا بهجت اسماعيل
قسم الرياضيات، كلية التربية للعلوم الصرفة – ابن الهيثم، جامعة بغداد، بغداد، العراق

الخلاصة

أصبحت لبعض المباريات التبولوجية والمعرفة بالاعتماد على المجموعات الناعمة اهمية في تقنيات الحياة الواقعية،وسوق العمل وخاصة في الاقتصاد على سبيل المثال، في هذا العمل تم تعريف أنواعًا جديدة من المجموعات الناعمة، وهي المجموعات المفتوحة الناعمة من النمط– ῐ –g ودراسة بعض خواصها. بالإضافة إلى ذلك، تم تعريف و دراسة بعض الأنواع الجديدة من بديهيات الفصل الناعمة . وأخيرا، تم دراسة أنواع عديدة من المباريات التبولوجية الناعمة التي تعتمد على بديهيات الفصل الناعمة وتوضيح الشروط المهمة لتحديد الاستراتيجيات الرابحة لاي من اللاعبين في المباريات المعرفة.

1. Introduction and preliminaries

Many topics and studies in the field of mathematics have dealt with problems that do not have precise data in an attempt to obtain solutions to these problems like; fuzzy sets and approximation sets. Molodtsov [1] explained that all of these topics cannot find solutions to these problems accurately, so he presented a concept soft theory which deals with problems with inaccurate data to obtain accurate solutions. Subsequently, Maji et al. [2, 3], generalaization soft set theory of Molodtsov and investigated fuzzy sets via soft theory in some decision-making real-life problems. Recently, soft sets were studied in many directions in soft topological spaces in (see [4]-[9]).

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Game theory in topological spaces have been premeditated for numerous years. And then many games followed, which were known later. A lot of standard the information given by El-Atik et al. [10] are now obtainable in a formula of a selection games belief the improvement of viewing exactly solutions overdue those the information.
Recently, there is an alternative of topological games and show effect of the topology may arise in the certain covering such as separation axioms and compactness and many topologists used open sets and weak open sets to propose new types of soft topological property and new games (see [11]- [23]).

The concepts of game theory via topological spaces has introduced and studied by Berge [15]. Many authors used it to solve some topological problems (e.g. [12]-[14]). Shabir et. Al [11] initiated the concept of separation axioms and connectedness via soft sets by topological properties and studied their properties. Soft sets are used in many applications (see [4]-[11]).
The purpose of the work is to use soft sets and soft separation axiom for defining and studying some new types of topological games. In Section 1, some basic definitions which can be used throughout this paper will be introduced. In Section 2, we give a generalization for soft open sets via the $s\dot{\mathfrak{l}}$g-open soft collections. In the third section, a lot of kinds in soft separation axioms on soft-$\dot{\mathfrak{l}}$-g-spaces, say, $s\dot{\mathfrak{l}}g$-$Ţ_m$ -space $m \in \{0,1,2\}$ will be stablished and some of their characterizations are studied. Finally, soft topological games, namely, $\hat{S}G(Ţ_0, X)$ and $\hat{S}G(Ţ_0, \dot{\mathfrak{l}})$ with perfect information on soft ideal $s\dot{\mathfrak{l}}g$-$Ţ_i$ -spaces will be applied in terms of $s\dot{\mathfrak{l}}g$-sets and their generalizations.
Throughout this work, $(\mathcal{U})^c$ will denote to the complement of $\mathcal{U}$ w. r. t. $\widetilde{X}$ to avoid the confusion.

Definition 1.1. [2],[3] Let $X \neq \emptyset$, Q be a set of parameters. Such that is $\mathfrak{p}(X)$ the collection of X and $\mathcal{P} \neq \emptyset$ such that $\mathcal{P} \subseteq Q$. $(Ғ, Q)$ (briefly, $Ғ_{\mathcal{H}}$) is a soft set over X when, Ғ is a function such that $Ғ: Q \to \mathfrak{p}(X)$. So, $Ғ_Q = \{ Ғ(q): q \in \mathcal{P} \subseteq Q,\ Ғ: Q \to \mathfrak{p}(X) \}$. The family of all soft sets (denoted $\hat{S}\hat{S}(X)_{\mathcal{H}}$).

Definition 1.2. [2],[3] Let $(Ғ, Q), (G, Q) \in \hat{S}\hat{S}(X)_Q$. Then, $(Ғ, Q)$ is a soft subset of, (G, Q), (briefly, $(Ғ, Q) \tilde{\sqsubseteq} (G, Q)$), if $Ғ(j) \tilde{\sqsubseteq} G(j)$, when $j \in Q$. Now, $(Ғ, Q)$ is a soft subset of (G, Q) so, (G, Q) is a soft super set of $(Ғ, Q)$, $(Ғ, Q) \tilde{\sqsubseteq} (G, Q)$.

Definition 1.3. [22] The complement of $(Ғ, Q)$ ($(Ғ, Q)'$, for short) $(Ғ, Q)' = (Ғ', Q)$, $Ғ': Q \to \mathfrak{p}(X)$ is a function such that $Ғ'(q) = X - Ғ(q)$, for all $q \in Q$ and $Ғ'$ is called the soft complement of Ғ.

Definition 1.4. [14] For any $(Ғ, Q)$ which is a soft over X and $x \in X$. Then, $x \tilde{\in} (Ғ, Q)$, whenever, $x \in Ғ(q)$ for each $q \in Q$.

Definition 1.5. [2],[3] $(Ғ, Q)$ called null soft set (denoted by, $\widetilde{\emptyset} \vee \emptyset_Q$) whenever, $\forall\ q \in Q$, $Ғ(q) = \emptyset$.

Definition 1.6. [2],[3] $(Ғ, Q)$ called absolute soft set (dented by, $\widetilde{X} \vee X_Q$), when $\forall q \in Q$, $Ғ(q) = X$.

Definition 1.7. [2],[3] Let $(Ғ, \mathcal{P})$, $(G, W) \in \hat{S}\hat{S}(X)$, $(\mathcal{K}, Ҿ) = (Ғ, \mathcal{P}) \tilde{\cup} (G, W)$ where, $Ҿ = \mathcal{P} \cup W$ and $\forall\ j \in Ҿ$,

$$\mathcal{K}(j)=\begin{cases} Ғ(j),\ j \in \mathcal{P} - W, \\ G(j),\ j \in W - \mathcal{P}, \\ Ғ(j) \cup G(j),\ j \in \mathcal{P} \cap W. \end{cases}$$

Definition 1.8. [2],[3] Let $(Ғ,\mathcal{P}),(G,W) \in \hat{S}\hat{S}(X)$, $(\mathcal{K},Ҿ) = (Ғ,\mathcal{P}) \tilde{\cap} (G,W)$ such that $Ҿ = \mathcal{P} \cap W$, $\forall\ j \in Ҿ$. Then, $\mathcal{K}(j) = Ғ(j) \cap G(j)$.

Definition 1.9. [22] For any subfamily of soft sets Ţ on Ŗ with same Q, then $Ţ \in \hat{S}\hat{S}(Ŗ)_Q$ is a soft topology on Ŗ if the conditions are held.

(i) $\tilde{Ŗ}, \tilde{\emptyset} \in Ţ$ when, $\tilde{\emptyset}(q) = \emptyset$ and $\tilde{X}(q) = X$, for all $q \in Q$,

(ii) $\tilde{\bigcup}_{\alpha\in\Lambda} (Ʊ_\alpha, Q) \in Ţ$ when, $(Ʊ_\alpha, Q) \in Ţ$ for all $\alpha \in \Lambda$,

(iii) $((Ғ,Q) \tilde{\cap} (G,Q)) \in Ţ$ for each $(Ғ,Q), (G,Q) \in Ţ$.

$(Ŗ, Ţ, Q)$ is a soft topological space if $(Ʊ,Q) \in Ţ$, so $(Ʊ,Q)$ is open-soft.

Definition 1.10. [22] The soft space $(X, Ţ, Q)$, $(Ғ,Q)$ is called a soft-closed set, if $(Ғ,Q)' \in Ţ$. The set of all soft-closed sets is symbolized by $Ţ^*$.

Definition 1.11. [22] For any soft space $(Ŗ, Ţ, Q)$, $(Ғ,Q)' \in \hat{S}\hat{S}(Ŗ)_{\mathbf{Q}}$. Then,

(i) $Cl((Ғ,Q)) = \tilde{\cap}\{(\omega,Q) : (\omega,Q) \in Ţ^*,\ (Ғ,Q) \tilde{\subseteq} (\omega,Q)\}$.

(ii) $Int(G,Q) = \tilde{\cup}\{(\omega,Q) : (\omega,Q) \in Ţ, (\omega,Q) \tilde{\subseteq} (G,Q)\}$.

are soft closure and soft interior of $(Ғ,Q)'$, respectively.

Definition 1.12. [22]Let $\dot{Ɩ} \neq \emptyset$. Then, $\dot{Ɩ} \tilde{\subseteq} \hat{S}\hat{S}(Ŗ)_{\mathbf{Q}}$ called soft ideal when,

(i) If $(Ғ,Q) \tilde{\in} \dot{Ɩ} \wedge (G,Q) \tilde{\in} \dot{Ɩ}$, then $(Ғ,Q) \tilde{\cup} (G,Q) \tilde{\in} \dot{Ɩ}$.

(ii) If $(Ғ,Q) \tilde{\in} \dot{Ɩ} \wedge (G,Q) \tilde{\subseteq} (Ғ,Q)$, then $(G,Q) \tilde{\in} \dot{Ɩ}$.

Definition 1.13. [1] Any $(Ŗ, Ţ, Q)$ via soft ideal $\dot{Ɩ}$ is said that a soft ideal topological space and symbolized by $(Ŗ, Ţ, Q, \dot{Ɩ})$.

Definition 1.14. [22] Suppose that the space $(Ŗ, Ţ, Q)$ be soft topological space on Ŗ. It is called a soft-$Ţ_0$ if for all $q_{\mathcal{M}}, q_{\mathcal{N}} \tilde{\in} \tilde{Ŗ}$ such that $q_{\mathcal{M}} \neq q_{\mathcal{N}}$, then $\exists$ soft-open $(Ʊ,Q)$ containing $q_{\mathcal{M}}$, but not containing $q_{\mathcal{N}}$ or $(Ʊ,Q)$ containing $q_{\mathcal{N}}$, but not containing $q_{\mathcal{M}}$.

