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# On Nano Ɽ-Space Application

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**Abstract**

This study introduces a novel topological space named the nano Ɽ-space, which is based on the principles of nano topological spaces. Thus, the reconstruction of this space and their properties are demonstrated and explored, accompanied by illustrative examples to aid in understanding its nature. Furthermore, the investigation of the practical implications of the nano Ɽ-space through a series of case studies involving pediatric patients diagnosed with leukemia. Then, by examining the symptoms exhibited by these patients and their impact on the equivalence relations within the nano Ɽ-space, the elucidate the applied aspect of this novel topological framework. the findings indicate that the presence of unexplained bruises on the body emerges as a fundamental and indispensable symptom, allowing us to definitively diagnose the presence of leukemia in a patient.

**Keywords:** Lower approximation, upper approximation, nano-topology, R- space.

# حول تطبيق الفضاء النانوي من النمط– Ɽ

**رنا بهجت اسماعيل**

قسم الرياضيات، كلية التربية للعلوم الصرفة – ابن الهيثم، جامعة بغداد، بغداد، العراق

**الخلاصة**

في هذه الدراسة، يتم تقديم فضاء تبولوجي جديد يُسمى "فضاء النانو من النمط Ɽ"، والذي يستند إلى مبادئ الفضاءات التبولوجية النانو. يتم استكشاف بناء هذا الفضاء وخصائصه، بالإضافة إلى تقديم أمثلة توضيحية للمساعدة في فهم طبيعته. وعلاوة على ذلك، يتم دراسة تأثيرات فضاء النانو من النمط Ɽ في الممارسة العملية من خلال سلسلة من الدراسات الحالية تشمل المرضى الأطفال المصابين بسرطان الدم. من خلال دراسة الأعراض التي يظهرها هؤلاء المرضى وتأثيرها على العلاقات المكافئة داخل فضاء النانو من النمط Ɽ، حيث يوضح الجانب التطبيقي لهذا الفضاء التبولوجي الجديد. تشير النتائج إلى أن وجود كدمات غير مفسرة على الجسم يعتبر احد الاعراض الأساسية والضرورية، مما يتيح لنا تشخيص وجود سرطان الدم بشكل قاطع لدى المريض.

## 1. Introduction

The concept of nano-topological spaces was first introduced by Thivagar in 2013 [1], building upon the definitions proposed by Pawlak [2,3]. This innovative framework

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introduced a new type of topological space, incorporating notions such as upper approximation, lower approximation, and boundary. Equivalence relations emerged as a central focus of investigation within this domain, captivating the attention of numerous researchers who sought to explore the properties of this space and establish novel types of weakly nano open sets. Extensive scholarly interest has been devoted to examining the applications of nano-topological spaces, particularly in fields such as medicine, engineering, and various related disciplines etc… [4-6].

Many recent studies have demonstrated the applied aspect of the concept of nano topological spaces and how to narrow them to benefit from them in many fields, especially medical fields. The concept was used with many weakly open sets to demonstrate the effect of the concept of nano topological spaces and approximation sets on the symptoms of various diseases, which was one of the most prominent; heart attack, blood sugar, dengue fever, measles, hepatitis and others, [7-12].

The concept of nano topology was used to find new types of generalizations of open sets and to study their topological properties, where the nano topological space is considered a special case of topological spaces, which is established based on an equivalence relationship with the knowledge of approximations sets, this topology is considered one of the types of finite topologies, which consists of five elements at most. Therefore, this concept has been developed by studying generalizations that would clarify the benefits of this concept. See [13-20].

In this research, we will present a new type of topological space and its medical applications by presenting leukaemia in children and the effect of nano space in identifying the important symptoms of this disease.

## 2. Preliminaries

**Definition 2.1: [2, 3]**

Let non-empty set $Z$ and $w$ as an equivalence relation on $Z$. Let $\boldsymbol{\upsilon}$ be a subset of $Z$.

