



Comparative Study between Classical and Fuzzy Filters for Removing Different Types of Noise from Digital Images

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Abstract

The aim of this paper is to compare between classical and fuzzy filters for removing different types of noise in gray scale images. The processing used consists of three steps. First, different types of noise are added to the original image to produce a noisy image (with different noise ratios). Second, classical and fuzzy filters are used to filter the noisy image. Finally, comparing between resulting images depending on a quantitative measure called Peak Signal-to-Noise Ratio (PSNR) to determine the best filter in each case.

The image used in this paper is a 512 * 512 pixel and the size of all filters is a square window of size 3*3. Results indicate that fuzzy filters achieve varying successes in noise reduction in image compared to classical filters. Matlab 2012b program is used to add noise to the original image and remove it because it has powerful tools to deal with digital images.

Keywords: fuzzy filters, noise model, classical filters, matlab

دراسة مقارنة بين المرشحات العادية و المضببة لأزالة أنواع مختلفة من الضوضاء من الصور الرقمية

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الخلاصة:

الهدف من هذا البحث هو المقارنة بين المرشحات العادية و المضببة لأزالة أنواع مختلفة من الضوضاء من الصور الرقمية. المعالجة المستخدمة تتكون من ثلاث خطوات. أولاً ، أنواع مختلفة من الضوضاء يتم إضافتها الى الصورة الأصلية لإنتاج الصورة المشوشة. ثانياً ، المرشحات العادية و المضببة يتم استخدامها لتتقية الصورة المشوشة. و اخيراً ، المقارنة بين الصور الناتجة بالأعتماد على مقياس كمي يسمى نسبة الإشارة الى التشويش لتحديد افضل مرشح في كل حالة.

الصورة المستخدمة في هذا البحث هي 512*512 نقطة و حجم جميع المرشحات هو نافذة مربعة ذات حجم 3*3. النتائج توشر الى أن المرشحات المضببة تنجز نجاحات متباينة في تقليل الضوضاء من الصورة بالمقارنة مع المرشحات العادية. تم استخدام برنامج الماتلاب 2012 ب لأضافة الضوضاء الى الصورة الأصلية و أزالته لأنه يمتلك ادوات قوية للتعامل مع الصور الرقمية.

Introduction

The principal sources of noise in digital images arise during image acquisition (digitization) and/or transmission. The performance of imaging sensors is affected by a variety of factors, such as

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environmental conditions during image acquisition, and by the quality of the sensing elements themselves. For instance, in acquiring images with a CCD camera, light levels and sensor temperature are major factors affecting the amount of noise in the resulting image. Images are corrupted during transmission principally due to interference in the channel used for transmission. For example, an image transmitted using a wireless network might be corrupted as a result of lightning or other atmospheric disturbance [1].

Six types of noise are used in this paper, these are Gaussian (Normal), Rayleigh, Gamma, Uniform, Exponential, and Salt and Pepper (Impulse). Noise filtering is a fundamental pre-processing step before further image processing techniques like image segmentation, image comparison and texture analysis can be performed [2].

Noise removal can be achieved using spatial filters or frequency filters. All filters used in this paper are spatial filters. The mechanics of spatial filtering are illustrated in figure - 1. The process consists simply of moving the filter mask from point to point in an image. At each point (x,y) , the response of the filter at that point is calculated using a predefined relationship [1].