Theorem 1.15. [22] The space $(Ŗ, Ţ, Q)$ is a soft-$Ţ_0$-space $\Leftrightarrow$ for all, $q_{\mathcal{N}} \tilde{\in} \tilde{Ŗ}$ such that $q_{\mathcal{M}} \neq q_{\mathcal{N}}$, there exists a soft closed set $(\mathcal{V},Q)$ s. t. $q \tilde{\in} (\mathcal{V},Q)$, $q_{\mathcal{N}} \tilde{\notin} (\mathcal{V},Q)$ or $q \tilde{\notin} (\mathcal{V},Q)$, $q_{\mathcal{N}} \tilde{\in} (\mathcal{V},Q)$.

Definition 1.16. [22] Let $(Ŗ, Ţ, Q)$ be a soft topological space on Ŗ. It is a soft-$Ţ_1$ if for all $q_{\mathcal{M}}, q_{\mathcal{N}} \tilde{\in} \tilde{X}$ such that $q_{\mathcal{M}} \neq q_{\mathcal{N}}$, there exists $(Ʊ,Q), (\mathcal{V},Q) \in Ţ$ such that $(Ʊ,Q)$ containing $q_{\mathcal{M}}$, not containing $q_{\mathcal{N}}$ and $(\mathcal{V},Q)$ containing $q_{\mathcal{N}}$ not containing $q_{\mathcal{M}}$.

Definition 1.17. [22] Let $(Ŗ, Ţ, Q)$ be a soft topological space on Ŗ. It is a soft-$Ţ_2$ if for all $q_{\mathcal{M}}, q_{\mathcal{N}} \tilde{\in} \tilde{Ŗ}$ such that $q_{\mathcal{M}} \neq q_{\mathcal{N}}$, there exists $(Ʊ,Q), (\mathcal{V},Q) \in Ţ$ such that $(Ʊ,Q)$ containing $q_{\mathcal{M}}$ and $(\mathcal{V},Q)$ containing $q_{\mathcal{N}}$ such that $(Ʊ,Q) \cap (\mathcal{V},Q) = \{\tilde{\emptyset}\}$.

Proposition 1.18. [22] Each soft-$Ţ_{m+1}$-space is a soft-$Ţ_m$ with $m \in \{0,1,2\}$. And so, soft-$Ţ_2 \Longrightarrow$ soft-$Ţ_1 \Longrightarrow$ soft-$Ţ_0$ and the converse may be not true, in general.

2. Soft-ḟ-g-open sets and their properties

The work in this section is a generalization of the concept of soft sets, which will be called, soft-ḟ-g-open sets and some of soft topological properties are investigated.

Definition 2.1. Let $(Ꝓ, Ţ, Q, ḟ)$ be soft ideal topological space and $(F, Q) \in \hat{S}\hat{S}(Ꝓ)_Q$. Then, (F, Q) is called to be a a soft-ḟ- closed set (briefly sḟg-closed) if $Cl(F, Q) - (U, Q) \in ḟ$ whenever, $(F, Q) - (U, Q) \in ḟ$ and $(U, Q) \in Ţ$. $(F, Q)^c$ is a soft-ḟ-g-open set (symbolized sḟg-open set) and its collection is symbolized by $sḟgo(Ꝓ)_Q$. The class of each sḟg-closed sets is denoted by $sḟg\text{-}c(Ꝓ)_Q$.

Example 2.2. In the space $(Ꝓ, Ţ, Q, ḟ)$, such that $Ꝓ = \{1,2\}$, $Q = \{q_1, q_2\}$, $Ţ = \{\widetilde{\emptyset}, \widetilde{Ꝓ}, F\}$, $ḟ = \{\widetilde{\emptyset}, \mathcal{K}\}$ such that $(F, Q) = \{(q_1, \{2\}), (q_2, Ꝓ)\}$ and $(\mathcal{K}, Q) = \{(q_1, \{\emptyset\}), (q_2, \{1\})\}$ then $sḟ - gc(Ꝓ)_Q = \{\widetilde{\emptyset}, \widetilde{Ꝓ}, (\mathcal{P}, Q), (W, Q), (\mathcal{Z}, Q), (\mathcal{D}, Q), (\mathcal{E}, Q), (\mathcal{N}, Q), (G, Q)\}$ whenever $(\mathcal{P}, Q) = \{(q_1,\{1\}), (q_2,\{1\})\}$, $(W, Q) = \{(q_1, Ꝓ), (q_2,\{\emptyset\})\}$, $(\mathcal{Z}, Q) = \{(q_1, Ꝓ), (q_2,\{1\})\}$, $(\mathcal{D}, Q) = \{(q_1, Ꝓ), (q_2,\{2\})\}$, $(\mathcal{E}, Q) = \{(q_1,\{1\}), (q_2,\{\emptyset\})\}$, $(\mathcal{N}, Q) = \{(q_1,\{1\}), (q_2,\{2\})\}$ and $(G, Q) = \{(q_1,\{1\}), (q_2, \{2\})\}$.

Proposition 2.3. From any soft ideal $(Ꝓ, Ţ, Q, ḟ)$, therefore

(i) The closed-soft set is sḟg-closed.

(ii) The open-soft set is sḟg-open.

Proof. (i) Suppose that $(\mathcal{P}, Q)$ be any closed-soft set in $(Ꝓ, Ţ, Q, ḟ)$ and (U, Q) be a soft-open set and $(\mathcal{P}, Q) - (U, Q) \in ḟ$, but $Cl(\mathcal{P}, Q) = (\mathcal{P}, Q)$, since $(\mathcal{P}, Q)$ is a closed soft set. So, $Cl(\mathcal{P}, Q) - (U, Q) = (\mathcal{P}, Q) - (U, Q) \in ḟ$ .Then, $(\mathcal{P}, Q)$ is a soft-ḟ-g-closed soft.

(ii) Let (U, Q) be any open-soft set in $(Ꝓ, Ţ, Q, ḟ)$ then $(U, Q)^c$ is a closed-soft set this implies by (i) $(U, Q)^c$ is a $sḟ - g$ colsed set. Therefore, (U, Q) is a sḟg-open soft.

Note that the opposite of the above may not be true.

Example 2.4. From the same space in Example 2.2

(i) Let $(\mathcal{P}, Q) = \{(q_1, \{1\}), (q_2, \{1\})\}$ be a sḟg-closed. It is clear that $(\mathcal{P}, Q)$ is not closed-soft set.

(ii) Let $(\mathcal{P}, Q) = \{(q_1, \{2\}), (q_2, \{2\})\}$ be a sḟg-open. But $(\mathcal{P}, Q) \notin Ţ$.

3. Soft separation axioms on soft-ḟ-g-spaces

The work in this section is a generalization of the concept of soft separation axioms via soft-ḟ-g-open sets with some properties are studied.

Definition 3.1. A space $(Ꝓ, Ţ, Q, ḟ)$ is a $sḟg$-$Ţ_0$, if for each $q_{\mathcal{M}} \neq q_{\mathcal{N}}$ and $q_{\mathcal{M}}, q_{\mathcal{N}} \widetilde{\in} \widetilde{Ꝓ}$, $\exists$ $(U, Q) \in sḟsg\text{-}o(Ꝓ)_Q$ such that (U, Q) containing $q_{\mathcal{M}}$, but not containing $q_{\mathcal{N}}$ or (U, Q) containing $q_{\mathcal{N}}$, but not containing $q_{\mathcal{M}}$.

Example 3.2. Let $Ꝓ = \{1,2,3\}$, $Q = \{q_1, q_2\}$, $Ţ = \{\widetilde{\emptyset}, \widetilde{Ꝓ}, (\mathcal{P}, Q), (W, Q)\}$ whenever, $(\mathcal{P}, Q) = \{(q_1,\{1\}), (q_2, \{1\})\}$, $(W, Q) = \{(q_1, \{1,2\}), (q_2, \{1,2\})\}$ and $ḟ = \{\widetilde{\emptyset}\}$. Then, $sḟg\text{-}c(Ꝓ)_Q = \{\widetilde{\emptyset}, \widetilde{X}, (\mathcal{P}', Q), (W', Q)\}$ and $sḟg\text{-}o(Ꝓ)_Q = Ţ$. Hence , $(Ꝓ, Ţ, Q, ḟ)$ is $sḟg$-$Ţ_0$-space, since $\forall$ $q_{\mathcal{M}} \neq q_{\mathcal{N}}$, $\exists$ $(U, Q) \in sḟg\text{-}o(Ꝓ)_Q$ such that containing $q_{\mathcal{M}}$, but not containing $q_{\mathcal{N}}$ or (U, Q) containing $q_{\mathcal{N}}$, but not containing $q_{\mathcal{M}}$.

Proposition 3.3. Suppose that $(Ꝓ, Ţ, Q)$ is soft-$Ţ_0$, then $(Ꝓ, Ţ, Q, ḟ)$ is a $sḟg$-$Ţ_0$.

Proof. Let $q_{\mathcal{M}}$, $q_{\mathcal{N}}$ $\widetilde{\in}$ $\widetilde{Ʀ}$ whenever $q_{\mathcal{M}} \neq q_{\mathcal{N}}$. Since $(Ʀ, Ț, Q)$ is soft-$Ț_0$, then $\exists (\mho, Q) \in Ț$ such that $(\mho, Q)$ containing $q_{\mathcal{M}}$, but not containing $q_{\mathcal{N}}$ or $(\mho, Q)$ containing $q_{\mathcal{N}}$, but not containing $q_{\mathcal{M}}$. By Proposition 2.3, $(\mho, Q)$ is $s\dot{\mathfrak{l}}g$-open and satisfies the required condition.

Theorem 3.4. $(X, Ț, Q, \dot{\mathfrak{l}})$ is $s\dot{\mathfrak{l}}g$-$Ț_0$ $\Leftrightarrow$ for each $q_{\mathcal{M}} \neq q_{\mathcal{N}}$ there is a $s\dot{\mathfrak{l}}g$-closed set (V, Q) s such that (V, Q) containing $q_{\mathcal{M}}$, but not containing $q_{\mathcal{N}}$ or (V, Q) containing $q_{\mathcal{N}}$, but not containing $q_{\mathcal{M}}$.