**i.** The upper approximation of $\boldsymbol{\upsilon}$ for w is a symbolizes by $\overline{\mathrm{w}(\upsilon)}$, such that

$$\overline{\mathrm{w}(\upsilon)} = \cup_{f\in Z} \{ \mathrm{w}(f) : \mathrm{w}(f) \cap \boldsymbol{\upsilon} \neq \emptyset \}.$$

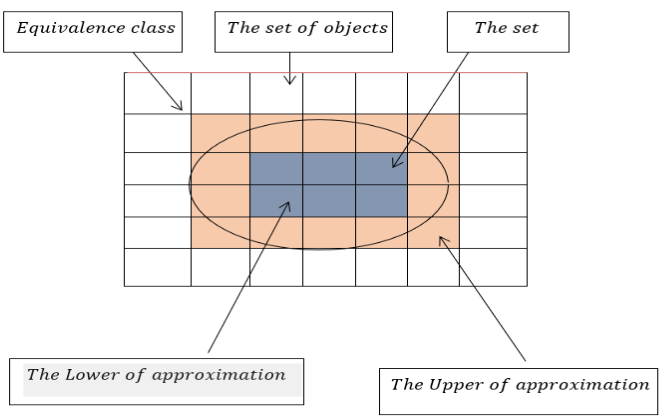
**ii.** The lower approximation of $\boldsymbol{\upsilon}$ for $\bar{Ṛ}$ is symbolizes by $\underline{\mathrm{w}(\upsilon)}$, such that

$$\underline{\mathrm{w}(\upsilon)} = \cup_{f\in Z} \{ \mathrm{w}(f) : \mathrm{w}(f) \subseteq \boldsymbol{\upsilon} \}.$$

**iii.** The boundary of $\boldsymbol{\upsilon}$ for $\bar{Ṛ}$ is symbolizes by $B_{\mathrm{w}(\upsilon)}$, such that

$$B_{\mathrm{w}(\upsilon)} = \overline{\mathrm{w}(\upsilon)} - \underline{\mathrm{w}(\upsilon)}$$

**Approximation of $\boldsymbol{\upsilon}$**



**Definition 2.2:** [1]

Let $Z \neq \emptyset$, $\boldsymbol{\omega}$ be an equivalence relation on $Z$ and $ŧ_{w(\upsilon)} = \{ Z, \emptyset, \overline{w(\upsilon)}, \underline{w(\upsilon)}, B_{w(\upsilon)}\}$, whenever $\boldsymbol{\upsilon} \subseteq Z$. Then $ŧ_{w(\upsilon)}$ is a topology on Z named nano topology for $\boldsymbol{\upsilon}$; $(Z, ŧ_{w(\upsilon)})$ is named nano topological space. The subsets of $ŧ_{w(\upsilon)}$ are named nano-open sets symbolizes by ka-open sets, and its complement called nano-closed set symbolizes by ka -closed set.

## 3. Nano Ʀ-Space

**Definition 3.1:**

Let $Z \neq \emptyset$, $\omega$ be an equivalence relation on Z and $\upsilon_r \subseteq Z, \forall r \epsilon J$ such that $\{ŧ_{w(\upsilon_r)}\}_{r\epsilon J}, r \geq 2$ be a nano-topologies on $Z$;

$$ƦO_Z = \left\{ M \subseteq Z: M = \emptyset \ or \ \exists\, \mathcal{T} \in \bigcap_{r\epsilon J} ŧ_{w(\upsilon_r)} \ni \emptyset \neq \mathcal{T} \subseteq M \right\}.$$

It's clear that $ƦO_Z$ satisfying the following conditions:

**1.** $Z, \emptyset \in ƦO_Z$.
**2.** $\bigcup_{i\in I} M_i \in ƦO_Z$, $\forall \{M_i\}_{i\in I} \in ƦO_Z$.
**3.** $\bigcap_{i=1}^{n} M_i \in ƦO_Z$, $\forall \{M_i\}_{i=1}^{n} \in ƦO_Z$.

Then $(Z, ƦO_Z)$ is named nano Ʀ-space and the subsets of $ƦO_Z$ are called Ʀ − open sets and the complement of Ʀ − open set is Ʀ − closed set. The collection of all Ʀ − closed sets of $Z$ symbolizes by $ƦC_Z$.

Then $(Z, ƦO_Z)$ as the nanoƦ-space, where $Z$ represents a set and $ƦO_Z$ denotes the collection Ʀ-open sets defined on $Z$. In this context, Ʀ-open sets refer to subsets of $Z$ that possess certain characteristics determined by the nano topological structure. Consequently, the complement of a Ʀ-open set is referred to as a Ʀ-closed set. We denote the collection of all Ʀ-closed sets of $Z$ as $ƦO_Z$, symbolizing the overall set Ʀ-closed sets associated with the nanoƦ-space $(Z, ƦO_Z)$.