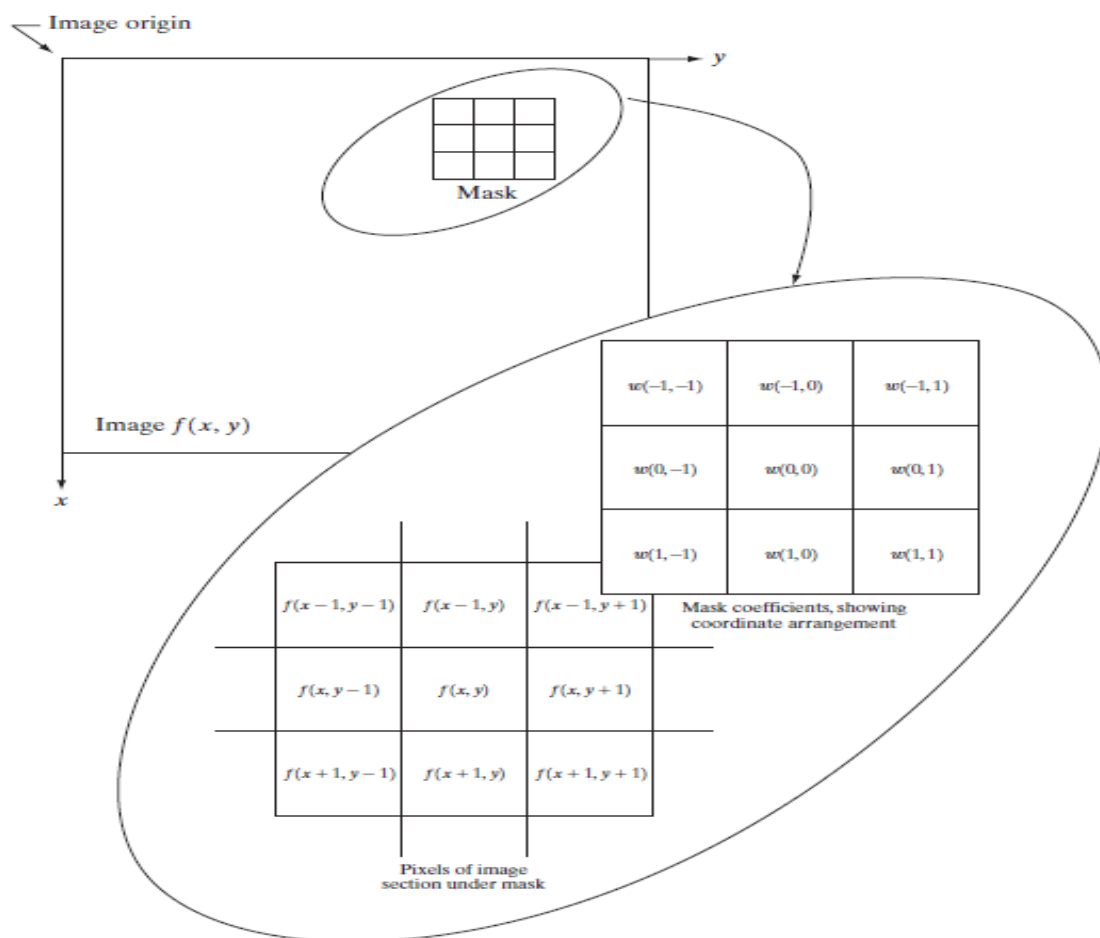


Figure 1- The mechanics of spatial filtering.

Six classical filters are used in this paper. Three of them are mean filters (arithmetic mean, geometric mean, and harmonic mean). The mean filters function by finding some form of an average with the $N * M$ window, using the sliding window concept to process the entire image. Mean filters have the disadvantage of blurring the image edges, or details. Two of them are order filters (median and midpoint). The order filters are implemented by arranging the neighborhood pixels in order from smallest to largest gray-level value and using this order to select the "correct" value. The order filters are nonlinear, so their results are sometimes unpredictable. The last one (Gaussian) used a Gaussian distribution to select the correct value [3].

Six fuzzy filters are used in this paper, these are Gaussian fuzzy filter with median center (GMED), Symmetrical triangular fuzzy filter with median center (TMED), Asymmetrical triangular fuzzy filter with median center (ATMED), Gaussian fuzzy filter with moving average center (GMAV), Symmetrical triangular fuzzy filter with moving average center (TMAV), and asymmetrical triangular fuzzy filter with moving average center (ATMAV). Each of these fuzzy filters applies a weighted membership function to an image within a window to determine the center pixel. These filters consisting of symmetrical and asymmetrical triangular membership functions with median center and moving average center have been applied to filtering of images contained with noise [4].

The paper is organized as follow: section 2 describes types of noise added to the original image. Section 3 explains the classical filters used to filter the noisy image. Section 4 explains the fuzzy filters used to filter the noisy image. The last section illustrates the results and compare between them and gives conclusions.

Noise Model

Adding noise to an image is described by the following equation:

$$g(x,y) = f(x,y) + \alpha n(x,y) \quad (1)$$

Where

$f(x,y)$ is the original image matrix.

$g(x,y)$ is the additive noise matrix.

$g(x,y)$ is the noisy image matrix.

α is the noise ratio, $0 \leq \alpha \leq 1$.

Noise can be considered as random variable characterized by a probability density function (PDF), so types of noise used here will be described using their PDFs as follow:

Gaussian Noise

Because of its mathematical tractability, Gaussian noise models are used frequently in practice. The PDF of a Gaussian random variable, z , is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mathcal{M})^2/2\sigma^2} \quad (2)$$

Where z represents gray level, μ is the mean of average value of z , and σ is its standard deviation [1].

Rayleigh Noise

The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases} \quad (3)$$

The mean and variance of this density are given by

$$\mu = a + \sqrt{\pi b/4} \quad \sigma^2 = \frac{b(4-\pi)}{4} \quad (4)$$

The Rayleigh density can be quite useful for approximating skewed histograms [1].