Proof. (⇒) Let $q_{\mathcal{M}}$, $q_{\mathcal{N}}$ $\widetilde{\in}$ $\widetilde{X}$ such that $q_{\mathcal{M}} \neq q_{\mathcal{N}}$. Since X is $s\dot{\mathfrak{l}}g$-$Ț_0$, then $\exists$ $(\mho, Q) \in s\dot{\mathfrak{l}}g$-$o(X)_Q$ such that $(\mho, Q)$ containing $q_{\mathcal{M}}$, but not containing $q_{\mathcal{N}}$ or $(\mho, Q)$ containing $q_{\mathcal{N}}$, but not containing $q_{\mathcal{M}}$. Then, $\exists$ $(V, Q) \in s\dot{\mathfrak{l}}g$-$c(X)_Q$ such that (V, Q) containing $q_{\mathcal{M}}$, but not containing $q_{\mathcal{N}}$ or (V, Q) containing $q_{\mathcal{N}}$, but not containing $q_{\mathcal{M}}$, where, $(V, Q) = (\mho, Q)^c$, and $(\mho, Q)^c$ is the complement of $(\mho, Q)$ w. r. t. $\widetilde{X}$.
(⇐) Let $q_{\mathcal{M}}$, $q_{\mathcal{N}}$ $\widetilde{\in}$ $\widetilde{X}$ such that $q_{\mathcal{M}} \neq q_{\mathcal{N}}$ and there is a $s\dot{\mathfrak{l}}g$-closed set (V, Q) when (V, Q) containing $q_{\mathcal{M}}$, but not containing $q_{\mathcal{N}}$ or (V, Q) containing $q_{\mathcal{N}}$, but not containing $q_{\mathcal{M}}$. Therefore, there is $s\dot{\mathfrak{l}}g$-open set $(V, Q) = (\mho, Q)^c$ which satisfies the required condition.

Definition 3.5. $(X, Ț, Q, \dot{\mathfrak{l}})$ is called $s\dot{\mathfrak{l}}g$-$Ț_1$, if for each $q_{\mathcal{M}}$, $q_{\mathcal{N}}$ $\widetilde{\in}$ $\widetilde{X}$ and $q_{\mathcal{M}} \neq q_{\mathcal{N}}$, there are $s\dot{\mathfrak{l}}g$-open sets (M_1,Q), (M_2,Q) whenever, $q_{\mathcal{M}} \widetilde{\in} (M_1,Q) - (M_2,Q)$ and $q_{\mathcal{N}} \widetilde{\in} (M_2,Q) - (M_1,Q)$.

Example 3.6. A space $(Ʀ, Ț, Q, \dot{\mathfrak{l}})$ such that Ʀ = Q = ₦, where ₦ the set of positive integers, $Ț = Ț_{Scof} = \{Ƒ_{\mathcal{A}}: Ƒ'(q) \text{ is finite } \forall\, q\} \widetilde{\cup} \{\widetilde{\emptyset}\}$ and $\dot{\mathfrak{l}} = \{\widetilde{\emptyset}\}$. Then, $(Ʀ, Ț, Q, \dot{\mathfrak{l}})$ is $s\dot{\mathfrak{l}}g$-$Ț_1$. If for each $q_{\mathcal{M}}$, $q_{\mathcal{N}}$ $\widetilde{\in}$ $\widetilde{Ʀ}$ and $q_{\mathcal{M}} \neq q_{\mathcal{N}}$, then, there are $s\dot{\mathfrak{l}}g$-open sets $(\widetilde{Ʀ} - M)$, $(\widetilde{Ʀ} - \mathcal{V})$ whenever $M \subseteq q_{\mathcal{M}}$, $\mathcal{V} \subseteq q_{\mathcal{N}}$ and M , $\mathcal{V}$ are finite sets whenever, $q_{\mathcal{M}} \widetilde{\in} \mathcal{V}^c$, $q_{\mathcal{N}} \notin \mathcal{V}^c$ and $q_{\mathcal{M}} \notin M^c$, $q_{\mathcal{N}} \widetilde{\in} M^c$ and $\mathcal{V}^c \cap M^c \neq \{\emptyset\}$.

Proposition 3.7. If $(Ʀ, Ț, Q)$ is soft-$Ț_1$, then $(Ʀ, Ț, Q, \dot{\mathfrak{l}})$ is $s\dot{\mathfrak{l}}g$-$Ț_1$.

Proof. Let $q_{\mathcal{M}}$, $q_{\mathcal{N}}$ $\widetilde{\in}$ $\widetilde{Ʀ}$ whenever $q_{\mathcal{M}} \neq q_{\mathcal{N}}$. Since $(Ʀ, Ț, Q)$ is soft-$Ț_1$, then $\exists$ (M_1,Q), (M_2,Q) $\in Ț$ such that $q_{\mathcal{M}} \widetilde{\in} (M_1,Q) - (M_2,Q)$ and $q_{\mathcal{N}} \widetilde{\in} (M_2,Q) - (M_1,Q)$. By Remark 2.3, (M_1,Q) and (M_2,Q) are $s\dot{\mathfrak{l}}g$-open.

Proposition 3.8. If $(X, Ț, Q, \dot{\mathfrak{l}})$ is $s\dot{\mathfrak{l}}g$-$Ț_1$, then it is $s\dot{\mathfrak{l}}g$-$Ț_0$.

Proof. Let $q_{\mathcal{M}}$, $q_{\mathcal{N}}$ $\widetilde{\in}$ $\widetilde{X}$ such that $q_{\mathcal{M}} \neq q_{\mathcal{N}}$. Since $(X, Ț, Q, \dot{\mathfrak{l}})$ is $s\dot{\mathfrak{l}}g$-$Ț_1$, then $\exists$ (M_1, Q) , $(M_2, Q) \in s\dot{\mathfrak{l}}g$-$o(X)_Q$ such that, $q_{\mathcal{M}} \widetilde{\in} (M_1,Q) - (M_2,Q)$ and $q_{\mathcal{N}} \widetilde{\in} (M_2,Q) - (M_1,Q)$. Then, $\exists$ $(M, Q) \in s\dot{\mathfrak{l}}g$-$o(X)_Q$-open such that (M, Q) containing $q_{\mathcal{M}}$, but not containing $q_{\mathcal{N}}$ or (M, Q) containing $q_{\mathcal{N}}$, but not containing $q_{\mathcal{M}}$.
Note that the opposite of Proposition 3.8 may not valid, in general. by Example 3.2. $(Ʀ, Ț, Q, \dot{\mathfrak{l}})$ is $s\dot{\mathfrak{l}}g$-$Ț_0$, but not $s\dot{\mathfrak{l}}g$-$Ț_1$. Because of $\exists$ $q_{\mathcal{M}} \neq q_{\mathcal{N}}$, $q_{\mathcal{M}} = \{1,2\}$ and $q_{\mathcal{N}} = \{3\}$, there is no $(M$,Q) and $(\mathcal{V}$,Q) when $q_{\mathcal{M}} \widetilde{\in} (M$,Q), $q_{\mathcal{N}} \widetilde{\notin} (M$,Q) and $q_{\mathcal{N}} \widetilde{\in} (\mathcal{V}$,Q), $q_{\mathcal{M}} \widetilde{\notin} (\mathcal{V}$,Q).

Theorem 3.9. A space $(Ʀ, Ț, Q, \dot{\mathfrak{l}})$ is $s\dot{\mathfrak{l}}g$-$Ț_1$ $\Leftrightarrow$ for each q , $q_{\mathcal{N}}$ $\widetilde{\in}$ $\widetilde{Ʀ}$ and $q_{\mathcal{M}} \neq q_{\mathcal{N}}$, there are two $s\dot{\mathfrak{l}}g$-closed sets (M_1, Q), (M_2, Q) such that $q_{\mathcal{M}}$ $\widetilde{\in}$ $(M_1, Q) \cap (M_2', Q)$ and $q_{\mathcal{N}}$ $\widetilde{\in}$ $(M_2, Q) \cap (M_1', Q)$.

Proof. (⇒) Let $q_{\mathcal{M}}$, $q_{\mathcal{N}}$ $\widetilde{\in}$ $\widetilde{Ʀ}$ such that $q_{\mathcal{M}} \neq q_{\mathcal{N}}$. Since $(Ʀ, Ț, Q, \dot{\mathfrak{l}})$ is soft-$Ț_1$, then $\exists$ $(\mho_1, Q), (\mho_2, Q) \in s\dot{\mathfrak{l}}g$-$o(Ʀ)_Q$whenever, q $\widetilde{\in}$ $(\mho_1,Q) - (\mho_2,Q)$ and q $\widetilde{\in}$ $(\mho_2,Q) - (\mho_1,Q)$. Then, there is a $s\dot{\mathfrak{l}}g$-closed sets (M_1, Q), (M_2, Q) whenever, $q_{\mathcal{M}} \widetilde{\in} (M_1, Q) - (M_2, Q)$ and $q_{\mathcal{N}}$ $\widetilde{\in}$ $((M_2, Q) - (M_1, Q))$ where, $(\mho_2, Q)^c = (M_2, Q)$ and $(\mho_1, Q)^c = (M_1, Q)$. Then, there are

two $s\dot{\imath}g$-closed sets (M_1, Q), (M_2, Q) such that $q_{\mathcal{M}} \,\tilde{\in}\, (M_1, Q) \cap (M_2', Q)$ and $q_{\mathcal{N}} \,\tilde{\in}\, ((M_2, Q) \cap (M_1', Q))$.
(⇐) Let $q_{\mathcal{M}}, q_{\mathcal{N}} \,\tilde{\in}\, \tilde{Ʀ}$ such that $q_{\mathcal{M}} \neq q_{\mathcal{N}}$ and there are $s\dot{\imath}g$-closed sets (M_1, Q), (M_2, Q) such that $q_{\mathcal{M}} \,\tilde{\in}\, (M_1, Q) \,\tilde{\cap}\, (M_2', Q)$ and $q_{\mathcal{N}} \,\tilde{\in}\, (M_2, Q) \,\tilde{\cap}\, (M_1', Q)$. Then, there are $s\dot{\imath}g$-open sets (U_1,Q), (U_2,Q) whenever, $q_{\mathcal{M}} \,\tilde{\in}\, (U_1,Q) - (U_2,Q)$ and $q_{\mathcal{N}} \,\tilde{\in}\, (U_2,Q) - (U_1,Q)$ where, $(M_2, Q)^c = (U_2,Q)$ and $(M_1, Q)^c = (U_1,Q)$.

Definition 3.10. $(Ʀ, Ţ, Q, \dot{f})$ is $s\dot{\imath}g$-$Ţ_2$, if for any $q_{\mathcal{M}} \neq q_{\mathcal{N}}$, there are $s\dot{\imath}g$-open sets (M_1,Q), (M_2,Q) such that $q_{\mathcal{M}} \,\tilde{\in}\, (M_1,Q)$, $q_{\mathcal{N}} \,\tilde{\in}\, (M_2,Q)$ and $(M_1,Q) \cap (M_2,Q) = \{\tilde{\emptyset}\}$.