**Example 3.2:**

Let $Z$= {$F_1$, $F_2$, $F_3$}, $\boldsymbol{\upsilon} \subseteq Z$, w= {($F_1$, $F_1$), ($F_2$, $F_2$), ($F_3$, $F_3$), ($F_1$, $F_2$), ($F_2$, $F_1$)}. Then $w_{(F_1)} = w_{(F_2)}$= {$F_1$, $F_2$}, $w_{(F_3)}$={$F_3$}, $Z/$ w= {{$F_1$, $F_2$}, {$F_3$}}. From table (3.1) can be calculated $\overline{w(\upsilon)}$, $\underline{w(\upsilon)}$, $B_{w(\upsilon)}$ and from these can be calculated $ŧ_{w(\upsilon)}$.

**Table 3.1**- Nano topology for υ

| $\boldsymbol{\upsilon}$ | $\overline{w(\upsilon)}$ | $\underline{w(\upsilon)}$ | $B_{w(\upsilon)}$ | $ŧ_{w(\upsilon)}$ |
|---|---|---|---|---|
| $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\{Z, \emptyset\}$ |
| $Z$ | $Z$ | $Z$ | $\emptyset$ | $\{Z, \emptyset\}$ |
| **{$F_1$}** | {$F_1$, $F_2$} | $\emptyset$ | {$F_1$, $F_2$} | {$Z$, $\emptyset$, {$F_1$, $F_2$}} |
| **{$F_2$}** | {$F_1$, $F_2$} | $\emptyset$ | {$F_1$, $F_2$} | {$Z$, $\emptyset$,{$F_1$, $F_2$}} |
| **{$F_3$}** | {$F_3$} | {$F_3$} | $\emptyset$ | {Z, $\emptyset$, {$F_3$}} |
| **{$F_1$, $F_2$}** | {$F_1$, $F_2$} | {$F_1$, $F_2$} | $\emptyset$ | {$Z$, $\emptyset$, {$F_1$, $F_2$}} |
| **{$F_1$, $F_3$}** | $Z$ | {$F_3$} | {$F_1$, $F_2$} | {$Z$, $\emptyset$,{$F_3$}, {$F_1$, $F_2$}} |
| **{$F_2$, $F_3$}** | $Z$ | {$F_3$} | {$F_1$, $F_2$} | {$Z$, $\emptyset$,{$F_3$},{$F_1$,$F_2$}} |

Now, if $\upsilon_{1=}$\{$F_1$}, $\upsilon_{2=}$\{$F_1$, $F_2$}and $\upsilon_{3=}$\{$F_1$, $F_3$}, therefore $ƦO_Z = \{Z, \emptyset, \{F_1, F_2\}\}$

## 4. Application in Nano Ɽ-Space: Leukaemia in Paediatric Patients

Leukaemia, a form of cancer that affects children, primarily impacts the blood cells and the tissues responsible for their production, such as the bone marrow. The two main types of blood cancer, leukaemia, can be classified as follows:

1. Acute Leukaemia: This variant exhibit rapid growth and poses a potential threat to life. In this type, the bone marrow produces an excessive amount of immature white blood cells known as blasts, which enter the bloodstream. Consequently, these immature cells overcrowd the normal cells in the bloodstream, impeding their functionality in combating infections, preventing bleeding, or maintaining proper levels of red blood cells, leading to weakness and fatigue. The most common types of leukaemia are:

a. Acute lymphoblastic leukaemia (ALL).

b. Acute myeloid leukaemia (AML).

2. Chronic Leukaemia: This form progresses slowly over time and may take a considerable duration before symptoms become apparent. Occasionally, chronic leukaemia is diagnosed during routine examinations, even before any symptoms manifest. In such cases, the cancer cells have reached a level of maturity where they can still function similarly to normal white blood cells, without exhibiting abnormal proliferation. The two primary types of chronic leukaemia are:

a. Chronic lymphocytic leukaemia (CLL).

b. Chronic myelogenous leukaemia (CML).