Gamma Noise

The PDF of gamma (Erlang) noise is given by

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \quad (5)$$

Where the parameters are such that $a > 0$, b is a positive integer, and "!" indicates factorial. The mean and variance of this density are given by [1]

$$\mu = \frac{b}{a} \quad \sigma^2 = \frac{b}{a^2} \quad (6)$$

Exponential Noise

The PDF of exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \quad (7)$$

Where $a > 0$. The mean and variance of this density function are given by [1]

$$\mu = \frac{1}{a} \quad \sigma^2 = \frac{1}{a^2} \quad (8)$$

Uniform Noise

The PDF of uniform noise is given by

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

The mean and variance of this density function are given by [1]

$$\mu = \frac{a+b}{2} \quad \sigma^2 = \frac{(b-a)^2}{12} \quad (10)$$

Impulse (Salt and Pepper) Noise

The PDF of impulse noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

If $a > b$, gray-level b will appear as light dot in the image. Conversely, level a will appear like a dark dot. If either P_a or P_b is zero, the impulse noise is called unipolar. If neither probability is zero, and especially if they are approximately equal, the impulse noise is called bipolar. Negative impulses appear as black (pepper) in an image while positive impulses appear white (salt) noise. For an 8-bit image this means that $a = 0$ (black) and $b = 255$ (white) [1]. Figure - 2 below shows the probability density function of noise types described previously.

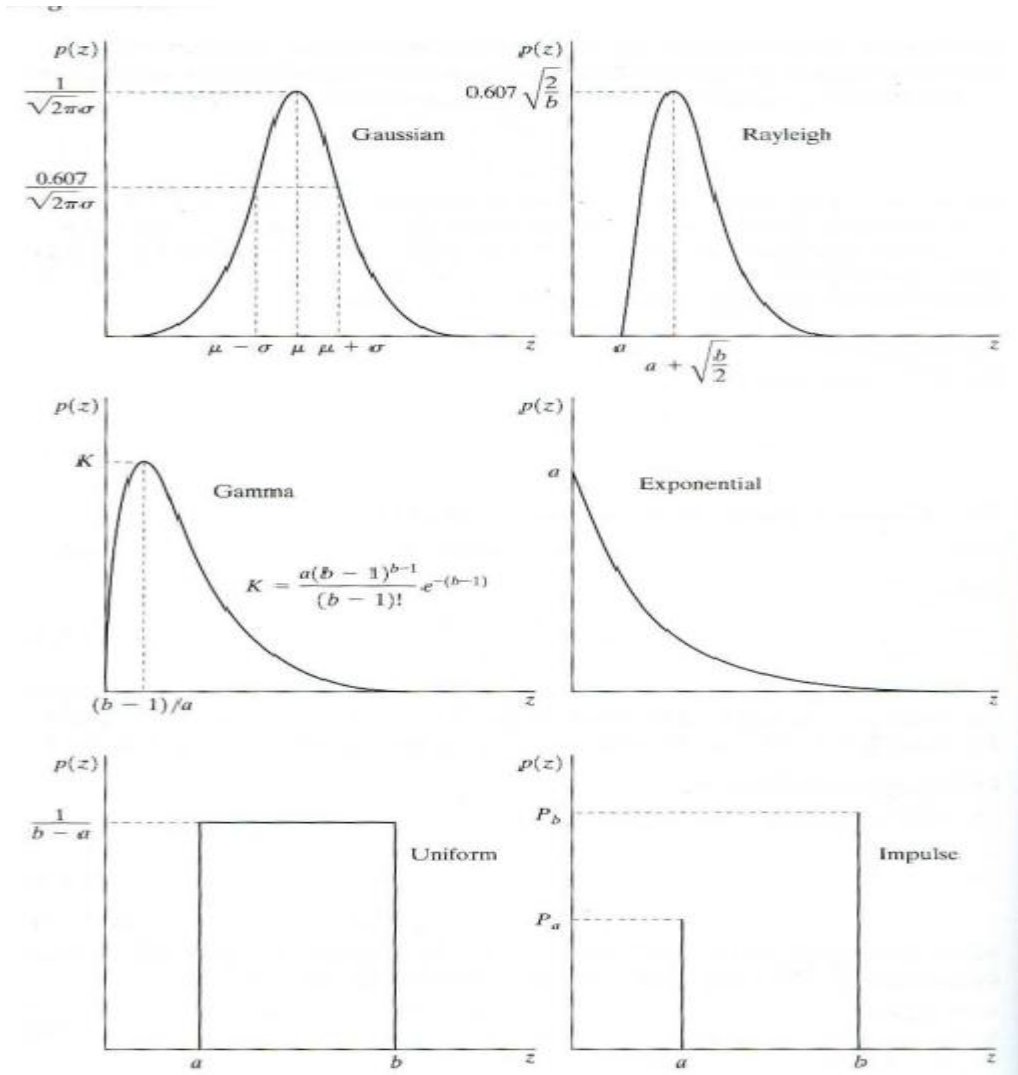


Figure 2- Some important probability density functions.