Example 3.11. Let $Ʀ = \{1, 2, 3\}$, $Ţ = \{\tilde{\emptyset}, \tilde{Ʀ}\}$ and $\dot{f} = \hat{S}\hat{S}(Ʀ)_Q$. Then, $s\dot{\imath}g\text{-}c(Ʀ)_Q = s\dot{\imath}g\text{-}o(Ʀ)_Q = \hat{S}\hat{S}(Ʀ)_Q$. Therefore, $(Ʀ, Ţ, Q, \dot{f})$ is $s\dot{\imath}g$-$Ţ_2$.

Corollary 3.12. Each $(X, Ţ, Q)$ soft-$Ţ_2$ is $(X, Ţ, Q, \dot{f})$ $s\dot{\imath}sg$-$Ţ_2$.

Proof. Let $q_{\mathcal{M}}, q_{\mathcal{N}} \,\tilde{\in}\, \tilde{X}$ and $q_{\mathcal{M}} \neq q_{\mathcal{N}}$. *For* $(X, Ţ, Q, \dot{f})$ $s\dot{\imath}sg$-$Ţ_2$, $\exists\, (M_1,Q), (M_2,Q) \in Ţ$ such that $q_{\mathcal{M}} \,\tilde{\in}\, (M_1,Q)$, $q_{\mathcal{N}} \,\tilde{\in}\, (M_2,Q)$ and $(M_1,Q) \,\tilde{\cap}\, (M_2,Q) = \{\tilde{\emptyset}\}$. By Remark 2.3, there are $s\dot{\imath}g$-open sets (M_1,Q), (M_2,Q), such that $q_{\mathcal{M}} \,\tilde{\in}\, (M_1,Q)$, $q_{\mathcal{N}} \,\tilde{\in}\, (M_2,Q)$ and $(MU_1,Q) \,\tilde{\cap}\, (M_2,Q) = \{\tilde{\emptyset}\}$.

Corollary 3.13. Each $(X, Ţ, Q, \dot{f})$ soft-$Ţ_2$ is $(X, Ţ, Q, \dot{f})$ $s\dot{\imath}sg$-$Ţ_1$.

Proof. Let $q_{\mathcal{M}}, q_{\mathcal{N}} \,\tilde{\in}\, \tilde{X}$ and $q_{\mathcal{M}} \neq q_{\mathcal{N}}$. Since $(X, Ţ, Q, \dot{f})$ is $s\dot{\imath}g$-$Ţ_2$, then there are $s\dot{\imath}g$-open sets (M_1,Q), (M_2,Q) such that $q_{\mathcal{M}} \,\tilde{\in}\, (M_1,Q)$, $q_{\mathcal{N}} \,\tilde{\in}\, (M_2,Q)$ and $(M_1,Q) \cap (M_2,Q) = \{\tilde{\emptyset}\}$. This implies that $q_{\mathcal{M}} \,\tilde{\in}\, (M_1,Q) - (M_2,Q)$ and $q_{\mathcal{N}} \,\tilde{\in}\, (M_2,Q) - (M_1,Q)$.
From Example 3.6. The reversable of Remark 3.13 does not verify.
Now, Definition 3.5 can be reformulated for $s\dot{\imath}g$-$Ţ_1$. If for each $q_{\mathcal{M}}, q_{\mathcal{N}} \,\tilde{\in}\, \tilde{X}$ and $q_{\mathcal{M}} \neq q_{\mathcal{N}}$, then, there are $s\dot{\imath}g$-open sets M^c, $\mathcal{V}^c$ whenever, $q_{\mathcal{M}} \,\tilde{\in}\, \mathcal{V}^c$, $q_{\mathcal{N}} \notin \mathcal{V}^c$ and $q_{\mathcal{M}} \notin M^c$, $q_{\mathcal{N}} \,\tilde{\in}\, \mathcal{U}^c$ and $\mathcal{V}^c \cap M^c \neq \{\emptyset\}$. $s\dot{\imath}g$-$Ţ_1$ is not necessity to be $s\dot{\imath}g$-$Ţ_2$. Because of for any $s\dot{\imath}g$-open sets (M_1,Q), (M_2,Q) such that $q_{\mathcal{M}} \,\tilde{\in}\, (M_1,Q)$, $q_{\mathcal{N}} \,\tilde{\in}\, (M_2,Q)$, $(M_1,Q) \cap (M_2,Q) \neq \tilde{\emptyset}$.
In general, each $s\dot{\imath}g$-$Ţ_i$ is $s\dot{\imath}g$-$Ţ_{i+1}$ $\forall\, i = \{0, 1, 2\}$. The reversable will be not true. This can be shown in Example3.14.

Example 3.14. $(Ʀ, Ţ, Q, \dot{f})$ is a $s\dot{\imath}g$-$Ţ_m$-space, $m \in \{0,1,2\}$, whenever $Ʀ = \{1,2,3\}$, $Ţ = \{\tilde{\emptyset}, \tilde{X}\}$ and $\dot{f} = \hat{S}\hat{S}(Ʀ)_Q$. Since $s\dot{\imath}g\text{-}C(Ʀ)_Q = s\dot{\imath}g\text{-}O(Ʀ)_Q = \hat{S}\hat{S}(Ʀ)_Q$. But $(Ʀ, Ţ, Q)$ is not soft-$Ţ_m$.

Soft-$Ţ_2$-*space* ⟶ **Soft-$Ţ_1$-*space*** ⟶ **Soft-$Ţ_0$-*space***
↓ ↓ ↓
$s\dot{\imath}sg$-$Ţ_2$-space ⟶ ***$s\dot{\imath}sg$-$Ţ_1$-space*** ⟶ ***$\dot{\imath}sg$-$Ţ_0$-space***

The reversible of the above diagram will be not true, in general. via Examples 3.2, 3.11 and 3.14.

4. Games in soft ideal topological spaces

In the fourth section, based on the concept of $s\dot{\imath}g$-open sets and $s\dot{\imath}g$-$Ţ_i$-spaces, $i \in \{0,1,2\}$ in soft ideal spaces, some numerous types of topological games will be presented and the comparison between them are discussed.

In this part, we will symbolize the first step and the second step at any stage of the game with the symbol step1 (respectively, step2); and we will also symbolize the first player and the second player in any game as $PL1$(respectively, $PL2$).

Definition 4.1. For any (Ʀ, ƫ, Q, İ) be a soft ideal space. A game $\hat{S}G$(ƫ$_0$, Ʀ) (resp., $\hat{S}G$(ƫ$_0$, İ) for $PL1$ and $PL2$ proceeds by playing an inning for all natural numbers in the r-th inning: in step1. $PL1$, will be choose $(q_{\mathcal{M}})_r \neq (q_{\mathcal{N}})_r$, whenever $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r$ $\tilde{\in}$ $\tilde{Ʀ}$. In step2, $PL2$ choose M_r a soft-open (resp., sİ-g-open set) containing only one of the two elements $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r$. Then, $PL2$ wins in the soft game $\hat{S}G$(ƫ$_0$, Ʀ) (resp., SG(ƫ$_0$, İ) if M = { M_1 , M_2 , M_3 ,… M_r,…..} be a family of a soft-open set (resp., sİg-open) set in Ʀ such that ∀ , $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r$ $\tilde{\in}$ Ʀ , ∃ $M_r \in M$ contains one of the following $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r$. Otherwise, Player 1 wins.
In the following, ↑ denotes to the winning and ↓ to the opposite (losing) strategy.

Example 4.2. Let $\hat{S}G$(ƫ$_0$, Ʀ) (resp. $\hat{S}G$(ƫ$_0$, İ)) be a soft game where, Ʀ = {1,2,3}, Q = { q_1, q_2}, ƫ= {$\tilde{\emptyset}$, $\tilde{Ʀ}$,($\mathcal{P}$, Q), (W, Q), ($\mathcal{Z}$, Q)} whenever, ($\mathcal{P}$, Q) ={(q_1,{1}),(W, Q) = (q_2,{1})}, {(q_1,{3}), (q_2,{3})}, ($\mathcal{Z}$, Q) = {(q_1,{1,3}),(q_2,{1,3})} and İ = {$\tilde{\emptyset}$} . Then sİg-c(Ʀ)$_Q$ = ƫ* and sİg-o(Ʀ)$_Q$ = ƫ. Then, the game will be run in the following innings.

In the first round: the step1, $PL1$ will be choose $q_{\mathcal{M}} \neq q_{\mathcal{N}}$ whenever, $q_{\mathcal{M}}$, $q_{\mathcal{N}}$ $\tilde{\in}$ $\tilde{Ʀ}$ such that $q_{\mathcal{M}}$ = {1} Λ $q_{\mathcal{N}}$ = {2}. In the second step, $PL2$ chosen ($\mathcal{P}$, Q) = {(q_1,{1}), (q_2,{1})} a soft-open (resp. sİg-open set)).

In the next round: the step1, $PL1$ will be choose $q_{\mathcal{M}} \neq q_{\mathcal{O}}$ where, $q_{\mathcal{M}}$, $q_{\mathcal{O}}$ $\tilde{\in}$ $\tilde{Ʀ}$ such that $q_{\mathcal{M}}$ = {1} Λ $q_{\mathcal{O}}$ = {3}. In the step2, $PL2$ chosen (W, Q) = {(q_1,{3}), (q_2,{3})} which is a soft-open (resp. sİg-open set).

In the next round: the step1, $PL1$ will be choose $q_{\mathcal{N}} \neq q_{\mathcal{O}}$ whenever, $q_{\mathcal{N}}$, $q_{\mathcal{O}}$ $\tilde{\in}$ $\tilde{Ʀ}$ such that $q_{\mathcal{N}}$ = {2} Λ $q_{\mathcal{O}}$ = {3}. In the step2, $PL2$ chosen (W, Q) = {(q_1,{3}), (q_2,{3})} which is a soft-open (resp., sİg-open set).

In the next round*:* the step1, $PL1$ will be choose $q_{\mathcal{M}} \neq q_{\mathcal{R}}$ whenever, $q_{\mathcal{M}}$, $q_{\mathcal{R}}$ $\tilde{\in}$ $\tilde{Ʀ}$ such that $q_{\mathcal{M}}$ = {1} Λ $q_{\mathcal{R}}$ = {2,3}. In the step2, $PL2$ chosen ($\mathcal{P}$, Q) = {(q_1,{1}), (q_2,{1})} which is a soft-open (resp. sİg-open set)).

In the next round: the step1, $PL1$ will be choose $q_{\mathcal{N}} \neq q_{\mathcal{S}}$ whenever, $q_{\mathcal{N}}$, $q_{\mathcal{S}}$ $\tilde{\in}$ $\tilde{Ʀ}$ such that $q_{\mathcal{N}}$ = {2} Λ $q_{\mathcal{S}}$ = {1,3}. In the step2, $PL1$ chosen ($\mathcal{Z}$, Q) = {(q_1, {1,3}), (q_2, {1,3})} which is a soft-open (resp., sİg-open set).

