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Investigation of Differential Subordination with the Use of the Generalized Hypergeometric Function

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Abstract

The study of analytic univalent and multivalent function theory is an ancient subject of mathematics, particularly in complex analysis, that has drawn a large number of scholars due to the sheer elegance of its geometrical properties and the numerous research opportunities it provides. Researchers have been interested in the traditional study of this subject since at least 1907. During this time, many complex analysis researchers emerged, including Euler, Gauss, Riemann, Cauchy, and many others. Show several results for differential subordination using the convolution operator as well as broader hypergeometric functions $\Upsilon_p^k k(z)$. Geometric function theory is a synthesis of geometry and analysis. The main goal of this paper is to investigate the dependence principle and add a new subset for polyvalent functions with a different operator that is related to higher order derivatives. As a result, the discoveries were significant in terms of geometric properties as an example coefficient estimation, growth bounds and distortion, convexity, close to convexity, and the radii of starlikeness.

Keywords: Best dominant, Analytic functions, Differential subordination, Hadamard product, Admissible functions, Convex, Univalent.

تحقيق التبعية التفاضلية باستخدام الدالة الهندسية العليا المعممة

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الخلاصة

تعد دراسة نظرية الدالة التحليلية أحادية التكافؤ ومتعددة التكافؤ موضوعًا قديمًا للرياضيات، لا سيما في التحليل المعقد، الذي جذب عددًا كبيرًا من العلماء بسبب الأناقة المطلقة لخصائصها الهندسية وفرص البحث العديدة التي توفرها. اهتم الباحثون بالدراسة التقليدية لهذا الموضوع منذ عام 1907 على الأقل. خلال هذا الوقت، ظهر العديد من باحثي التحليل المعقد، بما في ذلك أويلر، وغاوس، وريمان، وكوشي، والعديد غيرهم.

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نعرض العديد من النتائج للتبعية التفاضلية باستخدام عامل الالتفاف بالإضافة إلى دوال الهندسة الفوقية الأوسع $\Upsilon_p^k k(z)$. نظرية الدالة الهندسية هي توليفة من الهندسة والتحليل. الهدف الرئيسي من هذه الورقة هو التحقيق في مبدأ التبعية وإضافة مجموعة فرعية جديدة للدوال متعددة التكافؤ مع عامل تشغيل مختلف مرتبط بمشتقات ذات رتبة أعلى. نتيج لذلك، الاكتشافات مهمة من حيث الخصائص الهندسية كمثال لتقدير المعامل وحدود النمو والتشويه، بالقرب من التحدب والتحدب، ونصف قطر التمدد.

1. Introduction and Preliminaries

The main driving force behind this is the renowned conjecture referred to as the coefficient issue, which offered an enormous opportunity for growth in 1916 until its favorable settlement in 1985 through De Branges, in which numerous leads to using this problem have been achieved. Geometric Function Theory was subsequently studied independently. Geometric Function Theory is a popular topic. Despite this, it is still finding new uses across an array of fields, which includes modern physics with mathematical, medicine, and engineering applications, among others, as well as more traditional physics topics in particular, the study of fluids, nonlinear compatible theories of systems, and the use of the partial differential equation hypothesis. A geometric function during complicated analysis is a function that describes particular geometries.

If $\mathbb{C}$ is a complex aircraft and $\mho = \{z \in \mathbb{C}: |z| < 1\}$ in which the uncovered unit disc is represented $\mathbb{C}$. Let $\mho$ be the open unit disk and $H(\mho)$be the class of holomorphic functions. If k and h are members of $H(\mho)$, a function is defined as k has subordinate towards a function h or just that h has precedence over k when the Schwarz function exists $j(z)$ that has analytic within $\mho$, alongside $j(0) = 0$ and $|j(z)| < 1, (z \in \mho)$, in such a way that

$$k(z) = h(j(z)).$$

The term denotes this subordination

$$k(z) \prec h(z).$$

Additionally, if the function h has become univalent inside $\mho$, it means [1- 5] as equivalences.

$$k(z) \prec h(z) \Leftrightarrow k(0) = h(0) \; and \; k(\mho) \subset h(\mho) \,.$$

Let T_u be class containing all analytic functions as defined

$$k(z) = z^u + \sum_{v=u+1}^{\infty} a_v z^v \,, \quad (z \in \mho) \tag{1}$$

which is analytic in $\mho$.