Figure - 3 shows the original image and noisy images for different types of noise with different noise ratios.



(a) Original image



(b) Gaussian (ratio 30%)



(c) Gaussian (ratio 50%)



(d) Gaussian (ratio 80%)



(e) Rayleigh (ratio 30%)



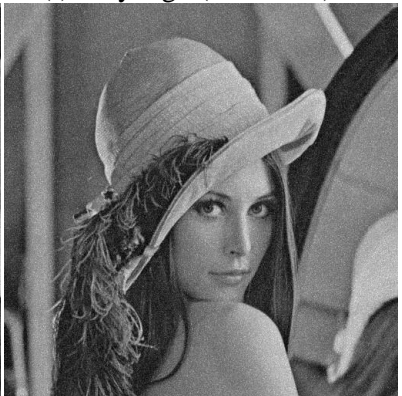
(f) Rayleigh (ratio 50%)



(g) Rayleigh (ratio 80%)



(h) Gamma (ratio 30%)



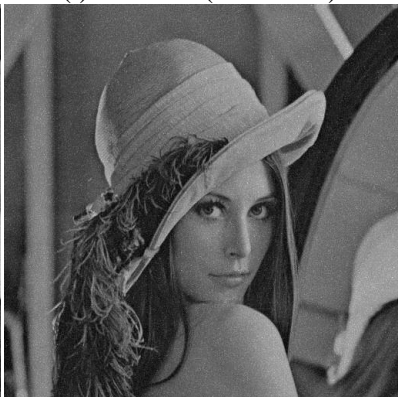
(i) Gamma (ratio 50%)



(j) Gamma (ratio 80%)



(k) Exponential (ratio 30%)



(l) Exponential (ratio 50%)