In the finally round*:* the step1, $PL1$ will be choose $q_{\mathcal{O}} \neq q_{\mathcal{L}}$ whenever, $q_{\mathcal{O}}$, $q_{\mathcal{L}}$ $\tilde{\in}$ $\tilde{Ʀ}$ such that $q_{\mathcal{O}}$= {3} Λ $q_{\mathcal{L}}$ = {1,2}. In the step2, $PL2$ chosen (W, Q) = {(q_1,{3}), (q_2,{3})} which is a soft-open (resp., sİg-open set)).
Therefore, M = {($\mathcal{P}$, Q) ,(W, Q) ,($\mathcal{Z}$, Q)} is the winning strategy for $PL2$ in $\hat{S}G$(ƫ$_0$, Ʀ) (resp., $\hat{S}G$(ƫ$_0$, İ)) so, $PL2$ ↑ $\hat{S}G$(ƫ$_0$, Ʀ) (resp., $\hat{S}G$(ƫ$_0$, İ)).

Example 4.3. Let $\hat{S}G$(ƫ$_0$, Ʀ) (resp. $\hat{S}G$(ƫ$_0$, İ)) is a game whenever,Ʀ = {1,2,3} , Q = {q_1, q_2}, ƫ = {$\tilde{\emptyset}$, $\tilde{Ʀ}$,(W, Q)} whenever, (W, Q) = {(q_1,{3}),(q_2,{3})} Λ İ = {$\tilde{\emptyset}$} then sİg-c(Ʀ)$_Q$= ƫ*Λ sİg-o(Ʀ)$_Q$= ƫ. Then, In the first round: the step1, $PL1$ will be choose $q_{\mathcal{M}} \neq q_{\mathcal{N}}$ whenever, $q_{\mathcal{M}}$, $q_{\mathcal{N}}$$\tilde{\in}$ $\tilde{Ʀ}$ since $q_{\mathcal{M}}$= {1} and $q_{\mathcal{N}}$ = {2}. In the step2, $PL2$ cannot find (U, Q) which is a soft-open (resp., sǵg-open set)) containing one of $q_{\mathcal{M}}$, $q_{\mathcal{N}}$. so, $PL1 \uparrow \hat{S}G(Ţ_0, X)$ (resp., $\hat{S}G(Ţ_0, \dot{f})$).

Corollary 4.4. In the space $(Ŗ, Ţ, Q, \dot{f})$, then
(i) $PL2 \uparrow \hat{S}G(Ţ_0, Ŗ)$ implies that $PL2 \uparrow \hat{S}G(Ţ_0, \dot{f})$.
(ii) $PL1 \uparrow \hat{S}G(Ţ_0, Ŗ)$ implies that $PL1 \uparrow \hat{S}G(Ţ_0, \dot{f})$.

Corollary 4.5. In the space $(Ŗ, Ţ, Q, \dot{f})$, if $PL2 \downarrow \hat{S}G(Ţ_0, Ŗ)$, then $PL2 \downarrow \hat{S}G(Ţ_0)$.
proposition 4.6. If the space $(Ŗ, Ţ, Q, \dot{f})$ is soft-$Ţ_0$ (resp., sǵg-$Ţ_0$) $\Leftrightarrow$ $PL2 \uparrow \hat{S}G(Ţ_0, Ŗ)$ (resp., $\hat{S}G(Ţ_0, \dot{f})$).

Proof. ($\Rightarrow$) From the $r\text{-}th$ round $PL1$ in $\hat{S}G(Ţ_0, Ŗ)$ (resp. $\hat{S}G(Ţ_0, \dot{f})$), will be choose $(q_{\mathcal{M}})_r \neq (q_{\mathcal{N}})_r$ whenever, $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r \tilde{\in} \tilde{Ŗ}$, $PL2$ in $\hat{S}G(Ţ_0, Ŗ)$ (resp. $\hat{S}G(Ţ_0, \dot{f})$) chosen $(Ʋ_r, Q)$ is a soft-open (resp. sǵg-open set) contains one of $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r$.
For $(Ŗ, Ţ, Q)$ soft-$Ţ_0$ (resp. sǵg-$Ţ_0$), if $M = \{(Ʋ_1, Q), (Ʋ_2, Q), (Ʋ_3, Q), \ldots, (Ʋ_r, Q), \ldots\}$ is the winning strategy for $PL2$ in $\hat{S}G(Ţ_0, Ŗ)$ (resp. $\hat{S}G(Ţ_0, \dot{f})$). Therefore, $PL2 \uparrow \hat{S}G(Ţ_0, Ŗ)$ (resp. $\hat{S}G(Ţ_0, \dot{f})$).
($\Leftarrow$) Follows in the same manner.

Corollary 4.7. In the space $(Ŗ, Ţ, Q)$,
(i) $PL2 \uparrow \hat{S}G(Ţ_0, Ŗ) \Leftrightarrow \forall\, q_{\mathcal{M}} \neq q_{\mathcal{N}}$ whenever, $q_{\mathcal{M}}, q_{\mathcal{N}} \tilde{\in} Ŗ \; \exists\, (\mathcal{A}, Q)$ is a closed-soft set whenever $q_{\mathcal{M}} \tilde{\in} (\mathcal{A}, Q)$ and $q_{\mathcal{N}} \tilde{\notin} (\mathcal{A}, Q)$.
(ii) $PL2 \uparrow \hat{S}G(Ţ_0, \dot{f}) \Leftrightarrow \forall\, q_{\mathcal{M}} \neq q_{\mathcal{N}}$ whenever, $q_{\mathcal{M}}, q_{\mathcal{N}} \tilde{\in} Ŗ \; \exists\, (\mathcal{B}, Q)$ is a sǵg-closed set whenever $q_{\mathcal{M}} \tilde{\in} (\mathcal{B}, Q)$ and $q_{\mathcal{N}} \tilde{\notin} (\mathcal{B}, Q)$.

Proof. (i) ($\Longrightarrow$) Let $q_{\mathcal{M}} \neq q_{\mathcal{N}}$ whenever, $q_{\mathcal{M}}, q_{\mathcal{N}} \tilde{\in} Ŗ$. Since $PL2 \uparrow \hat{S}G(Ţ_0, Ŗ)$, then, by Proposition 4.6, $(Ŗ, Ţ, Q)$ is soft-$Ţ_0$. Therefore, Theorem 1.16, is hold.
($\Longleftarrow$) By Theorem 1.16, $(Ŗ, Ţ, Q)$ is soft-$Ţ_0$. So, Proposition 4.6, is hold.
(ii) ($\Longrightarrow$) Let $q_{\mathcal{M}} \neq q_{\mathcal{N}}$ whenever $q_{\mathcal{M}}, q_{\mathcal{N}} \tilde{\in} Ŗ$. Since $PL2 \uparrow \hat{S}G(Ţ_0, \dot{f})$, this implies that, by proposition 4.6, $(Ŗ, Ţ, Q)$ is sǵg-$Ţ_0$. Then Theorem 3.4, is hold.
($\Longleftarrow$) By Theorem 3.4, the space $(Ŗ, Ţ, Q)$ is a sǵg-$Ţ_0$. Implies that Theorem 4.6, is hold.

Corollary 4.8.
(i) In the space $(Ŗ, Ţ, Q)$ is soft-$Ţ_0 \Leftrightarrow PL1 \nUparrow \hat{S}G(Ţ_0, Ŗ)$.
(ii) In the space $(Ŗ, Ţ, Q, \dot{f})$ is sǵg-$Ţ_0 \Leftrightarrow PL1 \nUparrow \hat{S}G(Ţ_0, \dot{f})$.

Proof: From Theorem 4.6, the proof is hold.

Theorem 4.9.
(i) In the space $(Ŗ, Ţ, Q)$ is not soft-$Ţ_0 \Leftrightarrow PL1 \uparrow \hat{S}G(Ţ_0, Ŗ)$.
(ii) In the space $(Ŗ, Ţ, Q, \dot{f})$ is not sǵg-$Ţ_0 \Leftrightarrow PL1 \uparrow \hat{S}G(Ţ_0, \dot{f})$.

Proof.
(i) ($\Longrightarrow$) From the r-th round $PL1$in $\hat{S}G(Ţ_0, Ŗ)$ will be choose $(q_{\mathcal{M}})_r \neq (q_{\mathcal{N}})_r$ whenever $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r \tilde{\in} \tilde{Ŗ}$, $PL2$in $\hat{S}G(Ţ_0, Ŗ)$ cannot find $(Ʋ_r, Q)$ which is a soft-open set $(q_{\mathcal{M}})_r \tilde{\in} (Ʋ_r, Q)$, $(q_{\mathcal{N}})_r \tilde{\notin} (Ʋ_r, Q)$ or $(q_{\mathcal{M}})_r \tilde{\notin} (Ʋ_r, Q)$, $(q_{\mathcal{N}})_r \tilde{\in} (Ʋ_r, Q)$. $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r$, since $(Ŗ, Ţ, Q)$ is not soft-$Ţ_0$. So, $PL1\hat{S}G(Ţ_0, Ŗ)$.

(⟸) Follows directly in the same manner.
(ii) (⟹) In the r-th round $PL1$in $\hat{S}G(Ţ_0, ḟ)$ will be choose $(q_{\mathcal{M}})_r \neq (q_{\mathcal{N}})_r$ whenever, $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r$ $\tilde{\in}$ $\tilde{Ɽ}$, $PL2$ in $\hat{S}G(Ţ_0, ḟ)$ cannot find $(Ʊ_r, Q)$ is a sḟg-open set $(q_{\mathcal{M}})_r \tilde{\in} (Ʊ_r, Q)$, $(q_{\mathcal{N}})_r$ $\tilde{\notin}$ $(Ʊ_r, Q)$ or $(q_{\mathcal{M}})_r \tilde{\notin} (Ʊ_r, Q)$, $(q_{\mathcal{N}})_r \tilde{\in}$ $(Ʊ_r, Q)$, since (Ɽ , Ţ, Q) is not sḟg-Ţ0. So, $PL1 \uparrow$ $\hat{S}G(Ţ_0, ḟ)$.
(⟸) Follows directly by the same manner.

From Theorem 4.9, it is easy to prove the following corollary. So, the proof will be omitted.

Corollary 4.10.
(i) In the space (Ɽ , Ţ, Q) is not soft-$Ţ_0$ ⟺ $PL2 \uparrow \hat{S}G(Ţ_0, Ɽ)$.
(ii) In the space (Ɽ , Ţ, Q, ḟ) is not sḟg-$Ţ_0$ ⟺ $PL2 \uparrow \hat{S}G(Ţ_0, ḟ)$.