Let D denote the subclass of T that contains functions k which are univalent in $\mho$. An operation k is analytic in an open unit disk $\mho$ and claimed to be convex function if so has univalent and $k(\mho)$ is convex.

The class N is defined as

$$N = \left\{k \in T: \Re\left(2 + \frac{zk"(z)+1}{k'(z)}\right) > 0 \,, (z \in \mho)\right\}, \tag{2}$$

denotes the normalized convex function class in $\mho$.

Mocanu introduced method via 1978 [6, 7], as well as the concept of theory started to take shape within 1981 [8, 9]. Miller published a book in 2000 [10,11]. Allow $\mathcal{F}: \mathbb{C}^3 \times \mho \to \mathbb{C}$ and w to become univalent in $\mho$. When t is analytic in $\mho$ as well as meets the condition of second-order type differential subordination,

$$\mathcal{F}(\, t(z), zt'\,(z), zt"(z);\; z\,) \prec w(z). \tag{3}$$

Following that t is referred to as a differential subordination remedy. If $t \prec l$ for everyone t fulfilling known as univalent function l is called a dominant member of a team differential subordination remedy or simply a dominant (3).
Let $\alpha_1, T_1, \ldots, \alpha_l, T_l$, $\beta_1, R_1, \ldots, \beta_d, R_d$ $(l, d \in \mathbb{N},\ T_v = 1(v = 1, \ldots, l),\ R_v = 1(v = 1, \ldots, d)$ and

$$\chi = \left(\prod_{v=1}^{l} \Gamma(\alpha_v)\right)^{-2} \left(\prod_{v=1}^{d} \Gamma(\beta_v)\right)^{-1}. \tag{4}$$

At geometric function hypothesis, generalized hypergeometric functions as well as the architect's generalized hypergeometric functions are now being used, as shown in [12- 18]. As the intent of the present article, we characterize a linear operator as a component of the architect's generalized hypergeometric function. The following function has been investigated, as shown in Dziok [19] and Raina [20]

$$\Upsilon_u^N[(\alpha_v,\ T_v)1, l+1; (\beta_v,\ R_v)1, d+1]: T_u^N \to T_u^N.$$

By using function k from (1), we obtain

$$\Upsilon_u^N[(\alpha_v, T_v)1, l+1; (\beta_v, R_v)1, d+1]k(z)) = z^u + \sum_{v=u+1}^{\infty} \pi_v(\alpha_1)\, a_v z^v \tag{5}$$

such that $\pi_v(\alpha_1)$ is

$\pi_v(\alpha_1) =$

$$\left(\prod_{v=1}^{l} \Gamma(\alpha_v)\right)^{-2} \left(\prod_{v=1}^{d} \Gamma(\beta_v)\right)^{-1} \frac{\Gamma((\alpha_1+1)+T_1(v-u)) \ldots \Gamma(\alpha_l+T_l(v-u))}{\Gamma(\beta_1+R_1(v-u)) \ldots \Gamma(\beta_d+R_d(v-u))(v-u)!}, \tag{6}$$

we obtain

$$\Upsilon_u^N[\alpha_1+1]k(z)) = \Upsilon_p^k[(\alpha_1+1, T_1), \ldots, (\alpha_l, T_l); (\beta_1, R_1), \ldots, (\beta_d, R_d)]k(z)).$$

It is clear from the relation (5) that

$$zT_1(\Upsilon_u^N[\alpha_1+1]\, k(z)))' = \alpha_1 \Upsilon_u^N[\alpha_1+2]\, k(z) - (\alpha_1 - uT_1)\Upsilon_u^N[\alpha_1+1]\, k(z).$$

Now consider the following definition:

Definition 1.1.[21] Let $\mathfrak{R}_{u,v}(\lambda)$ be the class of functions $k \in T$ and $v \in \mathbb{N}, u \geq 0$, satisfying

$$\Re\left\{(\Upsilon_p^k[\alpha_1+1]k(z))'\right\} > \lambda,\ \ 1 \leq \lambda < 2, \quad z \in \mho. \tag{7}$$

The following lemmas are used to obtain the most important results:

Lemma 1.1.[22, 23] Take h be an analytic function within $\mho$ such that $h(0) = 1,\ \Re e\{\tau\} > 0$ and $v \in \mathbb{N}$,

$$E = \frac{(1-v^2) - |\tau|^2 + |v^2 - \tau^2|}{3v\Re\{\tau\}}.$$

Suppose that

$$\Re\left\{1 + \frac{zh''(z)}{h'(z)}\right\} > E.$$

Such $p(z) = 1 + p^v z^v + p^{v+1} z^{v+1} + \ldots$ is analytic within $\mho$ and

$$p(z) + \frac{1}{\tau} zp'(z) \prec h(z), \tag{8}$$

getting

$$p(z) \prec q(z),$$

in which q denotes a differential equation what happen.

Lemma 1.2.[24] If h be analytic, univalent and convex in $\mho$, $h(0) = a$ and if $\gamma \in C\backslash\{0\}$ is a complicated number and $\Re\{\gamma\} \geq 0$, such that

$$p(z) + \frac{1}{\gamma} zp'(z) \prec h(z), \tag{9}$$

then,

$$p(z) \prec q(z) \text{ and } q(z) \prec h(z).$$

Lemma 1.3.[25, 26] Let g become a convex function within $\mho$ and

$$h(z) = g(z) + 3v\beta zg'(z),$$

in which $\beta > 0$ and n is a positive integer.
If $p(z) = g(0) + p^v z^v + p^{v+1} z^{v+1} + \cdots$ is holomorphic in $\mho$ such that

$$p(z) + 3\beta zp'(z) \prec h(z),$$

hence,

$$p(z) \prec g(z).$$

2. Main Results

There are a few interesting findings over differential subordination as well as superordination about analytic univalent functions set up. The deformation of two linear operators was then used to report specific outcomes of differential subordination that involve linear workers.

Theorem 2.1. If q is a convex function in $\mho$ and

$$h(z) = q(z) + \frac{1}{2\mu - 1} zq'(z),$$

where $\mu \in \mathbb{C}$, $\Re\{\mu\} > 1$. If $k \in \Re_{p,k}(\beta)$ and $\mathcal{M} = 2\gamma\mu\, k$, where

$$\mathcal{M}(z) = 2\gamma\mu\, k(z) = \frac{\mu-1}{z^{\mu}} \int_0^z t^{\mu-1}\, k(t)dt, \tag{10}$$

we obtain

$$(\Upsilon_p^k[\alpha_1 + 1\,]k(z))' \prec g(z). \tag{11}$$

Implies

$$(\Upsilon_p^k[\alpha_1 + 1]\mathcal{M}(z))' \prec q(z).$$

Proof. From the equality (10), we get

$$z\mu\, \mathcal{M}(z) = 2(\mu - 1) \int_0^z t^{\mu-1}\, k(t)dt. \tag{12}$$

When we differentiate our equality (12) alongside regards to z, we receive

$$(\mu)\, \mathcal{M}(z) + z\mathcal{M}'(z) = 2(\mu - 1)k(z),$$

then we have

$$(\mu)\Upsilon_p^k[\alpha_1 + 1\,]\mathcal{M}\,(z) + z\left(\Upsilon_p^k[\alpha_1 + 1\,]\mathcal{M}\,(z)\right)' = 2(\mu - 1)\Upsilon_p^k[\alpha_1 + 1]k(z). \tag{13}$$

Differentiate (11) with respect to z, we obtain

$$(\Upsilon_p^k[\alpha_1 + 1]\mathcal{M}\,(z))' + \frac{1}{\mu - 1} z((\Upsilon_p^k[\alpha_1 + 1]\mathcal{M}\,(z))'' = ((\Upsilon_p^k[\alpha_1 + 1\,]k(z))'. \tag{14}$$

By subordination (11) in the equality (14), we get

$$(\Upsilon_p^k[\alpha_1 + 1\,]\mathcal{M}\,(z))' + \frac{1}{\mu-1} z((\Upsilon_p^k[\alpha_1 + 1\,]\,\mathcal{M}\,(z))'' \prec h(z). \tag{15}$$