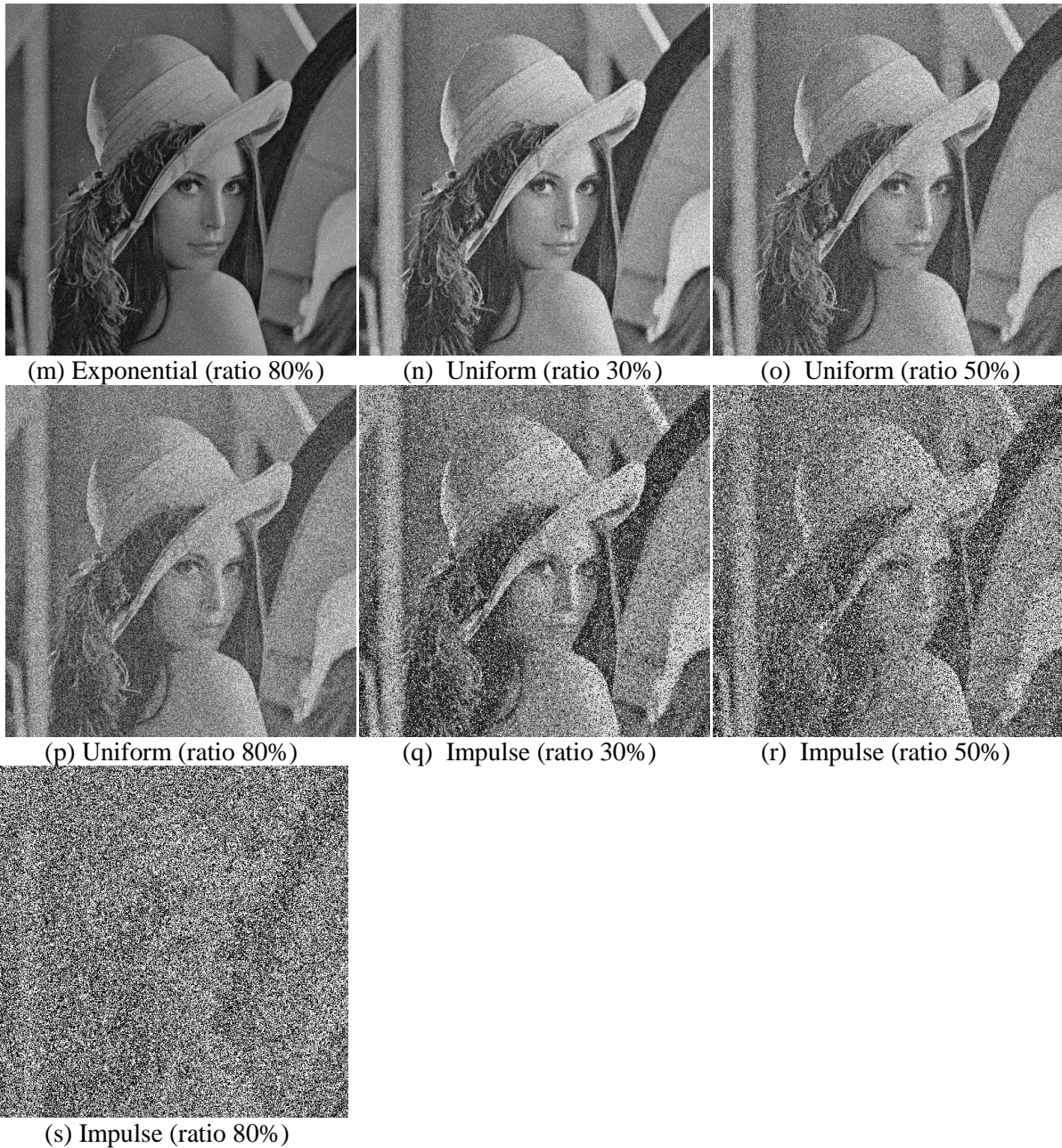


Figure 3- Original image and noisy images for different noise types (with different noise ratio).

Classical Filters

Six classical filters are used in this paper, these are:

Arithmetic Mean Filter (AMF)

It finds the arithmetic average of the pixel values in the window, as follow

$$\text{Arithmetic Mean} = \frac{1}{n*m} \sum_{(r,c) \in W} d(r,c) \quad (12)$$

The arithmetic mean smooth out local variations within an image. It can be implemented with a convolution mask where all the mask coefficients are $1/n*m$. it will tend to blur an image while mitigating the noise effects [3].

Geometric Mean Filter (GMF)

It is defined as the product of the pixel values within the window, raised to the $1/n*m$ power [3].

$$Geometric\ Mean = \prod_{(r,c) \in W} [d(r,c)]^{\frac{1}{n*m}} \tag{13}$$

Harmonic Mean Filter (HMF)

It is defined as follow: [3]

$$Harmonic\ Mean = \frac{n*m}{\sum_{(r,c) \in W} \frac{1}{d(r,c)}} \tag{14}$$

Median Filter (MF)

In this filter, the pixel values in the window W is first ordered from smallest to largest, as follows:

$$I_1 \leq I_2 \leq I_3 \leq \dots \leq I_{n*m} \tag{15}$$

Where { $I_1, I_2, I_3, \dots, I_{n*m}$ } are the gray-level values of the pixels in the $n*m$ window W (that is $(r,c) \in W$). Then, the median filter selects the middle pixel value from the ordered set [3].

Midpoint Filter (MPF)

After ordering the values in the $n*m$ window, W, midpoint filter is the average of the maximum and minimum values in the ordered set [3].

$$Midpoint = \frac{I_1 + I_{n*m}}{2} \tag{16}$$

Gaussian Filter (GF)

It uses Gaussian distribution to remove noise from the image:

$$h(x,y) = e^{\frac{-(x^2+y^2)}{2\sigma^2}} \tag{17}$$

$$h_1(x,y) = \frac{h(x,y)}{\sum_x \sum_y h(x,y)} \tag{18}$$

Fuzzy Filters

Let $x(i,j)$ be the input of a 2-dimensional fuzzy filter, the output of the fuzzy filter is defined as:

$$y(i,j) = \frac{\sum_{(r,s) \in A} F[x(i+r,j+s)].x(i+r,j+s)}{\sum_{(r,s) \in A} F[x(i+r,j+s)]} \tag{19}$$

$F[x(i,j)]$ is the general window function and A is the area of the window. For a square window of dimensions $N*N$, the range of r and s are: $-R \leq r \leq R$ and $-S \leq s \leq S$, where $N=2R+1=2S+1$ [4].

Review on fuzzy-type filters can be found in [5-7]. In [8], median filtering using fuzzy concept is described. The fuzzy filters used in this paper are:

GMED

It is defined as:

$$F[x(i+r,j+s)] = e^{-\frac{1}{2} \left[\frac{x(i+r,j+s) - x_{med}(i,j)}{\sigma(i,j)} \right]^2} \quad for \ r,s \in A \tag{20}$$

$X_{med}(i,j)$ and $\sigma(i,j)$ represent, respectively, the median value and the variance value of all the input values $x(i+r,j+s)$ for $r,s \in A$ in the window A at discrete indexes(i,j)[4].

TMED

It is defined as:

$$F[x(i+r,j+s)] = \begin{cases} 1 - |x(i+r,j+s) - x_{med}(i,j)|/x_{mm}(i,j) \\ for \ |x(i+r,j+s) - x_{med}(i,j)| \leq x_{mm}(i,j) \\ 1 \\ for \ x_{mm} = 0 \end{cases} \tag{21}$$

$x_{mm}(i, j) = \max[x_{max}(i, j) - x_{med}(i, j), x_{med}(i, j) - x_{min}(i, j)]$ (22)
 $x_{max}(i, j)$, $x_{min}(i, j)$ and $x_{med}(i, j)$ are, respectively, the maximum value, the minimum value, and the median value of all the input values $x(i+r, j+s)$ for $r, s \in A$ within the window A at discrete indexes (i, j) [4].