Definition 4.11. From the space (Ɽ , Ţ, Q, ḟ). A game $\hat{S}G(Ţ_1, Ɽ)$ (resp. $\hat{S}G(Ţ_1, ḟ)$) for $PL1 \wedge PL2$ proceeds by playing an inning with all natural numbers in the r-th round : the step1, $PL1$, will be choose $(q_{\mathcal{M}})_r \neq (q_{\mathcal{N}})_r$ whenever, $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r \tilde{\in} \tilde{Ɽ}$. In the step2, $PL2$ chosen $(\mathcal{A}_r, Q), (\mathcal{B}_r, Q)$ are soft-open (resp. sḟg-open) sets when $(q_{\mathcal{M}})_r \tilde{\in} ((\mathcal{A}_r, Q) - (\mathcal{B}_r, Q)) \wedge (q_{\mathcal{N}})_r \tilde{\in} ((\mathcal{B}_r, Q) - (\mathcal{A}_r, Q))$. So, $PL2$ wins in the soft game $\hat{S}G(Ţ_1, Ɽ)$ (resp. $\hat{S}G(Ţ_1, ḟ)$), if M $= \{\{(\mathcal{A}_1, Q), (\mathcal{B}_1, Q)\}, \{(\mathcal{A}_2, Q), (\mathcal{B}_2, Q)\}, \ldots, \{(\mathcal{A}_r, Q), (\mathcal{B}_r, Q)\}, \ldots\}$ be a collection of a soft-open (resp. sḟg-$open$) sets in Ɽ such that $\forall (q_{\mathcal{M}})_r \neq (q_{\mathcal{N}})_r$ whenever, $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r \tilde{\in} \tilde{Ɽ}$, $\exists \{(\mathcal{A}_r, Q), (\mathcal{B}_r, Q)\} \in$ M such that $(q_{\mathcal{M}})_r((\mathcal{A}_r, Q) - (\mathcal{B}_r, Q))$ and $(q_{\mathcal{N}})_r \tilde{\in} ((\mathcal{B}_r, Q) - (\mathcal{A}_r, Q))$. Others, $PL1$wins in the soft game $\hat{S}G(Ţ_1, Ɽ)$ (resp. $\hat{S}G(Ţ_1, ḟ)$).

Example 4.12. Let $\hat{S}G(Ţ_1, Ɽ)$ and $\hat{S}G(Ţ_1, ḟ)$) be games such that Ɽ = {1,2,3}, Q ={ q_1, q_2}, Ţ = $\hat{S}\hat{S}$(Ɽ)Q, ḟ = $\{\tilde{\emptyset}\}$. Therefore, sḟg-c(Ɽ)Q = sḟg-o(Ɽ)Q = $\hat{S}\hat{S}$(Ɽ)Q.

From the first round: the step1, $PL1$ will be choose $q_{\mathcal{M}} \neq q_{\mathcal{N}}$ whenever, $q_{\mathcal{M}}, q_{\mathcal{N}} \tilde{\in}$ Ɽ when $q_{\mathcal{M}}$ = {1} Λ $q_{\mathcal{N}}$ = {2}. In the step2, $PL2$ chosen $(\mathcal{A}, Q), (\mathcal{B}, Q)$ when $\mathcal{A}$(q) = {1}, $\mathcal{B}$(q) = {2} ∀ q which are soft-open (resp., sḟg-$open$) sets.

In the next round: the step1, $PL1$ will be choose $q_{\mathcal{M}} \neq q_{\mathcal{O}}$ whenever, $q_{\mathcal{M}}$, $q_{\mathcal{O}} \tilde{\in} \tilde{Ɽ}$ when $q_{\mathcal{M}}$ = {2} Λ $q_{\mathcal{O}}$ = {3}. In the step2, $PL2$ chosen $(\mathcal{B}, Q), (\mathcal{C}, Q)$ when $\mathcal{B}$(q) = {2}, $\mathcal{C}$(q) = {3} ∀ q which are soft-open (resp., sḟg-$open$) sets.

In the next round: the step1, $PL1$ will be choose $q_{\mathcal{N}} \neq q_{\mathcal{O}}$ whenever, $q_{\mathcal{N}}$, $q_{\mathcal{O}} \tilde{\in} \tilde{Ɽ}$ when $q_{\mathcal{N}}$ = {1} Λ $q_{\mathcal{O}}$ = {3}. In the step2, $PL2$ chosen $(\mathcal{A}, Q), (\mathcal{C}, Q)$ when $\mathcal{A}$(q) = {1}, $\mathcal{C}$(q) = {3} ∀ q which are soft-open (resp., sḟg-$open$) sets.

In the next round: the step1, $PL1$ will be choose $q_{\mathcal{M}} \neq q_{\mathcal{R}}$ whenever, $q_{\mathcal{M}}$, $q_{\mathcal{R}} \tilde{\in} \tilde{Ɽ}$ such that $q_{\mathcal{M}}$ = {1} Λ $q_{\mathcal{R}}$ = {2,3}. In step2, $PL2$ chosen $(\mathcal{A}, Q), (\mathcal{D}, Q)$ when $\mathcal{A}$(q) = {1}, $\mathcal{D}$(q) = {2,3} ∀ q which are soft open (resp., sḟg-$open$) sets.

In the next round: the step1, $PL1$ will be choose $q_{\mathcal{N}} \neq q_{\mathcal{S}}$ whenever, $q_{\mathcal{N}}$, $q_{\mathcal{S}} \tilde{\in} \tilde{Ɽ}$ such that $q_{\mathcal{N}}$ = {2} Λ $q_{\mathcal{S}}$ = {1,3}. In the step2, $PL2$ chosen $(\mathcal{B}, Q), (\mathcal{E}, Q)$ when $\mathcal{B}$(q) = {2}, $\mathcal{E}$(q) = {1,3} ∀ q which are soft-open (resp., sḟg-$open$) sets.

In the finally round: the step1, $PL1$ will be choose $q_{\mathcal{O}} \neq q_{\mathcal{L}}$ whenever, $q_{\mathcal{O}}, q_{\mathcal{L}} \tilde{\in} \tilde{Ʀ}$ such that $q_{\mathcal{O}} = \{3\} \wedge q_{\mathcal{L}} = \{1,2\}$. In the step2, $PL2$ chosen $(\mathcal{C},Q), (\mathcal{F},Q)$ when $\mathcal{C}(q) = \{3\}$, $\mathcal{F}(q) = \{1,2\}$ $\forall q$ which are soft-open (resp., sĩg-$open$) sets.
Then, M = $\{\{(\mathcal{A},Q),(\mathcal{B},Q)\},\{(\mathcal{B},Q),(\mathcal{C},Q)\},\{(\mathcal{A},Q),(\mathcal{C},Q)\},\{(\mathcal{A},Q),(\mathcal{D},Q)\},$ $\{(\mathcal{B},Q),(\mathcal{E},Q)\},\{(\mathcal{C},Q),(\mathcal{F},Q)\}\}$ is the winning strategy for $PL2$ in $\hat{S}G(Ţ_1, Ʀ)$ (resp. $\hat{S}G(Ţ_1, \dot{ĩ})$). So, $PL2 \uparrow \hat{S}G(Ţ_1, Ʀ)$(resp. $\hat{S}G(Ţ_1, \dot{ĩ})$). By the same way in Example 4.3, $PL1 \uparrow \hat{S}G(Ţ_1, Ʀ)$ and $PL1 \uparrow \hat{S}G(Ţ_1, \dot{ĩ})$.

Corollary 4.13. In the space $(Ʀ, Ţ, Q, \dot{ĩ})$, we have
(i) $PL2 \uparrow \hat{S}G(Ţ_1, Ʀ)$, implies that $PL2 \uparrow \hat{S}G(Ţ_1, \dot{ĩ})$.
(ii) $PL1 \uparrow \hat{S}G(Ţ_1, \dot{ĩ})$, implies that $PL1 \uparrow \hat{S}G(Ţ_1, Ʀ)$.

Corollary 4.14. In the space $(Ʀ, Ţ, Q, \dot{ĩ})$, if $PL2 \downarrow \hat{S}G(Ţ_1, Ʀ)$, then $PL2 \downarrow \hat{S}G(Ţ_1, \dot{ĩ})$.
Theorem 4.15. If the space$(Ʀ, Ţ, Q, \dot{ĩ})$, is soft-$Ţ_1$ (resp., sĩg-$Ţ_1$) $\Leftrightarrow$ $PL2 \uparrow \hat{S}G(Ţ_1, Ʀ)$ (resp., $\hat{S}G(Ţ_1, \dot{ĩ})$).

Proof. ($\Longrightarrow$) From the r-th round $PL1$ in $\hat{S}G(Ţ_1, Ʀ)$ (resp. $\hat{S}G(Ţ_1, \dot{ĩ})$), choose $\forall (q_{\mathcal{M}})_r \neq (q_{\mathcal{N}})_r$ whenever, $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r \tilde{\in} \tilde{Ʀ}$, $PL2$ in $\hat{S}G(Ţ_1, Ʀ)$ (resp., $\hat{S}G(Ţ_1, \dot{ĩ})$) will choose $(\mathcal{A}_r,Q), (\mathcal{B}_r,Q)$ are soft-open (resp., sĩg-open) sets such that $(q_{\mathcal{M}})_r \tilde{\in} ((\mathcal{A}_r,Q) - (\mathcal{B}_r,Q))$ $\wedge$ $(q_{\mathcal{N}})_r \tilde{\in} ((\mathcal{B}_r,Q) - (\mathcal{A}_r,Q))$. Since $(Ʀ, Ţ, Q)$ soft-$Ţ_1$ (resp., sĩg-$Ţ_1$), then M = $\{\{(\mathcal{A}_1,Q),(\mathcal{B}_1,Q)\},\{(\mathcal{A}_2,Q),(\mathcal{B}_2,Q)\},\dots,\{(\mathcal{A}_r,Q),(\mathcal{B}_r,Q)\},\dots\}$ is the winning strategy for $PL2$ in $\hat{S}G(Ţ_1, Ʀ)$ (resp., $\hat{S}G(Ţ_1, \dot{ĩ})$). So, $PL2 \uparrow \hat{S}G(Ţ_1, X)$ (resp., $\hat{S}G(Ţ_1, \dot{ĩ})$).
($\Longleftarrow$) Follows directly by the same manner.