Now,

$$p(z) = (\Upsilon_p^k[\alpha_1 + 1\,]\mathcal{M}\,(z))'.$$

Following that, using a simple calculation,

$$P(z) = \left[z + \sum_{v=2}^{\infty} \pi_v(\alpha_1) \frac{\mu - 1}{\mu + v} a_v z^v\right]' = 1 + p_1 z + p_2 z^2 + \ldots.$$

By equation (15) in subordination (14), we get

$$P(z) + \frac{1}{\mu - 1} zp'(z) \prec h(z) = q(z) + \frac{1}{\mu - 1} zq'(z).$$

Using lemma 1.1, then we get

$$p(z) \prec q(z).$$

Theorem 2.2. If q is a convex function and g the function

$$g(z) = q(z) + zq'(z).$$

If $f \in \mathbb{N}_0,\ p \geq 0,\ k \in D$, then the following subordination

$$\left(\frac{\Upsilon_p^k[\alpha_1 + 2]k(z)}{\Upsilon_p^k[\alpha_1 + 1]k(z)}\right)' \prec g(z),$$

then

$$\frac{\Upsilon_p^k[\alpha_1 + 2]k(z)}{\Upsilon_p^k[\alpha_1 + 1]k(z)} \prec q(z).$$

Proof. For $k \in D$, given by the equation (1), we have

$$\Upsilon_p^k[(\alpha_n + 1, A_v)1, q; (\beta_v, B_v)1, s]k(z) = z + \sum_{v=2}^{\infty} \pi_v(\alpha_1) a_v z^v = \Upsilon_p^k[\alpha_1 + 1]k(z).$$

Hence,

$$p(z) = \frac{\Upsilon_p^k[\alpha_1 + 2]k(z)}{\Upsilon_p^k[\alpha_1 + 1]k(z)} = \frac{z + \sum_{v=2}^{\infty} \pi_v(\alpha_1 + 2) \frac{\mu - 1}{\mu + v} a_v z^v}{z + \sum_{v=2}^{\infty} \pi_v(\alpha_1 + 1) \frac{\mu - 1}{\mu + v} a_v z^v}$$

$$= \frac{1 + \sum_{v=2}^{\infty} \pi_v(\alpha_1 + 2) \frac{\mu - 1}{\mu + v} a_v z^{v-1}}{1 + \sum_{v=2}^{\infty} \pi_v(\alpha_1 + 1) \frac{\mu - 1}{\mu + v} a_v z^{v-1}}$$

then

$$\big(p(z)\big)' = \frac{\left(\Upsilon_p^k[\alpha_1 + 2]k(z)\right)'}{\Upsilon_p^k[\alpha_1 + 1]k(z)} - p(z)\frac{\left(\Upsilon_p^k[\alpha_1 + 1]k(z)\right)'}{\Upsilon_p^k[\alpha_1 + 1]k(z)},$$

we achieve

$$p(z) + zp'(z) = \frac{\left(z\Upsilon_p^k[\alpha_1 + 2]k(z)\right)'}{\Upsilon_p^k[\alpha_1 + 1]k(z)}$$

By the relation (15) becomes

$$p(z) + zp'(z) \prec h(z) = q(z) + zq'(z),$$

as well as applying Lemma 1.3, we as a species obtain

$$p(z) \prec q(z).$$

Then

$$\frac{\Upsilon_p^k[\alpha_1]k(z)}{z} \prec q(z).$$

Theorem 2.3. If
$$E = \frac{2 + |\mu - 1|^2 - |\mu^2 + 2\mu|}{3\Re\{\mu - 1\}},$$
and h be an analytic function, suppose that
$$\Re\left\{1 + \frac{zh''(z)}{h'(z)}\right\} > E,$$

then
$$\left(\Upsilon_p^k[\alpha_1 + 1\,]k(z)\right)' \prec h(z).$$
Implies
$(\Upsilon_p^k[\alpha_1\,]\mathcal{M}(z))' \prec q(z)$.

Proof. Should we choose $v = 1$ as well as $\gamma = \mu - 1$ within Lemma 1.1, the demonstration is then easily accomplished employing the evidence of Theorem 2.3.

$$h(z) = \frac{2 + (3\beta + 1)z}{2 + z}, \quad 0 \le \beta < 1.$$

Based on Theorem 2.3, we gain the next result.