ATMED

It is defined as:

$$F[x(i+r, j+s)] = \begin{cases} 1 - [x_{med}(i, j) - x(i+r, j+s)]/[x_{med}(i, j) - x_{min}(i, j)] \\ \text{for } x_{min}(i, j) \leq x(i+r, j+s) \leq x_{med}(i, j) \\ 1 - [x(i+r, j+s) - x_{med}(i, j)]/[x_{max}(i, j) - x_{med}(i, j)] \\ \text{for } x_{med}(i, j) \leq x(i+r, j+s) \leq x_{max}(i, j) \\ 1 \text{ for } x_{med}(i, j) - x_{min}(i, j) = 0 \text{ or } x_{max}(i, j) - x_{med}(i, j) = 0 \end{cases} \quad (23)$$

The degree of asymmetry depends on the difference between $x_{med}(i, j) - x_{min}(i, j)$ and $x_{max}(i, j) - x_{med}(i, j)$. $x_{max}(i, j)$, $x_{min}(i, j)$ and $x_{med}(i, j)$ are, respectively, the maximum value, the minimum value, and the median value among all the input values $x(i+r, j+s)$ for $r, s \in A$ within the window A at discrete indexes (i, j) [4].

GMAV

It is defined as:

$$F[x(i+r, j+s)] = e^{-\frac{1}{2} \left[\frac{x(i+r, j+s) - x_{mav}(i, j)}{\sigma(i, j)} \right]^2} \quad \text{for } r, s \in A \quad (24)$$

$x_{mav}(i, j)$ and $\sigma(i, j)$ represent, respectively, the moving average value and the variance value of all the input values $x(i+r, j+s)$ for $r, s \in A$ in the window A at discrete indexes (i, j) [4].

TMAV

It is defined as:

$$F[x(i+r, j+s)] = \begin{cases} 1 - |x(i+r, j+s) - x_{mav}(i, j)|/x_{mv}(i, j) \\ \text{for } |x(i+r, j+s) - x_{mav}(i, j)| \leq x_{mv}(i, j) \\ 1 \\ \text{for } x_{mv} = 0 \end{cases} \quad (25)$$

$$x_{mv}(i, j) = \max[x_{max}(i, j) - x_{mav}(i, j), x_{mav}(i, j) - x_{min}(i, j)] \quad (26)$$

$x_{max}(i, j)$, $x_{min}(i, j)$ and $x_{mav}(i, j)$ represent, respectively, the maximum value, the minimum value, and the moving average value of all the input values $x(i+r, j+s)$ for $r, s \in A$ in the window A at discrete indexes (i, j) [4].

ATMAV

It is defined as:

$$F[x(i+r, j+s)] = \begin{cases} 1 - [x_{mav}(i, j) - x(i+r, j+s)]/[x_{mav}(i, j) - x_{min}(i, j)] \\ \text{for } x_{min}(i, j) \leq x(i+r, j+s) \leq x_{mav}(i, j) \\ 1 - [x(i+r, j+s) - x_{mav}(i, j)]/[x_{max}(i, j) - x_{mav}(i, j)] \\ \text{for } x_{mav}(i, j) \leq x(i+r, j+s) \leq x_{max}(i, j) \\ 1 \text{ for } x_{mav}(i, j) - x_{min}(i, j) = 0 \text{ or } x_{max}(i, j) - x_{mav}(i, j) = 0 \end{cases} \quad (27)$$

The degree of asymmetry depends on the difference between $x_{\max}(i,j) - x_{\min}(i,j)$ and $x_{\max}(i,j) - x_{\text{mav}}(i,j)$. $x_{\max}(i,j)$, $x_{\min}(i,j)$ and $x_{\text{mav}}(i,j)$ represent, respectively, the maximum value, the minimum value, and the moving average value of all the input values $x(i+r,j+s)$ for $r,s \in A$ in the window A at discrete indexes (i,j) [4].

Results and Discussions

Peak Signal to Noise Ratio (PSNR) is the measure of peak error. It is an expression used to depict the ratio of maximum possible power of image (signal) and the power of the corrupted noise that affects the quality of its representation. It is represented in terms of mean square error as: [9]

$$PSNR = 10 \log_{10} \frac{MAX^2}{MSE} \quad (28)$$

MAX is the maximum possible pixel value of the image. It is equal to 255 for 8 bit gray scale image. MSE is the mean square error which is the cumulative squared error between the final denoised image and the original image before introduction of noise. It is mathematically stated as: [9]

$$MSE = \frac{1}{n*m} \sum_{y=1}^m \sum_{x=1}^n [f(x,y) - g(x,y)]^2 \quad (29)$$

Table 1- below shows PSNR values for filters used in this paper for different types of noise with different noise ratios.