Corollary 4.16. In the space $(Ʀ, Ţ, Q, \dot{ĩ})$, we have
(i) $PL2 \uparrow \hat{S}G(Ţ_1, Ʀ)$ if $\forall q_{\mathcal{M}} \neq q_{\mathcal{N}}$ whenever $q_{\mathcal{M}}, q_{\mathcal{N}} \tilde{\in} \tilde{X}$, $\exists (\mathcal{A},Q), (\mathcal{B},Q)$ are soft-closed sets when $q_{\mathcal{M}} \tilde{\in} ((\mathcal{A},Q) - (\mathcal{B},Q)) \wedge q_{\mathcal{N}} \tilde{\in} ((\mathcal{B},Q) - (\mathcal{A},Q))$.
(ii) $PL2 \uparrow G(Ţ_1, \dot{ĩ})$ if $\forall q_{\mathcal{M}} \neq q_{\mathcal{N}}$ whenever $q_{\mathcal{M}}, q_{\mathcal{N}} \tilde{\in} \tilde{X}$, $\exists (\mathcal{A},Q), (\mathcal{B},Q)\}$ are sĩg-closed sets when, $q_{\mathcal{M}} \tilde{\in} ((\mathcal{A},Q) - (\mathcal{B},Q)) \wedge q_{\mathcal{N}} \tilde{\in} ((\mathcal{B},Q) - (\mathcal{A},Q))$.

Proof.
(i) ($\Longrightarrow$) Let $q_{\mathcal{M}} \neq q_{\mathcal{N}}$ whenever $q_{\mathcal{M}}, q_{\mathcal{N}} \tilde{\in} \tilde{Ʀ}$. Since $PL2 \uparrow \hat{S}G(Ţ_1, X)$, then by Theorem 4.15, $(Ʀ, Ţ, Q)$ is soft-$Ţ_1$. So, Theorem 1.18, is hold.
($\Longleftarrow$) By Theorem 1.18, $(Ʀ, Ţ, Q)$ is soft-$Ţ_1$. So, Theorem 4.15, is hold.
(ii) ($\Longrightarrow$) Let $q_{\mathcal{M}} \neq q_{\mathcal{N}}$ whenever $q_{\mathcal{M}}, q_{\mathcal{N}} \tilde{\in} \tilde{Ʀ}$. Since $PL2 \uparrow \hat{S}G(Ţ_1, \dot{ĩ})$, so by Theorem 4.15, the space $(Ʀ, Ţ, Q)$ is s$\dot{ĩ}$ − g-$Ţ_1$. Implies that, Theorem 3.9, is hold. ($\Longleftarrow$) By Theorem 3.9, $(Ʀ, Ţ, Q)$ is sĩg-$Ţ_1$. Therefore, Theorem 4.15, is hold.

Corollary 4.17.
(i) In the space $(Ʀ, Ţ, Q)$ is soft-$Ţ_1 \Leftrightarrow PL1 \Uparrow \hat{S}G(Ţ_1, Ʀ)$.
(ii) In the space $(Ʀ, Ţ, Q, \dot{ĩ})$ is sĩg-$Ţ_1 \Leftrightarrow PL1 \Uparrow \hat{S}G(Ţ_1, \dot{ĩ})$.

Proof: Via Theorem 4.15, the proof is obvious.

Theorem 4.18. Form any space $(Ʀ, Ţ, Q, \dot{ĩ})$, we have
(i) The space $(Ʀ, Ţ, Q)$ is not soft-$Ţ_1 \Leftrightarrow PL1 \uparrow \hat{S}G(Ţ_1, X)$.
(ii) The space $(Ʀ, Ţ, Q, \dot{ĩ})$ is not sĩg-$Ţ_1 \Leftrightarrow PL1 \uparrow \hat{S}G(Ţ_1, \dot{ĩ})$.

Proof.

(i) (⟹) In the r-th round $PL1$ in $\hat{S}G(Ţ_1, Ŗ)$ choose $(q_{\mathcal{M}})_r \neq (q_{\mathcal{N}})_r$ whenever, $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r \tilde{\in} \tilde{Ŗ}$, $PL2$ in $\hat{S}G(Ţ_1, Ŗ)$ cannot find $(\mathcal{A}_r, Q), (\mathcal{B}_r, Q)$ are soft-open sets when $(q_{\mathcal{M}})_r \tilde{\in} ((\mathcal{A}_r, Q) - (\mathcal{B}_r, Q)) \wedge (q_{\mathcal{N}})_r \tilde{\in} ((\mathcal{B}_r, Q) - (\mathcal{A}_r, Q))$, because $(Ŗ, Ţ, Q)$ is not soft-$Ţ_1$. Hence $PL1 \uparrow \hat{S}G(Ţ_1, Ŗ)$. (⟸) Follows directly in the same manner.

(ii) (⟹) In the r-th round $PL1$in $\hat{S}G(Ţ_1, \dot{ſ})$ choose $(q_{\mathcal{M}})_r \neq (q_{\mathcal{N}})_r$ whenever, $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r \tilde{\in} \tilde{Ŗ}$, $PL2$in $\hat{S}G(Ţ_1, \dot{ſ})$ cannot find $(\mathcal{A}_r, Q), (\mathcal{B}_r, Q)$ are two $s\dot{ſ}-$g-$open$ sets when $(q_{\mathcal{M}})_r \tilde{\in} ((\mathcal{A}_r, Q) - (\mathcal{B}_r, Q))$ and $(q_{\mathcal{N}})_r((\mathcal{B}_r, Q) - (\mathcal{A}_r, Q))$, since $(Ŗ, Ţ, Q)$ is not soft- $Ţ_1$. So, $PL1 \uparrow \hat{S}G(Ţ_1, \dot{ſ})$.
(⟸)Follows directly by the same manner.

Corollary 4.19.

(i) If a space $(Ŗ, Ţ, Q)$ is not soft-$Ţ_1$ ⟺ $PL2 \Uparrow \hat{S}G(Ţ_1, Ŗ)$.

(ii) If a space $(Ŗ, Ţ, Q, \dot{ſ})$ is not s$\dot{ſ}$g-$Ţ_1$ ⟺ $PL2 \Uparrow \hat{S}G(Ţ_1, \dot{ſ})$.

Proof: Similarity to the proof of Theorem 4.18.

Definition 4.20. For any space $(Ŗ, Ţ, Q, \dot{ſ})$. A game $\hat{S}G(Ţ_2, Ŗ)$ (resp. $\hat{S}G(Ţ_2, \dot{ſ})$) for $PL1$ and $PL2$ proceeds by playing an inning with all natural numbers in the r-th round: the step1, $PL1$ will be choose $(q_{\mathcal{M}})_r \neq (q_{\mathcal{N}})_r$ whenever, $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r \tilde{\in} \tilde{Ŗ}$. In the step2, $PL2$ choose $(\mathcal{A}_r, Q), (\mathcal{B}_r, Q)$ are soft-open (resp. s$\dot{ſ}$g-open) sets such that $(q_{\mathcal{M}})_r \tilde{\in} (\mathcal{A}_r, Q)$, $(q_{\mathcal{N}})_r \tilde{\in} (\mathcal{B}_r, Q)$ and $(\mathcal{A}_r, Q) \tilde{\cap} (\mathcal{B}_r, Q) = \{\tilde{\emptyset}\}$. Then, $PL2$ wins in the game $\hat{S}G(Ţ_2, Ŗ)$ (resp., $\hat{S}G(Ţ_2, \dot{ſ})$) if
$M = \{\{(\mathcal{A}, Q), (\mathcal{B}, Q)\}, \{(\mathcal{B}, Q), (\mathcal{C}, Q)\}, \{(\mathcal{A}, Q), (\mathcal{C}, Q)\}\}$ be a family of a soft-open (resp. s$\dot{ſ}$g-open) sets in Ŗ when $\forall (q_{\mathcal{M}})_r \neq (q_{\mathcal{N}})_r$ such that, $(q_{\mathcal{M}})_r$, $(q_{\mathcal{N}})_r \tilde{\in} Ŗ \exists \{(\mathcal{A}_r, Q), (\mathcal{B}_r, Q)\} \in M$ and $(q_{\mathcal{M}})_r \tilde{\in} (\mathcal{A}_r, Q)$ and $(q_{\mathcal{N}})_r \tilde{\in} (\mathcal{B}_r, Q)$ and $(\mathcal{A}_r, Q) \tilde{\cap} (\mathcal{B}_r, Q) = \{\tilde{\emptyset}\}$. Otherwise, $PL1$ wins in the game $\hat{S}G(Ţ_2, Ŗ)$ (resp., $\hat{S}G(Ţ_2, \dot{ſ})$). By Example 4.12., let there is a game $\hat{S}G(Ţ_2, Ŗ)$ (resp., $\hat{S}G(Ţ_2, \dot{ſ})$) be a game when, $Ŗ = \{1,2,3\}$, $Q = \{q_1, q_2\}$, $Ţ = \hat{S}\hat{S}(Ŗ)_Q$, $\dot{ſ} = \{\tilde{\emptyset}\}$. So, s$\dot{ſ}$g-$c(Ŗ)_Q$ = s$\dot{ſ}$g-$o(Ŗ)_Q = \hat{S}\hat{S}(Ŗ)_Q$. Then, $M = \{\{(\mathcal{A}, Q), (\mathcal{B}, Q)\}, \{(\mathcal{B}, Q), (\mathcal{C}, Q)\}, \{(\mathcal{A}, Q), (\mathcal{C}, Q)\}, \{(\mathcal{A}, Q), (\mathcal{D}, Q)\}, \{(\mathcal{B}, Q), (\mathcal{E}, Q)\}, \{(\mathcal{C}, Q), (\mathcal{F}, Q)\}\}$ is the winning strategy for $PL2$ in $\hat{S}G(Ţ_2, Ŗ)$ (resp. $\hat{S}G(Ţ_2, \dot{ſ})$). So, $PL2 \uparrow \hat{S}G(Ţ_2, Ŗ)$ (resp., $\hat{S}G(Ţ_2, \dot{ſ})$). By the same way in Example 4.3, $PL1 \uparrow \hat{S}G(Ţ_2, Ŗ)$ and $PL1 \uparrow \hat{S}G(Ţ_2, \dot{ſ})$.

Remark 4.21. From any space $(Ŗ, Ţ, Q, \dot{ſ})$, we have

(i) $PL2 \uparrow \hat{S}G(Ţ_2, Ŗ)$ implies that $PL2 \uparrow \hat{S}G(Ţ_2, \dot{ſ})$.

(ii) $PL1$ □ $\uparrow \hat{S}G(Ţ_2, \dot{ſ})$ implies that $PL1 \uparrow \hat{S}G(Ţ_2, Ŗ)$.

Corollary 4.22. In the space $(Ŗ, Ţ, Q, \dot{ſ})$, if $PL2 \downarrow \hat{S}G(Ţ_2, Ŗ)$ then $PL2 \downarrow \hat{S}G(Ţ_2, \dot{ſ})$.