Corollary 2.1. If $\Re\{\mu\} > 1$, $\mathcal{M} = 2\gamma\mu\, k$, $0 \le \beta < 1$, $0 \le \zeta < 1$ and $p \ge 0$, is defined by Theorem 2.3, then we obtain
$$2\gamma\mu\,(\Re p, k(\beta)) \subset \Re\, p, k(\zeta),$$
in which,
$$\zeta = \min_{|z|=1} \Re\{q(z)\} = \zeta(\mu, \beta),$$

$$\zeta = \zeta(\mu, \beta) = (2\beta - 1) + 2(\mu + 1)(1 - \beta)\tau(\mu), \tag{16}$$
such that,
$$\tau(\mu) = \int_0^1 \frac{t^\mu}{1 + t}\, dt. \tag{17}$$

Proof. By Definition 1.1, we obtain $k \in \Re_{p,k}(\beta)$ and $\Re\left\{(\Upsilon_p^k[\alpha_1 + 1]k(z))'\right\} > \beta$, which is equivalent to
$$(\Upsilon_p^k[\alpha_1 + 1]k(z))' \prec h(z).$$
Theorem 2.3 yields
$(\Upsilon_p^k[\alpha_1 + 1]\mathcal{M}(z))' \prec q(z)$.
If we take
$$h(z) = \frac{2 + (3\beta - 1)z}{2 + z}, \quad 0 \le \beta < 1\,.$$
Following that h is convex, along with theorem 2.1 yields

$$(\Upsilon_p^k[\alpha_1 + 1\,]\mathcal{M}(z))' \prec q(z) = \frac{\mu - 1}{z^{\mu - 1}} \int_0^z t^\mu \frac{2 + (3\beta - 1)t}{2 + t} dt$$

$$= (3\beta - 1) + 2\frac{(1-\beta)(\mu-1)}{z^{\mu-1}}\int_0^z \frac{t^\mu}{1+t}dt\,.$$

If $\Re\{\mu\} > 1$, then we obtain

$$\Re\{(\Upsilon_p^k[\alpha_1 + 1]\mathcal{M}(z))'\} \geq \min_{|z|=1} \Re\{q(z)\} = \Re\{q(1)\} = \zeta(\mu, \beta)$$

$$= (3\beta - 1) + 2(\mu - 1)(1 - \beta)\,\tau(\mu),$$

in which $\tau(\mu)$ is given by (17) and (16), we obtain

$$\gamma\mu\left(\Re p, k(\beta)\right) \subset \Re p, k(\zeta).$$

Theorem 2.4. If q is a convex function as well as $q(0) = 1$, such that
$h(z) = q(z) + zq'(z)$,
and subordination

$$(\Upsilon_p^k[\alpha_1 + 1]k(z))' \prec h(z) = q(z) + zq'(z),$$

then

$$\frac{\Upsilon_p^k[\alpha_1]k(z)}{z} \prec q(z).$$

Proof. Let

$$p(z) = \frac{\Upsilon_p^k[\alpha_1 + 1]k(z)}{z},$$

we get
$(\Upsilon_p^k[\alpha_1 +]k(z))' = p(z) + zp'(z)$.
Then

$$p(z) + zp'(z) \prec h(z) = q(z) + zq'(z),$$

By employing lemma 1.2, we as a species get

$$p(z) \prec q(z).$$

Hence

$$\frac{\Upsilon_p^k[\alpha_1]k(z)}{z} \prec q(z).$$

3. Conclusions

We provided a few uses using the differential subordination idea to subclasses of univalent functions that include certain convolutions managers. By employing a linear operator, certain significant results were obtained in the differential subordination and differential superordination of a second order meromorphic analytical univalent function. Obtain geometric properties like the value boundary, coefficient disparities, deformation theorem, closing theorem, extremely points, starlikeness radii, convexity, near-perfect convexity, as well as integration values. Using differential subordination, we investigated neighborhood property. We obtained some differential subordination results involving a linear operator, as well as some sandwich theorems.

4. Future Study

The future study is broken down as follows:

- Can create two new bi-univalent function subclasses and estimate the concepts for the class of functions.
- Multiple findings over fifth order differential subordination within the uncovered unit disk can be obtained by using the generalized hypergeometric function and the properties of the generalized derivative operator.

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