Table 1- PSNR values of original and filtered images.

Noise Type	Filter Type	PSNR		
		Noise Ratio 30%	Noise Ratio 50%	Noise Ratio 80%
Gaussian	Noisy Image	24.1921	19.8529	17.4010
	AMF	27.8902	21.8624	19.5860
	MF	27.7924	21.7453	19.4295
	GMF	22.9208	17.7647	15.9158
	HMF	23.3296	18.1731	16.4668
	MPF	26.8141	21.5600	19.2924
	GF	25.5690	21.8206	18.7134
	GMED	27.8548	21.8473	19.5670
	TMED	27.6865	21.6967	19.3676
	ATMED	27.8632	21.8032	19.5098
	GMAV	27.8554	21.8480	19.5673
	TMAV	27.9112	21.8018	19.4974
	ATMAV	27.8410	21.8260	19.5434
Rayleigh	Noisy Image	27.9439	22.1672	18.1219
	AMF	30.4616	25.5618	19.4702
	MF	30.4811	25.0354	19.1432
	GMF	27.1358	25.7097	25.4443

	HMF	27.1633	25.9981	26.0718
	MPF	29.2126	25.4103	19.7696
	GF	28.0767	23.4419	21.0573
	GMED	30.4371	25.5303	19.4583
	TMED	30.3394	24.8502	19.0421
	ATMED	30.5283	25.3295	19.3176
	GMAV	30.4402	25.5300	19.4588
	TMAV	30.5840	25.1603	19.1879
	ATMAV	30.4534	25.4615	19.4131
Gamma	Noisy Image	28.3250	22.3377	16.8926
	AMF	28.7285	23.3721	17.4189
	MF	28.8866	23.1829	17.2367
	GMF	27.0171	28.0155	27.4335
	HMF	27.3108	27.8031	28.0460
	MPF	27.9751	23.4343	17.6791
	GF	27.6374	24.2740	20.4006
	GMED	28.7198	23.3617	17.4144
	TMED	28.8233	23.1096	17.1801
	ATMED	28.6449	23.2955	17.3364
	GMAV	28.7217	23.3643	17.4152
	TMAV	28.8915	23.2147	17.2530
	ATMAV	28.7431	23.3380	17.3887
Exponential	Noisy Image	26.5875	17.4266	16.7541
	AMF	26.3356	17.4205	16.9438
	MF	26.4364	17.3809	16.7982
	GMF	26.4969	29.8733	29.8449
	HMF	26.6760	28.9680	28.9532
	MPF	25.9792	17.5119	17.2033
	GF	27.8259	21.2636	20.2366
	GMED	26.3301	17.4171	16.9400
	TMED	26.4049	17.3672	16.7579
	ATMED	25.8821	17.3734	16.8712

	GMAV	26.3321	17.4180	16.9418
	TMAV	26.4127	17.3845	16.8091
	ATMAV	26.2593	17.4098	16.9182
Uniform	Noisy Image	21.3122	18.0707	15.6512
	AMF	28.0833	23.9587	21.0487
	MF	26.5710	22.4244	19.5430
	GMF	26.1254	23.1060	21.1806
	HMF	26.7977	24.3527	22.1765
	MPF	27.4611	24.0199	21.4057
	GF	22.8950	19.9402	17.7636
	GMED	27.9717	23.8692	20.9669
	TMED	26.1135	22.0064	19.1578
	ATMED	27.2783	23.1262	20.2309
	GMAV	27.9725	23.8691	20.9668
	TMAV	27.0876	22.8935	19.9828
	ATMAV	27.7132	23.6043	20.7293
	Impulse	Noisy Image	10.8488	8.6562
AMF		18.9617	16.2256	13.4220
MF		23.7630	15.7131	8.7014
GMF		8.3825	7.0609	6.2356
HMF		7.6419	6.5648	5.9684
MPF		13.7275	14.0745	14.4163
GF		14.1464	11.9423	9.7886
GMED		18.7936	16.0802	13.3020
TMED		23.0072	15.6940	8.8054
ATMED		24.6880	17.8091	10.1734
GMAV		18.7936	16.0801	13.3020
TMAV		22.7688	16.3375	9.9742
ATMAV		25.6500	22.3741	15.2304

Shaded values mean that they are the best values for PSNR (dB).

The figure - 4, 5, 6, 7, 8, and 9 show the filtered images using classical and fuzzy filters for the six different types of noise used in this paper (taking noise ratio 50% for all types).



Figure 4- Filtered images for Gaussian noise (noise ratio is 50%).



Figure 5- Filtered images for rayleigh noise (noise ratio is 50%).



Figure 6- Filtered images for gamma noise (noise ratio is 50%).



Figure 7- Filtered images for exponential noise (noise ratio is 50%).