Theorem 4.23. The space $(Ŗ, Ţ, Q, \dot{ſ})$ soft-$Ţ_2$ (resp. $s\dot{ſ}g$-$Ţ_2$) ⟺ $PL2 \uparrow \hat{S}G(Ţ_2, Ŗ)$ (resp. $\hat{S}G(Ţ_2, \dot{ſ})$).

Proof: Follows directly in the same manner.

Corollary 4.24.

(i) The space $(Ŗ, Ţ, Q)$ is soft-$Ţ_2$ ⟺ $PL1 \Uparrow \hat{S}G(Ţ_2, Ŗ)$.

(ii) The space $(Ŗ, Ţ, Q, \dot{ſ})$ is s$\dot{ſ}$g-$Ţ_2$ ⟺ $PL1 \Uparrow \hat{S}G(Ţ_2, \dot{ſ})$.

Proof: via Theorem 4.23, the proof is obvious.

Theorem 4.25. For a space (X , Ţ, Q, ḟ), we have

(i) A space (Ɽ , Ţ, Q) is not soft-$Ţ_2$ ⟺ $PL1 \uparrow \hat{S}G(Ţ_2, Ɽ)$.

(ii) A space (Ɽ , Ţ, Q, ḟ) is not sḟg-$Ţ_2$ ⟺ $PL1 \uparrow \hat{S}G(Ţ_2, ḟ)$.

Proof:

Follows directly in the same manner.

Corollary 4.26.

(i) A space (Ɽ , Ţ, Q) is not soft-$Ţ_2$ ⟺ $PL2 \not\uparrow \hat{S}G(Ţ_2, X)$.

(ii) A space (Ɽ , Ţ, Q, ḟ) is not sḟg-$Ţ_2$ ⟺ $PL2 \not\uparrow \hat{S}G(Ţ_2, ḟ)$.

Proof: via Theorem 4.25, the proof is obvious.

Corollary 4.27. In the space (X , Ţ, Q, ḟ), we have

(i) If $PL2 \uparrow \hat{S}G(Ţ_{i+1}, X)$ (resp. $\hat{S}G(Ţ_{i+1}, ḟ)$), then $PL2 \uparrow \hat{S}G(Ţ_i, X)$ (resp. $\hat{S}G(Ţ_i, ḟ)$), where $i = \{0,1\}$.

(ii) If $PL2 \uparrow \hat{S}G(Ţ_i, X)$, then $PL2 \uparrow \hat{S}G(Ţ_i, ḟ)$, where $i = \{0,1,2\}$.

Corollary 4.27. The implication in Figure 2 clarifies the relation between Theorems 4.6, 4.15, 4.23 and

Corollary 4.28. For a space (X, Ţ, ḟ), we have

(i) if $PL1 \uparrow \hat{S}G(Ţ_i, X)$ (resp. $\hat{S}G(Ţ_i, ḟ)$) then $PL1 \uparrow \hat{S}G(Ţ_{i+1}, X)$ (resp. $\hat{S}G(Ţ_{i+1}, ḟ)$), where $i = \{0,1\}$.

(ii) if $PL1 \uparrow \hat{S}G(Ţ_i, ḟ)$ then $PL1 \uparrow \hat{S}G(Ţ_i, X)$, where $i = \{0,1,2\}$.

The implication in Figure 3 clarifies the relation between in Theorem 4.9, Theorem 4.18, Theorem 4.26 and Corollary 4.28.

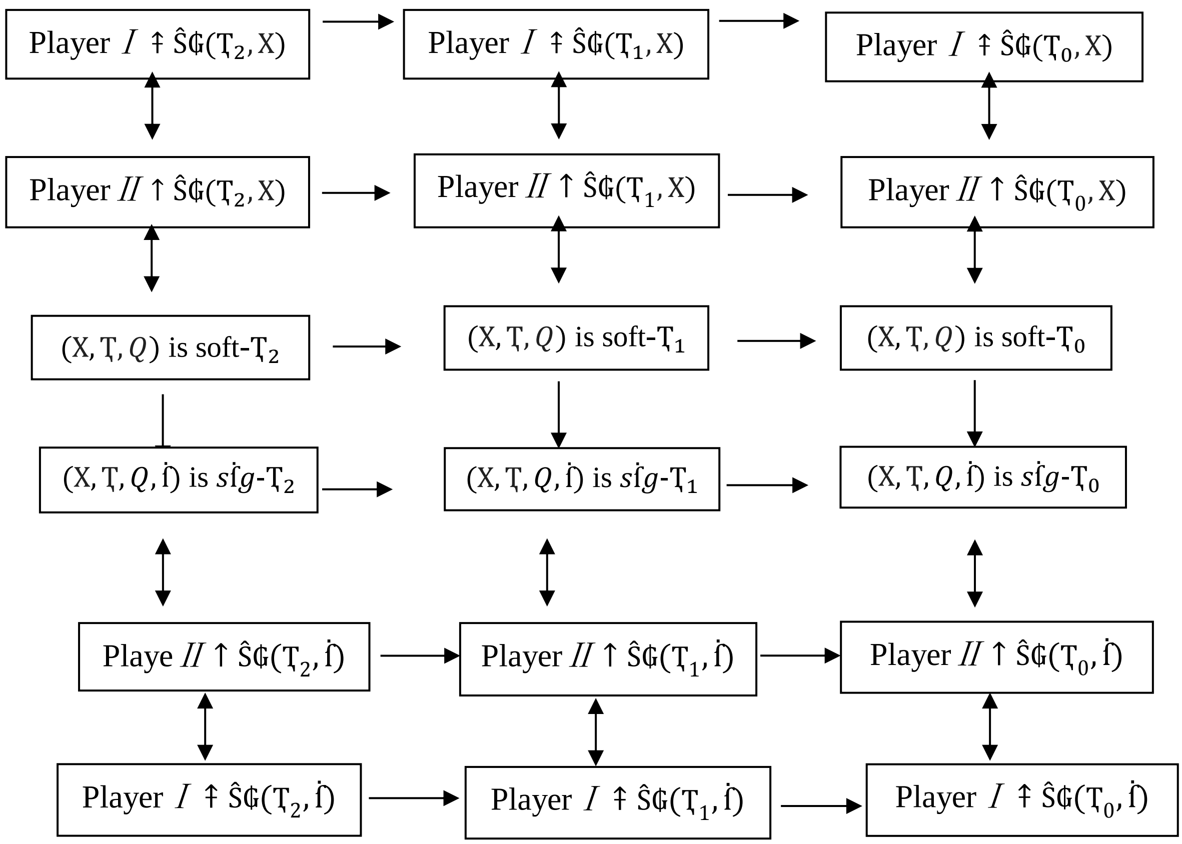

Figure 2: Relation between Theorems 4.6, 4.15, 4.23 and Corollary 4.27.

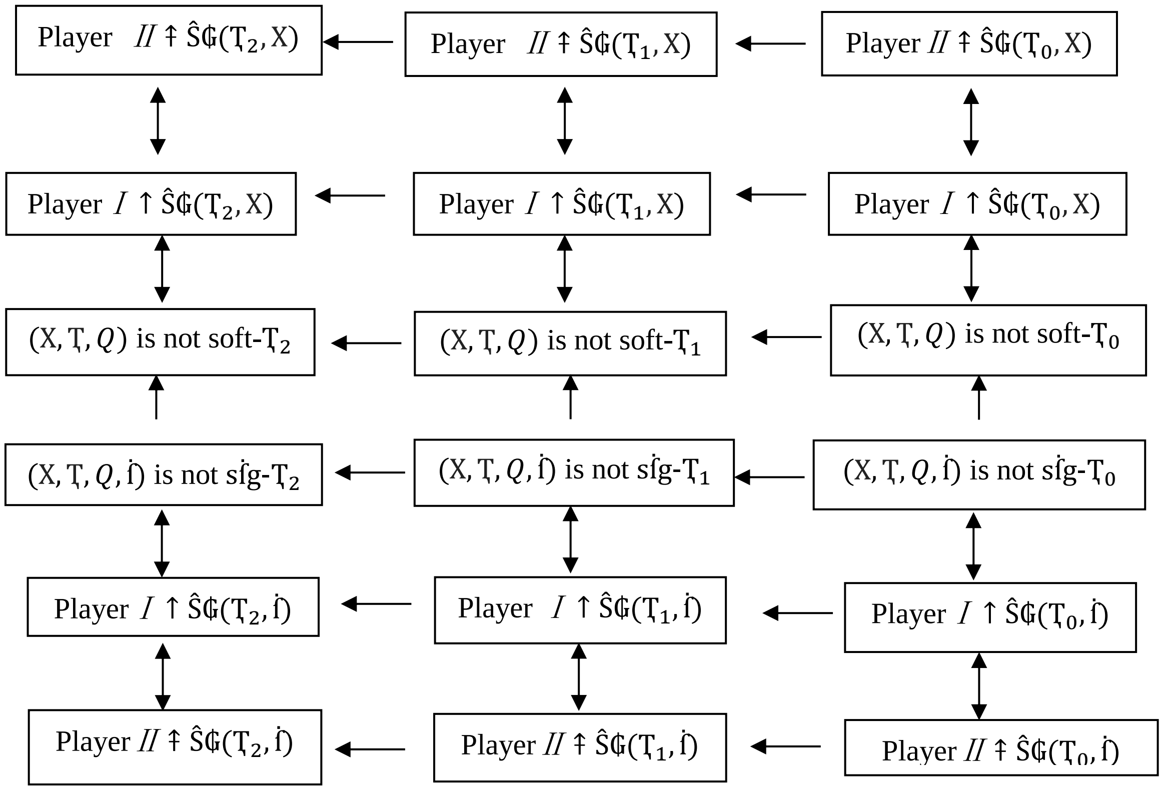

Figure 3: Relation between in Theorems 4.9, 4.18 and 4.26 and Corollary 4.28.

Conclusion

A combination between soft sets and soft topology has a significant role in studying of some classical applications and nonclassical logic. Depending on the new concept of sİg-open soft sets, some soft separation axioms, namely $s\dot{I}g$-$Ţ_i$ -space, $i \in \{0,1,2\}$ are given and of their comparisons are discussed in terms of soft point defined by Zorlutuna [22]. Soft topological games, called, $\hat{S}G(Ţ_0, X)$ and $\hat{S}G(Ţ_0, \dot{I})$ with perfect information on soft ideal $s\dot{I}g$-$Ţ_i$ -spaces will be applied to solve some problems that having uncertainties in engineering, medical, economics and in general machine systems of different sorts.

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