Causes of Leukaemia: In the majority of childhood leukaemia cases, the exact causes of the disease remain unknown.

to lower the chances of developing blood cancer, the following recommendations should be considered:

1. Maintain a nutritious diet: It is advisable to adopt a well-balanced eating plan that incorporates a variety of fruits, vegetables, and whole grains. It is important to steer clear of processed foods, saturated fats, and foods with low nutritional value.
2. Engage in regular physical activity: Participate in consistent physical exercises like brisk walking, jogging, and strength training to enhance physical fitness and reinforce the immune system.
3. Avoid smoking and alcohol consumption: Quit smoking and avoid excessive alcohol consumption, as these are known factors associated with an increased risk of blood cancer.
4. Safe handling of chemicals: Minimize exposure to toxic chemicals and follow appropriate safety guidelines when handling them. Use personal protective equipment and maintain a safe working environment.
5. Protection from radiation: Take necessary precautions to protect yourself from excessive exposure to radiation, whether it's from sunlight or X-rays and nuclear radiation. Wear protective clothing and use sunscreen when going out in sunny areas. Follow safety guidelines for the safe use of radiation.
6. Regular screenings and early detection: Undergo regular screenings and early detection measures to identify and manage potential blood cancer risks.

By following these guidelines, you can reduce the risk of developing blood cancer and promote overall health and well-being.

The following table provides information about four patients {PA1, PA2, PA3, PA4}. We will denote the presence of clear symptoms with the symbol (√) and indicate the absence of symptoms with the symbol (Χ).

**Table 4.1**- Information of Leukaemia in children

| Patients | Bruises on the body for no reason (B) | Breathing problems (P) | Anemia (M) | Nose bleeding (N) | Anorexia (A) | Leukemia |
|---|---|---|---|---|---|---|
| **PA1** | √ | √ | √ | √ | √ | √ |
| **PA2** | √ | √ | *X* | √ | *X* | √ |
| **PA3** | √ | √ | *X* | *X* | *X* | *X* |
| **PA4** | *X* | √ | *X* | *X* | *X* | *X* |

In this table let $Z$ = {PA$_1$, PA$_2$, PA$_3$, PA$_4$} be the set of patient's persons we think they have a Leukaemia, $\upsilon_{1=}$ {PA$_1$, PA$_3$}, $\upsilon_{2=}$ {PA$_4$}. let $\boldsymbol{\omega}$ be the equivalence relation on $Z$. Such that $\boldsymbol{\omega}$={(PA$_i$, PA$_j$); PA$_i$, PA$_j$∈ Z} such that PA$_i$, PA$_j$ In cases where identical symptoms manifest, the set of equivalence classes corresponding to the variable $\boldsymbol{\omega}$ can be expressed as follows:

$Z/$ w={{PA$_1$}, {PA$_2$}, {PA$_3$}, {PA$_4$}}, ŧ$_{w(\upsilon_1)}$={ $Z$, ∅, {PA$_1$, PA$_3$}}, and ŧ$_{w(\upsilon_2)}$={ $Z$, ∅, {PA$_4$}}. Therefore, Ʀ$O_Z$ = **{$Z$, ∅}**.

If the Bruises on the body for no reason (B) column was cancelled, so $Z/$ w − (B)= {{PA$_1$}, {PA$_2$}, {PA$_3$, PA$_4$}}, hence ŧ$_{w(\upsilon_1)-(B)}$= { $Z$, ∅, {PA$_1$}, {PA$_3$, PA$_4$}, {PA$_1$, PA$_3$, PA$_4$}}, and ŧ$_{w(\upsilon_2)-(B)}$= { $Z$, ∅, { PA$_3$, PA$_4$}}. Therefore Ʀ$O_Z$ = **{Z, ∅,** {PA$_3$, PA$_4$}**,** {PA$_1$, PA$_3$, PA$_4$}, {PA$_2$, PA$_3$, PA$_4$}**}**.

If the Breathing problems (P)column was cancelled, so $Z/$ w − (P)={{PA$_1$}, {PA$_4$}, {PA$_2$, PA$_3$}}. Therefore Ʀ$O_Z$ = **{$Z$, ∅}**.

If the Anaemia (M)column was cancelled, so $Z/$ w − (M) = {{PA$_1$}, {PA$_2$}, {PA$_3$}, {PA$_4$}}, Therefore, Ʀ$O_Z$ = **{$Z$, ∅}**.

If the Nose bleeding (N) column was cancelled, so $Z/$ w − (N) = {{PA$_1$}, {PA$_2$, PA$_3$}, {PA$_4$}}, hence ŧ$_{w(\upsilon_1)-(N)}$= { $Z$, ∅, {PA$_1$}, {PA$_2$, PA$_3$}, {PA$_1$, PA$_2$, PA$_3$}}and, ŧ$_{w(\upsilon_2)-(N)}$= { $Z$, ∅, { PA$_4$}}Therefore Ʀ$O_Z$ = **{$Z$, ∅}**.