Figure 8- Filtered images for uniform noise (noise ratio is 50%).

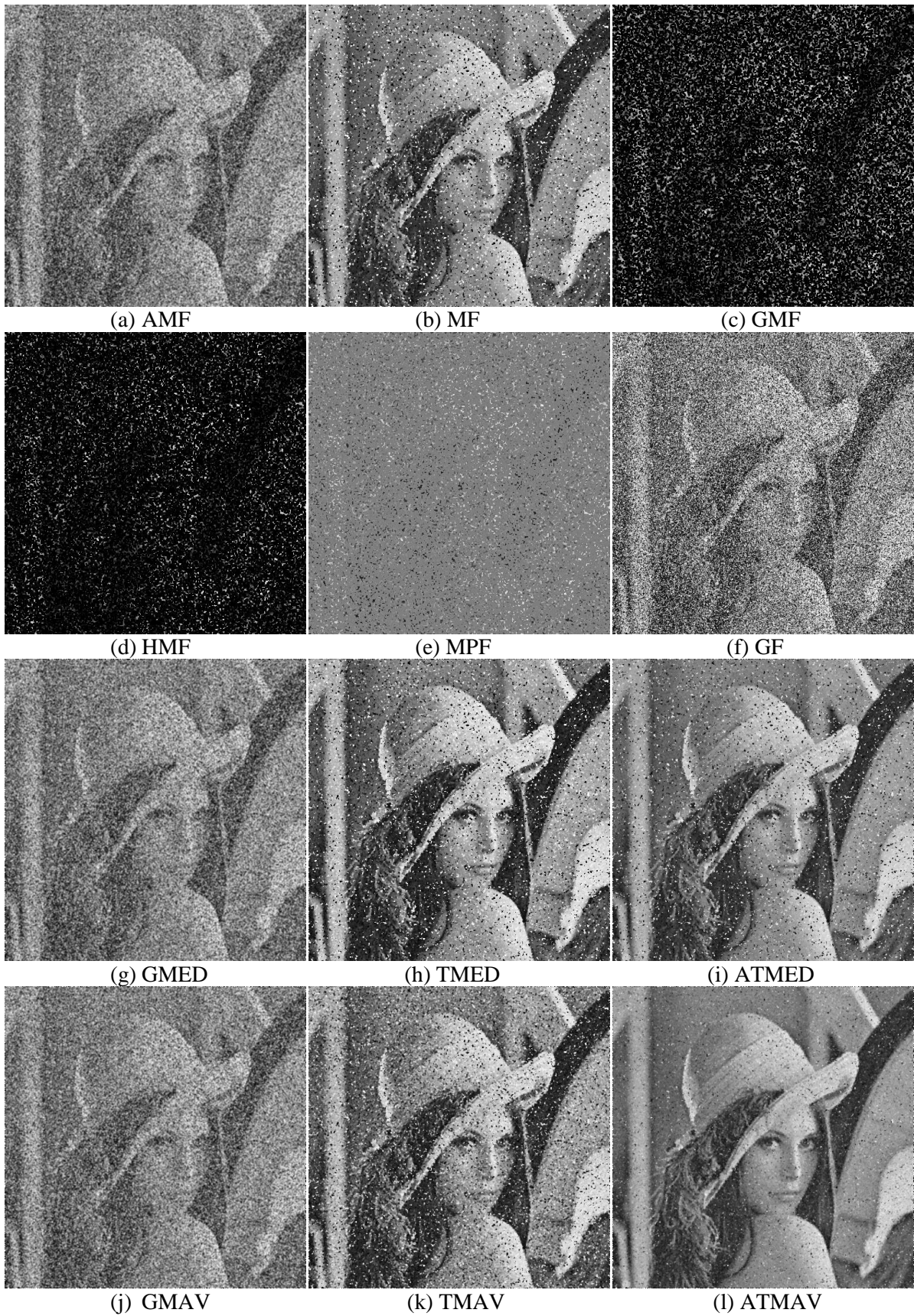


Figure 9- Filtered images for impulse noise (noise ratio is 50%).

Conclusions

From the table -1 and figures - 4, 5, 6, 7, 8, and 9, the following inferences can be drawn:

- In general, fuzzy filters are best than classical filters except some cases.
- For Gaussian, Rayleigh, and Gamma noises, when noise ratio is small, the best filter is TMAV.
- For Gaussian noise, while increasing the noise ratio, the best filter is AMF.
- For Rayleigh noise, while increasing the noise ratio, the best filter is HMF.
- For Gamma noise, while increasing the noise ratio, the best filter is GMF and HMF.
- For Exponential noise, while increasing the noise ratio, the best filter is GMF.
- For Uniform noise, while increasing the noise ratio, the best filter is HMF.
- For Impulse noise, the best filter in all cases is ATMAV.

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