In the event that the column labelled Anorexia (A) is removed, so $Z/$ w − (A) = {{PA$_1$}, {PA$_2$}, {PA$_3$}, {PA$_4$}}. Therefore, Ʀ$O_Z$ = **{Z, ∅}.**

## Conclusion

After careful analysis, it has been established that the core of the variable $\boldsymbol{\omega}$, referred to as core ($\boldsymbol{\omega}$), exclusively comprises element B. As a result, we can deduce that the presence of unexplained bruises on the body (B) is both necessary and sufficient for drawing the conclusion that a patient is afflicted with Leukaemia. To provide a concise summary of this information, it can be effectively represented in the following tabular format:

**Table 4.2**- Conclusion of the most important symptoms of Leukaemia

| The collection of equivalent classes | Nano topology on $\boldsymbol{\upsilon_1}$ | Nano topology on $\boldsymbol{\upsilon_2}$ | Ʀ$\boldsymbol{O_Z}$ |
|---|---|---|---|
| $\boldsymbol{Z/}$ **w**={{PA$_1$}, {PA$_2$}, {PA$_3$}, {PA$_4$}}. | ŧ$_{w(\upsilon_1)}$= {Z, ∅, {PA$_1$, PA$_3$}}. | ŧ$_{w(\upsilon_2)}$= {Z, ∅, {PA$_4$}}. | Ʀ$O_Z$ = {$Z$, ∅}. |
| $\boldsymbol{Z/}$ **w** −(B)= {{PA$_1$}, {PA$_2$}, {PA$_3$, PA$_4$}}. | ŧ$_{w(\upsilon_1)-(B)}$= {Z, ∅, {PA$_1$}, {PA$_3$, PA$_4$}, {PA$_1$, PA$_3$, PA$_4$}}. | ŧ$_{w(\upsilon_2)-(B)}$= {Z, ∅, {PA$_3$, PA$_4$}}. | Ʀ$O_Z$ = {$Z$, ∅, {PA$_3$, PA$_4$}, {PA$_1$, PA$_3$, PA$_4$}, {PA$_2$, PA$_3$, PA$_4$}}. |
| $\boldsymbol{Z/}$ **w** − (P)={{PA$_1$}, {PA$_4$}, {PA$_2$, PA$_3$}}. | ŧ$_{w(\upsilon_1)}$= {Z, ∅, {PA$_1$, PA$_3$}}. | ŧ$_{w(\upsilon_2)}$= {Z, ∅, {PA$_4$}}. | Ʀ$O_Z$ = {$Z$, ∅}. |
| $\boldsymbol{Z/}$ **w** −(M) = {{PA$_1$}, {PA$_2$}, {PA$_3$}, {PA$_4$}}, | ŧ$_{w(\upsilon_1)}$= {Z, ∅, {PA$_1$, PA$_3$}}. | ŧ$_{w(\upsilon_2)}$= {Z, ∅, {PA$_4$}}. | Ʀ$O_Z$ = {$Z$, ∅}. |

| | | | |
|---|---|---|---|
| $\boldsymbol{Z}/\,\mathbf{w}-$(N) = $\{\{PA_1\}, \{PA_2, PA_3\}, \{PA_4\}\}$. | ŧ$_{w(\upsilon_1)-(N)}$= $\{Z, \emptyset, \{PA_1\}, \{PA_2, PA_3\}, \{PA_1, PA_2, PA_3\}\}$. | ŧ$_{w(\upsilon_2)-(N)}$= $\{Z, \emptyset, \{PA_4\}\}$. | Ꝛ$O_Z = \{Z, \emptyset\}$. |
| $\boldsymbol{Z}/\,\mathbf{w}-$(A)=$\{\{PA_1\}, \{PA_2, PA_3\}, \{PA_4\}\}$. | ŧ$_{w(\upsilon_1)-(N)}$= $\{Z, \emptyset, \{PA_1\}, \{PA_2, PA_3\}, \{PA_1, PA_2, PA_3\}\}$. | ŧ$_{w(\upsilon_2)-(N)}$= $\{Z, \emptyset, \{PA_4\}\}$. | Ꝛ$O_Z = \{Z, \emptyset\}$. |

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